Economics 134 Fall 2023 Midterm 1 – Solutions

October 23, 2023

You have 75 minutes to complete this exam. Show your work.

QUESTION 1, PART I (50 points). Bob grows strawberries using fertilizer. Applying q units of fertilizer takes effort. It costs Bob q^2, but yields him $4q$ strawberries.

(a) If Bob can sell strawberries for $p = \frac{1}{2}$ per strawberry, and Bob maximizes his profits, how much fertilizer will he use?

> **Solution:** Bob's payoffs are $\pi(q) = p \cdot 4q - q^2$, which is maximized when $4p - 2q^* = 0$, or $q^* = 1$ if $p = \frac{1}{2}$.

(b) How would your answer to (a) change if strawberries double in price?

> **Solution:** Then $q^* = 2p = 2$, so he uses twice as much fertilizer.

(c) Suppose fertilizers hurt fish. There are nine fish. Each fish suffers q^2 when Bob uses q units of fertilizer. What is the efficient (or "first-best") level of strawberry production on Bob's farm?

> **Solution:** Damages here are $D(q) = 9q^2$. The first-best maximizes $\pi(q) - D(q)$, or $4pq - q^2 - 9q^2$, which is maximized when
>
> $$4p - 20q^{\mathrm{FB}} = 0$$
>
> implying that $q^{\mathrm{FB}} = \frac{1}{10}$ if $p = \frac{1}{2}$.

(d) Calculate the tax on Bob's fertilizer use that will incentivize him to produce the efficient quantity of strawberries. Could we get to the first-best by taxing Bob's strawberries instead of fertilizer? Explain.

Solution: (i) The tax will be the Pigouvian tax, $\tau = D'(q^{\text{FB}})$, which here equals $\tau = 18q^{\text{FB}} = 1.8$ per unit of fertilizer.

(ii) We could get to the first-best by taxing strawberries instead. To do this, note that 1 unit of fertilizer produces 4 strawberries. So the per-strawberry tax just needs to be 1/4 of the fertilizer tax; i.e., $\tau_{\text{strawberry}} = \frac{1}{4} \cdot \tau_{\text{fertilizer}} = \frac{1}{4} \cdot \frac{18}{10} = \frac{9}{20}$.

(e) What are the total environmental benefits (i.e., to the fish) from imposing the optimal tax on Bob?

Solution: Environmental harm to the fish without the tax was $D(q^*) = 9(q^*)^2 = 9(1)^2 = 9$. Environmental harm with the tax becomes $D(q^{\text{FB}}) = 9 \cdot (1/10)^2 = \frac{9}{100}$. The total benefits are then

$$D(q^*) - D(q^{\text{FB}}) = 9 - \frac{9}{100} = 8.91.$$

(f) Suppose that we shut down Bob's farm (i.e., mandate that $q = 0$). Will this improve welfare relative to the free market outcome? Is this efficient?

Solution: Yes, this will improve welfare. Welfare under the free market is $\pi(q^*) - D(q^*) = \frac{1}{2} \cdot 4 \cdot 1 - 1^2 - 9 \cdot 1^2 = 2 - 1 - 9 = -8$. Welfare without Bob's farm is $\pi(0) - D(0) = 0$. And clearly $-8 < 0$.

No, this is not efficient. As we showed in (c), the efficient (first-best) outcome involves nonzero fertilizer use ($q^{\text{FB}} = 0.1$).

QUESTION 1, PART II (20 points). Bob's friend, Evan.

Continue to assume everything from Question 1, Part I.

(a) Evan the fisherman moves downstream of Bob. Suppose Evan likes watching the fish. Specifically, Evan's utility depends on the amount of fertilizer that upsets the fish, and is given by $u(q) = 4 - q$. How will this change the optimal tax on Bob's fertilizer?

Solution: The new "damage" function is $D(q) = 9q^2 + q$. This creates an additional marginal damage of $-u'(q) = 1$, so the tax should be $\tau = D'(q^{\text{FB}}) = 18q^{\text{FB}} + 1$.

Note that the q^{FB} will also change, to maximize $4pq - q^2 - 9q^2 + 4 - q$, or solve

$$4 \cdot \frac{1}{2} - 2q^{\mathrm{FB}} - 18q^{\mathrm{FB}} - 1 = 0.$$

This means that $q^{\mathrm{FB}} = \frac{1}{20}$.

Consequently, the optimal tax should be $18 \cdot \frac{1}{20} + 1 = 1 + \frac{9}{10} = 1.9$

(b) Assume Evan and Bob both maximize their respective utilities and have full information. If Evan can contract costlessly with Bob over his fertilizer use, will they reach the efficient outcome? Explain.

Solution: No. The Coase theorem will apply to Evan and Bob, but the fish are not present for the negotiation.

You can also see this through calculation. Evan and Bob will maximize $4 \cdot \frac{1}{2} - q^2 + 4 - q$, which is maximized at $2 - 1 - 2\tilde{q} = 0$, or $\tilde{q} = \frac{1}{2}$. We know this is not the first-best level from our work in (a).

QUESTION 2 (30 points). Otters.

Teresa is now a successful musician. She plans to purchase a house in Los Angeles with an ocean view. There are two houses for sale, House A and House B, that are identical except that otters often swim by House A. No otters swim by House B.

The sale price of House A is \$2500. The sale price of House B is \$2200.

(a) Teresa sees the two houses and their prices. Assuming that she has taken Economics 134, what can she deduce about the average willingness to pay for living in a house from which one can see otters?

Solution: Let the average willingness to pay for living in an otter house equal θ. Suppose the price is $p_i = \theta t_i + \beta' x_i + \varepsilon_i$, for $i \in \{A, B\}$, where t_i is 1 if there is an otter in house i and 0 otherwise. We know that $\beta' x_A + \varepsilon_A = \beta' x_B + \varepsilon_B$ (because houses A and B are otherwise identical), so we can calculate θ as $p_A - p_B = \theta$ and $p_A - p_B = \$300$.

(b) Now suppose Teresa learns that House A also has one more room than House B. Using housing prices from 1,000,000 other similar homes, she finds that, on average, each additional room in a house increases its price by \$100. How would this change Teresa's answer to (a)?

Solution: Now $p_A - p_B = \beta_{\text{room}} + \theta$, so $\theta = \$300 - \$100 = \$200$.

(c) Suppose there are 100 houses with ocean views that include otters. Explain how to use the estimate from (a) to assess a policy that would spend \$10,000 to save these otters.

Solution: We have $\theta = \$300$, so if there are 100 houses, the total value of the views that include the otters is at least $\$300 \cdot 100 = \$30{,}000$. Therefore the policy would improve welfare by at least a net benefit of \$20,000.

QUESTION 3 (20 points). The platypus.

In Australia, some homeowners will use barrels to trap rainwater for their gardens. Suppose that the water that is not trapped will flow down to where a platypus (an endangered animal) lives.

The government of Melbourne is interested in protecting the platypus.

(a) Homeowners derive $8g - \gamma g^2$ utility from watering their garden with g gallons of water, where $\gamma = 1$. Suppose there are 10 homeowners, each who receive 5 gallons of rain. How much water will be left for the platypus?

Solution: The homeowner will water the garden to maximize $8g - \gamma g^2$, implying $g^* = 4/\gamma$, or $g^* = 4$ in the case in which $\gamma = 1$. Then there will be $5 - g^* = 1$ left per homeowner, or $1 \cdot 10 = 10$ gallons left over for the platypus.

(b) Suppose that protecting the platypus is worth \$1,000,000 to society (and that homeowners' utilities are in dollars). Suppose also that platypus will be protected if it obtains at least 20 gallons of rainwater. Give an example of a policy that could improve welfare (if any).

Solution: A tax, regulation, or assignment of property rights could improve welfare. For example, a regulation that limits water barrels to no more than $g^{\text{FB}} = 3$ (ensuring that at least $10 \cdot (5 - 3) = 20$ gallons flow to the platypus) would accomplish the first-best.

(c) (*Extra credit*). Suppose γ is unknown. The government of Melbourne offers \$4 per gallon to homeowners that don't trap all of their rainwater. The amount of water left over for the platypus increases by 20 gallons. If the government wanted to use this policy experiment to learn about γ, what would they find?

Solution: Before, the homeowner's optimal $g_0^* = 4/\gamma$. With the subsidy, their optimal g_1^* solves the first-order condition $8 + 4 - 2\gamma g_1^* = 0$, or $g_1^* = 4/\gamma + 2/\gamma$.

We know that $g_1^* - g_0^* = 2$ (because there are 10 homeowners each releasing 2 more gallons).

So $2 = g_1^* - g_0^* = 4/\gamma + 2/\gamma - 4/\gamma = 2/\gamma$, implying that $\boxed{\gamma = 1}$.