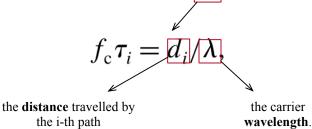
Rayleigh and Rician fading - Digital Communications 23/24

The simplest **probabilistic** model for the channel filter *taps* is based on:

Assumption — > large number of statistically *independent* reflected and scattered paths with random amplitudes in the delay window corresponding to a single tap.

The **phase** of the i-th path is $2\pi f_{\rm c} au_i \mod 2\pi$.



Since the reflectors and scatterers are **far away** relative to the carrier wavelength





the *phase* for each path is **uniformly** distributed between 0 and 2*pi and that the phases of different paths are **independent**.

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The **contribution** of each path in the tap gain $h_{\ell}[m]$:

$$a_i(m/W)e^{-j2\pi f_c\tau_i(m/W)}\operatorname{sinc}[\ell-\tau_i(m/W)W]$$

 $h_{\ell}[m]$ is a **sum** of a large number of such small independent *circular symmetric* random variables.

We assume that:

 $rac{r}{r} h_{\ell}[m]$ is $rac{\mathcal{CN}(0, \sigma_{\ell}^2)}{r}$ circular symmetric

Variance of $h_{\ell}[m]$, is a function of the tap ℓ , but independent of time m

With this assumption the **magnitude** $|h_{\ell}[m]|$ is a *Rayleigh* random variable with density:

$$\frac{x}{\sigma_{\ell}^2} \exp\left\{\frac{-x^2}{2\sigma_{\ell}^2}\right\}, \quad x \ge 0,$$

and squared magnitude $|h_{\ell}[m]|^2$ is exponentially distributed with density:

$$\frac{1}{\sigma_{\ell}^2} \exp\left\{\frac{-x}{\sigma_{\ell}^2}\right\}, \quad x \ge 0.$$

This **model**, which is called *Rayleigh fading*, is quite reasonable for scattering mechanisms where there are many small reflectors.



Assumption: the tap gains are *circularly symmetric complex* Gaussian random variables.

Alternative model: the *line-of-sight* path is large and has a known magnitude, and that there are also a large number of independent paths. So we can write:

$$h_{\ell}[m] = \sqrt{\frac{\kappa}{\kappa+1}} \frac{1}{\sigma_{\ell}} e^{j\theta} + \sqrt{\frac{1}{\kappa+1}} \frac{2}{\mathcal{C} \mathcal{N}} (0, \sigma_{\ell}^2)$$
DETERMINISTIC TERM SCATTERED PATHS

Where we have:

- (1): the *line-of-sight* path arriving with uniform phase theta
- (2): the aggregation of the large number of reflected and scattered paths, independent of theta
- (k): is the *ratio* of the energy in the line-of-sight path to the energy in the scattered paths

The **magnitude** of such a random variable is said to have a *Rician* distribution.