MACHINE LEARNING

EXERCISES

Elements of non-parametric techniques: the knn classifier

All the course material is available on the web site

Course web site: https://unica-ml.github.io/#

Exercise 1

Given the following patterns belonging to three different classes A, B, and C

A	1.1 1.3	1.7 1.4	1.2 2.0	1.6 1.9
B	2.7 1.4	2.6 1.2	2.2 2.0	2.2 1.3
C	1.4 2.5	1.2 2.4	1.8 2.6	1.5 2.9

We want to classify the unknown pattern:

x_t=(2; 2)'

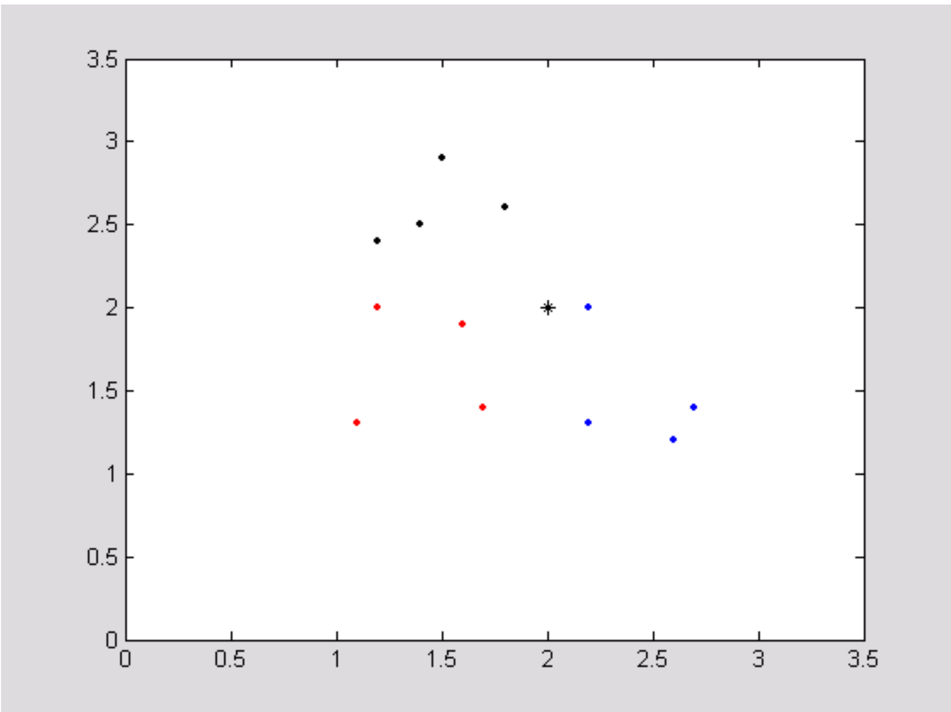
but we do not know from which probability distribution the pattern has been generated. Then, we can use a non-parametric method like the *k*-nn pattern classifier.

A) Classify the pattern x_t with values of *k*= 1, …, 4 using the *Euclidean* and the *Manhattan* distance. *Manhattan* distance: |x1 – x2| + |y1 – y2|.

B) Use the "*leave-one-out"* method to select the best value of the "k" parameter between k=1 and k=4, using the *Euclidean* distance. The "*leave-one-out"* method works as follows:

1) Given the training set D with *n* patterns (12 patterns in this exercise)
2) for i=1, …, *n,* use the training set {D -{x_i}} and then classify the pattern x_i left out.
3) Repeat the point (2)
4) Compute the error probability (number of errors for the classifications of the *n* patterns left out)

You should use the above "*leave-one-out"* method for k=1 and k=4 and then select the value of the k parameter that provides the minimum error.



Class A: red points; Class B: blue points; Class C: black points;

A) Classify the pattern x_t with values of k= 1, …, 4 using the *Euclidean* and the *Manhattan* distance.

Squared *Euclidean* distances

	a1	a2	a3	a4
x_t	1.3000	0.4500	0.6400	0.1700

	b1	b2	b3	b4
x_t	0.8500	1.0000	0.0400	0.5300

	c1	c2	c3	c4
x_t	0.6100	0.8000	0.4000	1.0600

Classification result

k	A	B	C	Classification
1		1		B
2	1	1		A-B*
3	1	1	1	A-B-C*
4	2	1	1	A

* -> you can do a random choice among the classes

Manhattan distances

	a1	a2	a3	a4
x_t	1.60	0.90	0.80	0.50

	b1	b2	b3	b4
x_t	1.30	1.40	0.20	0.90

	c1	c2	c3	c4
x_t	1.10	1.20	0.80	1.40

k	A	B	C	Classification
1		1		B
2	1	1		A-B *
3	2	1		A *
4	2	1	1	A

* -> you can do a random choice among the classes

B) Use the "*leave-one-out*" method to select the best value of the "k" parameter between k=1 and k=4, using the *Euclidean* distance.

In order to use the "*leave-one-out*" method, you should compute the Euclidean distances between each pattern and all the other ones, so that, for each value of k, k=1 or k=4, you can compute the classification of the pattern x_i left out ($\{D - \{x_i\}\}$).

Here the Euclidean distances between each pattern and all the other ones:

	b1	**b2**	**b3**	**b4**
a1	2.57	2.26	1.70	1.21
a2	1.0	0.85	0.61	0.26
a3	2.61	2.60	1.0	1.49
a4	1.46	1.49	0.37	0.72

	c1	**c2**	**c3**	**c4**
a1	1.53	1.22	2.18	2.72
a2	1.30	1.25	1.45	2.29
a3	0.29	0.16	0.72	0.90
a4	0.40	0.41	0.53	1.01

	c1	**c2**	**c3**	**c4**
b1	2.90	3.25	2.25	3.69
b2	3.13	3.40	2.60	4.10
b3	0.89	1.16	0.52	1.30
b4	2.08	2.21	1.85	3.05

	a1	**a2**	**a3**	**a4**
a1	0	0.37	0.50	0.61
a2	0.37	0	0.61	0.26
a3	0.50	0.61	0	0.17
a4	0.61	0.26	0.17	0

	b1	**b2**	**b3**	**b4**
b1	0	0.05	0.61	0.26
b2	0.05	0	0.80	0.17
b3	0.61	0.80	0	0.49
b4	0.26	0.17	0.49	0

	c1	**c2**	**c3**	**c4**
c1	0	0.05	0.17	0.17
c2	0.05	0	0.40	0.34
c3	0.17	0.40	0	0.18
c4	0.17	0.34	0.18	0

In the following tables, for each value of k (k=1 or k=4), we compute the classification of the pattern x_i left out ($\{D - \{x_i\}\}$).

- k=1

pattern	a1	a2	a3	a4	b1	b2	b3	b4	c1	c2	c3	c4
One nearest neighbor	**A**	**A**	**C**	**A**	**B**	**B**	**A**	**B**	**C**	**C**	**C**	**C**

Error = 2/12

- k=4

pattern	a1	a2	a3	a4	b1	b2	b3	b4	c1	c2	c3	c4
Classes of the 4 nearest neighbors	**AA AB**	**AA AB**	**AA CC**	**AA BC**	**AB BB**	**AB BB**	**AB BC**	**AB BB**	**AC CC**	**AC CC**	**BC CC**	**AC CC**
Classification	**A**	**A**	**A-C***	**A**	**B**	**B**	**B**	**B**	**C**	**C**	**C**	**C**

Error = between 0/12 and 1/12 depending on the random choice for "tie breaks" highlighted in yellow.

The best value of the parameter "k" is therefore *k*=4 (minimum error).

Exercise 2

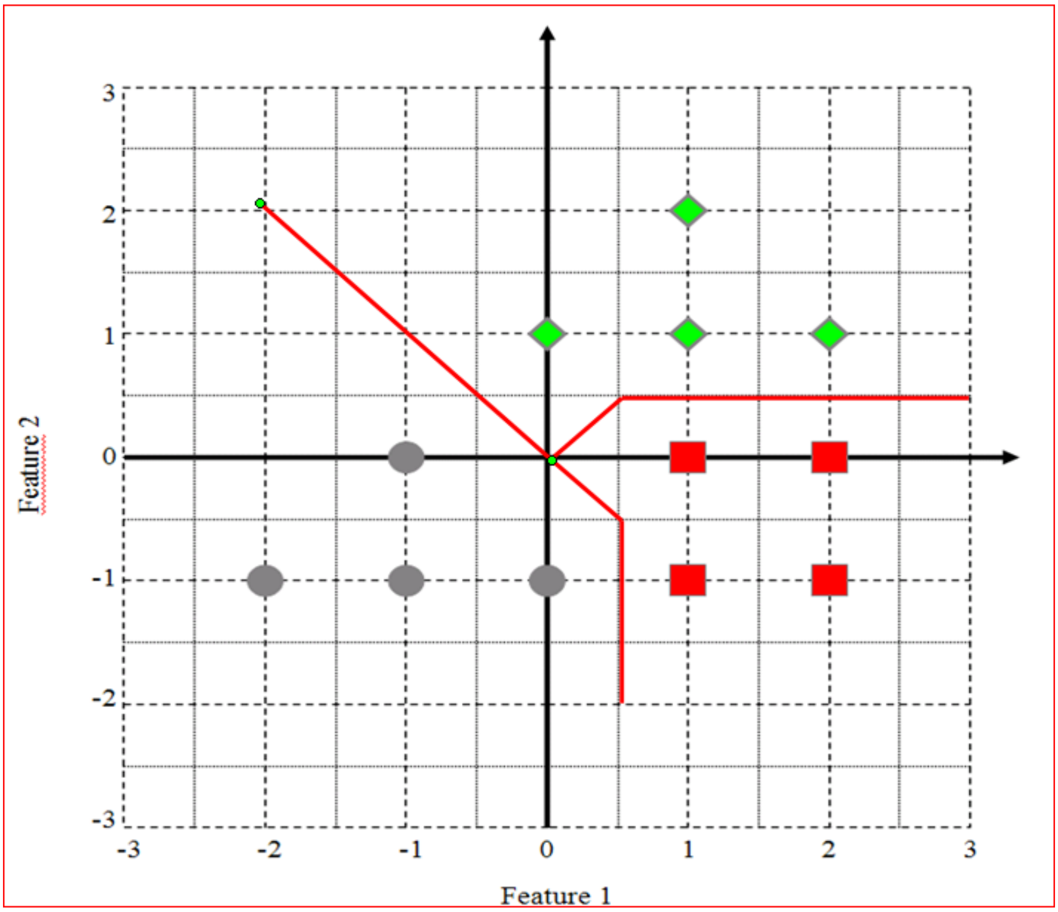
Given the patterns:

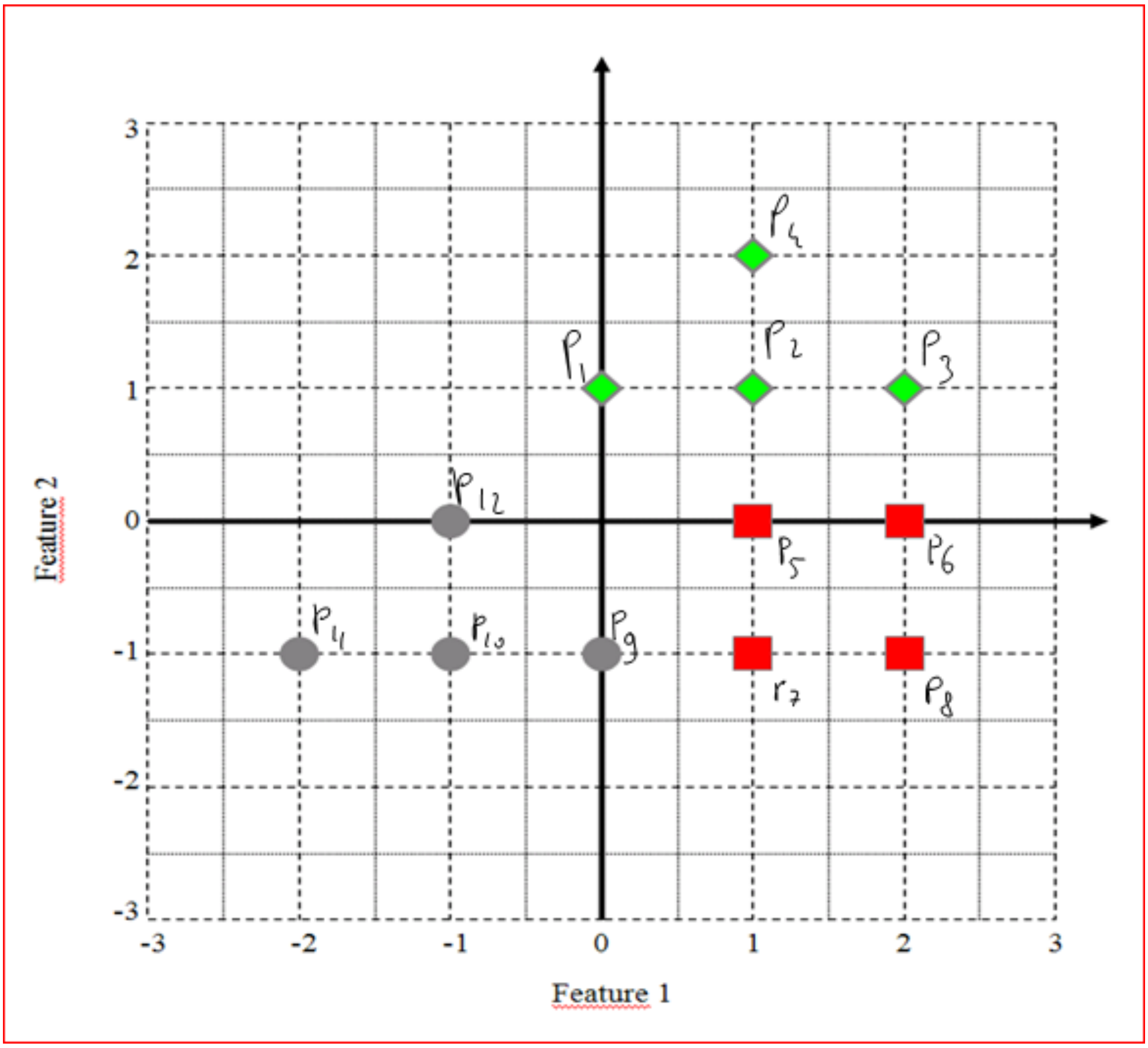
Class	ω_1				ω_2				ω_3			
	p1	p2	p3	p4	p5	p6	p7	p8	p9	p10	p11	p12
	0.0 1.0	1 1	2 1	1 2	1 0	2 0	1 -1	2 -1	0 -1	-1 -1	-2 -1	-1 0

1) Classify the unknown pattern $X_t = (0.5, 0.4)^t$ using the K-nn classifier and the Euclidean distance for k=1 and k=3.
2) Estimate the optimal value of k, between k=1 and k=3, using the Manhattan metric. Explain the used method.

[In the case the unknown pattern is equidistant from different training patterns, suppose that the used algorithm choose first those patterns with a lower index.]

Below, the Voronoi Diagram





Solution:
Question (1):

Evaluate the euclidean distance between the unknown pattern and the training patterns

Class	ω_1				ω_2				ω_3			
	p1	p2	p3	p4	p5	p6	p7	p8	p9	p10	p11	p12
X	**0.61**	**0.61**	2.61	2.81	**0.41**	2.41	2.21	4.21	2.21	4.21	8.21	2.41
	W1	W1			W2							

- K=1 x $\rightarrow \omega_2$
- K=3 x $\rightarrow \omega_1$

Question (2):

The optimal value of K can be estimated using the leave-one-out approach. According to this method it is possible classify each training pattern using all the remaining ones (a sort of N-fold Cross Validation).

Evaluate the distances between all the training patterns using the given metric:

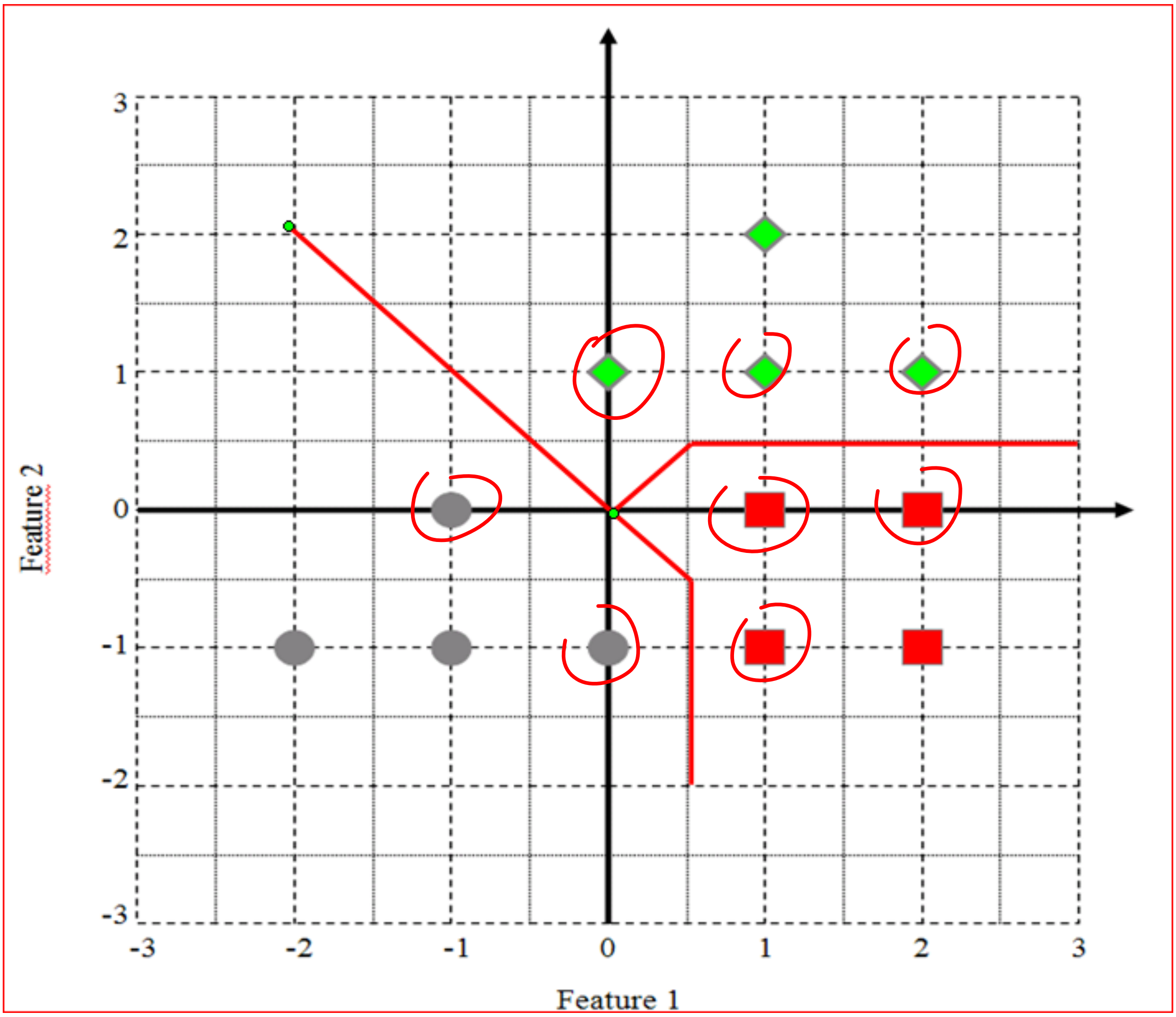
$$d(p_a, p_b) = |a_1 - b_1| + |a_2 - b_2|$$

	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12
P1	0	**1**	**2**	**2**	**2**	3	3	4	**2**	3	4	**2**
P2		0	**1**	**1**	**1**	2	2	3	3	4	5	3
P3			0	**2**	**2**	**1**	3	**2**	4	5	6	4
P4				0	**2**	3	3	4	4	5	6	4
P5					0	**1**	**1**	2	2	3	4	2
P6						0	2	**1**	3	4	5	3
P7							0	**1**	**1**	2	3	3
P8								0	**2**	3	4	5
P9									0	**1**	**2**	**2**
P10										0	**1**	**1**
P11											0	2
P12												0

Supposing that, in the case of equal distances or equal number of votes, the used algorithm chooses first the training patterns with a lower index, the result is:

True Class	Pattern	K=1 Nearest pattern	K=1 Assigned Class	K=3 Nearest pattern	K=3 Assigned Class
ω_1	P1	P2(ω_1)	ω_1	P2(ω_1), P3(ω_1), P4(ω_1)	ω_1
ω_1	P2	P1(ω_1)	ω_1	P1(ω_1), P4(ω_1), P5(ω_2)	ω_1
ω_1	P3	P2(ω_1)	ω_1	P2(ω_1) P6(ω_2) P1(ω_1)	ω_1
ω_1	P4	P2(ω_1)	ω_1	P2(ω_1) P1(ω_1), P3(ω_1)	ω_1
ω_2	P5	P2(ω_1)	$\boldsymbol{\omega_1}$	P2(ω_1) P6(ω_2) P7(ω_2)	ω_2
ω_2	P6	P3(ω_1)	$\boldsymbol{\omega_1}$	P3(ω_1) P5(ω_2) P8(ω_2),	$\boldsymbol{\omega_2}$
ω_2	P7	P5(ω_2)	ω_2	P5(ω_2) P8(ω_2) P9(ω_3)	ω_2
ω_2	P8	P6(ω_2)	ω_2	P6(ω_2) P7(ω_2) P3(ω_1),	ω_2
ω_3	P9	P7(ω_2)	$\boldsymbol{\omega_2}$	P7(ω_2) P10(ω_3) P1(ω_1)	$\boldsymbol{\omega_1}$
ω_3	P10	P9(ω_3)	ω_3	P9(ω_3) P11(ω_3) P12(ω_3)	ω_3
ω_3	P11	P10(ω_3)	ω_3	P10(ω_3) P9(ω_3) P12(ω_3)	ω_3
ω_3	P12	P10(ω_3)	ω_3	P10(ω_3) P1(ω_1), P5(ω_2)	$\boldsymbol{\omega_1}$
	ERR		3/12		2/12

The ‘leave-one-out’ method suggests to choose k=3



3
2
1
0
-1
-2
-3
Feature 2
-3
-2
-1
0
1
2
3
Feature 1

TABLES EXERCISE 3

X_{A1}=(0.4 0.5 12 10)^T ; X_{A2}=(5 11 10 10)^T

X_{B1}=(1 1 0 6)^T ; X_{B2}=(5 2 0.6 0.6)^T

X_t= (0.4, 0.5, 0.4, 0.5)^T

Distances between X_t and all the other patterns.

	d(xa1,xt)^2	d(xa2,xt)^2	d(xb1,xt)^2	d(xb2,xt)^2
Feature = 1	0	21.16	0.36	21.16
Feature = 1,2	0	131.41	0.61	23.41
Feature = 1,2,3	134.56	223.57	0.77	23.45
Feature = 1,2,3,4	224.81	313.82	31.02	23.46