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Pay-On-Exercise Options

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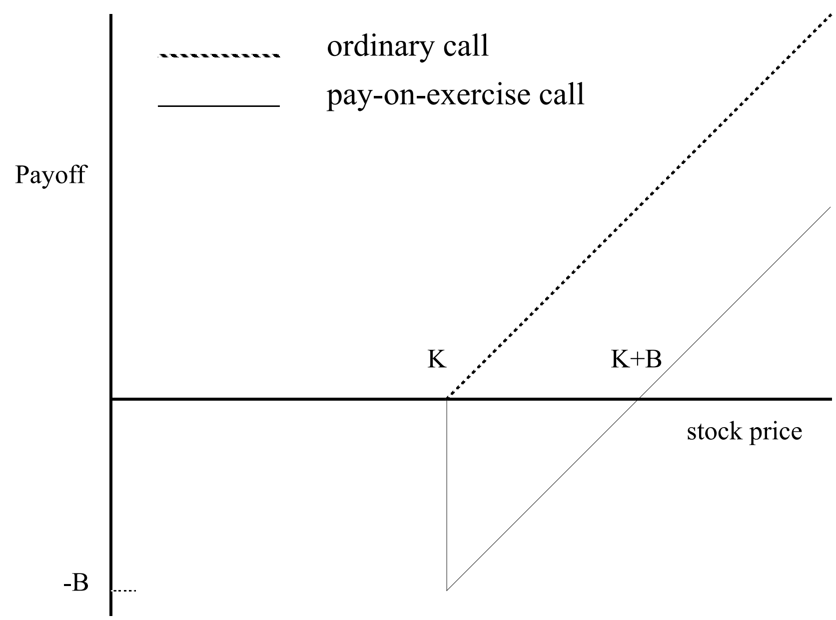
PAY-ON-EXERCISE OPTIONS

A pay-on-exercise option with payment *B* and strike *K* is an ordinary option with strike *K*, with the additional requirement that the option holder pay the option writer the amount *B* if the option expires in the money. In this note we consider only European-style pay-on-exercise options. At the end of this paper we point out some generalizations.

Let's look at a pay-on-exercise call. The holder at expiration of a pay-on-exercise call with strike *K* receives the payoff of an ordinary European call when the stock price is above K, but must pay the option writer an amount *B*. Figure 1 shows the values at expiration of an ordinary call and the pay-on-exercise call.

FIGURE 1. Expiration values for an ordinary call and a pay-on-exercise call with strike K and payment B



QUALITATIVE BEHAVIOR

You can understand the behavior of the value of pay-on-exercise call by comparing its payoff in Figure 1 with that of an ordinary call, whose value you know from the Black-Scholes[1] formula.

The holder of a pay-on-exercise option always receives less money at expiration than the holder of an ordinary option. A pay-on-exercise call is always worth less than an ordinary call.

For very large stock prices, the value of a pay-on-exercise call with strike K and payment B approaches that of an ordinary call with strike $K+B$.

The holder of the ordinary call receives a positive net payoff as long as the option expires with the stock price greater than K; otherwise he receives (and pays) nothing.The holder of the pay-on-exercise call receives a positive net payoff only if the stock price at expiration is greater than $K+B$. For stock prices between K and $K+B$ he must make a net payment to the option writer.

If the payment B is large enough, the theoretical value of the pay-on-exercise call can be zero, or even negative. When B is chosen such that the value of the call at issue is zero, the buyer of the call apparently pays nothing . However, if the pay-on-exercise call expires with the stock price between K and K+B, the option holder will have to pay the option writer.

When the stock price is zero, an ordinary call is worth zero. For stock prices much greater than K, where it is almost certain that the option will finish deep-in-the-money, an ordinary call has the approximate value of a forward with delivery price K. As the stock price increases from zero, the expected payoff becomes larger and the value of an ordinary call increases. The hedge ratio Δ is always positive.

The pay-on-exercise call is also worth zero when the stock price is zero. For stock prices much greater than K+B, the value of a pay-on-exercise option approaches that of an ordinary call with strike K+B. However, as the stock price approaches the region near K from below, the value of the call decreases because there is an increasing probability that the option will expire with the stock price between K and K+B where the option holder pays more than he receives. In this region the hedge ratio Δ is negative.

See the THEORETICAL VALUE section later in this note for a derivation of the formula for the value of a pay-on-exercise call.

1. F. Black and M. Scholes, *Journal of Political Economy* 81 (May/June 1973), pp 637-654.

AN EXAMPLE

Let's look at a one-year call struck at 110 on a non-dividend-paying stock with current price equal to 100. Assume that the riskless one-year interest rate is 10%, compounded annually, and that the volatility is 10%.

An ordinary (Black-Scholes) European-style call on this stock is worth 3.99. A pay-on-exercise call with the same expiration and a payment B of 9.14 is worth zero. We have chosen the payment to make the pay-on-exercise call have zero value.

In Figure 2 we show how the value of the pay-on-exercise call varies with the stock price at different times. You can see there is always a region of stock prices in the range between 80 and 110 in which the value of the call decreases. This means that, in contrast to ordinary calls, this call can have a negative Δ. As the time to expiration diminishes, the call value changes more rapidly in the region of the strike, and continuous hedging becomes more difficult.

FIGURE 2. Value of a pay-on-exercise call as a function of stock price at various times before expiration. Strike=110. Riskless interest rate = 10%. Volatility=10%.

THEORETICAL VALUE

In deriving the fair value of the pay-on-exercise call, we make the standard Black-Scholes assumptions and use the notation below.

S	-	current stock price

d	-	continuous dividend rate per year
t	-	time to expiration in years
K	-	strike price
B	-	payment to be made on exercise
σ	-	annual volatility in percent
r	-	continuously compounded annual riskless interest rate

Let's start with a forward on the stock with strike K and delivery time t. Its value today is $Se^{-dt} - Ke^{-rt}$, and the forward price is $S_F = Se^{(r-d)t}$.

Now look at an ordinary call option. The Black-Scholes formula for its value is

$$C = e^{-rt}(S_F N(x) - KN(x-v)) \qquad \textbf{(EQ 1)}$$

where $x = (ln(S_F/K) + v^2/2)/v$ and $v = \sigma\sqrt{t}$. *N()* is the cumulative normal distribution. **EQ 1** represents the fair value for the right at expiration to pay the strike *K* and receive the value *S*. Notice that the strike *K* appears in **EQ 1** in two places. First it multiplies *N(x-v),* where it represents the payment of the strike *K* by the option holder. Second, it appears in *x* within *N(x)* and *N(x-v)* themselves, where it is used to compute the probability of the option expiring in the money.

Finally, look at the pay-on-exercise call with payment *B*. The probability of expiring in the money is still given by the same *N(x-v)* as in **EQ 1**, but the payment that has to be made by the option holder is *K+B*. You can obtain the correct formula for the value of the pay-on-exercise call by replacing only the multiplier *K* in the second term of **EQ 1** by *K+B*.

$$C_{\text{pay-on-exercise}} = e^{-rt}(S_F N(x) - (K+B)N(x-v)) \qquad \textbf{(EQ 2)}$$

You can solve **EQ 2** for *B* to structure the pay-on-exercise call to have zero value at issue:

(EQ 3)

In EQ 3, x_0 and v_0 are the values at the time the call is issued. From then on, the value of the payment B is fixed by the contract.

From **EQ 2** you can calculate the hedge ratio for the pay-on-exercise call:

$$\Delta = e^{-dt}\left[N(x) - \frac{B}{K}\frac{e^{-x^2/2}}{\sqrt{2\pi v}}\right] \qquad \textbf{(EQ 4)}$$

You can calculate the value of a pay-on-exercise put, P, by using the put-call parity relation $P - C = (K + B)e^{-rt} - Se^{-dt}$.

GENERALIZATIONS

Any option, not only an ordinary put or call, can be modified so that the option holder has to make a payment if it expires in the money. You can structure pay-on-exercise spreads, knockouts, anything an investor is interested in, such that their initial value is zero or any other amount.

Let C be the value of an option of any kind with time to expiration equal to t. The pay-on-exercise version of this option, which requires a payment B to be made at expiration only if the stock price is greater than some limit K, is worth

$$C_{\text{pay-on-exercise}} = C - Be^{-rt}N(x - v) \qquad \textbf{(EQ 5)}$$

If the payment B is made only if the stock price is less than K, it is worth

$$C_{\text{pay-on-exercise}} = C - Be^{-rt}N(-x + v) \qquad \textbf{(EQ 6)}$$

You can choose B to give the option any value you like at issue.