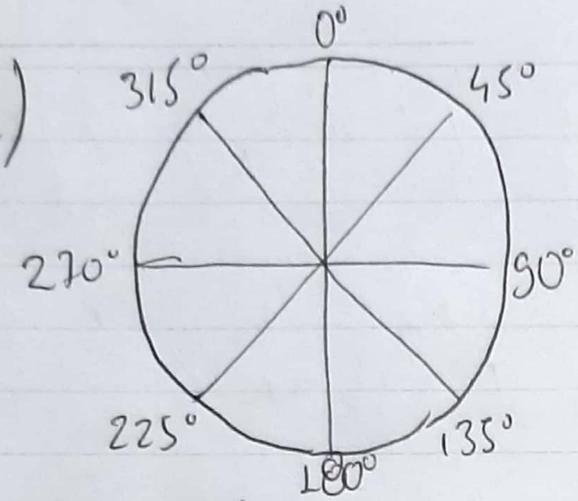
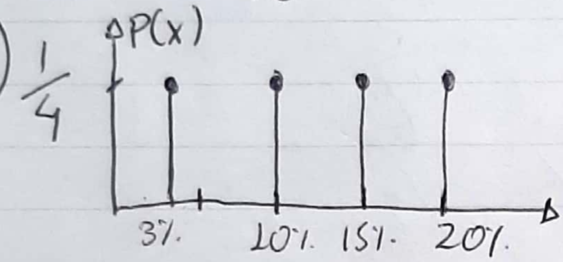
Q1)



X → desconto

$$X(\theta) = \begin{cases} 3\% & 0 \leq \theta < 45° \text{ ou } 180 \leq \theta < 225 \\ 15\% & 45 \leq \theta < 90 \text{ ou } 225 \leq \theta < 270 \\ 20\% & 90 \leq \theta < 135 \text{ ou } 270° \leq \theta < 315 \\ 10\% & 135 \leq \theta < 180 \text{ ou } 315° \leq \theta < 360 \end{cases}$$

b)



c) $E[x] = 1/4[3 + 15 + 20 + 10] = \underline{\underline{12\%}}$

$VAR[x] = E[x^2] - E[x]^2$

$E[x^2] = 1/4[3^2 + 15^2 + 20^2 + 10^2] = 183{,}5$

$VAR[x] = 183{,}5 - 144 = \underline{\underline{39{,}5\%}}$

No.

Q2) $d_1 = 3$ $\quad p(x|\omega_1) \sim N(d_1, 1)$
b) $\quad p(x|\omega_2) \sim N(d_1+2, 1)$

$p(x|\omega_1) \sim N(3,1)$
$p(x|\omega_2) \sim N(5,1)$
$P_r(\omega_1) = P_r(\omega_2) = 1/2$

$$\frac{p(x|\omega_1)}{p(x|\omega_2)} > \frac{P_r(\omega_2)}{P_r(\omega_1)}$$

$$\frac{\cancel{\frac{1}{\sqrt{2\pi}}} \exp\left(\frac{-(x-3)^2}{2}\right)}{\cancel{\frac{1}{\sqrt{2\pi}}} \exp\left(\frac{-(x-5)^2}{2}\right)} > \cancel{\frac{1/2}{1/2}}$$

$$\exp\left(\frac{-(x-3)^2}{2}\right) > \exp\left(\frac{-(x-5)^2}{2}\right)$$

$$\ln\left(\exp\left(\frac{-(x-3)^2}{2}\right)\right) > \ln\left(\exp\left(\frac{-(x-5)^2}{2}\right)\right)$$

$$-\frac{(x-3)^2}{2} > -\frac{(x-5)^2}{2}$$

$$(x-3)^2 < (x-5)^2$$

$$x^2 - 6x + 9 < x^2 - 10x + 25$$

$$-6x + 10x < 25 - 9$$

$$4x < 16$$

$\therefore$ se $x < 4 \longrightarrow$ escolha ω_1

No.

Q3) $d_2 = 2$ $\quad p(x|\omega_1) \sim N(d_2, 1)$

b) $\quad p(x|\omega_2) \sim N(d_2, 4)$

$p(x|\omega_1) \sim N(2, 1)$

$p(x|\omega_2) \sim N(2, 4)$

$P_r(\omega_1) = P_r(\omega_2) = 1/2$

$$\frac{p(x|\omega_1)}{p(x|\omega_2)} > \frac{P_r(\omega_2)}{P_r(\omega_1)}$$

$$\frac{\cancel{\frac{1}{\sqrt{2\pi}}} \exp\left(\frac{-(x-2)^2}{2}\right)}{\frac{1}{2\cancel{\sqrt{2\pi}}} \exp\left(\frac{-(x-2)^2}{2 \cdot 4}\right)} > \frac{\cancel{1/2}}{1/2}$$

$$2 \exp\left(-\frac{(x-2)^2}{2}\right) > \exp\left(-\frac{(x-2)^2}{8}\right)$$

$$\ln\left(2 \exp\left(-\frac{(x-2)^2}{2}\right)\right) > \ln\left(\exp\left(-\frac{(x-2)^2}{8}\right)\right)$$

$$\ln(2) + \left(-\frac{(x-2)^2}{2}\right) > -\frac{(x-2)^2}{8}$$

$$-8\ln(2) + 4(x-2)^2 < (x-2)^2$$

$$-8\ln(2) + 4(x^2 - 4x + 4) < x^2 - 4x + 4$$

$$-8\ln(2) + 3x^2 - 12x + 12 < 0 \longrightarrow \text{raíz } \pm\tfrac{2}{3}\left(3 \pm \sqrt{\ln(64)}\right)$$

$\therefore$ se $2/3\left(3 - \sqrt{\ln(64)}\right) < x < 2/3\left(3 + \sqrt{\ln(64)}\right)$

se $0{,}64 < x < 3{,}36$ escolha ω_1

Q4) $d_2 = 2$ $\quad p(x|w_1) \sim N(d_2, 1)$

a) $\quad p(x|w_2) \sim N(d_2+3, 1)$

$p(x|w_1) \sim N(2,1)$

$p(x|w_2) \sim N(5,1)$

$Pr(w_1) = 0{,}99$

$Pr(w_2) = 0.01$

$$\frac{p(x|w_1)}{p(x|w_2)} > \frac{Pr(w_2)}{Pr(w_1)}$$

$$\frac{\frac{1}{\sqrt{2\pi}} \exp\left(\frac{-(x-2)^2}{2}\right)}{\frac{1}{\sqrt{2\pi}} \exp\left(\frac{-(x-5)^2}{2}\right)} > \frac{0.01}{0.99}$$

$$99 \exp\left(\frac{-(x-2)^2}{2}\right) > \exp\left(\frac{-(x-5)^2}{2}\right)$$

$$\ln\left(99 \exp\left(\frac{-(x-2)^2}{2}\right)\right) > \ln\left(\exp\left(\frac{-(x-5)^2}{2}\right)\right)$$

$$\ln(99) + \left(-\frac{(x-2)^2}{2}\right) > -\frac{(x-5)^2}{2}$$

$$-\ln(99) + \frac{(x-2)^2}{2} < \frac{(x-5)^2}{2}$$

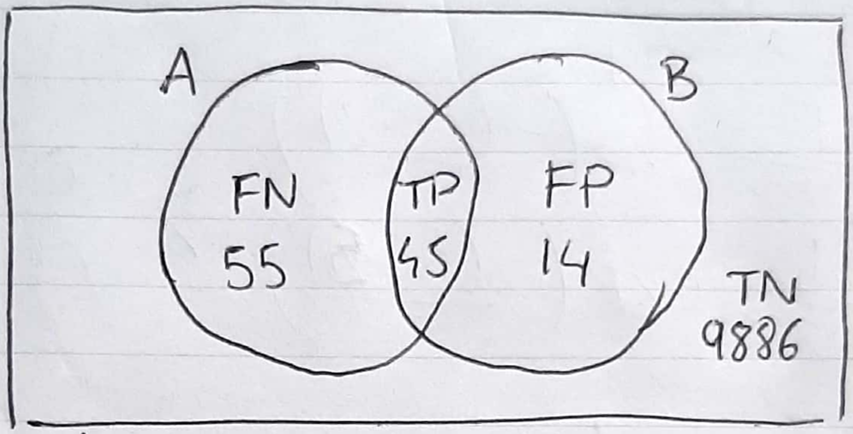
$$-2\ln(99) + \cancel{x^2} - 4x + 4 < \cancel{x^2} - 10x + 25$$

$$-4x + 10x < 25 - 4 + 2\ln(99)$$

$$6x < 21 + 2\ln(99)$$

$\therefore$ se $x < \dfrac{21 + 2\ln(99)}{6}$ escolha w_1

Q4) c)



A: pessoa doente

B: resultado positivo

d) $P(\omega_2 | \alpha_2) = \frac{TP}{TP+FP} = \frac{45}{45+14} = 76{,}27\%$

e) $P(\omega_1 | \alpha_1) = \frac{TN}{TN+FN} = \frac{9886}{9886+55} = 99{,}45\%$

No.

Q4) f) $\alpha_1 \to$ teste negativo $\quad \omega_1 \to$ saudável
$\alpha_2 \to$ teste positivo $\quad \omega_2 \to$ doente

$$R(\alpha_1|x) = \lambda_{11} P(\omega_1|x) + \lambda_{12} P(\omega_2|x)$$
$$R(\alpha_2|x) = \lambda_{21} P(\omega_1|x) + \lambda_{22} P(\omega_2|x)$$

$$\begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix} = \begin{bmatrix} 0 & 100 \\ 1 & 0 \end{bmatrix} \qquad \frac{p(x|\omega_1)}{p(x|\omega_2)} > \frac{(\lambda_{12}-\lambda_{22}) P(\omega_2)}{(\lambda_{21}-\lambda_{11}) P(\omega_1)}$$

$$P(x|\omega_1) \sim N(2,1) \quad P_r(\omega_1) = 0.99 \quad \lambda_{12} = 100$$
$$P(x|\omega_2) \sim N(5,1) \quad P_r(\omega_2) = 0.01 \quad \lambda_{21} = 1$$

$$\frac{\frac{1}{2\pi} \exp\left(\frac{-(x-2)^2}{2}\right)}{\frac{1}{2\pi} \exp\left(\frac{-(x-5)^2}{2}\right)} > \frac{100 \cdot 0.01}{1 \cdot 0.99} = \frac{100}{99}$$

$$99 \exp\left(\frac{-(x-2)^2}{2}\right) > 100 \exp\left(\frac{-(x-5)^2}{2}\right)$$

$$\ln(99) - \left(\frac{(x-2)^2}{2}\right) > \ln(100) - \left(\frac{(x-5)^2}{2}\right)$$

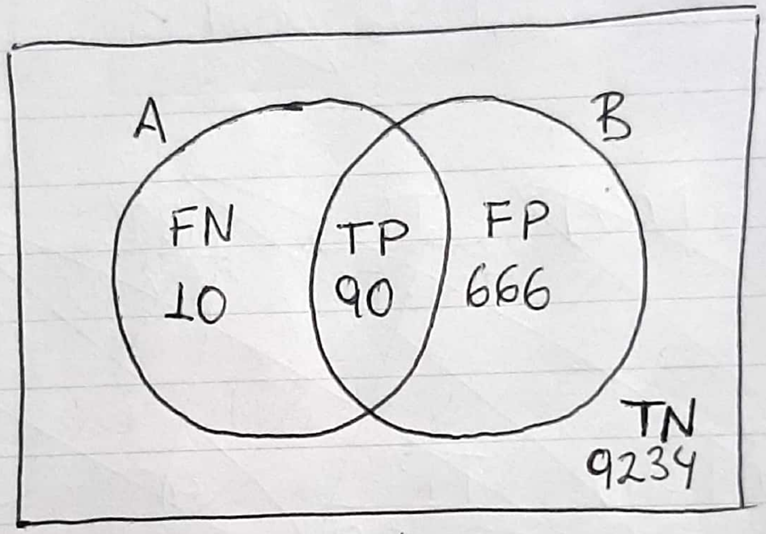
$$-2\ln(99) + (x^2-2) < -2\ln(100) - (x-5)^2$$

$$-2\ln(99) + x^2 - 4x + 4 < -2\ln(100) + x^2 - 10x + 25$$

$$-4x + 10x < 25 - 4 + 2\ln(99) - 2\ln(100)$$
$$6x < 21 + 2\ln(99) - 2\ln(100)$$

$$\therefore \text{ se } x < \frac{21 - 2\ln\left(\frac{99}{100}\right)}{6} \text{ escolhe } \omega_1$$

Q4) f)



A: pessoa Doente
B: resultado positivo

$$P(\omega_2|\alpha_2) = \frac{TP}{TP+FP} = \frac{90}{90+666} \cong 11,90\%$$

$$P(\omega_1|\alpha_1) = \frac{TN}{TN+FN} = \frac{9234}{9234+10} \cong 99,89\%$$

No.

Q5) a)

$d_1 : 3 \quad P(\bar{x}|w_1) \sim N(\mu_1, I) \quad \mu_1 = [3,2]^T$

$d_2 : 2 \quad P(x|w_2) \sim N(\mu_2, I) \quad \mu_2 = [0,1]^T$

$d_3 : 0$

$d_4 : 1 \quad P_r(w_1) = P_r(w_2) = 1/2$

Distribuição Normal Multivariada (Caso 1)

$\Sigma_i = \sigma^2 I \qquad \sigma^2 = 1 \qquad w_i = \frac{1}{\sigma^2}\mu_i$

$g_i(x) = w_i^T x + w_{i0} \qquad w_{i0} = \frac{-1}{2\sigma^2}\mu_i^T \mu_i + \ln(P(w_i))$

$g_1(x) = [3,2]x - \frac{1}{2}[3,2]\begin{bmatrix}3\\2\end{bmatrix} + \ln(1/2)$

$g_2(x) = [0\ 1]x - \frac{1}{2}[0\ 1]\begin{bmatrix}0\\1\end{bmatrix} + \ln(1/2)$

$g_1(x) = [3\ 2]x - 13/2 - \ln(2)$ fronteira

$g_2(x) = [0\ 1]x - 1/2 - \ln(2)$ $g_1 = g_2$

$[3\ 2]x - 13/2 = [0\ 1]x - 1/2$

$([3\ 2] - [0\ 1])x = 6$

$[3\ 1]x = 6 \rightarrow 3x_1 + x_2 = 6$

$x_2 = -3x_1 + 6$

No.

Q5) b) $d_1 = 3$ $p(x|\omega_1) \sim N(\mu_1, I)$ $\mu_1 = [3,2]^T$
$d_2 = 2$ $p(x|\omega_2) \sim N(\mu_2, I)$ $\mu_2 = [0,1]^T$
$d_3 = 0$ $Pr(\omega_1) = 1/3$
$d_4 = 1$ $Pr(\omega_2) = 2/3$

$$g_1 = [3,2]x - \frac{1}{2}[3,2]\begin{bmatrix}3\\2\end{bmatrix} + \ln(1/3)$$

$$g_2 = [0,1]x - \frac{1}{2}[0,1]\begin{bmatrix}0\\1\end{bmatrix} + \ln(2/3)$$

$$g_1 = [3\ 2]x - 13/2 + \ln(1/3)$$
$$g_2 = [0,1]x - 1/2 + \ln(2/3)$$

fronteira
$g_1 = g_2$

$$[3\ 2]x - 13/2 + \ln(1/3) = [0\ 1]x - 1/2 + \ln(2/3)$$

$$([3\ 2] - [0\ 1])x = -1/2 + 13/2 - \ln(1/3) + \ln(2/3)$$

$$[3\ 1]x \cong 6{,}69 \rightarrow 3x_1 + x_2 = 6{,}69$$

$$x_2 = -3x_1 + 6{,}69$$

No.

Q5) c)

$d_1 : 3$ $\quad P(x|w_1) \sim N(\mu_1, \Sigma)$ $\quad \mu_1 = [3\ 2]^T$

$d_2 : 2$ $\quad P(x|w_2) \sim N(\mu_2, \Sigma)$ $\quad \mu_2 = [0\ 1]^T$

$d_3 : 0$

$d_4 : 1$ $\quad P_r(w_1) = P_r(w_2) = 1/2$ $\quad \Sigma = \begin{bmatrix} 1 & 0.3 \\ 0.3 & 2 \end{bmatrix}$

Distribuição Normal Multivariada (Caso 2)

$g_i(x) = w_i^T x + w_{i0}$ $\qquad w_{i0} = -\frac{1}{2} \mu_i^T \Sigma^{-1} \mu_i + \ln(P(w_i))$

$w_i = \Sigma^{-1} \mu_i$ $\qquad \Sigma = \begin{bmatrix} 1 & 0.3 \\ 0.3 & 2 \end{bmatrix}$ $\qquad \Sigma^{-1} = \begin{bmatrix} \frac{200}{191} & \frac{-30}{191} \\ \frac{-30}{191} & \frac{100}{191} \end{bmatrix}$

$$g_1 = \frac{100}{191} \begin{bmatrix} 2 & -0.3 \\ -0.3 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} x - \frac{50}{191} [3\ 2] \begin{bmatrix} 2 & -0.3 \\ -0.3 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} - \ln(2)$$

$$g_1 = \left(\begin{bmatrix} 540 \\ 110 \end{bmatrix} \underline{x} - 920 \right) \cdot \left(\frac{1}{191} \right) - \ln(2)$$

$$g_2 = \frac{100}{191} \begin{bmatrix} 2 & -0.3 \\ -0.3 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} x - \frac{50}{191} [0\ 1] \begin{bmatrix} 2 & -0.3 \\ -0.3 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} - \ln(2)$$

$$g_2 = \left(\begin{bmatrix} -30 \\ 100 \end{bmatrix} \underline{x} - 50 \right) \cdot \left(\frac{1}{191} \right) - \ln(2)$$

$\boxed{g_1 = g_2}$

$$\begin{bmatrix} 540 \\ 110 \end{bmatrix} x - 920 = \begin{bmatrix} -30 \\ 100 \end{bmatrix} x - 50$$

$$\begin{bmatrix} 570 \\ 10 \end{bmatrix} x - 870 = 0$$

$$570 x_1 + 10 x_2 - 820 = 0$$

$$10 x_2 = -570 x_1 + 870$$

$$\rightarrow x_2 = -57 x_1 + 87 \leftarrow$$

No.

Q6) $p(x|w_1) \sim N(\mu_1, \Sigma_1)$ $\quad \mu_1 = [0,0]^T$ $\quad d1 = 3$

$p(x|w_2) \sim N(\mu_2, \Sigma_2)$ $\quad \mu_2 = [0,4]^T$

$$\Sigma_1 = \begin{bmatrix} 23 & 0 \\ 0 & 1 \end{bmatrix} \quad \Sigma_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \Sigma_1^{-1} = \begin{bmatrix} 1/23 & 0 \\ 0 & 1 \end{bmatrix} \quad |\Sigma_1| = 23$$

$$w_{10} = \cancel{-\frac{1}{2}[0\ 0]\begin{bmatrix} 1/23 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \end{bmatrix}} - \frac{1}{2}\ln(23) - \ln(2)$$

$$w_{10} = -\frac{1}{2}\ln(23) - \ln(2) \quad W_1 = -\frac{1}{2}\begin{bmatrix} 1/23 & 0 \\ 0 & 1 \end{bmatrix} \quad \cancel{w_1 = \begin{bmatrix} 1/23 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \end{bmatrix}}$$

$$w_{20} = -\frac{1}{2}[0\ 4]\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 4 \end{bmatrix} \cancel{-\frac{1}{2}\ln(1)} - \ln(2)$$

$$w_{20} = -8 - \ln(2) \quad W_2 = -\frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad w_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

$$x^T W_1 x + \cancel{w_1^T x} + w_{10} = x^T W_2 x + w_2^T x + w_{20}$$

$$[x_1\ x_2]\begin{bmatrix} -1/46 & 0 \\ 0 & -1/2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - \frac{1}{2}\ln(23) \cancel{- \ln(2)} = [x_1\ x_2]\begin{bmatrix} -1/2 & 0 \\ 0 & -1/2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [0\ 4]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - 8 \cancel{- \ln(2)}$$

$$\left[-\frac{1}{46}x_1\ \ -\frac{1}{2}x_2\right]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - \frac{1}{2}\ln(23) = \left[-\frac{1}{2}x_1\ \ -\frac{1}{2}x_2\right]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [0\ 4]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - 8$$

$$-\frac{1}{46}x_1^2 \cancel{-\frac{1}{2}x_2^2} - \frac{1}{2}\ln(23) = -\frac{1}{2}x_1^2 \cancel{-\frac{1}{2}x_2^2} - 8 + 4x_2 \quad (\times 46)$$

$$-x_1^2 - 23\ln(23) = -23x_1^2 - 368 + 184x_2$$

$$-23\ln(23) - x_1^2 + 23x_1^2 + 368 = 184x_2$$

$$22x_1^2 - 23\ln(23) + 368 = 184x_2$$

$$x_2 = \frac{22x_1^2 - 23\ln(23) + 368}{184}$$