Computational Logic - Assignment 1

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Prelude

I have never used LaTeX before, please bear with my plausibly horrible usage.

Exercise 2

Disjunction Using De Morgan's law, one can define the OR operator as such:

$$A \lor B \equiv \neg(\neg A \land \neg B)$$

A	B	$A \lor B$
T	T	T
T	F	T
F	T	T
F	F	F

A	B	$\neg A$	$\neg B$	$\neg A \land \neg B$	$\neg(\neg A \land \neg B)$
T	T	F	F	F	T
T	F	F	T	F	T
F	T	T	F	F	T
F	F	T	T	T	F

Implication One can define IMPLIES as such:

$$A \to B \equiv \neg A \lor B$$

A	B	$A \to B$
T	T	T
T	F	F
F	T	T
F	F	T

A	B	$\neg A$	$\neg A \lor B$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

*Arianna, per avermi stimolato ad usare LaTeX

Material equivalence One can define IFF as such:

$$A \leftrightarrow B \equiv (A \land B) \lor (\neg A \land \neg B)$$

A	B	$A \leftrightarrow B$
T	T	T
T	F	F
F	T	F
F	F	T

A	B	$A \land B$	$\neg A$	$\neg B$	$\neg A \land \neg B$	$(A \land B) \lor (\neg A \land \neg B)$
T	T	T	F	F	F	T
T	F	F	F	T	F	F
F	T	F	T	F	F	F
F	F	F	T	T	T	T

Note that IFF yields the same result as XNOR, which is opposite of XOR.

Exclusive disjunction One can define XOR as such:

$$A \oplus B \equiv (A \land \neg B) \lor (\neg A \land B)$$

A	B	$A \oplus B$
T	T	F
T	F	T
F	T	T
F	F	F

A	B	$\neg A$	$\neg B$	$A \land \neg B$	$\neg A \land B$	$(A \land \neg B) \lor (\neg A \land B)$
T	T	F	F	F	F	F
T	F	F	T	T	F	T
F	T	T	F	F	T	T
F	F	T	T	F	F	F

NAND is sufficient to define all simple logical operators

$\boldsymbol{\neg p}$ can be defined as $p \uparrow p$ ($\uparrow$ is the symbol for NAND), this is equivalent to $p \uparrow p \leftrightarrow \neg(p \land p) \leftrightarrow \neg p$. In natural language this can be referred to as "not both p and p".

$\boldsymbol{p \land q}$ can be defined as $\neg(p \uparrow q)$