# Economics 134 Final Exam, Fall 2023

December 9, 2023, 11.30am–2.30pm

Write your name and UID in the upper right corner of each page to help the robots conduct OCR.

You have three hours to complete this exam. Show your work.

**QUESTION 1** (40 points). True or false.

(a) Suppose that economists revise their prediction for future consumption growth from $g = 2\%$ to $g = 1.5\%$. According to Ramsey's rule, we should now discount future consumption flows at a lower rate as long as our marginal utility falls with greater levels of consumption.

**Solution:** True. When our marginal utility falls with greater levels of consumption, this implies the elasticity of marginal utility with respect to consumption ($\theta$) exceeds zero, and Ramsey's rule for the discount rate for future consumption is $r = \rho + \theta g$.

(b) Bob's mill produces $q^* = 4$ tons of flour to maximize profits of $\pi(q) = 8q - q^2$. If environmental damage from operating the mill is $D(q) = \frac{1}{3}q^3$, then a per-ton tax of $\tau = D'(q^*) = (q^*)^2 = 16$ on Bob's flour will attain the first-best.

**Solution:** False — the Pigouvian tax is $D'(q^{\text{FB}})$, not $D'(q^*)$.

(c) If we know that government subsidies will cause a reduction in future costs for renewable energy, then we always want to both (i) impose the optimal Pigouvian tax on the externalities of non-renewable energy use and (ii) subsidize renewable energy.

**Solution:** False — the reduction in costs needs to be due to an innovation externality (e.g., spillovers across firms), not simply economies of scale or within-firm learning-by-doing.

(d) Despite the negative environmental externalities from producing oil, the U.S. might still want to subsidize oil production because OPEC is a cartel with a global market share of roughly 40%.

**Solution:** True.

(e) Uncertainty about a firm's costs to produce an externality can make the optimal tax strictly worse for welfare than the optimal quantity mandate.

**Solution:** True.

(f) Some economic research shows that, in some—but not all—places of the United States, the environmental harm from electric vehicles exceeds that of gas vehicles.

**Solution:** True.

(g) Wetlands are ecosystems that consist of marshes or swamps. A study finds that places with disappearing wetlands also seem to be places with increasing events of extreme flooding: its authors calculate that a place with 10 more disappearing wetland acres has, on average, $1,000,000 greater flood damage. This implies that, on average, we should levy a tax on developers who destroy wetlands equal to $100,000 per wetland wetland acre.

**Solution:** False! Correlation is not causation.

In this case, there are many reasons why flood damages are already high in places with more wetlands disappearing, for example, in low-lying coastal zones that are disproportionately exposed to hurricanes, we also find more wetlands disappearing. (Why are wetlands wetlands? Because they are wet. Why are floods floods? Because they make land wet.)

(h) One lesson of Econ 134 is that UCLA should subsidize the completion of student course evaluations (e.g., by giving out Amazon gift cards to a random subset of students who complete their evaluations). This is because these evaluations are a public good for future UCLA students.

**Solution:** True.

**QUESTION 2** (50 points). Santa Claus.

(a) Santa Claus obtains profits $\pi(q) = 12q - q^2$ from deploying $q \geq 0$ reindeers in the delivery of toys to children. How many reindeer will Santa deploy to maximize his profits? What will be Santa's total profit?

**Solution:** Equilibrium reindeers solve $\pi'(q^*) = 0$, or $q^* = 6$. Santa's profits equal $\pi(q^*) = 12 \cdot 6 - 6^2 = 36$.

(b) Santa's reindeers cause environmental damage equal to $D(q) = 5q^2$, where $q$ is still the number of reindeer deployed to deliver presents and $D(\cdot)$ is in the same units as Santa's profits. Calculate the first-best number of reindeers delivering presents.

**Solution:** First-best reindeers maximize $12q - q^2 - 5q^2$, or $12 - 12q^{\text{FB}} = 0$, so that $q^{\text{FB}} = 1$ reindeer.

(c) Calculate the optimal per-reindeer tax on Santa's operation and the total avoided environmental damage from the tax.

**Solution:** The optimal tax is $D'(q^{\text{FB}}) = 10q^{\text{FB}} = 10$ per reindeer.

The total environmental benefit is $D(q^*) - D(q^{\text{FB}}) = 5 \cdot 6^2 - 5 \cdot 1^2 = 175$.

(d) What would we need to give to Santa in order to make the reindeer tax that you derived in (c) a Pareto improvement?

**Solution:** A lump-sum transfer equal to at least Santa's forgone profits, which are $\pi(q^*) - \left[\pi(q^{\text{FB}}) - \tau q^{\text{FB}}\right] = 36 - (12 \cdot 1 - 1^2 - 10 \cdot 1) = 36 - (11 - 10) = 35$.

(e) Suppose the environmental damage of the reindeers is confined to Mrs. Claus' garden. If Mrs. Claus and Santa can negotiate costlessly, have perfect information, and maximize their respective utility and profit, would we still need the reindeer tax to implement the first-best?

**Solution:** No, we no longer need the tax. Coase's theorem will apply to ensure that Santa and Mrs. Claus reach the first-best.

**QUESTION 3** (60 points). Public goods.

A town has $N = 9$ residents who want a park, which requires labor. Suppose it costs each resident $3g_i^2$ in utility to work $g_i$ hours to build the park. The park is a public good: each resident obtains $\theta G$ of utility from the total work done to build the park, $G = \sum_{i=1}^{N} g_i$, in proportion to $\theta > 0$.

(a) If each resident maximizes their utility, will any resident contribute any positive amount of work to build the park? Explain.

**Solution:** Yes. Each resident maximizes $-3g_i^2 + \theta \sum_{i=1}^{N} g_i$, so they contribute $g_i^* = \frac{1}{6}\theta > 0$.

(b) If the residents work together to maximize the total utility of all people in the town, how much work should each resident contribute (as a function of $\theta$)?

**Solution:** First-best maximizes $\sum_{i=1}^{N} \left[-3g_i^2 + \theta G\right]$, leading to the first-order condition for each $i$ of $g_i^{\text{FB}} = \frac{1}{6}N\theta$. When $N = 9$, we obtain $g_i^{\text{FB}} = \frac{3}{2}\theta$.

(c) Does moving from (a) to (b) create a Pareto improvement?

**Solution:** Yes. Each citizen is better off! The first-best $G^{\text{FB}} = \sum_{i=1}^{N} g_i^{\text{FB}} = \sum_{i=1}^{N} \frac{3}{2}\theta = \frac{27}{2}\theta$.

(d) The town mayor sets the budget priorities for the town. What optimal per-hour subsidy could the mayor pay to each citizen to obtain the first-best level of the public good?

**Solution:** The optimal subsidy per hour worked is $\sum_{j \neq i} \theta$, or $8\theta$. Including this subsidy in the first order condition in (a) will lead to $6g_i = 9\theta$ or $g_i = \frac{3}{2}\theta$.

(e) Suppose that the town finds a contractor who can provide $G^{\text{FB}}$ units of total work for a one-time cost. How much would the residents be willing to pay the contractor if they otherwise live in the world of part (b)? Express your answer as a function of $\theta$.

**Solution:** In part (b), the residents work together. If they pay the contractor, they will avoid the total cost of their contributions, which equal $\sum_{i=1}^{N} 3 \cdot (\frac{3}{2}\theta)^2$, or $9 \cdot 3 \cdot \frac{9}{4} \cdot \theta^2 = \frac{243}{4} \cdot \theta^2$.

(f) How much would the residents be willing to pay the same contractor from part (e) if they otherwise live in the world of part (a)?

**Solution:** In part (a), the residents do not work together, so paying the contractor not only avoids the cost of having to contribute as in (e), but also solves the problem of private under-provision of the public good. Hence they would be willing to pay the net benefit of moving to the first-best, plus the avoided costs from having to contribute themselves.

Total surplus in (a) is the utility evaluated

$$\sum_{i=1}^{N}\left[-3\cdot\left(\frac{\theta}{6}\right)^2+\theta\cdot G^*\right]=N\cdot\left(-\frac{1}{12}+\frac{3}{2}\right)\cdot\theta^2=N\cdot\frac{17}{12}\cdot\theta^2,$$

since $G^*=\sum_{i=1}^{N}\frac{\theta}{6}=9\cdot\frac{\theta}{6}=\frac{3}{2}\cdot\theta$.

The total surplus from hiring the contractor (before paying them) is $\sum_{i=1}^{N}\theta G^{\mathrm{FB}}=N\cdot\theta\cdot G^{\mathrm{FB}}=N\cdot\frac{27}{2}\cdot\theta^2$.

So in the case of (a), the residents will pay the contractor up to

$$N\cdot\frac{27}{2}\cdot\theta^2-N\cdot\frac{17}{12}\cdot\theta^2=N\cdot\frac{145}{12}\cdot\theta^2=\frac{3}{4}\cdot 145\cdot\theta^2.$$

since $N/12=\frac{9}{12}=\frac{3}{4}$. Alternatively, a calculator can be used for this calculation. (What was important for full credit was to correctly write the appropriate equation to be evaluated.)

**QUESTION 4** (40 points). Climate change policy.

In this question, you will derive some implications of a social cost of carbon equal to \$50/ton $CO_2$. Suppose that we do not discount utility in future periods, that our elasticity of marginal utility with respect to consumption is $\theta=2$, and that we expect future consumption growth of $g=1\%$.

(a) Assume that, from 2024 to 2100, the world will emit 35 billion tons of $CO_2$ per year. Calculate the net present discounted value of immediately moving to zero carbon permanently in 2023.

**Solution:** 35 billion $\times$ \$50 = \$1,750 billion of damages per year.

Here, $r=\rho+\theta g=0+2\cdot 0.01=0.02$. So we have

$$\sum_{t=2024}^{2100}\frac{1,750\text{ billion}}{(1+0.02)^{t-2023}}\approx 68\text{ trillion}.$$

(b) Suppose that each additional ton of $CO_2$ emitted increases the probability of the polar ice caps melting by $\frac{1}{200\text{ billion}}$. Further assume that the polar ice caps melting is equivalent to \$10 trillion of foregone consumption in 2100. How will these new facts change today's social cost of carbon?

**Solution:** We divide \$10 trillion by 200 billion to obtain \$50, then discount to 2100 with $1/(1+r)^{2100-2023} = (1+r)^{-77} \approx 1/4.6$ for $r = 0.02$, so today's social cost of carbon increases by about \$10.90/ton.

(c) Another consequence of carbon dioxide emissions is ocean acidification, which destroys coral reefs. Suppose that we discover today that climate change may irreversibly destroy reefs that will otherwise deliver $\overline{u} > 0$ units of utility in every year from now until forever. How much should society be willing to pay (in utils) to prevent climate change?

**Solution:** An infinite amount: the series $\sum_{t=0}^{\infty} \beta^t \overline{u}$ diverges when $\beta = 1/(1+\rho) = 1$ for $\rho = 0$.

If you mistakenly used $r$ instead of $\rho$, then you can obtain partial credit from noting that, if $\beta = \frac{1}{1+r}$, we have $\sum_{t=1}^{\infty} \beta^t \overline{u} = \frac{\beta}{1-\beta}\overline{u} = 1/0.02 = 50\overline{u}$.

(d) An entrepreneur offers to sell us a technology that will reduce $v$ billion tons of $CO_2$, where $v$ is a random variable uniformly distributed on $(0, 1)$. The entrepreneur's cost of the project is perfectly correlated with $v$ and equals $40v$ billion dollars. Calculate the expected social value of the project's carbon reduction if the social cost of carbon is \$50 and we offer the entrepreneur a subsidy equal to $50 \cdot \mathbb{E}[v] = 25$ billion dollars to complete the project. Does this exceed the cost of the subsidy?

**Solution:** $\mathbb{E}[50v|c \leq 25] = \mathbb{E}[50v|40v \leq 25] = \mathbb{E}\left[50v\ |v \leq \frac{5}{8}\right] = 50 \cdot \frac{5}{16} = 15.625$.

No, the subsidy of 25 costs more than the expected social value of 15.625. Adverse selection!

**QUESTION 5** (40 points). The Colorado River.

Water from the Colorado River flows into California through Imperial Valley, where farmers use water to grow crops. Some of this water can instead be diverted to San Diego with an aqueduct.

San Diego's value of using $w$ million acre-feet of water from the Colorado River is $u_s(w) = w - \frac{1}{2}w^2$. The value to the farmers in the Imperial Valley is $u_f(w) = 3w - \frac{1}{2}w^2$.

The Imperial Valley farmers have "senior" appropriative water rights, meaning that they can take as much water as they want before San Diego is allowed to use any.

(a) In a normal year, four million acre-feet of water flow into California from the Colorado River. Calculate the allocation of water under the water rights regime. Is the allocation efficient?

**Solution:** Yes, it will be efficient. Marginal water values are

$$u'_s(w) = 1 - w$$

and $u_f(w) = 3 - w$, so we will end up with $w_s = 1$ and $u_f = 3$ with $u'_s(w_s) = u'_f(w_f) = 0$.

(b) In a drought year, three million acre-feet of water flow into California from the Colorado River. Calculate the allocation of water under the water rights regime. Is the allocation efficient?

**Solution:** The allocation is inefficient. Imperial Valley will use all of the water because $u'_f(w) > 0$ for $w \in [0, 3)$, but it would have been more productive to split the water between San Diego and Imperial Valley the last million acre-foot more productively.

(c) Suppose that the water used in Imperial Valley to grow crops, $w_f$, also helps to sustain a desert ecosystem inhabited by various endangered species. Society (but not the Imperial Valley farmers) values this protection according to $u_d(w_f) = 2w_f$.

Given this environmental externality, calculate the first-best allocation of water in a normal year (four million acre-feet of water) and in a drought year (three million acre-feet of water). When, if ever, is the senior water rights regime efficient?

**Solution:** Now the value of water to Imperial is $u_f(w_f) + u_d(w_f) = 3w_f - \frac{1}{2}w_f^2 + 2w_f$, so that the marginal value becomes $5 - w_f$.

For a normal year, the social value of Imperial always exceeds San Diego's marginal value, so we should allocate $w_f = 4$ and $w_s = 0$.

The senior water rights regime is no longer efficient in a normal year (because $w_f = 3$ but $w_f^{\text{FB}} = 4$) but becomes efficient in a drought year (because $w_f = 3$ is also the first-best).