Economics 134 Final Exam, Fall 2022 – Solutions

December 7, 2022, 8–11am

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You have three hours to complete this exam. Show your work.

QUESTION 1 (21 points). True or false.

(a) A Pigouvian tax can have many different distributional outcomes, depending on how the tax revenues are used.

Solution: True.

(b) Allowing firms to trade rights to pollution may both lower pollution and increase total firm profits relative to a uniform mandate, depending on the design of the pollution permit market and the characteristics of the polluting firms.

Solution: True.

(c) An *LA Times* reporter argues that USC graduates must care more about the environment than the average Californian resident, based on a statistic that indicates people who own new electric cars are more likely to have attended USC than the average Californian. The reporter's inference is an example of omitted variable bias because (i) new electric cars can be more expensive than other cars and (ii) people who attend USC have higher incomes than the average Californian.

Solution: True.

(d) A certain species of eel will become extinct unless we pay a one-time cost of $c > 0$ today at $t = 0$ to restore their habitat. Assuming that the continued existence of the eel species delivers utility of v each period from $t = 1$ to ∞, and that we discount future utility by a factor of β, then we should protect the species if $c > \sum_{t=1}^{\infty} \beta^t v$.

Solution: False. The inequality is reversed! It should be $c < \sum_{t=1}^{\infty} \beta^t v$.

(e) If we assign relatively more weight to future generations' utility than current generations' utility, expect the economy to grow more slowly, or have a smaller elasticity of marginal utility with respect to consumption, then our optimal climate policy should involve more aggressive action today to protect against future global warming.

Solution: True. Ramsey's formula is $r = \rho + \theta g$, and (i) corresponds to a smaller ρ, (ii) corresponds to a smaller g, and (iii) corresponds to a smaller θ. Each will lower r.

(f) Oil prices might be higher than marginal drilling costs either because OPEC restricts supply, because oil is a finite resource, or both.

Solution: True.

(h) "Prior appropriation" water property rights, such as those found in some parts of the western United States, can be inefficient during a drought even if they are efficient in most years.

Solution: True.

QUESTION 2 (40 points). Textbook pollution externality.

A firm pollutes $q \geq 0$ and profits $\pi(q)$ from the pollution, with

$$\pi(q) = 3q - \frac{1}{4}q^2.$$

(a) Calculate the level of pollution that maximizes the firm's profits, q^*, and the profits they obtain.

Solution: $3 - \frac{1}{2}q^* = 0$ or $q^* = 6$, for profits of $\pi(q^*) = \pi(6) = 18 - \frac{1}{4} \cdot 36 = 9$.

(b) Suppose that pollution creates environmental damage, $D(q) = \frac{1}{4}\alpha \cdot q^2$, where $\alpha > 0$ is some positive number. What is the first-best level of pollution that maximizes profits net of environmental damage, as a function of α?

Solution: This gives $3 - \frac{1}{2}q^{\text{FB}} - \frac{1}{2}\alpha q^{\text{FB}} = 0$ or $6 = q^{\text{FB}} + \alpha q^{\text{FB}}$, or $q^{\text{FB}} = 6/(1+\alpha)$.

(c) Suppose that $\alpha = 1$. What per-unit tax, if any, should we impose on the firm in order to attain the first-best outcome?

Solution: The tax should be $D'(q^{\text{FB}}) = \frac{1}{2}\alpha \cdot q^{\text{FB}} = \frac{1}{2}\alpha \cdot \frac{6}{1+\alpha} = \frac{1}{2} \cdot \frac{6}{2} = \frac{3}{2}$.

(d) The firm has been taken over by some activist investors who claim to be environmentally responsible. The investors plan to use 50% of the firm's total profits to build a device that lowers environmental damages. Specifically, the device will cause $\alpha = \frac{1}{2}$ instead of $\alpha = 1$. Will the activist investors improve welfare relative to the free market outcome? Explain.

Solution: The costs of the device will be $0.5 \cdot \pi(q^*) = 9/2$. The avoided pollution damages will be $\left(1 - \frac{1}{2}\right)\frac{1}{4}(q^*)^2 = \frac{36}{8} = 9/2$. So, while damages fall, the investors leave welfare the same as before. (If you do not assume that the money we spend to build the device disappears, then you will conclude that welfare has been improved relative to the free market. This answer also received full credit.)

A common mistake on this question was to assume that the activist investors would pollute at the first-best level, rather than producing at q^*.

(e) Will (d) attain the first-best outcome?

Solution: No; we know from (d) that welfare under the activist investors is no greater than that under the free market. The firm will still overproduce pollution because it does not face its marginal social cost of production, which is still positive after the device is built.

Alternatively, you can calculate welfare under first-best outcome and contrast that with the payoffs under the market with the technology ($\alpha = 1/2$). This contrasts $W(q^{\text{FB}}) = \pi(q^{\text{FB}}) - D(q^{\text{FB}}) = 9 - \frac{9}{4} - \frac{1}{4} \cdot 3^2 = \frac{9}{2}$ using $q^{\text{FB}} = 3$, with payoffs under the market with the technology equaling $\pi(6) - \frac{1}{8}(q^*)^2 - \frac{9}{2} = 9 - \frac{1}{8} \cdot 36 - \frac{9}{2} = 9 - 9 = 0 < W(q^{\text{FB}})$.

(f) Suppose that we want to maximize welfare and that we can choose between shutting down the firm entirely or allowing the firm to operate in the manner described in (d). What should we do?

Solution: The firm's profits were $\pi(6) = 9$ from (a).

Damages were $D(q^*) = \frac{1}{4}\alpha \cdot 6^2 = \alpha \cdot \frac{36}{4} = 9$. Damages with the new $\alpha = \frac{1}{2}$ are $\frac{9}{2}$.

Welfare without the firm is just zero.

Welfare with the firm becomes $\pi(q^*) - \tilde{D}(q^*) = 9 - \frac{9}{2} > 0$. Note that you could also have subtracted the costs of the device, 9/2, from welfare, if you thought that those resources were used up to produce the device. Then welfare will not change!

So, we should keep the firm or be indifferent to it (in the final case).

QUESTION 3 (40 points). One fish, two fish.

Amartya and Bob each own a fishing boat. When both of them go fishing at the same time, the fish become suspicious and harder to catch.

In particular, Amartya can spend $a \geq 0$ hours trying to catch fish, with profits

$$\pi_a(a) = a - \frac{1}{2}a^2 - \theta ab,$$

where $\theta > 0$ is some positive number. Bob can spend $b \geq 0$ hours trying to fish with profits

$$\pi_b(b) = b - \frac{1}{2}b^2 - \theta ab,$$

for the same $\theta > 0$.

(a) Suppose that Amartya maximizes his profits and believes that Bob will spend some known $\tilde{b}$ hours fishing. How long should Amartya spend fishing, as a function of $\tilde{b}$ and θ?

Solution: Given $\tilde{b}$, Amartya solves

$$1 - a^* - \theta\tilde{b} = 0$$

or $a^* = 1 - \theta\tilde{b}$.

(b) Suppose that Amartya and Bob each maximize their own profits. How long will they each spend trying to catch fish?

Solution: Given b^*, Amartya solves

$$1 - a^* - \theta b^* = 0$$

Given a^*, Bob solves

$$1 - b^* - \theta a^* = 0$$

Solving the system of equations, or using symmetry, we have that $1 - a^* - \theta a^* = 0$ or $a^* = \frac{1}{1+\theta}$. Note that this is the same as $1 - \theta \frac{1}{1+\theta} = \frac{1}{1+\theta}$.

(c) Suppose that you care about maximizing the sum of Amartya's and Bob's profits ("welfare"). How much time would you recommend that Amartya and Bob each spend fishing?

Solution: Welfare equals

$$W = a + b - \frac{1}{2}a^2 - \frac{1}{2}b^2 - 2\theta ab,$$

so $\frac{\partial W}{\partial a} = 1 - a^{\mathrm{FB}} - 2\theta b^{\mathrm{FB}} = 0$ and, symmetrically, $\frac{\partial W}{\partial b} = 1 - b^{\mathrm{FB}} - 2\theta a^{\mathrm{FB}} = 0$, so that $a^{\mathrm{FB}} = \frac{1}{1+2\theta}$.

(d) Suppose that $\theta = 1$. Calculate the profits that Amartya and Bob obtain in the case of (b). Contrast with the total profits in (c), and discuss. Would moving from (b) to (c) create a Pareto improvement?

Solution:

For (b), profits are evaluated at $a^* = 1/(1 + \theta) = 1/2$, so they are

$$\pi_a(a^*) = \frac{1}{2} - \frac{1}{2}\frac{1}{4} - \frac{1}{4} = \frac{1}{8}.$$

They are the same for Bob, since the profit function is symmetric.

For (c), profits are evaluated at $a^{\mathrm{FB}} = 1/(1 + 2\theta) = 1/3$, or

$$\frac{1}{3} - \frac{1}{2}\frac{1}{9} - \frac{1}{9} = \frac{1}{3} - \frac{1}{9} = \frac{1}{3}.$$

The common-pool externality makes the market outcome is inefficient. Amartya and Bob are both better off in the first-best from (c). Moving from (b) to (c) will therefore be a Pareto improvement.

(e) Suppose that you can impose a uniform mandate on both Amartya and Bob that neither fish more than $\overline{q}$ hours. Will this improve on the free market outcome in (b)? Will it attain the first-best outcome in (c)?

Solution: Yes, the mandate (or "command-and-control") improves the free market outcome (strictly better, by Lecture 11 on environmental markets).

Yes, it will attain the first-best—unlike in Lecture 11, both Amartya and Bob have the same production technology, so there is no need to differentiate the rule across them.

QUESTION 4. (40 points). Road trip.

Driving from Los Angeles to San Francisco involves about 380 miles of driving, or 19 gallons of gas for a car that averages 20 miles per gallon (mpg). According to the U.S. Environmental Protection Agency, each gallon of gas generates approximately 0.01 tons of CO_2.

(a) Suppose the true social cost of carbon is \$50/ton CO_2 (approximately the U.S. government social cost of carbon). Suppose also that someone obtains surplus of \$20 from driving to LA to SF, all else equal. Is it efficient for them to drive?

Solution: The externality is 19 gallons $\cdot$ 0.01 tCO_2/gallons $\cdot$ \$50/tCO_2 or $19 \cdot 0.5 = \$9.50$. This is less than their surplus; they should drive.

(b) Suppose instead that the true social cost of carbon is \$200/ton CO_2. Would your answer to (a) change?

Solution: The externality is 19 gallons $\cdot$ 0.01 ton/gallons $\cdot$ \$200/tCO_2 or $19 \cdot \$2 = \$38$. This reverses the conclusion of (a).

(c) If the social cost of carbon is \$200/ton CO_2, what tax should California impose on a gallon of gas in order to correct the climate change externality associated with driving?

Solution: The climate change externality would be 0.01 tCO_2/gallons $\cdot$ \$200/tCO_2; implying a Pigouvian tax of \$2/gallon.

(d) California charges about 41 cents/gallon due to concerns about climate change. What marginal social cost of carbon could justify this tax as an optimal Pigouvian tax?

Solution: We know that a gallon of gas generates 0.01 tCO_2, so this implies a marginal social cost estimate of carbon equal to $0.41/0.01 = \$41$ per ton.

(e) Gas in California is produced by small number of oil refineries. Suppose that these refineries occasionally reduce total output below the competitive level to increase their profits. Without doing any math, how will this change the optimal tax that you calculated in (c) or (d)?

> **Solution:** Yes, this would change the answer—our tax should be lower than before. (The exact calculation would depend on the markups as discussed in Lecture 15.)

(f) Elon Tusk promises to deliver you an electric car for Christmas. What is the total avoided climate damage from Elon Tusk's present to you? Use a social cost of carbon of \$50/ton CO_2 and assume (for simplicity) electric vehicles emit no carbon. Also assume that if you do not receive the car, you instead will drive a gas-powered car for 150,000 miles (averaging 22 miles per gallon). Compare your calculation with the \$7,500/car federal subsidy for an electric vehicle, and discuss.

> **Solution:** $(150,000/22) \cdot 0.01 \cdot 50 = \$3,409$. This is less than half of the federal subsidy.