

# Approximating the Invariant Wealth Distribution

## 1 Preliminaries

This is an exercise to implement a list of different methods to approximate the invariant wealth distribution of ABH (Aiyagari-Bewley-Huggett) type models. The methods to be implemented are the following:

1. Eigen Vector Method
2. Monte Carlo Simulation
3. Discretization of the CDF
4. Discretization of the PDF
5. Piece Wise Linear interpolation (Still to be done)
6. Collocation (Still to be done)

These methodologies of the listed approximation methods are described in detail in Violante (2015), and Winberry (2018).

In the next section I describe the model to be solved. This example is taken from Sargent and Stachurski QuantEcon website .<sup>1</sup>.

## 2 The model

### Households

Infinitely lived households solve the following maximization problem:

$$\max E \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{1}$$

Subject to

$$\begin{aligned} a_{t+1} + c_t &\leq w z_t + (1+r)a_t \\ c_t &\geq 0 \quad \text{and} \quad a_t \geq 0 \end{aligned}$$

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<sup>1</sup>Since the objective of this exercise is to show the different methods of approximation the General Equilibrium is not computed

- $c_t$  is current consumption.
- $a_t$  is assets.
- $z_t$  exogenous component of labor income.
- $w$  is the wage rate.
- $r$  is the interest rate
- Agents are not allowed to borrow.

The exogenous process  $z_t$  follows a finite state Markov chain with a given stochastic matrix. The wage rate and interest rate are given. Households supply labor in elastically.

## Firms

Firms produce output by hiring capital and labor, with a constant returns to scale technology:

$$Y_t = AK_t^\alpha N^{1-\alpha} \quad (2)$$

With:

- $A = 1$  and  $\alpha$  are parameters.
- $K_t$  is aggregate capital.
- $N$  is total labor supply.

Then the firms maximization problem is:

$$\max_{K,N} \left[ AK_t^\alpha N^{1-\alpha} - (r + \delta)K - wN \right] \quad (3)$$

Where  $\delta$  is the depreciation rate.

## References

- Sargent, T. and Stachurski, J. Lectures quantecon the aiyagari model. <https://lectures.quantecon.org/py/aiyagari.html>.
- Violante, G. (2015). Nyu teaching, lecture11 distribution slides. [http://www.econ.nyu.edu/user/violante/NYUTeaching/QM/Fall15/Lectures/Lecture11\\_Distribution\\_Slides.pdf](http://www.econ.nyu.edu/user/violante/NYUTeaching/QM/Fall15/Lectures/Lecture11_Distribution_Slides.pdf).

Winberry, T. (2018). A method for solving and estimating heterogeneous agent macro models. *Quantitative Economics*, 9(3):1123–1151.