ASE 389P-7
Problem set 3

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1 Problem 8

1.1 Code

```
function [pZ_H0,pZ_H1,lambda0,Pd,ZVec] = performAcqHypothesisCalcs(s)
% performAcqHypothesisCalcs : Calculate the null-hypothesis and alternative
%                             hypothesis probability density functions and
%                             the decision threshold corresponding to GNSS
%                             signal acquisition with the given inputs.
%
% Z is the acquisition statistic:
%
%         N  |      |^2
% Z =    sum |  Sk  |
%        k=1 |      |
%
%
%
%        N   |                |
%   =  sum   |  Ik^2 + Qk^2   |
%       k=1  |                |
%
%
%
% where Sk = rhok + nk = Ik + j*Qk
%
% and nk = nIk + j*nQk
%
% with nIk ~ N(0,1), nQk ~ N(0,1), and E[nIk nQi] = 1 for k = i and 0 for k !=
% i. The amplitude rhok is related to familiar parameters Nk, Abark, and
% sigma_IQ by rhok = (Nk*Abark)/(2*sigma_IQ), i.e., it is the magnitude of the
% usual complex baseband phasor normalized by sigma_IQ.
%
% Under H0, the statistic Z is distributed as a chi square distribution with
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% 2*N degrees of freedom; under H1, it is distributed as a noncentral chi
% square distribution with lambda = N*rhok^2 and 2*N degrees of freedom.
%
% The total number of cells in the search grid is assumed to be nCells =
% nCodeOffsets*nFreqOffsets, where nFreqOffsets = 2*fMax*Ta and Ta = Na*T is
% the total coherent accumulation time. Here, Na is the average value of the
% number of samples in each accumulation, Nk.
%
% INPUTS
%
% s ---------- A structure containing the following fields:
%
% C_N0dBHz --------- Carrier to noise ratio in dB-Hz.
%
% Ta --------------- Coherent accumulation interval, in seconds.
%
% N ---------------- The number of accumulations summed noncoherently to
%                    get Z.
%
% fMax ------------- Frequency search range delimiter. The total
%                    frequency search range is +/- fMax.
%
% nCodeOffsets ---- Number of statistically independent code offsets in
%                    the search range.
%
% PfaAcq ----------- The total acquisition false alarm probability.
%                    This is the probability that the statistic Z
%                    exceeds the threshold lambda in any one of the
%                    search cells under the hypothesis H0. One can
%                    derive the false alarm probability for *each*
%                    search cell from PfaAcq. This procedure is
%                    straightforward if we assume that the detection
%                    statistics from the search cells are independent
%                    of one another.
%
% ZMax ------------- The maximum value of Z that will be considered.
%
% delZ ------------- The discretization interval used for the
%                   independent variable Z. The full vector of Z
%                   values considered is thus ZVec = [0:delZ:ZMax].
%
%
% OUTPUTS
%
% pZ_H0 ------------ The probability density of Z under hypothesis H0.
```

```
%
% pZ_H1 ------------ The probability density of Z under hypothesis H1.
%
% lambda0 ---------- The detection threshold.
%
% Pd --------------- The probability of detection.
%
% Zvec ------------- The vector of Z values considered.
%
%+------------------------------------------------------------------------------+
% References:
%
%
%+==============================================================================+
dof = 2*s.N;
ZVec = 0:s.delZ:s.ZMax;
nFreqOffsets = 2*s.fMax*s.Ta;
nCells = s.nCodeOffsets * nFreqOffsets;
rho_squared = 10^(s.C_N0dBHz/10) * 2 * s.Ta;
lambda = s.N * rho_squared;

% compute the probability density of Z under hypothesis H0 and H1
pZ_H0 = chi2pdf(ZVec, dof)';
pZ_H1 = ncx2pdf(ZVec, dof, lambda)';

% Calculate the threshold that ensures that the probability of false
% acquisition is below the user-defined value
Pf = 1 - (nthroot((1-s.PfaAcq), nCells));
nuStar = chi2inv(1-Pf, dof);
lambda0 = nuStar;

% Calculate the probability of detection
Pd = 1 - ncx2cdf(nuStar, dof, lambda);
```

1.2 Results

1.2.1 Part A

In part A, N was adjusted to reach a probability of detection in the neighborhood of 95% for signals with different carrier-to-noise ratios, this is shown in Figure 1.

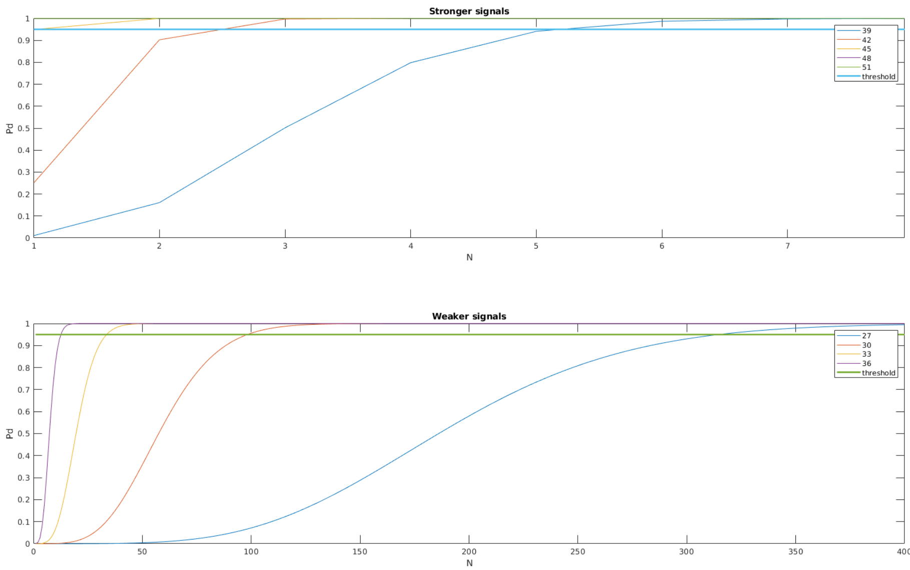

Figure 1: Probability of detection vs. number of accumulation periods for different C/N0 ratios (Non-coherent accumulation).

In Figure 2 it is possible to observe a particular case of the hypothesis testing for GNSS acquisition. In particular, it is taylored to detect a 30dBHz C/N0 signal with 95% probability and 0.01% chance of false alarm by performing non-coherent integration.

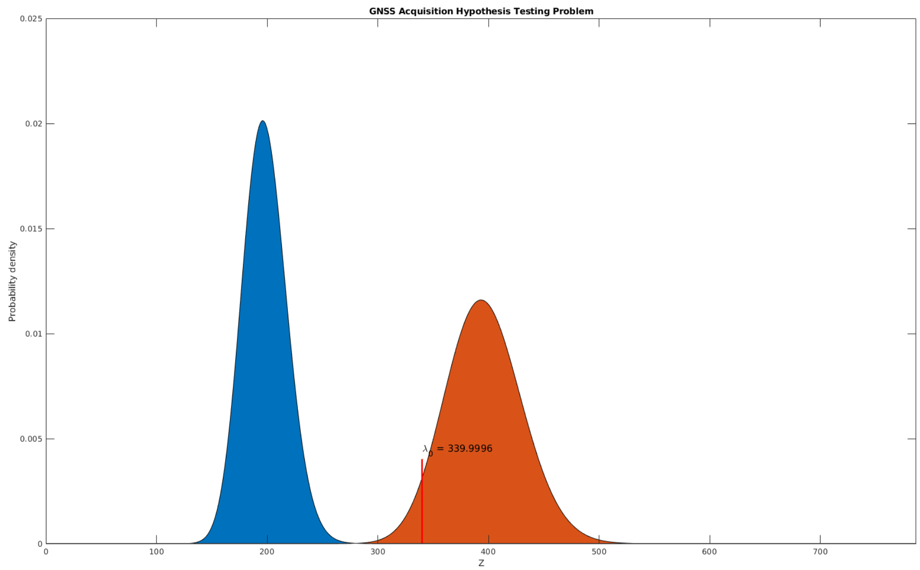

Figure 2: Hypothesis testing for the case where a 30dBHz carrier to noise ratio signal. The detection threshold was set to have a probability of false alarm 0.0001. Then, using $Ta = 1ms$, N was adjusted to get a probability of detection greater than 95%. Thus, $N = 99$.

1.2.2 Part B

In part B, Ta was adjusted to reach a probability of detection in the neighborhood of 95% for signals with different carrier-to-noise ratios, this is shown in Figure 3.

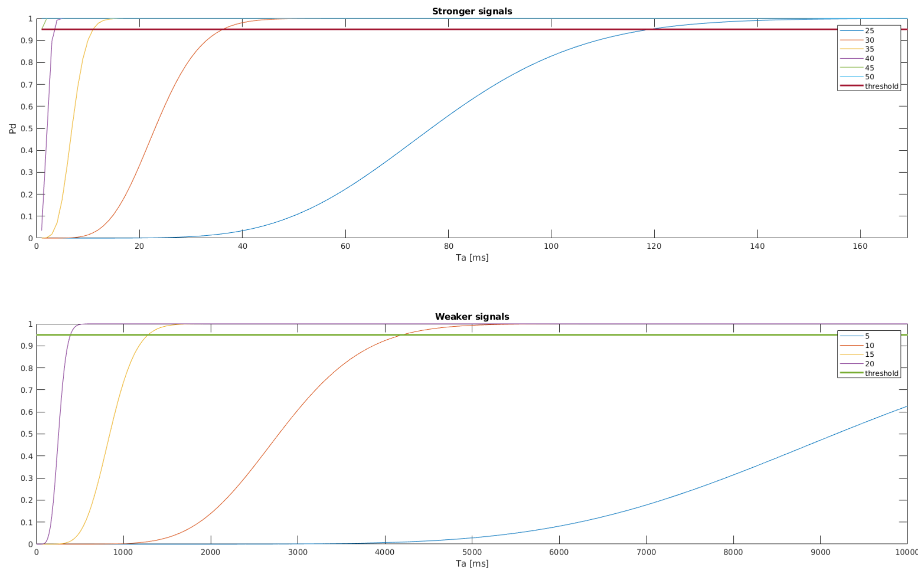

Figure 3: Probability of detection vs. accumulation period for different C/N0 ratios (Coherent accumulation).

In Figure 4 it is possible to observe a particular case of the hypothesis testing for GNSS acquisition. In particular, it is taylored to detect a 30dBHz C/N0 signal with 95% probability and 0.01% chance of false alarm by performing coherent integration.

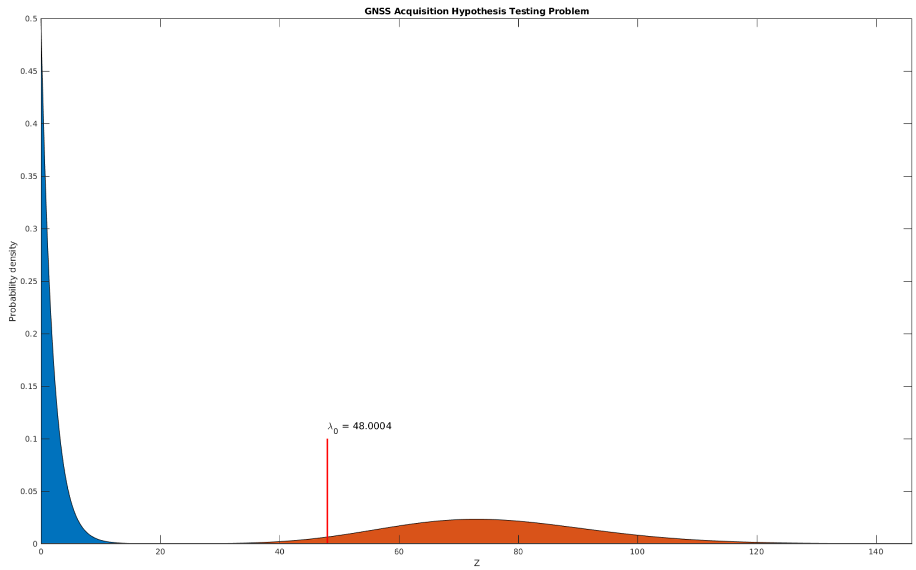

Figure 4: Hypothesis testing for the case where a 30dBHz carrier to noise ratio signal. The detection threshold was set to have a probability of false alarm 0.0001. Then, using $N = 1$, Ta was adjusted to get a probability of detection greater than 95%. Thus, $Ta = 37ms$.

Analizing Figure 2 and 4 its is possible to see that coherent integration leads to a lower overall data usage. since it needs 37ms of the signal while the non-coherent integration need to process 99 intervals of 1ms each.