

MIMO Signal Procession - Digital Communications 23/24

We should consider the following expression:

$$\mathbf{x}_k = \mathbf{G}_C \mathbf{s}_k + \mathbf{w}_k$$

is the $N_{Rc} \times 1$ vector of received signals

is the $N_{Rc} \times N_{Tx}$ channel matrix

is the $N_{Tx} \times 1$ transmitted signal vector

is the $N_{Rc} \times 1$ noise vector

$$N_{S,max} \triangleq \text{rank } \mathbf{G}_C \quad \circ \text{-----} \circ \quad N_{S,max} \leq \min(N_{Rc}, N_{Tx})$$

for the definition of the rank

The singular value decomposition (SVD):

$$\mathbf{G}_C = \mathbf{U}_{ext} \mathbf{\Sigma}_{ext} \mathbf{V}_{ext}^H$$

“H” stand for complex and transposed

$N_{Rc} \times N_{Rc}$ unitary matrix

$N_{Rc} \times N_{Tx}$ diagonal matrix

$N_{Tx} \times N_{Tx}$ unitary matrix

$$\mathbf{\Sigma}_{ext} = \begin{bmatrix} \mathbf{\Sigma} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \longrightarrow \mathbf{\Sigma} = \text{diag} \left([\varsigma_1, \dots, \varsigma_{N_{S,max}}]^T \right)$$

The **capacity** for the MIMO NB AWGN channel is:

$$C_{[bit/s]} = \frac{1}{T} \sum_{n=1}^{N_{S,max}} \log_2 (1 + \Gamma_n) \longrightarrow \Gamma_n = \mathbf{M}_n \mathbf{\Theta}_n, \longrightarrow \mathbf{\Theta}_n = \frac{|\varsigma_n|^2}{\frac{N_0}{2} 2B}$$

SNR on stream n

With N_{Tx} inputs and N_{Rc} outputs we can transmit up to $N_{S,max}$ independent data streams, thus exploiting the multiplexing capabilities of the MIMO channel. Since $N_{S,max} \leq \min(N_{Rc}, N_{Tx})$, we observe that a larger number of streams can be transmitted only when both N_{Tx} and N_{Rc} **increase**.

The **optimum** architecture:

- ① We introduce the extended generated data vector $\mathbf{a}_{ext} = [\mathbf{a}^T, \mathbf{0}]^T$, where the dimension of \mathbf{a} is $N_{S,max}$.
- ② We apply the precoder $\mathbf{s} = \mathbf{V}_{ext}\mathbf{a}_{ext}$.
- ③ At the receiver we apply the combiner \mathbf{U}_{ext}^H obtaining

$$\mathbf{y}_{ext} = \mathbf{U}_{ext}^H \mathbf{G} \mathbf{C} \mathbf{s} + \tilde{\mathbf{w}}_{ext} = \mathbf{U}_{ext}^H \mathbf{U}_{ext} \Sigma_{ext} \mathbf{V}_{ext}^H \mathbf{V}_{ext} \mathbf{a}_{ext} + \tilde{\mathbf{w}}_{ext} = \Sigma_{ext} \mathbf{a}_{ext} + \tilde{\mathbf{w}}_{ext}$$

We have $N_{S,max}$ parallel AWGNs and we know how to deal with AWGN