Rayleigh and Rician fading - Digital Communications 23/24

The simplest **probabilistic** model for the channel filter *taps* is based on:

Assumption $\longrightarrow$ **large** number of statistically *independent* reflected and scattered paths with random amplitudes in the delay window corresponding to a single tap.

The **phase** of the i-th path is $2\pi \boxed{f_c \tau_i}$ modulo 2π.

$$f_c \tau_i = \boxed{d_i} / \boxed{\lambda},$$

the **distance** travelled by the i-th path

the carrier **wavelength**.

Since the reflectors and scatterers are **far away** relative to the carrier wavelength

$$d_i \gg \lambda,$$

$\xrightarrow{\text{assume}}$ the *phase* for each path is **uniformly** distributed between 0 and 2*pi and that the phases of different paths are **independent**.

The **contribution** of each path in the tap gain $h_\ell[m]$:

$$a_i(m/W)e^{-j2\pi f_c \tau_i(m/W)} \text{sinc}[\ell - \tau_i(m/W)W]$$

$h_\ell[m]$ is a **sum** of a large number of such small independent *circular symmetric* random variables.

We **assume** that:

☞ $h_\ell[m]$ is $\mathcal{CN}(0, \sigma_\ell^2)$ circular symmetric

☞ Variance of $h_\ell[m]$, is a function of the tap *l*, but independent of time *m*

↳ With this assumption the **magnitude** $|h_\ell[m]|$ is a *Rayleigh* random variable with density:

$$\boxed{\frac{x}{\sigma_\ell^2} \exp\left\{\frac{-x^2}{2\sigma_\ell^2}\right\}, \quad x \geq 0,}$$

and **squared magnitude** $|h_\ell[m]|^2$ is *exponentially* distributed with density:

$$\boxed{\frac{1}{\sigma_\ell^2} \exp\left\{\frac{-x}{\sigma_\ell^2}\right\}, \quad x \geq 0.}$$

This **model**, which is called *Rayleigh fading*, is quite reasonable for scattering mechanisms where there are many small reflectors.

↓

Assumption: the tap gains are *circularly symmetric complex* Gaussian random variables.

Alternative model: the *line-of-sight* path is large and has a known magnitude, and that there are also a large number of independent paths. So we can write:

$$\boxed{h_\ell[m] = \underset{\text{DETERMINISTIC TERM}}{\overset{1}{\sqrt{\frac{\kappa}{\kappa+1}}\sigma_\ell e^{j\theta}}} + \underset{\text{SCATTERED PATHS}}{\overset{2}{\sqrt{\frac{1}{\kappa+1}}\mathcal{CN}(0, \sigma_\ell^2)}}}$$

Where we have:

- **(1):** the *line-of-sight* path arriving with uniform phase theta
- **(2):** the aggregation of the large number of reflected and scattered paths, independent of theta
- **(k):** is the *ratio* of the energy in the line-of-sight path to the energy in the scattered paths

The **magnitude** of such a random variable is said to have a *Rician* distribution.