

with (Linear Algebra) :

Laser pose in robot coordinates:

$$x_L, y_L, \theta_L$$

Line features in world coordiantes:

$$\alpha_w, r_w$$

Predicted robot pose in world coordinates:

$$x, y, \theta$$

World to robot:

$$\alpha_r = \alpha_w - \theta :$$

$$r_r = r_w - x \cdot \cos(\alpha_w) - y \cdot \sin(\alpha_w) :$$

Robot to laser:

$$\alpha_L = \alpha_r - \theta_L :$$

$$r_L = r_r - x_L \cdot \cos(\alpha_r) - y_L \cdot \sin(\alpha_r) :$$

Combining them so that  $h_i = (z, p_R, p_L) :$

$$\alpha_L := \alpha_w - \theta - \theta_L :$$

$$r_L := r_w - x \cdot \cos(\alpha_w) - y \cdot \sin(\alpha_w) - x_L \cdot \cos(\alpha_w - \theta) - y_L \cdot \sin(\alpha_w - \theta) :$$

On earth form:

$$x_{LC} := (r_w - x \cdot \cos(\alpha_w) - y \cdot \sin(\alpha_w) - x_L \cdot \cos(\alpha_w - \theta) - y_L \cdot \sin(\alpha_w - \theta)) \cdot \cos(\alpha_w - \theta - \theta_L) :$$

$$y_{LC} := (r_w - x \cdot \cos(\alpha_w) - y \cdot \sin(\alpha_w) - x_L \cdot \cos(\alpha_w - \theta) - y_L \cdot \sin(\alpha_w - \theta)) \cdot \sin(\alpha_w - \theta - \theta_L) :$$

$$\theta_{LC} := \arctan\left(\frac{y_{LC}}{x_{LC}}\right)$$

$$\theta_{LC} := -\arctan\left(\frac{\sin(-\alpha_w + \theta + \theta_L)}{\cos(-\alpha_w + \theta + \theta_L)}\right) \quad (1)$$

$$\frac{\partial}{\partial x}(x_{LC}) \quad -\cos(\alpha_w) \cos(-\alpha_w + \theta + \theta_L) \quad (2)$$

$$\frac{\partial}{\partial y}(y_{LC}) \quad \sin(\alpha_w) \sin(-\alpha_w + \theta + \theta_L) \quad (3)$$

$$\frac{\partial}{\partial \theta}(\theta_{LC})$$

$$-1 \tag{4}$$

We then have:

$$\nabla h = \begin{bmatrix} 1 & 0 & -\cos(\alpha_w) \cos(-\alpha_w + \theta + \theta_L) \\ 0 & 1 & \sin(\alpha_w) \sin(-\alpha_w + \theta + \theta_L) \\ 0 & 0 & -1 \end{bmatrix} :$$

$$\frac{\partial}{\partial \theta}(\alpha_L) \tag{5}$$

$$x_L \sin(-\alpha_w + \theta) + y_L \cos(-\alpha_w + \theta)$$

$$\nabla h = \begin{bmatrix} \frac{\partial}{\partial x}(\alpha_L) & \frac{\partial}{\partial y}(\alpha_L) & \frac{\partial}{\partial \theta}(\alpha_L) \\ \frac{\partial}{\partial x}(r_L) & \frac{\partial}{\partial y}(r_L) & \frac{\partial}{\partial \theta}(r_L) \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & -1 \\ -\cos(\alpha_w) & -\sin(\alpha_w) & x_L \sin(-\alpha_w + \theta) + y_L \cos(-\alpha_w + \theta) \end{bmatrix} \tag{6}$$

$$diff(r_L, \theta)$$

$$x_L \sin(-\alpha_w + \theta) + y_L \cos(-\alpha_w + \theta) \tag{7}$$

$$(0.25, 0, 0)$$