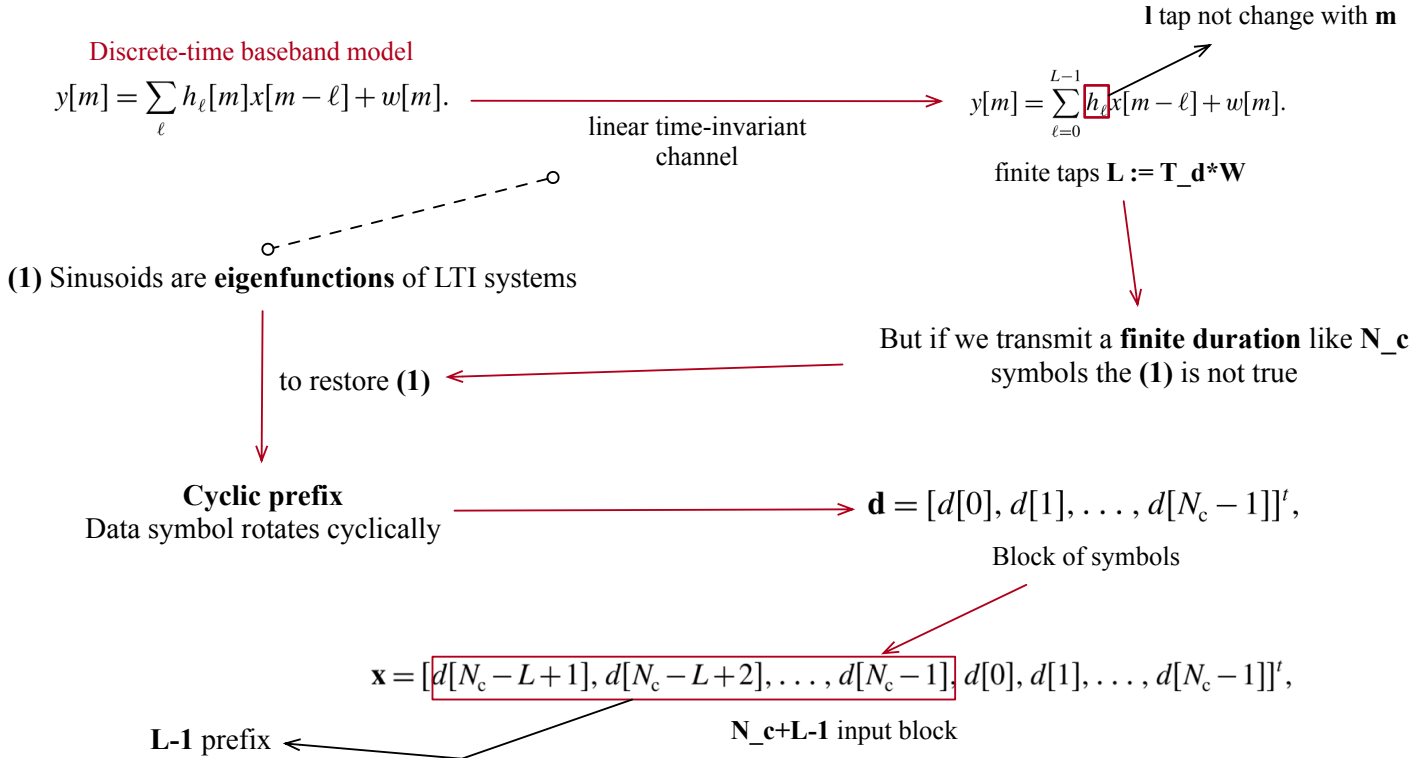
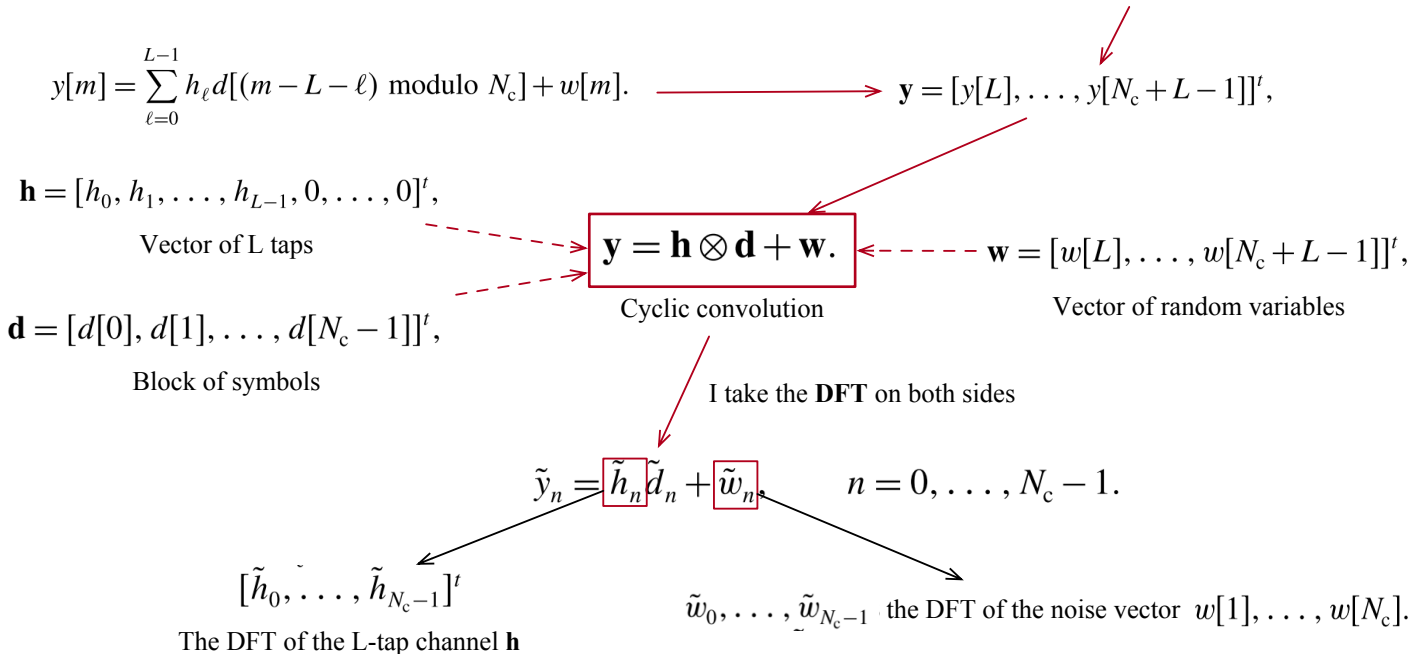


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If the channel is **underspread** and is **time-invariant** for a long time-scale the transformation in **frequency domain** can be useful to communication over *frequency-selective* channels.



Due to the additional cyclic prefix the **output** over this time interval (for avoid ISI) $m \in [L, N_c + L - 1]$.



We can redo everything in terms of **matrices**

$$\mathbf{u} = \mathbf{h} \otimes \mathbf{d} \longrightarrow \mathbf{u} = \mathbf{C}\mathbf{d}, \longrightarrow \mathbf{C} \text{ is the } \textit{circulant} \text{ matrix}$$

Circular convolution

Linear transformation

$$\mathbf{C} = \mathbf{U}^{-1} \mathbf{\Lambda} \mathbf{U}.$$

is the diagonal matrix with diagonal entries
sqrt(N_c) times the DFT of \mathbf{h}

$$\Lambda_{nn} = \tilde{h}_n := \left(\sqrt{N_c} \mathbf{U} \mathbf{h} \right)_n, \quad n = 0, \dots, N_c - 1.$$

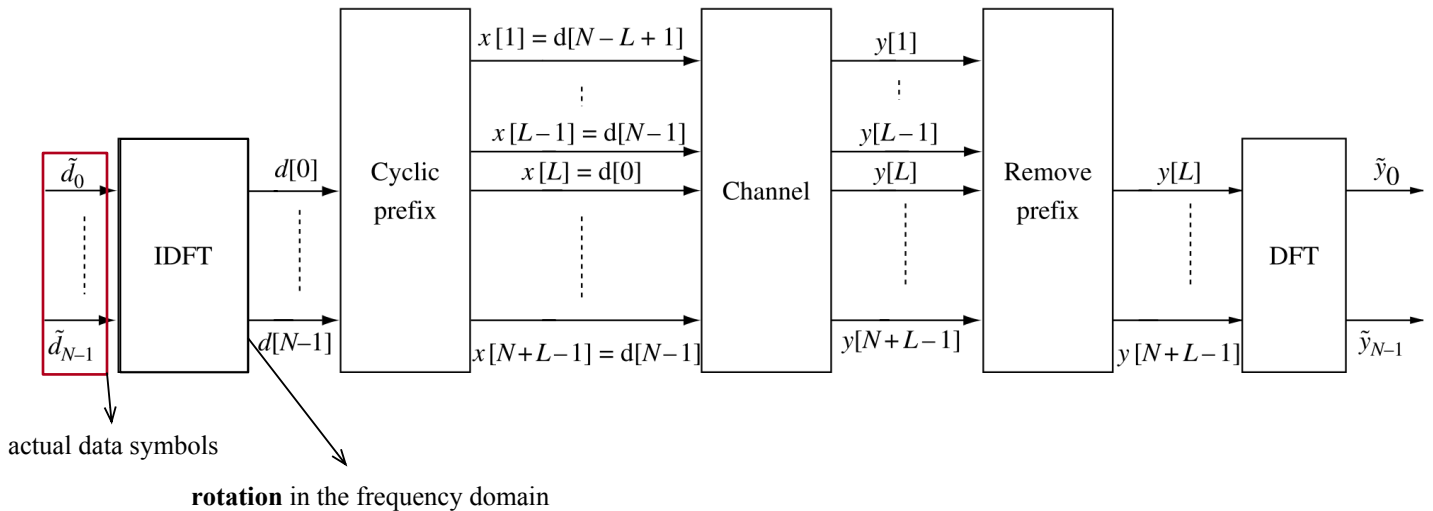
is a unitary matrix with its (k, n)th entry equal to

$$\frac{1}{\sqrt{N_c}} \exp\left(\frac{-j2\pi kn}{N_c}\right), \quad k, n = 0, \dots, N_c - 1.$$

For what we saw, we can write in a matrix form:

$$\mathbf{y} = \mathbf{h} \otimes \mathbf{d} + \mathbf{w}. \longrightarrow \mathbf{y} = \mathbf{C}\mathbf{d} + \mathbf{w} = \mathbf{U}^{-1} \mathbf{\Lambda} \mathbf{U} \mathbf{d} + \mathbf{w}.$$

This representation suggests a natural **rotation** at the input and at the output to *convert* the channel to a set of non-interfering channels with no ISI.



The data symbols modulate N_c sub-carriers, which occupy the bandwidth W and are *uniformly* separated by W/N_c . The data symbols on the sub-carriers are then converted (through the IDFT) to time domain. The procedure of introducing the cyclic prefix before transmission allows for the removal of ISI.

The receiver converts the N_c symbols back to the frequency domain through a DFT. The data symbols on the sub-carriers are maintained to be **orthogonal** as they propagate through the channel and hence go through narrowband *parallel* sub-channels