Economics 134 Problem Set 2 Solutions

November 6, 2025

1. Global warming

Suppose there are several countries, $i \in \{1, 2, \ldots, N\}$. Each can emit carbon k_i into the atmosphere and obtain profits

$$\pi_i(k_i) = k_i - \frac{\gamma_i}{2} k_i^2.$$

(a) Consider country 1 and suppose $\gamma_1 = 4$. What level of emissions maximize its profits?

Solution: The first-order condition to maximize $\pi_i(k_i)$ is $1 - \gamma_i k_i = 0$, implying $k_i = 1/\gamma_i$ or $1/4$ for $\gamma_i = 4$.

(b) Now suppose that total carbon emissions, $K = \sum_{i=1}^{N} k_i$, affect the climate, and that each country incurs damages

$$D_i(K) = \theta_i K,$$

with $\theta_i \in (0, 1)$, so that country i's welfare is $\pi_i(k_i) - D_i(K)$. What level of carbon emissions will maximize country i's welfare?

Solution: The first-order condition to maximize $\pi_i(k_i) - D_i(K)$ is now

$$\frac{\partial \pi_i}{\partial k_i} - \frac{\partial D_i(K)}{\partial k_i} = 0.$$

Since $K = \sum_{i=1}^{N} k_i$, we know that $\frac{\partial K}{\partial k_i} = 1$, which means that $\frac{\partial D_i(K)}{\partial k_i} = \theta_i$. Consequently, the first-order condition simplifies to $1 - \gamma_i k_i - \theta_i = 0$, or $k_i^* = \frac{1-\theta_i}{\gamma_i}$.

This resembles the free market outcome for the tragedy of the commons problem (Lecture 5, slides 9–10). Each country (in lecture, each firm) takes into account the externality on itself (θ_i), but not its externality on others ($\sum_{j \neq i} \theta_j$).

(c) Assume that $\gamma_i = \gamma$ for all i, and that $\sum_{i=1}^{N} \theta_i = \overline{\theta}$. What will be the total emissions if each country maximizes their welfare independently? Call this the "free market" outcome.

Solution: Adding up our answer to (b), we obtain $K^* = \sum_{i=1}^N k_i^* = \frac{N-\overline{\theta}}{\gamma}$.

(d) Define the "first-best" outcome as maximizing total welfare across all countries. If $\overline{\theta} < 1$, what is the first-best level of total emissions?

Solution: The first-best emissions level for each country i is $k^{\text{FB}} = (1-\overline{\theta})/\gamma$. So, total first-best emissions are

$$K^{\text{FB}} = \sum_{i=1}^{N} k_i^{\text{FB}} = \frac{N(1-\overline{\theta})}{\gamma}.$$

As in the solution to the common-pool problem in lecture (L5, slides 11–12), the first-best emission level ("extraction") is much lower than the free market / open access outcome.

(e) If $\overline{\theta} > 1$, what is the first-best level of emissions?

Solution: $K^{\text{FB}} = 0$. We should stop all emissions, since $\sum_{i=1}^N [\pi_i(k_i) - D_i(K)]$ is negative for all positive values of k_i.

(f) Suppose that $\theta_i = 0.2$ for each i, $\gamma = 4$, and $N = 4$ (the U.S., Russia, China, and the E.U.). Use your answers in (c) and (d) to calculate total welfare under the free market outcome, total welfare under the first-best, and the net benefits of solving this common-pool externality.

(*Remark. If you multiply total profits under the free market outcome by* 10^{14}*, you will get a number a bit less than the combined GDP of these countries in 2021.*)

Solution: Under the free market, $k_i^* = (1-0.2)/4 = 0.2$, so that $\pi_i^* = 0.2 - 2 \cdot 0.2^2 = 0.2 - 2 \cdot .04 = 0.12$, and $4\pi_i^* = 0.48$. Damages will be $0.8 \cdot 0.8 = 0.64$, so total welfare is -0.16.

Under the first-best, we obtain $k_i^{\text{FB}} = (1-0.8)/4 = 0.05$. Profits are $4 \cdot (0.05 - 2 \cdot 0.05^2) = 0.18$. Damages are $0.8 \cdot 0.2 = 0.16$. So welfare is 0.02.

The net benefits are $0.02 - (-0.16) = 0.18$.

2. Public goods

Many people find ospreys to be very beautiful. Ospreys can occasionally be seen in the Ballona wetlands in West Los Angeles. Suppose that there are people indexed by $i \in \{1, 2, \ldots, N\}$, each of whom obtain some delight or utility $v_i > 0$ from knowing that ospreys can be seen in West LA. Suppose also that maintaining the osprey habitat in Ballona requires a fixed conservation cost C.

(a) Explain why the existence of ospreys in this example might be considered a public good.

Solution: When there are ospreys, we can't prevent people from knowing they exist (nonexcludable). Also, no person's delight from knowing that ospreys can be seen in West LA detracts from any other person's delight from knowing this fact (nonrivalrous).

(b) Now suppose that individuals can choose to contribute a fund for ospreys to cover the fixed conservation cost C.

If the first $M < N$ individuals each donate b_i, and that $\sum_{i=1}^{M} b_i = C$, what will the $(M+1)^{\text{th}}$ person donate to the conservation fund? Suppose that they maximize their utility $u(b_i) = v_i s - b_i$, where $v_i > 0$ is some real number and $s = 1$ if there are ospreys in Ballona and 0 otherwise.

Solution: 0, because the donation will not affect s (we already know that $s = 1$ based on the prior donations) and $u(b_i)$ is strictly decreasing in b_i.

(c) If person $(M+1)$ expects no one else to contribute to the osprey fund, what will they contribute?

Solution: If $v_i < C$, person $M + 1$ will contribute nothing: they do not believe that their contribution will ensure that there are ospreys, and their utility $u(b_i)$ is strictly decreasing in b_i. If $v_i > C$, then they will contribute C to enjoy the ospreys. If $v_i = C$, they are indifferent between contributing and not contributing.

(d) If person $(M + 1)$ expects that everyone else will contribute $b_i = v_i$ to the osprey fund, what will they contribute?

Solution: If $\sum_{i=1}^{M} v_i \geq C$, then person $(M + 1)$ will contribute zero for the same reason as in (b).

If $\sum_{i=1}^{M} v_i < C$, but $v_{M+1} + \sum_{i=1}^{M} v_i \geq C$, then person $M+1$ will donate the difference to make sure the ospreys are conserved.

Otherwise, they will contribute zero.

(e) Deduce from (b), (c), and (d) that in this model, the circumstances in which person $(M+1)$ will contribute to the osprey fund are limited. Argue that person $(M+1)$ will contribute some $b_i > 0$ if and only if they expect that their contribution will be pivotal for osprey habitat conservation (that is, if the habitat will be conserved with the contribution, but not otherwise).

Solution: This follows from (b), (c), and (d).

(f) Argue that the free-riding above limits the private provision of public goods and makes coordination through local government potentially useful for such problems.

Solution: This follows from (e); coordination could be useful. For example, a local government could tax everyone some small amount, and then use it to fund the conservation. The key difference from the private outcome is that individuals cannot opt out of paying the tax.

3. Estimating climate damages

Suppose that you observe data on agricultural yields Y_i and temperature T_i for a set of counties i. You estimate the econometric model

$$Y_i = \alpha_0 + \alpha_1 T_i + \varepsilon_i$$

over all i to obtain an ordinary least squares (OLS) estimate of $\hat{\alpha}_1$.

(a) Argue that if T_i is randomly assigned (so that T_i is independently distributed from ε_i), then your estimate of $\hat{\alpha}_1$ will give the average causal effect of temperature on agricultural yields.

Solution: The random assignment of T_i makes the OLS estimate consistent for the causal effect.

(b) Explain why, when ε_i is not independent of T_i—for example, due to an omitted variable like farmers' investments in better technology to adapt to higher temperatures—your estimate of $\hat{\alpha}_1$ will not necessarily reflect the causal effect of temperature on agricultural yields.

Solution: Any omitted variable will confound the estimate of $\hat{\alpha}_1$, such as the potentially omitted variable given in the question.

(c) Now suppose you observe also controls X_i for each county i. If ε_i is independent of T_i conditional on X_i, then will your estimate of α_1 in the following equation,

$$Y_i = \alpha_0 + \alpha_1 T_i + \beta' X_i + \varepsilon_i,$$

correspond to the average causal effect of temperature on yields?

Solution: Yes, because we will have controlled for the omitted variables!

(d) Suppose that there are 1,000 counties and each unit Y_i of agricultural output is worth \$100 million in social surplus. If you interpret $\hat{\alpha}_1$ as the true causal relationship between temperature and yields, and you estimate $\hat{\alpha}_1 = -0.3$, then what is the lost social surplus due to the effects of a temperature increase of 2°C on the agricultural industry?

Solution: (\$100 million) $\cdot$ 1000 $\cdot\ \hat{\alpha}_1 \cdot 2 = -0.3 \cdot 2 = -0.6 = -$\$60 billion, or 60 million per county.

4. Valuing the future

(a) What is the Ramsey formula for the discount rate r defined in lecture?

Solution: $r = \rho + \theta g$.

(b) True or false:

- There is a moral argument that we should set $\rho = 0$.
- The discount rate r measures how much we care about utility tomorrow relative to utility today.
- If inflation makes \$10 today worth \$9 tomorrow, then this would imply that the real discount rate between today and tomorrow is given by $10/(1 + r) = 9$, or $r = 1/9$.
- The "prescriptive" approach to discounting prescribes ρ, θ, and g to then determine r.
- The "descriptive" approach to discounting takes data on r, θ, and g to then calibrate ρ.

Solution:

- True. For example, the quotation from Ramsey in the lecture (L7, slide 14).
- False. The discount rate measures the value of consumption tomorrow relative to today, not utility.
- False. The real discount rate does not include inflation.
- True.
- True.

(c) Nordhaus and Stern each reach very different conclusions about the marginal social cost of carbon. Which estimates do you find more compelling, if any?

Solution: Any answer to this question is correct.

(d) Suppose that it is currently 2022, and that damage from climate change is predicted to be 0 for every year until 2030, at which point climate change is predicted to destroy \$100 billion of consumption each year until 2100. If we use an annual discount rate of $r = 0.05$, how much would we be willing to pay to entirely prevent climate change from happening?

Solution: Using the formula for net present discounted value (Lecture 7, slide 3), we obtain $\sum_{t=2030}^{2100}(1+r)^{2022-t}100 = 13.77 \cdot 100$ billion or approximately 1.3 trillion USD.

More explicitly, we have $T = 2100 - 2022 = 78$. Then we obtain

$$\sum_{t=0}^{T} \frac{B_t}{(1+r)^t} = \sum_{t=0}^{7} \frac{0}{(1+r)^t} + \sum_{t=8}^{T} \frac{100}{(1+r)^t} = 13.77 \cdot 100 \text{ billion}$$

or approximately 1.3 trillion USD.

(e) Recalculate your answer to (d) with $r = 0.02$, and contrast.

Solution: 3.29 trillion USD, which is larger. A lower discount rate makes future consumption more valuable today. Climate change will destroy future consumption. So, lowering the discount rate makes us willing to pay more today to avoid the same loss in future consumption.

(f) Suppose that we can fund some scientists today to conduct research to invent a costless method to take all of the carbon out of the atmosphere. If funded, the research will succeed with probability $p = 0.1$. We will learn of its success or failure in 2025. How much should we be willing to pay today to fund the research project, if climate damages and the discount rate are the same as in (d)?

Solution: By the definition of expected utility (L8, slide 6), the expected value of the research is the probability that it successfully avoids climate change times the damages of climate change. So, using the answer to (d), we obtain $p \cdot 1.377$ trillion USD, or \$137.7 billion when $p = 0.1$.

(g) Suppose that we are in the world of (f), but in addition that if the project fails, we can then instead undertake evasive action that prevents global warming for a one-time payment of \$1 trillion. How much should we be willing to pay today to fund the research project?

Solution: We know that the total damages under (d) are greater than \$1 trillion from the viewpoint of 2022. Damages will be even larger from the viewpoint of 2025. This implies that in 2025, if we learn the project fails, we will undertake evasive action (for \$1 trillion). This option value makes the willingness to pay for the research project the probability that we avoid the \$1 trillion payment in 2025. We have to discount the payment to 2022, so we obtain $p \cdot \frac{1}{(1+r)^3} \cdot 1$, or approximately 0.0864, or \$86.4 billion.

5. More rabbits

(a) Suppose that you have the utility function $u(c) = \ln(c)$. Suppose that you can choose between taking $c = 2$ for sure or a rabbit that is worth either $c = 1$ or $c = 8$ with probability 2/3 and 1/3, respectively. Which would you prefer?

Solution: You are indifferent. $\frac{2}{3}\ln 1 + \frac{1}{3}\ln 8 = \ln 2$.

(b) Suppose that you have the same choice as above, but that your utility is $u(c) = 1 + \frac{c^{1-\theta}}{1-\theta}$, where $\theta = 3$. Which do you prefer now?

Solution: Utility is now $1 - \frac{1}{2c^2}$, so we either obtain $1 - \frac{1}{8}$ or $1 - \frac{2}{3}\frac{1}{2} - \frac{1}{3}\frac{1}{128}$. Hence we prefer the $c = 2$ for sure because it gives strictly higher utility than the random rabbit.

(c) Which utility function is more "risk-averse"?

Solution: The case in which $\theta = 3$ is more risk-averse.

(d) Recall that the logarithmic utility function corresponds to $\theta = 1$ and that there is a debate over what specific value θ should take in the evaluation of climate change, even if most agree that it likely lies in $[1, 3]$. Explain how, when the dangers of climate change are uncertain and occur in the distant future, a higher value for θ could either increase or decrease the expected net present discounted value of preventing climate change.

Solution: On one hand, as we learned in parts (a)–(c), a higher θ makes uncertainty more costly. So, all else equal, this will raise the expected value of preventing climate change.

On the other hand, higher θ raises the discount rate. So, all else equal, this will lower the net present discounted value of preventing climate change.

The combined effect of these two forces could either increase or decrease the expected net present discounted value.