ASE 389P-7
Exam 1

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1 Problem 1

1.1 Instruction

Write a function in Matlab that simulates the train-horn-Doppler scenario discussed in lecture. Assume that the train tracks are rectilinear.

Be sure to account for the nonzero time of flight δt_{TOF} as discussed in lecture. The effect of $\delta t_{TOF} > 0$ is that the stationary observer will discern an f_D at time t_k that relates to the train's line-of-sight velocity at time $t_k - \delta t_{TOF}$. More precisely, the apparent frequency of the train horn at the location of the observer at time t_k is given by

$$f_r(t_k) = \frac{f_c}{1 + \frac{v_{los}(t_k)}{v_s}} \tag{1}$$

where f_c is the nominal horn frequency, $v_{los}(t_k)$ is the line-of-sight velocity at t_k, and v_s is the speed of the signal in the medium. Note that the line-of-sight geometry used to calculate $v_{los}(t_k)$ is between the observer at time t_k and the horn at time $t_k - \delta t_{TOF}$.

Download the audio file trainout.wav from Canvas. This file was created with the following input argument values:

```
fh = 440;
vTrain = 20;
t0 = 0;
x0 = 0;
delt = 0.01;
N = 1000;
vs = 343;
```

Set up your simulator with these same values. Estimate the values of xObs and dObs by adjusting them in your simulation until you get an apparent received frequency profile that matches the one in the audio file.

1.2 MATLAB code

```
function [fDVec,tVec] = ...
simulateTrainDoppler(fc, vTrain, t0, x0, xObs, dObs, delt, N, vs)
% simulateTrainDoppler : Simulate the train horn Doppler shift scenario.
%
% INPUTS
%
% fc ------- train horn frequency, in Hz
%
% vTrain -- constant along-track train speed, in m/s
%
% t0 ------- time at which train passed the along-track coordinate x0, in
%            seconds
%
% x0 ------- scalar along-track coordinate of train at time t0, in meters
%
% xObs ----- scalar along-track coordinate of observer, in meters
%
% dObs ----- scalar cross-track coordinate of observer, in meters (i.e.,
%            shortest distance of observer from tracks)
%
% delt ----- measurement interval, in seconds
%
% N -------- number of measurements
%
% vs ------- speed of sound, in m/s
%
%
% OUTPUTS
%
% fDVec ---- N-by-1 vector of apparent Doppler frequency shift measurements as
%            sensed by observer at the time points in tVec
%
% tVec ----- N-by-1 vector of time points starting at t0 and spaced by delt
%            corresponding to the measurements in fDVec
%
%+------------------------------------------------------------------------------+
% References:
%
%
% Author:
%+==============================================================================+
   tVec = zeros(N,1);
   fDVec = zeros(N,1);

   t = t0;
```

```
    for h = 1:N
        % Find rRX(t) and vRX(t)
        rRX = positionRX(t, t0, xObs, dObs);
        vRX = velocityRX(t, t0);

        % Calculate TOF
        TOF = calculateTOF(t, rRX, t0, x0, vTrain, vs);

        % Find rTX(t-TOF) and vTX(t-TOF)
        rTX = positionTX(t-TOF, t0, x0, vTrain);
        vTX = velocityTX(t-TOF, t0, vTrain);

        % Find the line-of-sight
        los = (rRX - rTX) / norm(rRX - rTX);

        % Project the velocity of TX to the line-of-sight
        vTX_los =  dot(vTX, los);
        % Project the velocity of RX to the line-of-sight
        vRX_los =  dot(vRX, los);
        % Calculate the line-of-sight velocity
        v_los = vRX_los - vTX_los;

        % Calculate the apparent Doppler frequency shift as sensed by the
        % observer
        beta = v_los / vs;
        fr = fc / (1 + beta);
        fDVec(h) = fr - fc;
        tVec(h) = t;
        t = t + delt;
    end
end

function [rRX] = positionRX(t, t0, x0, y0)
    rRX = [x0, y0];
end

function [vRX] = velocityRX(t, t0)
    vRX = [0, 0];
end

function [rTX] = positionTX(t, t0, x0, v0)
    x = x0 + v0*(t-t0);
    rTX = [x, 0];
end
```

```
function [vTX] = velocityTX(t, t0, v0)
    vTX = [v0, 0];
end

function [TOF] = calculateTOF(t, rRX, t0, x0, vTrain, vs)
    % Initialize with a guess
    TOF = norm(rRX - positionTX(t, t0, x0, vTrain))/vs;
    error = vs*TOF - norm(rRX - positionTX(t-TOF, t0, x0, vTrain));

    % Iterate until convergence
    while ( abs(error) > 1e-3 )
        TOF = TOF - error/vs;
        error = vs*TOF - norm(rRX - positionTX(t-TOF, t0, x0, vTrain));
    end
end

% topSimulateTrainDoppler.m
%
% Top-level script for train Doppler simulation


clear; clc; close all;
%----- Setup
fc = 440;
vTrain = 20;
t0 = 0;
x0 = 0;
delt = 0.01;
N = 1000;
vs = 343;
xObs = 56.8;
dObs = 10;

%----- Simulate
[fDVec,tVec] = simulateTrainDoppler(fc,vTrain,t0,x0,xObs,dObs,delt,N,vs);
fApparentVec = fDVec + fc;

%----- Plot
plot(tVec,fDVec + fc, 'r');
xlabel('Time (seconds)');
ylabel('Apparent horn frequency (Hz)');
grid on;
shg;

%----- Generate a sound vector
T = delt*N;                       % simulation time (sec)
```

```
fs = 22050;                    % sample frequency (Hz)
deltSamp = 1/fs;               % sampling interval (sec)
Ns = floor(T/deltSamp);        % number of samples
tsamphist = [0:Ns-1]'*deltSamp;
Phihist = zeros(Ns,1);
fApparentVecInterp = interp1(tVec,fApparentVec,tsamphist,'spline');
for ii=2:Ns
  fii = fApparentVecInterp(ii);
  Phihist(ii) = Phihist(ii-1) + 2*pi*fii*deltSamp;
end
soundVec = sin(Phihist);

% %----- Play the sound vector
%sound(soundVec, fs);

%----- Write to audio file
audiowrite('my_trainout.wav',soundVec,fs);

%----- Write frequency time history to output file
save trainData fApparentVec tVec

%% Extra Points
close all
% I use an spectrogram to better understand the original audio signal
[y, fs] = audioread('trainout.wav');

Nspec = 2^(nextpow2(length(y)) - 7);
wspec = hamming(Nspec);
Noverlap = Nspec/2;
figure()
% spectrogram(y, 512, 128, 128, fs, 'yaxis');
spectrogram(y, wspec, Noverlap, Nspec, fs, 'yaxis');
colormap jet
title("original signal")

set(gca,'Linewidth',1.2,'FontSize',36)
set(gcf,'Position',[2500 100 1550 800])

% Then, I manually analyze my simulated audio signal to make it look as
% similar to the original as posible by changing the position of the
% receiver
% [y, fs] = audioread('my_trainout.wav');
%
% figure()
% spectrogram(y, 512, 120, 128, fs, 'yaxis');
```

```
% colormap jet
% title("My signal")
%
% set(gca,'Linewidth',1.2,'FontSize',36)
% set(gcf,'Position',[2500 100 1550 800])
```

1.3 Result

Analizing the given signal it's possible to manually tag the moment in which the train passed in front of the receiver. Figure 1 shows how the power density is concentrated at higher frequency before 3.069 sec and drops rapidly after that. This indicates the exact moment at which the train passed infront of the receiver.

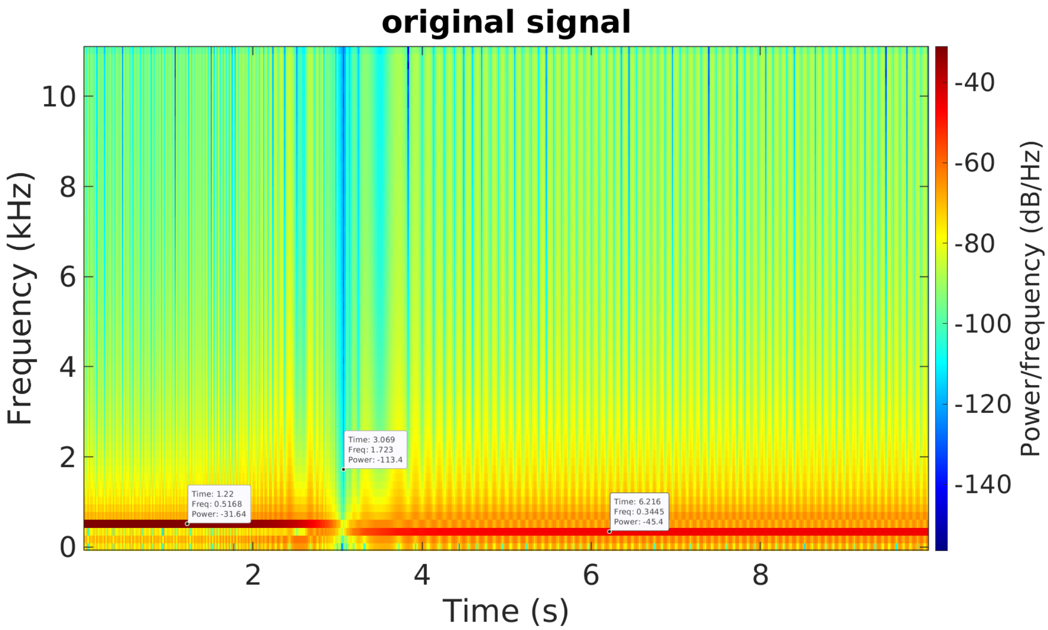

Figure 1: Spectrogram of the given signal.

This, approach lead to $xObs = 56.8$ and $dObs = 10$.

2 Exercise 2

2.1 Instruction

In this problem, we'll examine the effects of clock frequency bias on Doppler derived from frequency downconversion and sampling.

Usually, we don't measure signal properties directly at the signal's incoming frequency. Instead, we convert incoming signals to a lower center frequency by a process called frequency conversion. To understand frequency conversion, recall that

$$cos(x)cos(y) = \frac{1}{2}[cos(x-y) + cos(x+y)] \tag{2}$$

Hence, by multiplying an incoming signal $x(t) = cos(2\pi f_c t)$ by a local signal $x_l(t) = 2cos(2\pi f_l t)$ and then low-pass filtering the result to eliminate the high frequency $f_c + f_l$ component, we convert the high frequency signal down to a lower frequency $f_b = f_c - f_l$. Processes such as amplification, transmission, filtering, delaying, recording, and sampling are easier to do at lower frequencies. Figure 2 shows a diagram of the frequency conversion process.

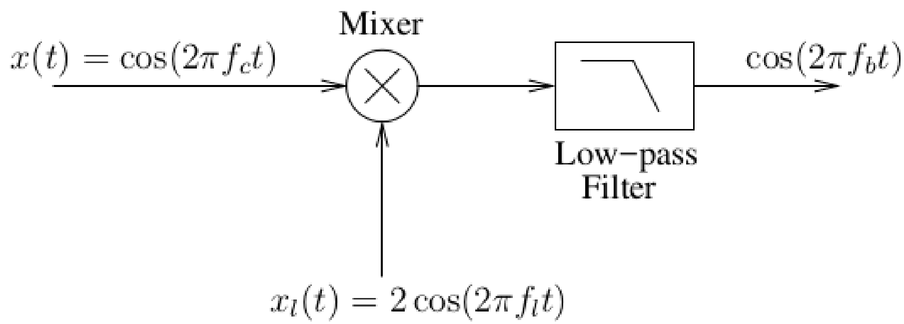

Figure 2: A single-stage frequency conversion (mixing) operation.

Assume that some oscillator with a perfect clock produces a pure sinusoid of the form $x(t) = cos(2\pi f_c t)$. Suppose that you receive this signal and make a measurement f_m of the signal's frequency by (1) performing frequency conversion to translate the signal to a nominal center frequency $f_{b,nom} = 0Hz$ and then (2) sampling the signal with nominal sampling interval Δt and observing the number of samples per period. Assume there is no motion between your receiver and the signal transmitter. Suppose that the local clock you're using to perform frequency conversion and sampling has a fractional frequency error of $\Delta f/f < 0$.

Derive an expression for the apparent Doppler f_D resulting from frequency conversion and sampling with a clock that has this fractional frequency error. Express your result in terms of f_c and $\Delta f/f$. How does this expression differ from the one derived in problem 3?

Note that if $f_l < f_c$, as will be the case for our problem because $\Delta f/f < 0$, then $f_b > 0$. This is an example of low-side mixing. But if $f_l > f_c$, then $f_b < 0$; this is high-side mixing. What would be the meaning of a signal for which $f_b < 0$? Upon sampling a signal with $f_b < 0$ to determine the period, would we measure a negative frequency?

2.2 Result

$$x(t) * x_l(t) = 2cos(2\pi f_c t)cos(2\pi f_l t)) \tag{3}$$

$$x(t) * x_l(t) = cos(2\pi(f_c - f_l)t) + cos(2\pi(f_c + f_l)t) \tag{4}$$

After applying the low-pass filter we get

$$x_{LPF}(t) = cos(2\pi(f_c - f_l)t) \tag{5}$$

Let's clarify:

$$f_b = f_c - f_l = f_c - (f_{l,nom} + \Delta f) \tag{6}$$

We know that $f_{b,nom} = 0Hz$. Therefore, $f_{l,nom} = f_c$.

Now, it's necessary to focus on the last part of the processing (sampling). For that, lets write the following relationships.

$$\frac{\Delta f}{f_{l,nom}} = \frac{f_l - f_{l,nom}}{f_{l,nom}} = \frac{f_l}{f_{l,nom}} - 1 = \frac{\Delta t_{l,nom}}{\Delta t_l} - 1 \tag{7}$$

$$\Delta t_{l,nom} = (\frac{\Delta f}{f_{l,nom}} + 1)\Delta t_l \tag{8}$$

Since the objective at this stage is to measure the frequency of the signal that comes from the low-pass filter, it is safe to say that we are trying to measure f_b. Thus, $f_b = \frac{1}{N*\Delta t_l}$.

$$f_m = \frac{1}{N * \Delta t_{l,nom}} = \frac{1}{N * (\frac{\Delta f}{f_{l,nom}} + 1)\Delta t_l} = \frac{f_b}{\frac{\Delta f}{f_{l,nom}} + 1} \tag{9}$$

Since it was stablished that $f_{l,nom} = f_c$. Then, $\Delta f = f_c \frac{\Delta f}{f_{l,nom}}$

$$f_m = \frac{\frac{\Delta f}{f_{l,nom}} f_c}{\frac{\Delta f}{f_{l,nom}} + 1} \tag{10}$$

Finally, it is possible to reason that the measured frequency is the very apparent Doppler f_D (with a changed sign). This is because the frequency conversion already applied the following substraction $f_b = f_c - f_l$, which is almost the same substraction needed for $f_D = f_l - f_c$.

$$f_D = -\frac{\frac{\Delta f}{f_{l,nom}} f_c}{\frac{\Delta f}{f_{l,nom}} + 1} \tag{11}$$

2.3 Q&A

Q: How does the expression differ from the one derived in problem 3? A: It doesn't. It's the same relationship.

Q: What would be the meaning of a signal for which $f_b < 0$? A: It would mean that the apparent Doppler would be inverted. Therefore, we would think that the TX and RX are getting further and further appart.

Q: Upon sampling a signal with $f_b < 0$, would we measure a negative frequency? A: No, we would measure the absolute value of f_b.

3 Exercise 3

3.1 Instruction

This problem will walk you through a noise analysis for the UT Radionavigation Laboratory (RNL) GNSS receiver setup. A block diagram of the setup is shown in Fig. 2. A rooftop Trimble Geodetic Zephyr II antenna is followed by cables, a bias tee, and a splitter, with parameters as follows:

- $T_A = 100K$: The temperature of the passive antenna element within the Trimble antenna.
- $L_1 = 1$ dB: Before being amplified, signals from the passive antenna element pass through a short transmission line and a low-insertion-loss bandpass filter. L_1 is the combined loss of the line and filter. Because L_1 enters before any gain is applied to the signals, it is very important to make L_1 as low as possible.
- $G_2 = 50$ dB; $F_2 = 1.5$ dB: The Trimble antenna's built-in low-noise amplifier (LNA) has extraordinary gain and a low noise figure.
- $L_3 = 6.48$ dB: Signals are routed from the active Trimble antenna to the RNL via 127 feet of C400 cable. The cable is fairly low loss: at 1.5 GHz (near the GPS L1 frequency), the cable loss is 5.1 dB per 100 feet.
- $L_4 = 0.6$ dB: A bias tee feeds the active antenna with a 5 V supply by biasing the center line of the coaxial cable coming from the antenna to 5 V DC. The DC supply voltage does not affect the high-frequency signals arriving from the antenna. From the perspective of the high-frequency signals, the bias tee looks like a passive component with a 0.6 dB insertion loss.
- $L_5 = 9.8$ dB: Signals exiting the bias tee are split by a passive 8-way splitter to provide GNSS signals for the many receivers in the RNL. For an ideal splitter, the signal power exiting any one of the 8 splitter outputs is a factor of 8 (9 dB) lower than the input power. In the RNL splitter, the input signal sees an additional 0.8 dB of insertion loss.

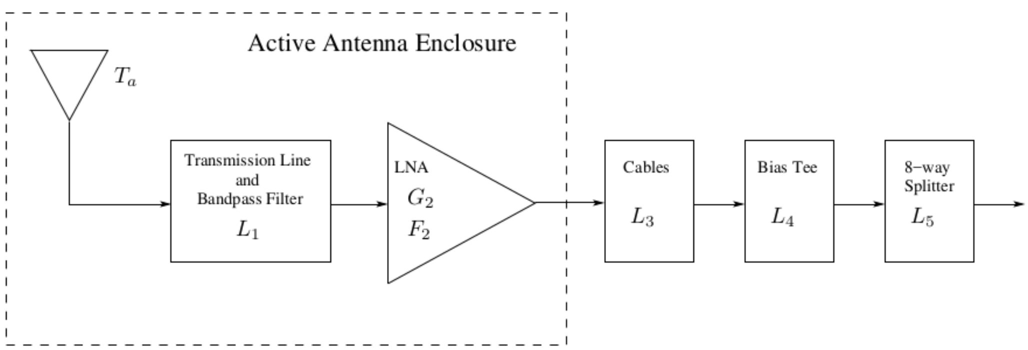

Figure 3: Radionavigation Laboratory GNSS receiver setup.

3.2 Results

3.2.1 Item a

Calculate the noise figure F_1 and effective input temperature T_1 corresponding to L_1. Assume an input temperature $T_{in} = T_0 = 290K$.

Since L_1 is a passive element

$$F = \frac{1}{G} > 1 \tag{12}$$

$$T_E = \frac{1 - G}{G} T_0 \tag{13}$$

Therefore, $F_1 = 1.259$ and $T_1 = 75.11K$.

3.2.2 Item b

Calculate the effective input temperature T_2 corresponding to F_2. Assume F_2 is taken from a device data sheet that assumes $T_{in} = T_0 = 290K$.

$$T_2 = (F_2 - 1)T_0 \tag{14}$$

Therefore, since $F_2 = 1.5dB = 1.413$ the effective input temperature $T_1 = 119.64K$.

3.2.3 Item c

Lump losses L_3, L_4, and L_5 into a single loss L_{345}. Calculate an effective input temperature T_{345} corresponding to L_{345}. Assume an input temperature $T_{in} = T_0 = 290K$.

Since, $L_3 = 6.48dB$, $L_4 = 0.6dB$, and $L_5 = 9.8dB$ the lump losses are $L_{345} = 16.88dB$. The equivalent system is passive. Thus, $F = L = 48.75$.

$$T_{345} = (F_{345} - 1)T_0 \tag{15}$$

Consequently, $T_{345} = 13,848.3K$

3.2.4 Item d

Use the Friis formula to calculate the system temperature $T_S = T_A + T_R$, expressed in degrees K. How many degrees K do the combined losses L_{345} contribute to the system temperature?

$$T_R = T_1 + \frac{T_2}{G_1} + \frac{T_{345}}{G_1 G_2} \tag{16}$$

$$T_R = 75.11 + \frac{119.64}{0.794} + \frac{13,848.3}{0.794 * 100000} = 75.11 + 150.68 + 0.17 = 293.28K \tag{17}$$

Therefore, $T_S = T_A + T_R = 393.28K$. Notice that the combined losses L_{345} contribute $0.17K$ to the overall system temperature.

3.2.5 Item e

The effective noise floor of the whole cascade, N_0, is related to the system temperature T_S by $N_0 = k * T_S$, where k is Boltzmann's constant, equal to $-228.6dBW/K - Hz$. The quantity N_0 is the one used in calculating C/N_0, that all-important parameter in GNSS receiver design. For purposes of calculating C/N_0, you can think of the receiver cascade as a chain of ideal gain blocks (no internal noise) with an effective noise density N_0 at the beginning of the cascade (i.e., at the output of the passive antenna element just before the L_1 block). This is precisly the point where the signal power C is defined, so it makes sense to calculate the ratio C/N_0 here. Calculate the value of N_0 in dBW/Hz. For an expected GPS L_1 C/A received signal power C ranging from -162.5 to $-154.5dBW$, calculate the expected range of carrier-to-noise ratio C/N_0 values. Express your result in $dB-Hz$.

If one thinks the receiver cascade as a chain of ideal gain blocks (no internal noise). Then $T_S = T_A = 20dBK$

$$N_0 = k * T_S = -228.6\frac{dBW}{K-Hz} + 20dBK = -208.6\frac{dBW}{Hz} \tag{18}$$

$$46.1dB-Hz = -162.5 - (-208.6) < \frac{C}{N_0} < -154.5 - (-208.6) = 54.1dB-Hz \tag{19}$$

3.2.6 Item f

What is the noise floor $N_{0,sp}$ at the output of the 8-way splitter? Express your answer in dBW/Hz.

Since $T_S = 393.28K = 25.947dBK$

$$N_0 = k * T_S = -228.6\frac{dBW}{K-Hz} + 25.947dBK = -202.65\frac{dBW}{Hz} \tag{20}$$

4 Problem 4

4.1 Instruction

To measure the receiver temperature T_R of a GNSS receiver and antenna setup, a friend recommends placing the receiver's antenna in a RF test enclosure such as the one in the Radionavigation Laboratory (seen here https://ramseytest.com/forensic-test-enclosures), but cryogenically cooled down to 5 K, and then measuring the noise power in the raw samples generated by the receiver. The enclosure effectively isolates the antenna from environmental noise. The antenna is an active antenna consisting of a patch element, a (passive) filter, and an amplifier. Is this a valid approach for measuring TR ? Why or why not?

4.2 Result

The cryogenically cooled enclosure would only allow us to reduce the noise of the receiver but not T_A. Therefore one could measure the noise power with and without the cryogenically cooled enclosure and get an estimate of the T_R by substracting the results.

It seems like this is not a good idea though because if one would like to make the best use of their money for the general usage of the GNSS receiver. Then, buying the cryogenically cooled enclosure would only buy you a couple dBs better noise performance. The main source of noise (the other GNSS signals one is not interested in when aquiring from a particular satellite) would not be attenuated. Therefore, one could measure the desired magnitude but that's it. No noticible benefit would come from this purchase.