Economics 134 Problem Set 3 Solutions

November 26, 2025

1. Cap-and-trade

(a) Suppose that the campuses of UCLA and USC each use energy q to produce ideas worth

$$\pi_i(q) = pq - \frac{c_i}{2}q^2,$$

where $i = 1$ indexes UCLA and $i = 2$ indexes USC. If $p = 1$, $c_1 = 1$, and $c_2 = 4$, what will q_1 and q_2 be if UCLA and USC each maximize the value of their ideas?

Solution: For UCLA, $1 - q_1 = 0$ or $q_1^* = 1$. For USC, $1 - 4q_2 = 0$ or $q_2^* = 1/4$.

(b) Environmentalists have learned that the total energy used by both campuses, $q_1 + q_2$, creates a negative externality, $D(q_1 + q_2) = \frac{1}{2} \cdot (q_1 + q_2)$. What are the first-best levels, q_1^{FB} and q_2^{FB}?

Solution: For UCLA, $1 - q_1 - \frac{1}{2} = 0$, or $q_1^{\text{FB}} = \frac{1}{2}$. For USC, $q_2^{\text{FB}} = 1/8$.

(c) With your answer from (b), calculate the total first-best quantity, $Q^{\text{FB}} = q_1^{\text{FB}} + q_2^{\text{FB}}$. Suppose that we require UCLA and USC each to produce exactly $Q^{\text{FB}}/2$. Will this attain the first-best outcome? Explain.

Solution: $1/2 + 1/8 = 5/8$, so each school is required to produce $5/16$.

No, it will not attain the first-best outcome. There are two ways of showing this. First, we can observe that $q_1^{\text{FB}} \neq 5/16$ and $q_2^{\text{FB}} \neq 5/16$, so we can improve on this allocation by allowing UCLA to produce a bit more and USC to produce a bit less, keeping damages at $D(5/8)$ but increasing profits from ideas. Alternatively, you can calculate $\pi_1(5/16) + \pi_2(5/16) - D(5/8)$ and show that it is strictly less than the first-best outcome, $\pi_1(1/2) + \pi_2(1/8) - D(5/8)$.

(d) Suppose we issue $Q^{\mathrm{FB}}/2$ permits to UCLA and $Q^{\mathrm{FB}}/2$ permits to USC, and allow them to trade. Each school cannot use more energy than they have permits. Calculate how many permits USC will sell to UCLA. Does this yield the first-best outcome?

Solution: USC will sell permits to UCLA up to the point at which UCLA's marginal value for using energy equals USC's, which gives the first-best outcome. Using UCLA's first-best energy consumption of 1/2, and UCLA's initial endowment (5/16), we can calculate that USC will sell $1/2 - 5/16 = 3/16$ of permits to UCLA. This is what we described in Lecture 10 as a "cap-and-trade."

2. Carbon offsets and adverse selection

(a) Some engineers in Finland have developed a carbon capture technology that costs them nothing to build, but will capture $v \geq 0$ tons of carbon. The engineers know v, but we believe that v is distributed uniformly on $[0, 1]$. If the externality of carbon is \$20/ton, what is the most that we (society) should be willing to pay for the project?

Solution: $\mathbb{E}[20v] = 20 \cdot \frac{1}{2} = 10$.

(b) Now suppose that that the cost of the carbon capture technology is given by some $c \geq 0$, distributed uniformly on $[0, 20]$ and uncorrelated with v. What should society be willing to pay for the carbon capture project? What is the expected value to society of offering this as a payment, s, to the Finnish engineers in exchange for building the carbon capture device? (Assume that the engineers know c, and will reject the offer if $s < c$.)

Solution: Society should be willing to pay $\mathbb{E}[20v|c \leq 10]$. We know that v is uncorrelated with c, so $\mathbb{E}[v|c \leq 10] = \mathbb{E}[v]$. Hence society is willing to pay the same as it was in part (a), $\mathbb{E}[20v] = 20 \cdot \frac{1}{2} = 10$.

The expected value of avoided carbon from offering the contract is the probability the engineers accept, denoted $\mathbb{P}(c \leq 10)$, times the willingness to pay. This is $\mathbb{P}(c \leq 10) \cdot 10 = \frac{1}{2} \cdot 10 = 5$.

Optional. You could also have included the cost of building the technology in the expected value of the contract. In this case, you obtain

$$\mathbb{P}(c \leq 10) \cdot (10 - \mathbb{E}[c|c \leq 10]),$$

and $\mathbb{E}[c|c \leq 10] = 5$, giving $\frac{1}{2} \cdot (10 - 5) = \frac{5}{2}$.

(c) Now suppose that the cost c from part (b) is correlated with v. Specifically, c is no longer uniformly distributed on $[0, 20]$, but instead is given by $c = \frac{2}{3} \cdot 20 \cdot v$. Continue to assume that

$v \sim \text{Unif}(0,1)$ and the carbon externality is \$20/ton. If we offer the engineers the same contract as in (b), what is its expected value? Should society offer the engineers the contract?

Solution: This makes this question identical to the example studied in class, up to a factor of 20.

In that case, the contract from before offers the engineers \$10 in exchange for building the carbon capture technology. The engineers will accept the contract if $c \leq 10$. This implies that the value of projects that will accept the contract will satisfy $20v \leq \frac{3}{2} \cdot 10$, because we know that $c = \frac{2}{3} \cdot 20 \cdot v$. But the expected value of v conditional on being less than $3/4$ is $3/8$. This implies the value of the carbon capture technology, conditional on it being built, is

$$20 \cdot \mathbb{E}\left[v \middle| v \leq \frac{3}{4}\right] = 20 \cdot \frac{3}{8} = \$7.50.$$

This is less than the cost of the contract (conditional on the project being built), which is \$10. So (in this case), society should not offer the engineers this contract!

(d) If we can pay \$2 to buy satellite data from a private contractor that will allow us to learn the exact values of v and c, does your answer to (c) change?

Solution: Yes. From (c), we know that the asymmetric information means we would not like to offer the contract. However, if we can pay \$2 to learn v and c, then we can offer a contract that pays exactly c. Then we will obtain

$$\mathbb{E}\left[20v - c\right] = 20 \cdot \mathbb{E}\left[v - \frac{2}{3}v\right] = 20 \cdot \frac{1}{3}\mathbb{E}[v] = 20 \cdot \frac{1}{3} \cdot \frac{1}{2} = \frac{10}{3}$$

and this is larger than 2, so, in this case, it is worthwhile to buy the data and then offer the engineers c to build their carbon capture technology.

(e) Finally, suppose that the project will be built no matter what. However, there is a risk (probability p) that the Finnish carbon capture technology fails catastrophically, causing damage H to the engineers. The engineers know that, in the event of a catastrophe, the Finnish Government will pay f to reduce this damage from H to zero. Is this efficient? How much should the Finnish Government charge the engineers for a permit to undertake this project?

Solution: Here, there is moral hazard, which leads to inefficiency for identical reasons as in Lecture 16. The Finnish government should charge the engineers the expected cost of the defensive expenditures, $p \cdot f$, so that the engineers' incentives align perfectly with society.

3. Dune

Arrakis is the only planet that produces spice in the known galaxy, giving it a monopoly over the intergalactic spice market. It can produce $q \geq 0$ dekagrams of spice each year for a cost $c(q) = 3q$.

Spice is useful to travel through space. The galaxy's marginal willingness to pay for the q^{th} dekagram of spice—that is, its inverse demand curve—is $P(q) = 6q^{-1/\theta}$.

(a) Suppose that Arrakis produces spice to maximize its profits, and that $\theta = 2$. How much spice will it produce per year? What markup does Arrakis charge above its marginal cost?

Solution: The formula derived in lecture takes the monopolist's first-order condition $P'(q)q - P(q) - c'(q) = 0$ and rearranges it, using the fact that $P'(q)q = -\frac{1}{\theta}P(q)$, to obtain

$$P(q) = \left(1 - \frac{1}{\theta}\right)^{-1} c'(q).$$

Evaluating this for $\theta = 2$ and $c(q) = 3q$ or $c'(q) = 3$, and $P(q) = 6q^{-1/\theta}$, we obtain

$$6 \cdot (q^m)^{-1/\theta} = \left(1 - \frac{1}{2}\right)^{-1} 3 = 2 \cdot 3 = 6$$

so that $q^m = 1$.

The markup above marginal cost is a factor of $\left(1 - \frac{1}{2}\right)^{-1} = 2$, or 100%.

(b) Calculate the first-best level of spice production that maximizes total surplus, defined as spice consumer surplus, $\int_0^q P(x)dx$, net of Arrakis production costs, $3q$. How much spice is Arrakis withholding from the market relative to the first-best?

Solution: Using the formula from lecture, or calculating the first order condition directly with the Leibniz integral rule, the first-best must satisfy

$$P(q^{\text{FB}}) = c'(q^{\text{FB}}).$$

Here, that implies that $6 \cdot (q^{\text{FB}})^{-1/\theta} = 3$, or $q^{\text{FB}} = (1/2)^{-\theta} = (1/2)^{-2} = 4$.

So, Arrakis supplies the market with 25% of the first-best level of spice, or equivalently, efficient spice production should be 400% of what Arrakis supplies as a monopolist.

(c) Suppose that $\theta = 4$. Recalculate Arrakis' profit-maximizing level of spice production, its markup above its marginal cost, and the first-best level of spice production.

Solution: For $\theta = 4$, Arrakis maximizes profits when

$$6 \cdot (q^m)^{-1/\theta} = \left(1 - \frac{1}{4}\right)^{-1} 3 = \frac{4}{3} \cdot 3 = 4,$$

or $q^m = (2/3)^{-4} = 81/16$.

The resulting monopoly price is a factor of $\left(1 - \frac{1}{4}\right)^{-1} = \frac{4}{3}$ of the cost, or a markup of $33\frac{1}{3}\%$.

The first-best level solves $6 \cdot (q^{\text{FB}})^{-1/\theta} = 3$, so that $q^{\text{FB}} = (1/2)^{-\theta} = 2^4 = 8$. So Arrakis is now withholding half of the first-best level of spice.

(d) Suppose that the sand worms do not like spice harvesters. Assume environmental damages of $D(q) = 3q$ from q dekagrams of spice. For $\theta = 2$, what level of spice production now maximizes this new measure of total surplus, defined as spice consumer surplus net of Arrakis production costs and environmental damage? Using the formula in lecture, what optimal tax could attain this objective?

Solution: Now we want

$$P(q^{\text{FB}}) = c'(q^{\text{FB}}) + D'(q^{\text{FB}}).$$

Here, that implies that $6 \cdot (q^{\text{FB}})^{-1/\theta} = 3 + 3$, or $q^{\text{FB}} = 1^{-\theta} = 1^{-2} = 1$.

Either using the formula in lecture, or noting that $q^{\text{FB}} = q^m$, the optimal tax should be zero! The distortion from Arrakis' monopoly and the distortion from Arrakis neglecting the sand worm welfare exactly counterbalance one another.

(e) Suppose that we have now learned that sand worms are a keystone species sustaining the ecosystem, and updated our damages estimate to $D(q) = 6q$. Recalculate the first-best level of spice production and the optimal tax on Arrakis. Explain why the optimal tax on spice harvesting is lower than the marginal damage to the sand worms.

Solution: Now $6 \cdot (q^{\text{FB}})^{-1/\theta} = 3 + 6$, or $q^{\text{FB}} = (3/2)^{-\theta} = (3/2)^{-2} = 4/9$. The tax that accomplishes this can be obtained from the formula in lecture as

$$\tau = (1 - \frac{1}{\theta})D'(q^{\text{FB}}) - \frac{1}{\theta}c'(q^{\text{FB}}) = (1 - \frac{1}{2}) \cdot 6 - \frac{1}{2} \cdot 3,$$

or $\tau = 3/2$. This is less than marginal damages (as it was also in part (d)) because we also do not want Arrakis to undersupply the intergalactic spice market. Conceptually, we are taxing Arrakis equal to its marginal damage on worms, then subsidizing Arrakis to correct for its incentive to withhold supply given its market power. This leads to a "lower" net optimal tax.

4. Wilderness conservation

(a) Suppose society obtains value $v \cdot W$ from a wilderness region in proportion to its area, W, where $v > 0$ is some constant. If society discounts utility in future periods by a factor of $\beta = 1/(1+\rho)$, what is the net-present discounted value of conserving this wilderness region from $t = 1$ to $t = \infty$?

Solution: The value of the wilderness conservation is

$$\sum_{t=1}^{\infty} \beta^t vW = vW \cdot \sum_{t=1}^{\infty} \beta^t = \frac{\beta v W}{1-\beta},$$

using the fact that

$$\sum_{t=1}^{\infty} \beta^t = \frac{\beta}{1-\beta}$$

derived in lecture.

(b) Suppose that it is possible to obtain p units of profits today by converting 1 unit of wilderness land permanently into development. For what values of β should we ban all such land development?

Solution: We would like to compare the marginal (net-present discounted) value of conservation, $\beta v/(1-\beta)$, with the profits we could obtain today, p (assuming we obtain value from the wilderness in the period that we develop it; otherwise, we would compare the marginal value of conservation with with $p - v$).

The former quantity exceeds the latter when

$$\frac{\beta v}{1-\beta} \geq p$$

or

$$\beta v \geq (1-\beta)p$$

which simplifies to $\beta \cdot (v + p) \geq p$, or $\beta \geq \frac{p}{v+p}$.