Economics 134 Fall 2022 Midterm 2

November 9, 2022

Write your name and UID in the upper right corner of each page.

You have 75 minutes to complete this exam. Show your work.

QUESTION 1 (25 points). Long-run climate change.

Recent scientific research has indicated that about 3.3% of the Greenland Ice Sheet will melt by 2100, raising the sea level by about ten inches.

Suppose that you believe that the annual rate of pure time preference is $\rho = 1\%$, our elasticity of marginal utility of consumption is $\theta = 2$, and consumption will grow by $g = 1\%$ per year.

(a) Calculate the real discount rate r appropriate for policy evaluation.

Solution: Using Ramsey's formula, $r = \rho + \theta g$, or $r = 1\% + 2 \cdot 1\% = 3\%$.

(b) Suppose all of the damages from sea level rise happen in the year 2100. In particular, sea level rise will lower the world's consumption in 2100 from \$200 trillion to \$94 trillion. From the perspective of 2022, what is the net present discounted value of the damage from sea level rise?

Solution: We obtain $\frac{1}{(1+r)^{2100-2022}} \cdot (200 - 94) = \frac{106}{(1+r)^{2100-2022}} = 10.57$ trillion for $r = 0.03$.

(c) Suppose that the damages happen gradually from 2050 until 2100. If sea level rise will destroy \$2 trillion of consumption every year from 2050 to 2100, what is the net present discounted value of the damage from sea level rise from the viewpoint of 2022?

Solution: We obtain $\sum_{t=2050}^{2100} \frac{2}{(1+r)^{t-2022}} = \sum_{t=2050}^{2100} \frac{2}{(1+0.03)^{t-2022}}$ trillon. It is not necessary to solve this expression for an exact number to get full credit.

(d) Imagine that marginal utility falls less quickly with consumption, so that $\theta = 1$. How does this change your answers to (b) and (c)?

Solution: Now $r = 1\% + 1 \cdot 1\% = 2\%$. Our answers to (b) and (c) will increase.

QUESTION 2 (50 points). International climate collaboration.

We have N countries, each with profits $\pi(k_i) = k_i - \frac{1}{4}k_i^2$ from carbon emissions $k_i \geq 0$. Each country i also experiences damages from climate change, $D_i(K) = \theta_i K$, where $\theta_i < 1$ is their value of climate protection and $K = \sum_{i=1}^{N} k_i$ are the total carbon emissions.

(a) Suppose each country i chooses emissions k_i to maximize just its profits. How much will each country i emit?

Solution: Maximizing profits gives $1 - \frac{1}{2}k_i = 0$ or $k_i^* = 2$.

(b) Suppose each country i chooses emissions k_i to maximize its welfare (defined as that country's profits minus its environmental damages). How much will each country i emit?

Solution: Maximizing country i's welfare gives $1 - \frac{1}{2}\tilde{k}_i - \theta_i = 0$ or $\tilde{k}_i = 2(1 - \theta_i)$.

(c) Define global welfare as total profits minus total damages across all countries. If we would like to maximize global welfare, what should each country i emit? Call this the "efficient" outcome.

Solution: Total welfare is

$$W = \sum_{i=1}^{N} \left[k_i - \frac{1}{4}k_i^2 - \theta_i K \right].$$

This is maximized when $k_i^{\text{FB}} = 2 - 2\sum_{i=1}^{N} \theta_i$.

(d) Suppose that $\theta_i = \frac{1}{2}$ for every i and that $N = 10$. If each country maximizes its own welfare (but not the welfare of any other country), will this lead to the efficient outcome?

Solution: No. If each country maximizes their own welfare, then they will choose $\tilde{k}_i = 2(1 - \theta_i) = 2(1 - \frac{1}{2}) = 1$. But the efficient outcome is for $k_i^{\text{FB}} = 0$ for all i.

(e) Now suppose that $\theta_i = 1$ for every i and that $N = 10$. If each country maximizes its own welfare (but not the welfare of any other country), will this lead to the efficient outcome?

Solution: Yes! This is because if $\theta_i = 1$, then $k_i^* = 0$. This is the same as the first-best.

(f) Suppose that it is now possible for countries to choose negative carbon emissions ($k_i < 0$), e.g., by planting trees. Will this change your answer to (e)?

Solution: Yes, it will change, as the outcome in (e) is no longer efficient because the first-best will involve negative emissions: we maximize

$$W = \sum_{i=1}^{N} \left[k_i - \frac{1}{4} k_i^2 - \theta_i K \right]$$

so that

$$1 - \frac{1}{2} k_i^{\text{FB}} - \sum_{i=1}^{N} \theta_i = 0$$

or

$$1 - \frac{1}{2} k_i^{\text{FB}} - 10 = 0$$

so that $k_i^{\text{FB}} = -18$ giving payoffs of $-18 - \frac{1}{4} 18^2 - (-18 \cdot 10) = 81$ for each i.

QUESTION 3 (25 points). Study groups.

Three students from Econ 134 have formed a group to create a study guide. With three or more hours of work, the students will produce a study guide that will improve their grade significantly. For fewer than three hours, the study guide will be incomplete and therefore useless.

Each student's utility is $u(\ell, s) = \frac{4}{3} s - \ell$, where $s = 1$ if there is a useful study guide and 0 otherwise, and ℓ is the number of hours the student spends working on the guide.

(a) Suppose the first two students have already contributed three hours of work. If the third student in the study group maximizes his utility, how much should he work?

Solution: Zero, because $s = 1$ no matter what he chooses, and his utility is decreasing in ℓ. Free-riding!

(b) Suppose the third student contributes nothing and that the first two students each choose ℓ_1 and ℓ_2, respectively, to maximize their respective utilities. Will the first two students contribute

three hours of work? Does it matter if the first two students instead act to maximize the sum of their utilities, $u(\ell_1, s) + u(\ell_2, s)$?

Solution: If student three does no work, then the first two have to contribute a total of at least $\ell_1 + \ell_2 = 3$. But their combined payoff would be only $\frac{8}{3} - 3$ which is less than three. So the students will not contribute.

(c) Now there are four students in the study group. If each student contributes one hour of work to the study guide, is this a Pareto improvement relative to none contributing? If every student thinks that the other three will contribute one hour of work, what will they each contribute?

Solution: Yes, it is a Pareto improvement, because everyone gets $\frac{4}{3} - 1 > 0$.

If a student thinks the other three will contribute, they will contribute nothing for the same reason as in (a). If all students think this, the total contribution will be zero.

(d) Is the study guide a public good for the Econ 134 students? Explain.

Solution: Yes, provided that it is nonexcludable.