Economics 134 L6. Common-Pool Externalities and Public Goods

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October 15, 2025

Plan for today

General principle: when people and/or firms do not perfectly internalize the effects of their actions, **markets fail**.

So far, externalities between two agents (the "firm" and "neighbor").

Now, economies with more participants.

To this end, today we introduce two separate but related concepts:

1. Common-pool externalities
2. Public goods

I. Common-pool externalities
    Overfishing
    Example

II. Public goods
    Definition
    Example
    Policy solutions
    Informational issues

Tragedy of the commons

Consider a collection of individuals (e.g., firms, people), each of whom draw from a common resource.

A **common-pool externality** occurs when the actions of each individual affect everyone's value of the remaining resource.

Examples:

- groundwater extraction
- oil drilling
- fisheries

Simple example: Overfishing

Suppose there is some large number of prospective fishing boats.

- Suppose that each boat can choose to go fishing for a cost $c > 0$.
- If a boat goes fishing, it obtains

$$f(N)$$

  fish, where N is the total number of boats and $f', f'' < 0$ (i.e., with more fishing boats, each catches fewer fish).
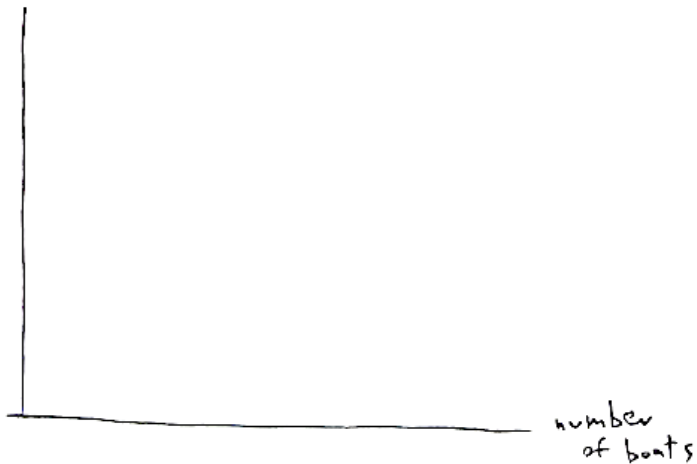
- Then the fish can be sold for some price p.

I. Free market outcome. If anyone can go fish, boats will go fishing as long as they expect to make a profit, i.e., if
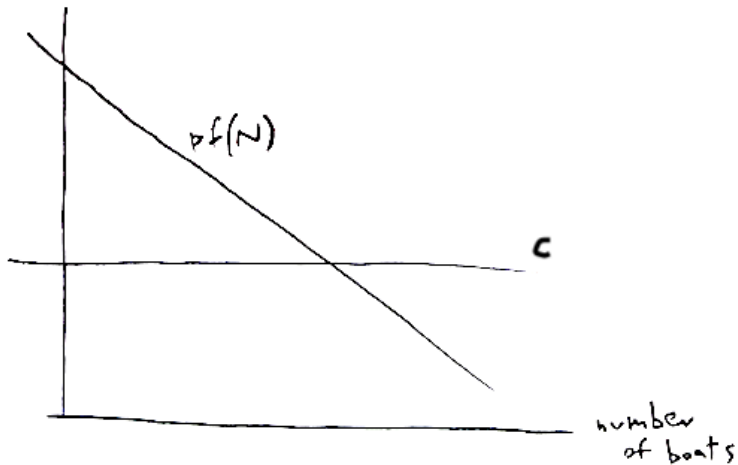
$$p \cdot f(N) - c \geq 0,$$

and with free entry, firms will enter until $p \cdot f(N) - c = 0$.
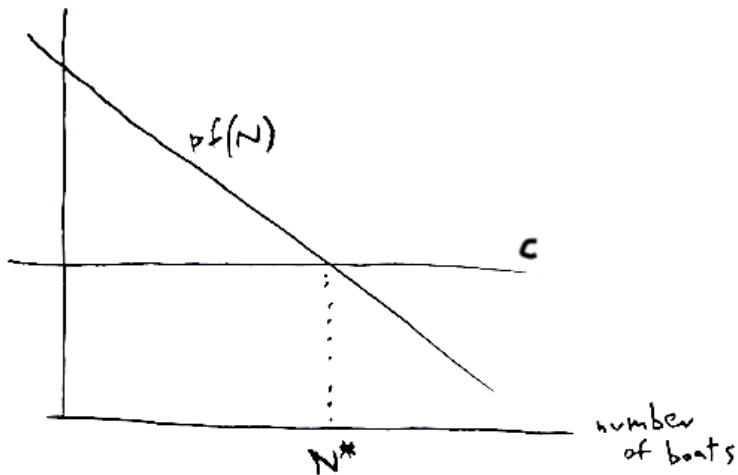
Free entry

number
of boats

Free entry

$pf(N)$

c

number
of boats

Free entry

Simple example: Overfishing, cont'd

II. First-best. In contrast, the efficient allocation chooses N boats to maximize
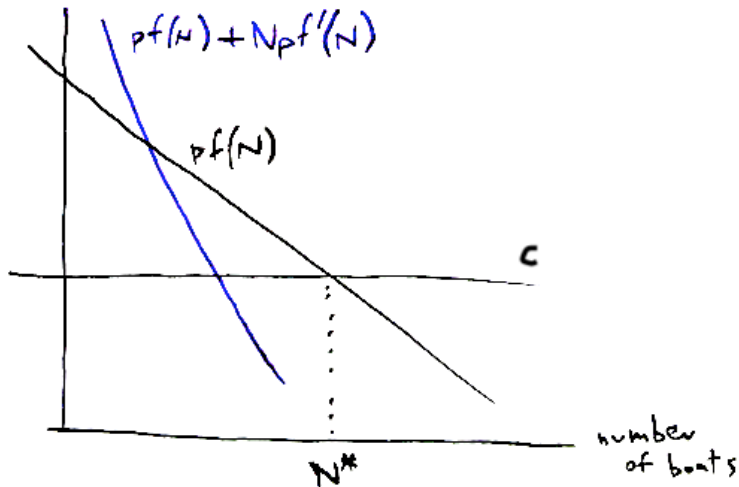
$$N \cdot [p \cdot f(N) - c].$$

This leads to the condition that

$$\underbrace{\left[p \cdot f(N^{\text{FB}}) - c\right]}_{\text{free market}} + \underbrace{N^{\text{FB}} p \cdot f'(N^{\text{FB}})}_{\text{social cost}} = 0$$
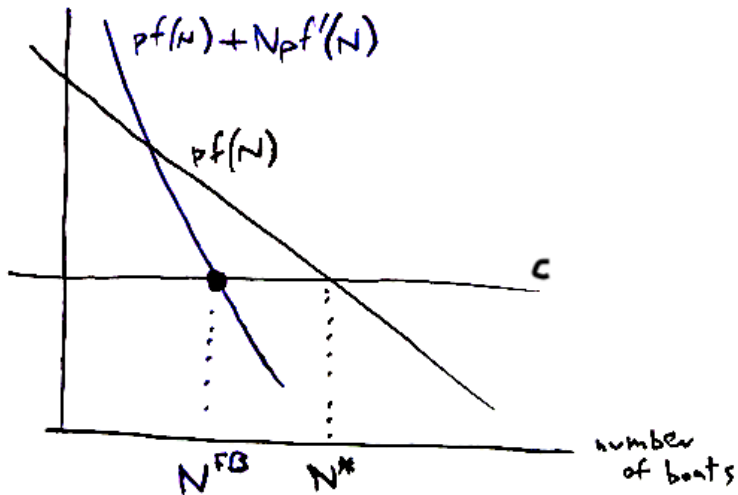
implying that we should **restrict** the number of fisherman who go fishing (or charge them an optimal tax to go fishing).

$\hookrightarrow$ The common-pool externality reverses the usual presumption in economics that free entry is desirable!

Efficient entry

$pf(N) + Npf'(N)$

$pf(N)$

c

N^*

number of boats

Efficient entry

Common-pool externalities

We can extend this simple example to a broad class of environmental problems.

<u>Basic idea</u>: each firm's cost depends on its own level of extraction **and** the total level of everyone else's extraction

<u>In math</u>: N firms, indexed by i, each using q_i of some resource.

The sum of all firms' extraction is $Q = \sum_{j=1}^{N} q_j$.

Each firm obtains

$$\pi(q_i; Q) = pq_i - c(q_i; Q),$$

with the cost of extraction $c(\cdot; \cdot)$ depending on both

- the firm's **individual** q_i and
- the **aggregate** quantity of extraction Q.

Common-pool externalities

For example, consider, convex costs that increase in total extraction:

$$c(q_i; Q) = \frac{\gamma}{2} q_i^2 + \overline{\gamma} q_i \cdot \underbrace{\sum_j q_j}_{Q}.$$

(For example, pumping costs rise as groundwater levels fall...)

Open access

Suppose $c(q_i; Q) = \frac{1}{2}\gamma q_i^2 + \overline{\gamma} q_i \sum_j q_j$. Then firm i's marginal cost is

$$\frac{\partial c}{\partial q_i} = \gamma q_i + \overline{\gamma} \cdot (Q + q_i).$$

I. Free market ("open access") outcome. The firm maximizes profits; in particular, they equate price with marginal cost:

$$p = \frac{\partial c}{\partial q_i} = \gamma q_i + \overline{\gamma} \cdot (Q + q_i)$$

which leads them to extract

$$q_i^* = \frac{p - \overline{\gamma}Q}{\gamma + \overline{\gamma}}.$$

<u>Remark 1</u>. Extraction q_i^* is increasing in price p. Not surprising.

<u>Remark 2</u>. Extraction q_i^* is decreasing in $\overline{\gamma}Q$: both the strength of the externality (measured by $\overline{\gamma}$) and the total level of extraction Q.

Open access, cont'd

Note that we haven't quite "solved" for the free market outcome yet, because Q is a function of q_i^*!

But, note that q_i^* is the same for all i. So, let $q_i^* = q^*$. By symmetry, $Q = Nq^*$.

With symmetry, the first-order condition becomes

$$p = \gamma q^* + \overline{\gamma} \cdot (Nq^* + q^*),$$

which gives us equilibrium extraction of

$$q^* = \frac{p}{\gamma + (N+1) \cdot \overline{\gamma}}$$

for each i under the market.

Efficient extraction, cont'd

II. First-best outcome. Now, we want to maximize total profits:

$$p \sum_{j=1}^{N} q_j - \sum_{j=1}^{N} c(q_j; Q).$$

Using symmetry, all $q_i = q$, and $\sum_j q_j = Nq$. Therefore

$$\sum_{i=1}^{N} \pi(q; Q) = N \cdot \pi(q; Q) = N \cdot \left[pq - \frac{1}{2}\gamma q^2 - \overline{\gamma} \cdot q \cdot Nq \right]$$

which is maximized when

$$p - \gamma q^{\mathsf{FB}} - \overline{\gamma} \cdot 2Nq^{\mathsf{FB}} = 0$$

or

$$q^{\mathsf{FB}} = \frac{p}{\gamma + 2N \cdot \overline{\gamma}}.$$

Tragedy of the commons

Comparing the free market ("open access") versus first-best, we see

$$\frac{p}{\gamma + (N+1) \cdot \overline{\gamma}} = q^* \geq q^{\mathsf{FB}} = \frac{p}{\gamma + 2N \cdot \overline{\gamma}}.$$

$\therefore$ There is **overextraction** as long as $N > 1$!

- The problem gets worse as N grows ("tragedy of the commons")

Policy solutions

This is an externality, so we can apply our previous work:

1. Regulation?
   - restrict users to no greater than q^{FB}
   - need to be able to monitor and enforce the restriction

2. Tax?
   - the optimal tax on users is equal to the marginal damage

3. Coase?
   - many agents! Defining property rights may be difficult; transaction costs possibly large
   - though, see Ostrom (2009) for some counterexamples!

4. Merger?
   - similar issues to Coase; costs of coordinating the merger is likely large unless N is very small
   - may introduce other concerns with monopoly power in the product market...

Some definitions

A standard economic good is **excludable**.

What do we mean by this?

Definition

A good is **excludable** if consumers can be excluded from consuming the good, and **nonexcludable** otherwise.

✓ Cannot price something that you cannot prevent people from consuming.

This was the key problem with the commons—users could not be excluded from using the common-pool resource (prior to regulation).

Some definitions, cont'd

In addition to excludable, private goods are also rivalrous:

Definition

A good is **rivalrous** if one person's consumption of that good affects the amount available for others, and **nonrivalrous** otherwise.

Example: common-pool resources are rivalrous (taking some of the resource leaves less for others).

Basic economic problem with nonrivalrous goods in competitive markets:

$\hookrightarrow$ zero cost of providing nonrivalrous goods to additional consumers

$\hookrightarrow$ if price equals marginal cost, the price should also be zero

$\hookrightarrow$ but then revenues will be less than costs, so no supplier will enter the market!

Some definitions, concluded

A private good is both excludable and rivalrous.

A public good is neither:

Definition (Public good)

A "**public good**" is a good that is both nonrival and nonexcludable.

Possible examples of **public** goods:

- peace / national defense
- national forests
- government-backed currency
- clean air

In each example, it's hard to exclude anyone from benefiting from these resources, and one person's use has a negligible effect on others' use.

Example

Suppose there are N individuals, each of whom have utility

$$u(g_i; G) = -\frac{1}{2}g_i^2 + \theta_i G,$$

defined over the cost of their individual contribution g_i and their value θ_i for the the public good, $G = \sum_{i=1}^{N} g_i$ (the sum of individual contributions).

I. First-best. The first-best level of the public good solves

$$\max_{\{g_i\}_{i=1}^{N}} \sum_{i=1}^{N} \left[\theta_i G - \frac{1}{2}g_i^2\right]$$

or the first-order condition $-g_i + \sum_i \theta_i = 0$, so that

$$g_i^{\text{FB}} = \sum_{i=1}^{N} \theta_i$$

for each i. That is, the efficient level of the public good equates the marginal cost of provision with the sum of marginal benefits for all (Samuelson, 1954).

Example

II. Free market. But the privately optimal contribution to the public good is just
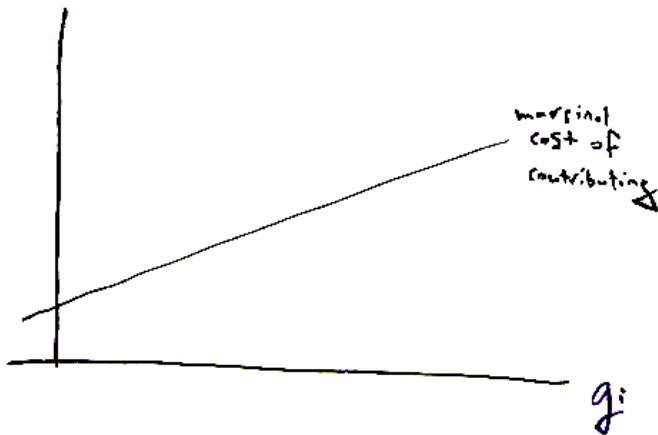
$$g_i^* = \theta_i.$$

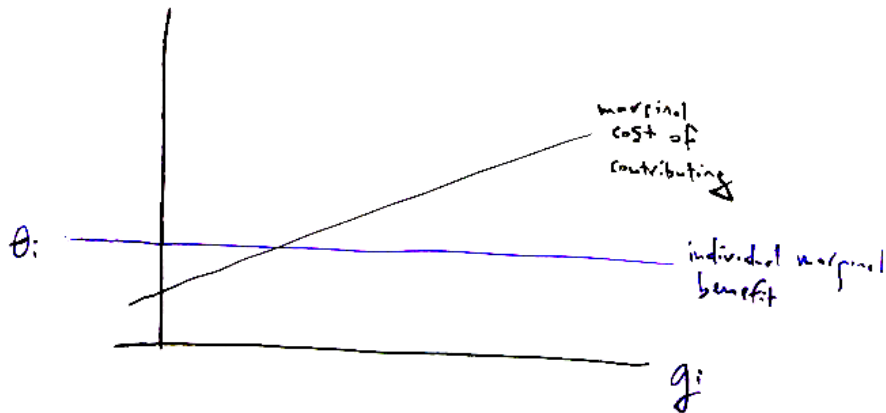That is, there is **private underprovision** of the public good.

Think about our prior examples:

- peace: free-riding in NATO; provided by the US
- forests: taxpayer-funded U.S. Forest Service
- fiat currency: automated subsidies for participants who allocate computing resources to ensure continuous, decentralized encryption (e.g., bitcoin miners)
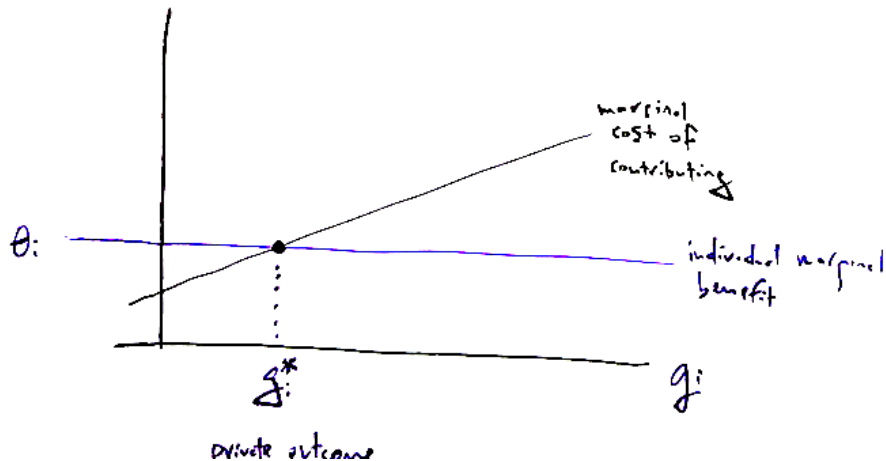
Public goods

marginal
cost of
contributing

g_i

Public goods

marginal
cost of
contributing

θ_i

individual marginal
benefit

g_i

Public goods: Private provision

marginal
cost of
contributing

θ_i

individual marginal
benefit

g_i^*

g_i

private outcome

Public goods: Efficient level
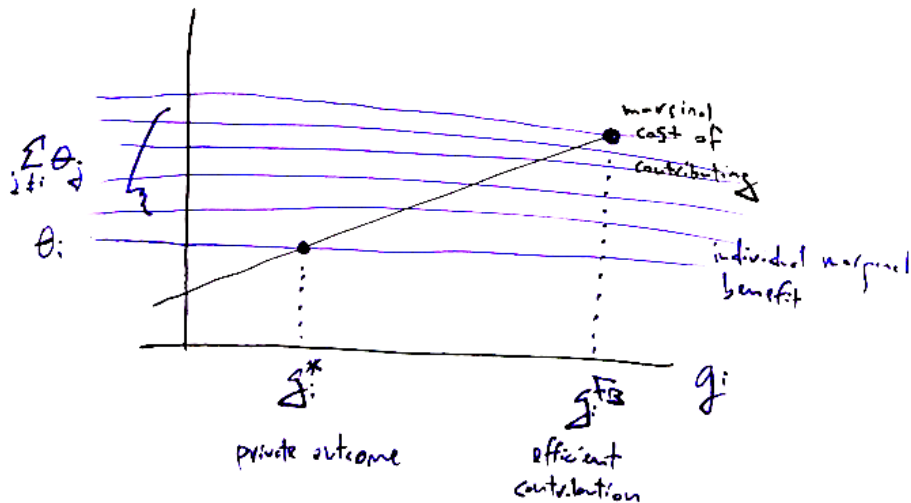
Discussion

Remark. Mathematically, very similar to the common-pool resource problem.
Instead of over-extraction, we have under-provision!

- for a common-pool resource,
    - if one firm raises their extraction, this raises the costs for others,
    - causing the others to slightly decrease their extraction

- for a public good,
    - if one person contributes more to the public good, everyone else benefits
    ↪ this raises the concern of **free-riding**

Key difference is that the public good is <u>nonrivalrous</u>.

Policy solutions

How can we incentivize people to contribute the optimal level of the public good?

Optimal subsidy needs to push individuals to contribute more:

$$g_i^* = \theta_i + s_i$$

with

$$s_i = \sum_{j \neq i} \theta_j.$$

$\hookrightarrow$ Our example has an important feature: people have different values θ_i for the good! There is no one optimal tax; everyone should face a **personalized** price.

Aside. This system of personalized pricing is sometimes called the **Lindahl solution** to public goods.

Public goods and informational constraints

Observing θ_i is often a challenge. This raises additional concerns.

For example, suppose that each i knows their value θ_i, but that the government knows only the total, $\Theta = \sum_i \theta_i$.

If the government knew each individual's i, then they would like to subsidize them

$$s_i = \sum_{j \neq i} \theta_j = \Theta - \theta_i.$$

Basic problem is that individuals **may not wish to report truthfully**.

- If you admit to having a high θ_i, and you know the government will give you

$$s_i = \Theta - \theta_i$$

  then you know you will receive a lower subsidy from the government.

- So, you can get a better deal if you misrepresent your preferences by pretending to have a lower θ_i!

Next time

- First midterm next Monday (10/20)
- Covers topics from the first six lectures and the problem set.
- You can bring up to one 8.5" $\times$ 11" sheet of paper with typed or handwritten notes (can be double-sided)
- In-class