Goldman Sachs

Quantitative Strategies Research Notes

November 1997

Predicting the Response of Implied Volatility to Large Index Moves

An October 1997 S&P 500 Case Study

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SUMMARY

In this note, we explain the various hypotheses and theories regarding the way implied volatilities of index options respond to sudden large index moves. One of these theories, the implied tree model developed at Goldman Sachs, uses the forward volatilities implicit in the volatility skew to predict subsequent moves in implied volatility. Using listed options data from both before (October 24) and after (October 28, 29 and 30) the recent market decline, we show that the rate at which implied volatility varied as the index level fluctuated was consistent with the predictions embedded in the volatility skew prior to the decline.

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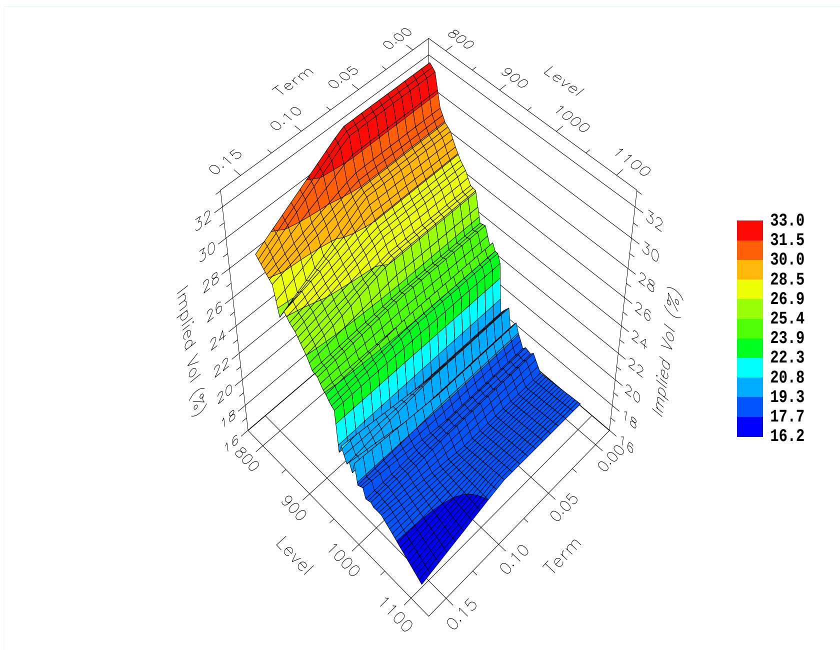
INTRODUCTION: How Does the Implied Volatility Surface Move?

Since the 1987 crash, the implied volatility of S&P 500 index options has displayed a strong variation with both option strike and expiration. At any instant, you can interpolate the available implied volatilities to construct an implied volatility surface. Figure 1 displays the mid-market implied volatility surface for listed options on the S&P 500 at market close on Friday, October 24, 1997, the last business day before the 7% drop in the market the following Monday, October, 27. You can see that implied volatilities increased steadily as strikes decreased, a so-called "negative" skew.

FIGURE 1. The mid-market S&P 500 implied volatility surface at the market close on Friday, October 24, 1997.



When the index moves, what happens to the implied volatility of each individual option on the surface? Is there some pattern to the way in which individual volatilities evolve? How does the whole surface move? We are interested here in what happens over the short term, days or weeks. In the longer run, excitement eventually fades away and volatilities tend to revert to some average level.

During the past few years, there have been many guesses or theories about how implied volatilities change. Clearly, there's a random component to these changes, independent of index level, as reflected in models of stochastic volatility. In smooth or tranquil markets, this random component is important. But here we are interested in how volatility changes during sharp market moves, and so we will ignore the random fluctuations, and instead try to model the short-term change in implied volatility that is correlated with short-term changes in index level.

In what follows, since we are interested in short-term changes in index and skew, we ignore changes in expiration. All the following comments therefore apply to what happens to the implied volatility of options of a *fixed* expiration as the index changes.

"STICKINESS" AND THE SKEW

People often find it easier to state laws of motion in terms of what *doesn't* change rather than what does. In physics, quantities that don't change during the motion of a system are called *invariants.* Many of the laws of physics can be simply stated by listing the invariants, like momentum or energy. In the same spirit, volatility traders have tried to theorize about the behavior of the implied volatility surface by stating which quantities are, in their terminology, *"sticky"* – that is, invariant as the index moves. Mostly, market participants have fallen into two camps about stickiness and implied volatilities.

The Sticky Strike Camp

The first camp claim that implied volatilities stick to strike level, and refer to the structure as "sticky strike." By this they mean that if, before an index move, an option with a certain strike and expiration has a certain implied volatility, then, even when the market moves so that the option's moneyness changes, that option's implied volatility remains unchanged. In other words, over the short term, implied volatility depends mainly on strike, independent of market level.

The Sticky Delta Camp

The second camp maintains that implied index volatilities stick to the degree of moneyness. That is, an option that is, for example, 10% out of the money after a market move will then have the implied volatility of the (different) option that was 10% out of the money before the market move. An option's moneyness is the ratio of its strike to its index level, and its Black-Scholes hedge ratio, delta, is a function of moneyness, so that sticky moneyness if roughly equivalent to sticky delta.

For small volatilities and short expirations, delta is approximately equal to the probability of the option expiring in the money in a Black-Scholes risk-neutral world. Therefore, in this style of stickiness, an option's implied volatility is regarded as depending primarily on the probability that it will be exercised.

A THIRD VIEW: STICKY LOCAL OR FORWARD VOLATILITY

Over the past few years, we have developed a new model for valuing options in a skewed environment. This *implied tree* model [1] can also be used for estimating how implied volatilities change with index level. In the terminology of stickiness, it takes the following view:

> Over the short term, what is truly sticky is the option market's expectation of forward volatility.

When index markets change sharply in an options market with a steep implied volatility skew, we believe that the forward volatilities in the skew remain approximately unchanged.

We have to be a little more precise now. What do we really mean by forward volatility? In the fixed income world, because rates depend on maturity, forward rates depend on future time. In the equity index options world, because implied volatilities depend on expiration *and* strike, so-called forward volatilities depend on future time *and* future index level. If you think of forward rates as the bond market's best guess at future short-term interest rates, you should think of forward volatility as the options market's best guess at future short-term volatility at each index levels.

A paragraph about nomenclature. Because forward volatility varies with future time and future index level, it really varies as you move *forward* (in time) and *sideways* (in index level). So, what we're really talking about is the "forward and sideways" volatility implicit in the implied volatility surface. Because of this dual dependency on time and index level, it's common to more precisely refer to "forward" volatility as *local volatility.* Local volatility is the options market's expectation for the future short-term index volatility at each future time and index level. So, the model we're discussing here is sometimes called the "sticky local volatility" model.

The local or forward volatility at a given future time and index level is also the option market's best guess for future, short-term, at-the-money volatility if the market is at that future level and time. That's so because that's the volatility that would be relevant in valuing an at-the-money forward-starting option at that index level.

1. See *Riding on a Smile* by E. Derman and I. Kani, RISK, Vol. 7, No. 2, 1994, *The Local Volatility Surface: Unlocking the Information in Index Option Prices* by E. Derman, I. Kani and J. Zou, Financial Analysts Journal, Vol. 52, No. 4, 1996 and *Trading and Hedging Local Volatility* by E. Derman, I. Kani and M. Kamal, The Journal of Financial Engineering, Vol. 6, No. 3, 1997.

Therefore, another way to think of sticky local volatility is to say that what stays invariant in the short-run during index moves is expected, at-the-money future volatility, or, equivalently, the price of forward start at-the-money options at all future market levels and times.

From now on, we will frequently employ the phrase "local volatility." It will help to think of this as short-hand for "expected future short-term at-the-money volatility."

The sticky local volatility or implied tree model uses the negative skew of the S&P 500 implied volatility market to extract its consistent local volatility as a function of index level and time. The process by which our model does this is analogous to, but more complicated than, the way in which you extract forward rates from bond yields in the fixed income world. We'll illustrate a simplified version of this method below. But first, in order to develop some intuition for local volatilities, we briefly remind readers of some characteristics of forward rates.

Extracting Forward Rates

Bond yields are averages over forward rates. Suppose the one-year bond yield is 10% and the two-year bond yield is 11%. Then the one-year rate one year forward must be approximately 12%, because the average of 10% earned the first year plus 12% earned the following year is approximately equivalent to 11% per year over two years.

Notice that if the two-year yield (11%) is *one percentage point* greater than the one-year yield (10%), then the forward rate one year from now (12%) is two percentage points greater than the current one-year rate (10%). If you thought that the bond market's expectations were reflected in forward rates, then the market's current guess for the one-year rate one year from now would be 12%. Of course, one year is a long and unrealistic time over which to assume that no new interest-rate relevant information will arrive.

Extracting Local Volatilities

We want to apply the same logic, over a short time period, to local volatility. Actual equity index volatility skews are often roughly linear at low strikes. Consider the sample linear skew of Table 1 for three-month options.

TABLE 1. A sample skew for three-month options. Index = 100.

Strike	Implied Volatility (%)
100	20%
99	21%
98	22%
97	23%

Just as a bond yield is an average over forward rates, the implied volatility of an option of a given strike can be regarded as a linear average over future local volatilities between the current index level and the strike[2]. You can then extract these local volatilities to get the results in Table 2.

Notice that the at-the-money implied volatilities in Table 2 increase *twice as fast* with index level implied volatilities increase with strike in Table 1. As with forward rates, the reason is the averaging. For example, when the index level is 100, an implied volatility of 21% for a strike of 99 in Table 1 is the average of a local volatility of 20% at an index level of 100 and 22% at a level of 99 in Table 2.

TABLE 2. The local volatilities extracted from the skew in Table 1.

Index Level	Local volatility (%)
100	20%
99	22%
98	24%
97	26%

2. In reality, the average is not truly linear except for strikes close to spot, and can also depend upon the term structure of short-term local volatilites. To take account of these details, see the implied tree model of Derman and Kani in footnote 1.

Sticky Local Volatility and When to Use It

At some instant, we extract local volatilities from the volatility surface. For linear skews, these local volatilities increase twice as fast with index level as the skew grew with strike. We then regard the local volatility at each index level as a predictor of future at-the-money implied volatility at that level. We hope this forecast will be good over the short-term (several days to a week or two), when the index changes appreciably. Since implied volatility is a measure of the options market's appetite for volatility, who knows better than the market itself how it's appetite will change as the index moves?

TESTING OUR IMPLIED VOLATILITY FORECASTS DURING THE MARKET DECLINE OF OCTOBER 1997

All options data used in this section are taken from listed options prices as reported by Reuters.

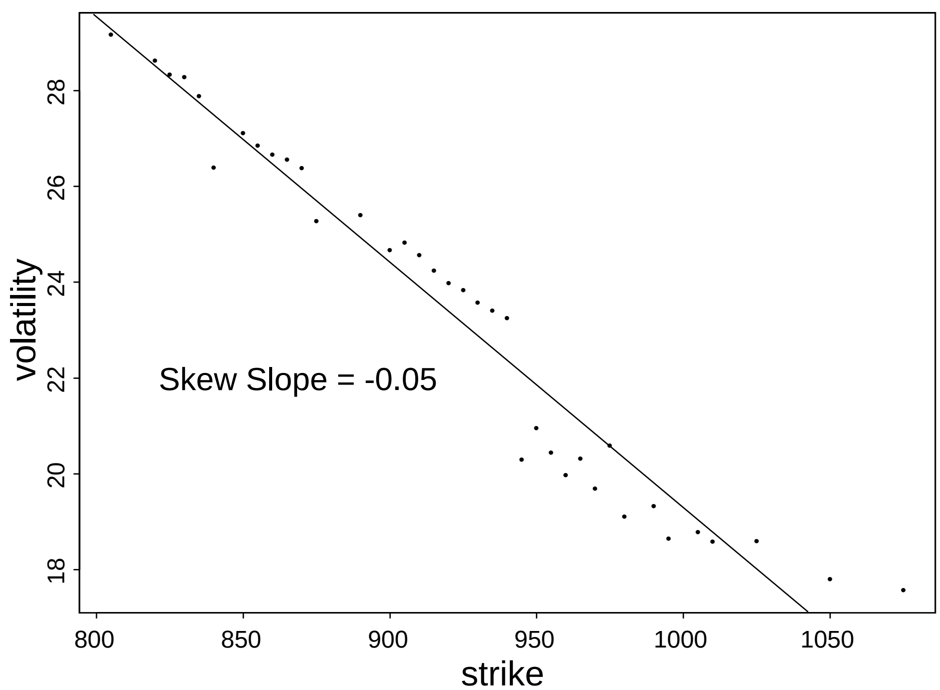
The S&P 500 index level closed at 941.64 on Friday, October 24, 1997. Figure 2 shows the skew at market close for S&P 500 index options expiring on December 20, 1997. The skew slope was approximately linear, and about -0.05 volatility points per strike point. According to our implied tree or sticky volatility model, at-the-money implied volatility should move by -0.10 volatility points per one point move in the index, that is at twice the rate of the skew slope.

At market close on Friday, October 24, the *average* mid-market implied volatility for S&P 500 index options was about 21.3%. (This value was recorded by one of our in-house computer programs that collect options prices from Reuters. The average is taken over all strikes and expirations, with a substantially greater weighting given to at-the-money options, and is meant to give an indication of overall implied volatility.)

According to our model, if local volatility is indeed sticky, at-the-money volatility should move at a rate of -0.10 volatility points per index point when the index moves. Figure 3 shows the average S&P 500 implied volatility, as recorded by our computer program, from Tuesday, October 28 through Thursday, October 30. The individual points represent observations of volatility taken at approximately half-hour intervals. The solid line is a least squares fit to the observed average volatilities as a function of index level, and its slope is -0.1, consistent with our prediction.

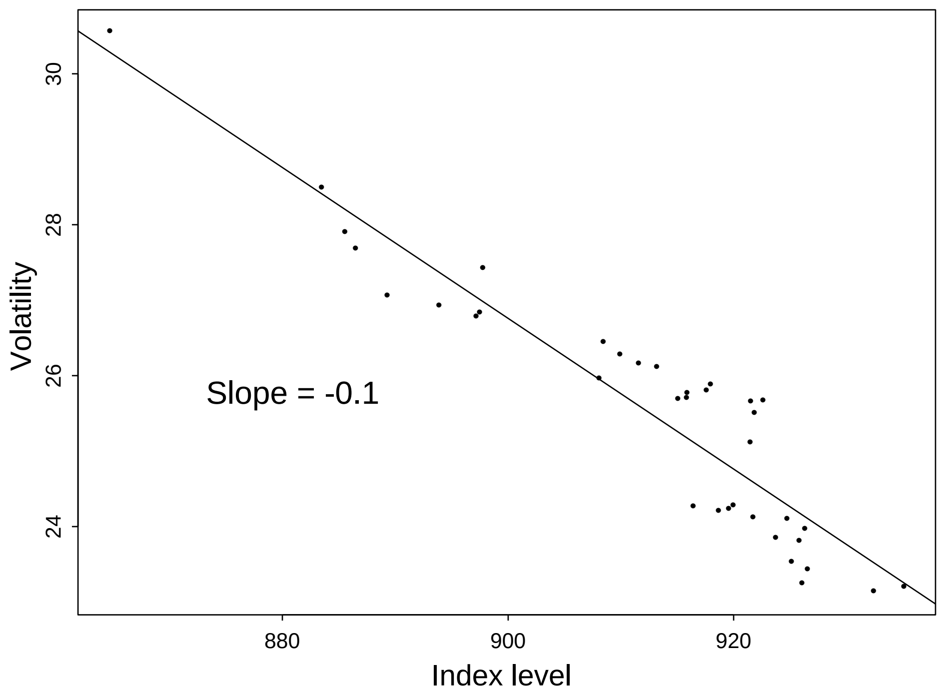
We conclude that the sticky local volatility model accurately predicted the rate at which the average implied volatility of index options changed with index level directly following the market decline on Monday, October 27. Volatilities rose as the index fell, and vice versa, approximately in accord with the skew of the previous Friday. Apparently, the option market's view, before the decline, of future at-the-money implied volatility was a good predictor of the rate at which post-decline at-the-money implied volatility changed with index level. These conclusions are in agreement with more general studies we have carried out on Nikkei 225 and S&P 500 implied volatilities over the past two years[3], and also correspond to similar observations made on post-decline Nikkei 225 implied volatilities. Although each individual option's implied volatility may move somewhat differently, on average, they moved consistently with our model's forecasts.

FIGURE 2. The implied volatility skew of December S&P 500 index options as of October 24, 1997.



3. See *Rolling Along the Volatility Surface: Options Hedging in Markets with a Skew,* Quantitative Strategies Research Notes, forthcoming (1998).

FIGURE 3. Mid-market average implied volatility for S&P 500 index options from October 28 through October 30, displayed as a function of the contemporaneous index level. Individual points correspond to implied volatility observations at approximately half-hour intervals. The solid line represents a least squares fit to the observed volatilities; its slope of -0.1 matches the implied tree (sticky local volatility) forecast for the variation of implied volatility with index level based on the skew of October 24.



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