Tarea 6 Algando Orozco

$$G(s) = \frac{20\,(s+2)}{s\,(s+1)\,(s+4)} \quad \left\{ \begin{array}{l} OS\% = 9,5\% \\ t_s = 0,745 \text{ seg} \end{array} \right.$$

$$U(s) \longrightarrow \boxed{\frac{1}{s^3+5s^2+4s}} \xrightarrow{X_1(s)} \boxed{0s^2+20s+100} \longrightarrow Y(s)$$

$$\frac{X_1(s)}{U(s)} = \frac{1}{s^3+5s^2+4s} \qquad (s^3+5s^2+4s)\,X_1(s) = U(s)$$

$$\dddot{X}_1 + 5\ddot{X}_1 + 4\dot{X}_1 = U$$

$$X_1(s) = X_1$$
$$X_2 = \dot{X}_1$$
$$\dot{X}_2 = \dot{X}_2 = \ddot{X}_1$$
$$\dot{X}_3 = \dddot{X}_1$$

$$\dot{X}_3 + 5X_3 + 4X_2 = U$$
$$\dot{X}_3 = -5X_3 - 4X_2 + U \quad (1)$$

$$Y(s) = (b_2 s^2 + b_1 s + b_0)\,X_1(s)$$
$$(0s^2 + 20s + 100)\,X_1(s)$$
$$(20s + 100)\,X_1(s) \xrightarrow[\mathcal{L}^{-1}]{} 20\dot{X}_1 + 100X_1$$
$$20X_2 + 100X_1$$

$$y = 20X_2 + 100X_1$$

$$\begin{vmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -4 & -5 \end{vmatrix} \begin{vmatrix} X_1 \\ X_2 \\ X_3 \end{vmatrix} + \begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix} U$$

$$y = \begin{vmatrix} 100 & 20 & 0 \end{vmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix}$$

$$\%OS = e^{-\left(\zeta\pi/\sqrt{1-\zeta^2}\right)} \times 100$$

$$0,095 = e^{-\left(\zeta\pi/\sqrt{1-\zeta^2}\right)}$$

$$\ln(0,095) = \ln e^{-\left(\zeta\pi/\sqrt{1-\zeta^2}\right)}$$

$$-2,3539 = -\frac{\zeta\pi}{\sqrt{1-\zeta^2}} \rightarrow \left(-2,3539\sqrt{1-\zeta^2}\right)^2 = (-\zeta\pi)^2$$

$$5,5407 - 5,5407\zeta^2 = \zeta^2\pi^2$$

$$5,5407 = \zeta^2\pi^2 + 5,5407\zeta^2$$

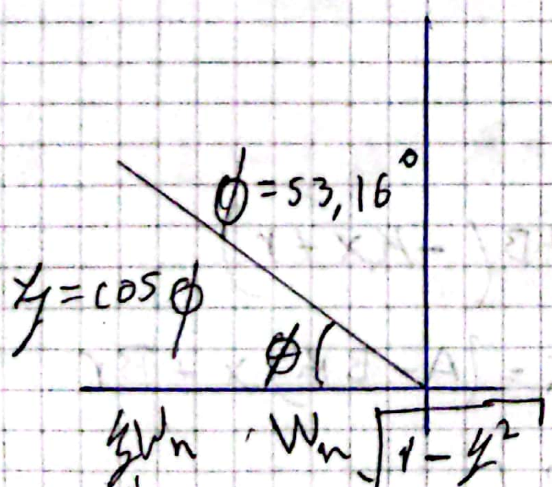
$$\zeta^2 \cdot (\pi^2 + 5,5407) = 5,5407$$

$$\zeta^2 = \frac{5,5407}{\pi^2 + 5,5407} \rightarrow \zeta = \sqrt{\frac{5,5407}{\pi^2 + 5,5407}}$$

$$\zeta = 0,5996$$

$$s = \sigma + j\omega_d$$

$$\mathrm{arccos}(0,5996)$$



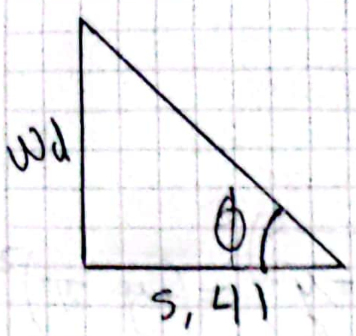
$$t_s = 0,74 \qquad 0,74 = \frac{4}{\sigma}$$

$$\sigma = \frac{4}{0,74} = 5,405$$

$$\sigma = \zeta\omega_n \qquad \omega_n = 9,02 \ rad/s$$

$$s = \sigma + j\omega_d$$

$\sigma = \zeta \omega_n$



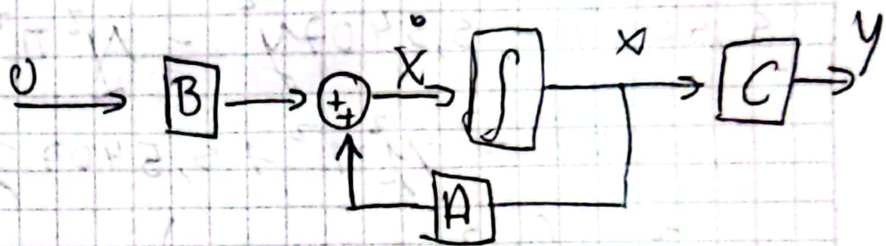
$$\tan\phi = \frac{w_d}{5,41}$$

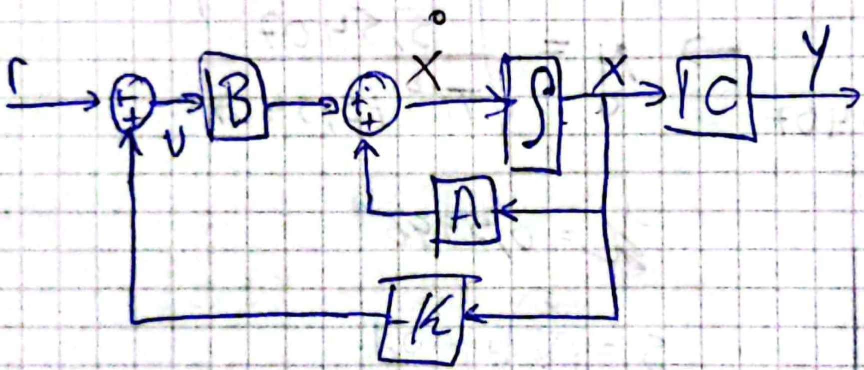
$$\tan(53,16)\, 5,41 = w_d$$

$$w_d = 7,21$$

$$\dot{x} = Ax + Bu$$

$$y = Cx$$



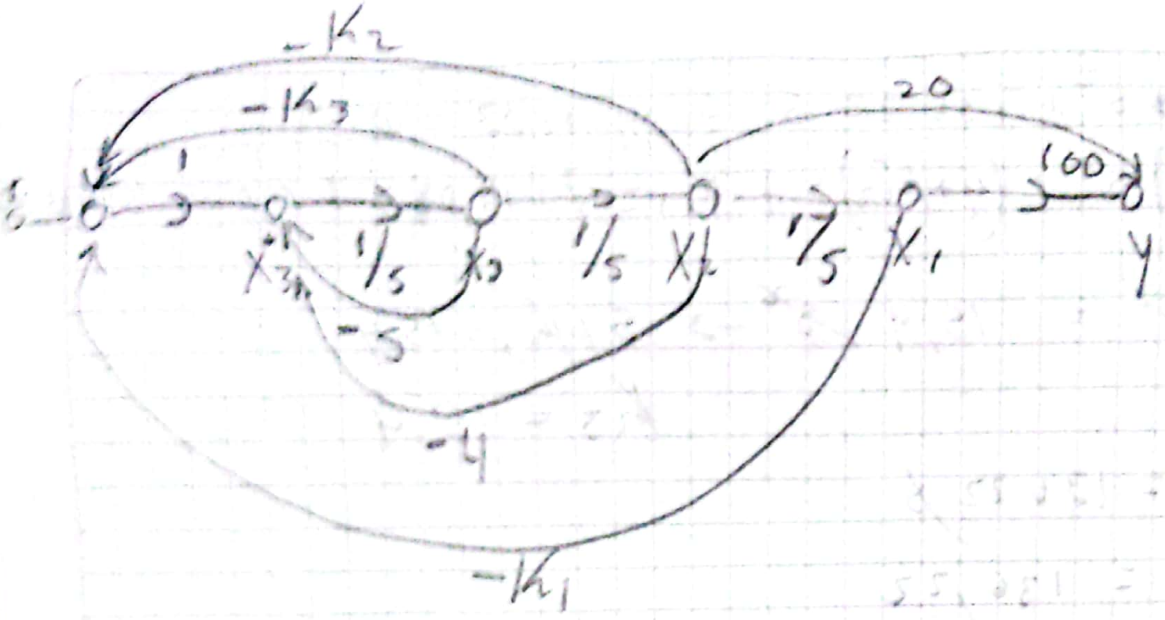


$$\dot{x} = Ax + Bu \rightarrow \dot{x} = Ax + B(-Kx + r)$$

$$\dot{x} = -BKx + Br + Ax \rightarrow \dot{x} = (A - BK)x + Br$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -4 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 100 & 20 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$



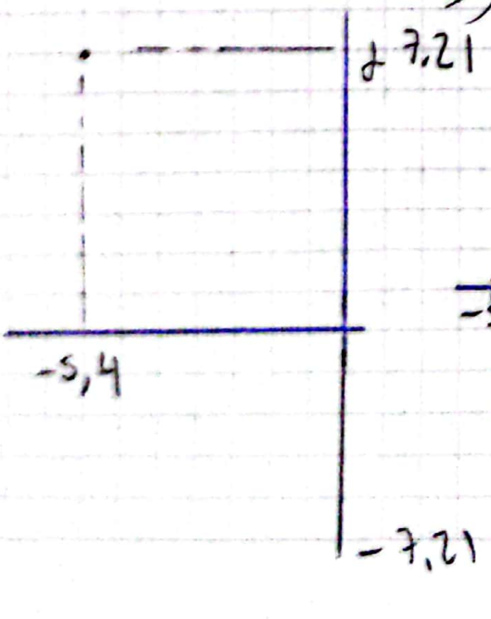
$$\dot{X}_3 = -4X_2 - 5X_3 + U$$

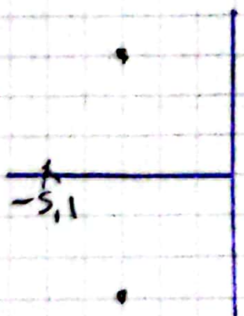
$$= -4X_2 - 5X_3 - K_3X_3 - K_2X_2 - K_1X_1 + r$$

$$= -K_1X_1 - (4+K_2)X_2 - (5+K_3)X_3 + r$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -K_1 & -(4+K_2) & -(5+K_3) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r$$

$$\det(sI - (A - BK)) = s^3 + (5+K_3)s^2 + (4+K_2)s + K_1 = 0$$





$(s+5{,}4-j7{,}2)(s+5{,}4+j7{,}2)$
$(s+5{,}1)$

$$s^3 + 15,9\, s^2 + 136\,22\, s + 413,83 = 0$$

$$s^3 + (5 + k_3)\, s^2 + (4 + k_2)\, s + k_1 = s^3 + 15,9\, s^2 + 136,22\, s + 413,83$$

$$(5 + k_3) \cancel{s^2} = 15,9\, \cancel{s^2} \rightarrow 5 + k_3 = 15,9$$

$$k_3 = 10,9$$

$$(4 + k_2) \cancel{s} = 136\,22\, \cancel{s}$$

$$4 + k_2 = 136,22$$

$$k_2 = 132,22$$

$$k_1 = 413,83$$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -413,8 & -136,22 & -15,9 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} r$$