MIMO Signal Procession - Digital Communications 23/24

We should consider the following expression:

$$\boxed{\mathbf{x}_k} = \boxed{\mathcal{G}_{\mathbf{C}}}\boxed{\mathbf{s}_k} + \boxed{\mathbf{w}_k}$$

is the **N_Rc x 1** *vector* of received signals

is the **N_Rc x 1** noise *vector*

is the **N_Rc x N_Tx** channel *matrix*

is the **N_Tx x 1** transmitted signal *vector*

$$N_{S,max} \triangleq \text{rank}\, \mathcal{G}_{\mathbf{C}} \quad \circ\!-\!-\!-\!-\!-\!\circ \quad N_{S,max} \leq \min\left(N_{Rc}, N_{Tx}\right)$$

for the definition of the rank

The singular value decomposition (**SVD**): $\mathcal{G}_C = \boxed{\mathbf{U}_{ext}}\boxed{\mathbf{\Sigma}_{ext}}\boxed{\mathbf{V}^H_{ext}}$

N_Rc x N_Rc
unitary matrix

N_Tx x N_Tx
unitary matrix

N_Rc x N_Tx
diagonal matrix

$$\mathbf{\Sigma}_{ext} = \begin{bmatrix} \mathbf{\Sigma} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \longrightarrow \mathbf{\Sigma} = \text{diag}\left(\left[\varsigma_1, \ldots, \varsigma_{N_{S,max}}\right]^T\right)$$

The **capacity** for the MIMO NB AWGN channel is:

$$C_{[bit/s]} = \frac{1}{T}\sum_{n=1}^{N_{S,max}} \log_2\left(1 + \boxed{\Gamma_n}\right) \longrightarrow \Gamma_n = \mathtt{M}_n\,\Theta_n\,, \longrightarrow \Theta_n = \frac{|\varsigma_n|^2}{\frac{N_0}{2}2B}$$

SNR on stream n

With N_Tx inputs and N_Rc outputs we can transmit up to **N_S,max** independent data streams, thus exploiting the multiplexing capabilities of the MIMO channel. Since N_S,max ≤ min(N_Rc, N_Tx), we observe that a larger number of streams can be transmitted only when both N_Tx and N_Rc **increase**.

The **optimum** architecture:

1. We introduce the extended generated data vector $\mathbf{a}_{ext} = \left[\mathbf{a}^T, \mathbf{0}\right]^T$, were the dimension of $\mathbf{a}$ is $N_{S,max}$.
2. We apply the precoder $\mathbf{s} = \mathbf{V}_{ext}\mathbf{a}_{ext}$.
3. At the receiver we apply the combiner $\mathbf{U}_{ext}^H$ obtaining $\mathbf{y}_{ext} = \mathbf{U}_{ext}^H \mathcal{G}_\mathbf{C}\mathbf{s} + \tilde{\mathbf{w}}_{ext} = \mathbf{U}_{ext}^H \mathbf{U}_{ext}\boldsymbol{\Sigma}_{ext}\mathbf{V}_{ext}^H \mathbf{V}_{ext}\mathbf{a}_{ext} + \tilde{\mathbf{w}}_{ext} = \boldsymbol{\Sigma}_{ext}\mathbf{a}_{ext} + \tilde{\mathbf{w}}_{ext}$

We have **N_S,max** parellel AWGNs and we know how to deal with AWGN