OFDM - Digital Communications 23/24

If the channel is **underspread** and is **time-invariant** for a long time-scale the transformation in **frequency domain** can be useful to communication over *frequency-selective* channels.

Discrete-time baseband model

$$y[m] = \sum_{\ell} h_\ell[m]x[m-\ell] + w[m].$$

linear time-invariant channel

$$y[m] = \sum_{\ell=0}^{L-1} \boxed{h_\ell} x[m-\ell] + w[m].$$

l tap not change with **m**

finite taps **L := T_d*W**

(1) Sinusoids are **eigenfunctions** of LTI systems

But if we transmit a **finite duration** like **N_c** symbols the **(1)** is not true

to restore **(1)**

Cyclic prefix
Data symbol rotates cyclically

$$\mathbf{d} = [d[0], d[1], \ldots, d[N_c-1]]^t,$$

Block of symbols

$$\mathbf{x} = [\boxed{d[N_c-L+1], d[N_c-L+2], \ldots, d[N_c-1]}, d[0], d[1], \ldots, d[N_c-1]]^t,$$

L-1 prefix

N_c+L-1 input block

Due to the additional cyclic prefix the **output** over this time interval (for avoid ISI) $m \in [L, N_c+L-1]$.

$$y[m] = \sum_{\ell=0}^{L-1} h_\ell d[(m-L-\ell) \text{ modulo } N_c] + w[m].$$

$$\mathbf{y} = [y[L], \ldots, y[N_c+L-1]]^t,$$

$$\mathbf{h} = [h_0, h_1, \ldots, h_{L-1}, 0, \ldots, 0]^t,$$

Vector of L taps

$$\boxed{\mathbf{y} = \mathbf{h} \otimes \mathbf{d} + \mathbf{w}.}$$

Cyclic convolution

$$\mathbf{w} = [w[L], \ldots, w[N_c+L-1]]^t,$$

Vector of random variables

$$\mathbf{d} = [d[0], d[1], \ldots, d[N_c-1]]^t,$$

Block of symbols

I take the **DFT** on both sides

$$\tilde{y}_n = \boxed{\tilde{h}_n}\tilde{d}_n + \boxed{\tilde{w}_n}, \qquad n = 0, \ldots, N_c - 1.$$

$$[\tilde{h}_0, \ldots, \tilde{h}_{N_c-1}]^t$$

The DFT of the L-tap channel **h**

$\tilde{w}_0, \ldots, \tilde{w}_{N_c-1}$ the DFT of the noise vector $w[1], \ldots, w[N_c]$.

We can redo everything in terms of **matrices**

$$\mathbf{u} = \mathbf{h} \otimes \mathbf{d} \longrightarrow \mathbf{u} = \mathbf{C}\mathbf{d}, \longrightarrow \mathbf{C} \text{ is the } \textit{circulant} \text{ matrix}$$

Circular convolution

Linear transformation

$$\mathbf{C} = \mathbf{U}^{-1}\boxed{\mathbf{\Lambda}}\boxed{\mathbf{U}}.$$

$\mathbf{\Lambda}$ is the *diagonal* matrix with diagonal entries sqrt(N_c) times the DFT of **h:**

$$\Lambda_{nn} = \tilde{h}_n := \left(\sqrt{N_c}\,\mathbf{U}\mathbf{h}\right)_n, \qquad n = 0, \ldots, N_c - 1.$$

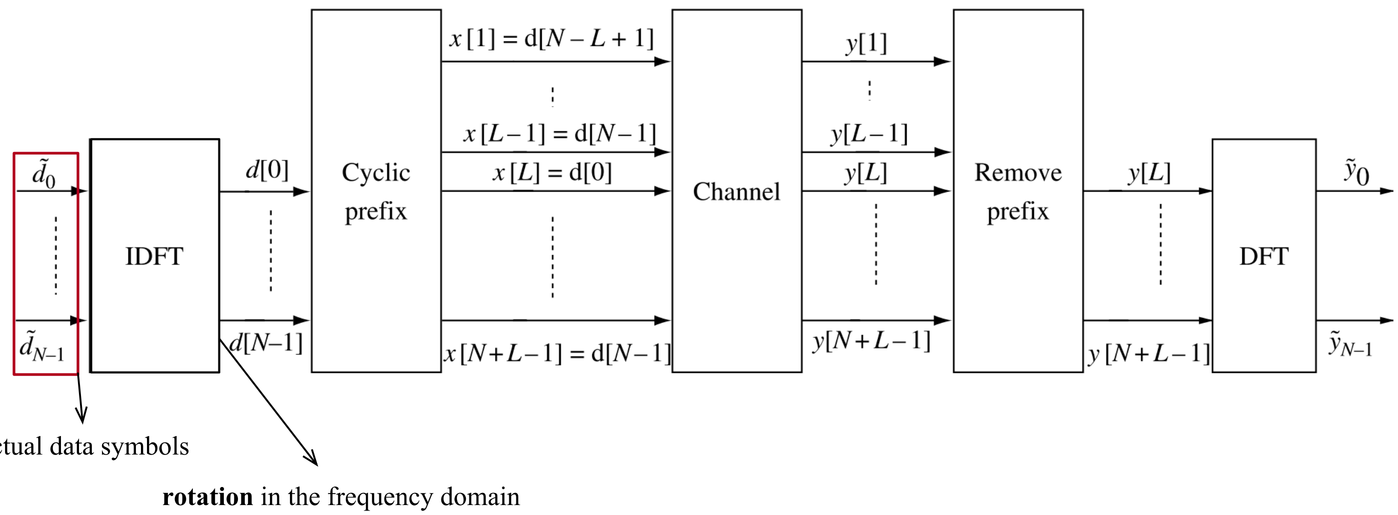
$\mathbf{U}$ is a *unitary* matrix with its (k, n)th entry equal to:

$$\frac{1}{\sqrt{N_c}} \exp\left(\frac{-j2\pi kn}{N_c}\right), \qquad k, n = 0, \ldots, N_c - 1.$$

For what we saw, we can write in a matrix form:

$$\mathbf{y} = \mathbf{h} \otimes \mathbf{d} + \mathbf{w}. \longrightarrow \mathbf{y} = \mathbf{C}\mathbf{d} + \mathbf{w} = \mathbf{U}^{-1}\mathbf{\Lambda}\mathbf{U}\mathbf{d} + \mathbf{w}.$$

This representation suggests a natural **rotation** at the input and at the output to *convert* the channel to a set of non-interfering channels with no ISI.



The data symbols modulate **N_c** *sub-carriers*, which occupy the bandwidth W and are *uniformly* separated by W/N_c. The data symbols on the sub-carriers are then converted (through the IDFT) to time domain. The procedure of introducing the cyclic prefix before transmission allows for the removal of ISI.
The receiver converts the **N_c** symbols back to the frequency domain through a DFT. The data symbols on the sub-carriers are maintained to be **orthogonal** as they propagate through the channel and hence go through narrowband *parallel* sub-channels