$x \in [0,1]$

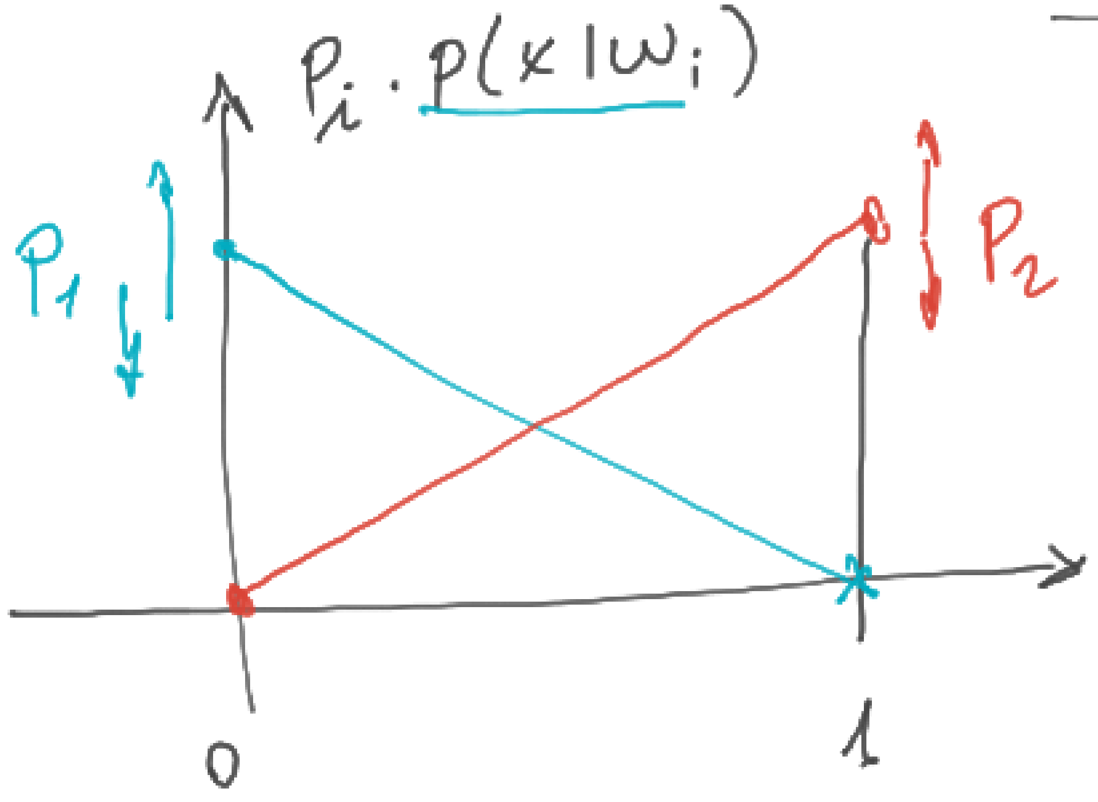
$p(x|\omega_1) = 2 - 2X$

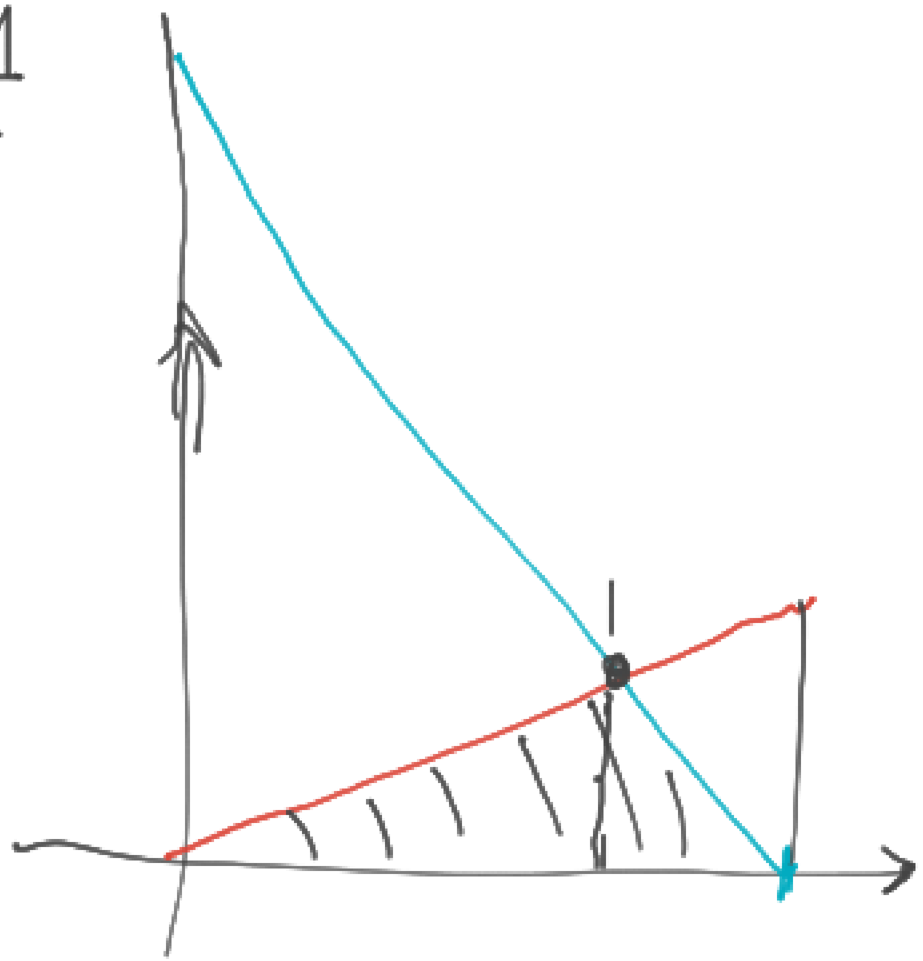
$p(x|\omega_2) = 2X$

$P(\omega_1) = ?$

$P(\omega_2) =$

$P_1 + P_2 = 1$



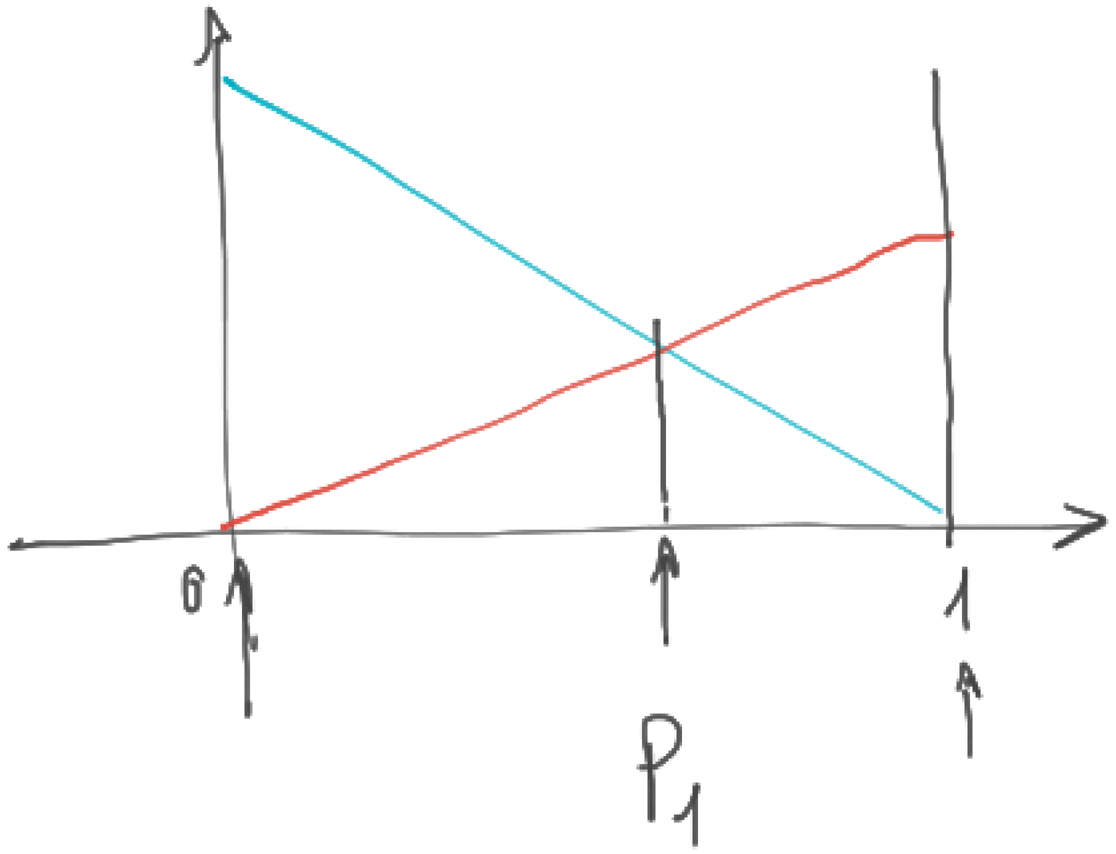


$$P_1 \cdot \overbrace{p(x|\omega_1)}^{2-2x} = P_2 \, \overbrace{p(x|\omega_2)}^{2x}$$

$$P_2 = 1 - P_1$$

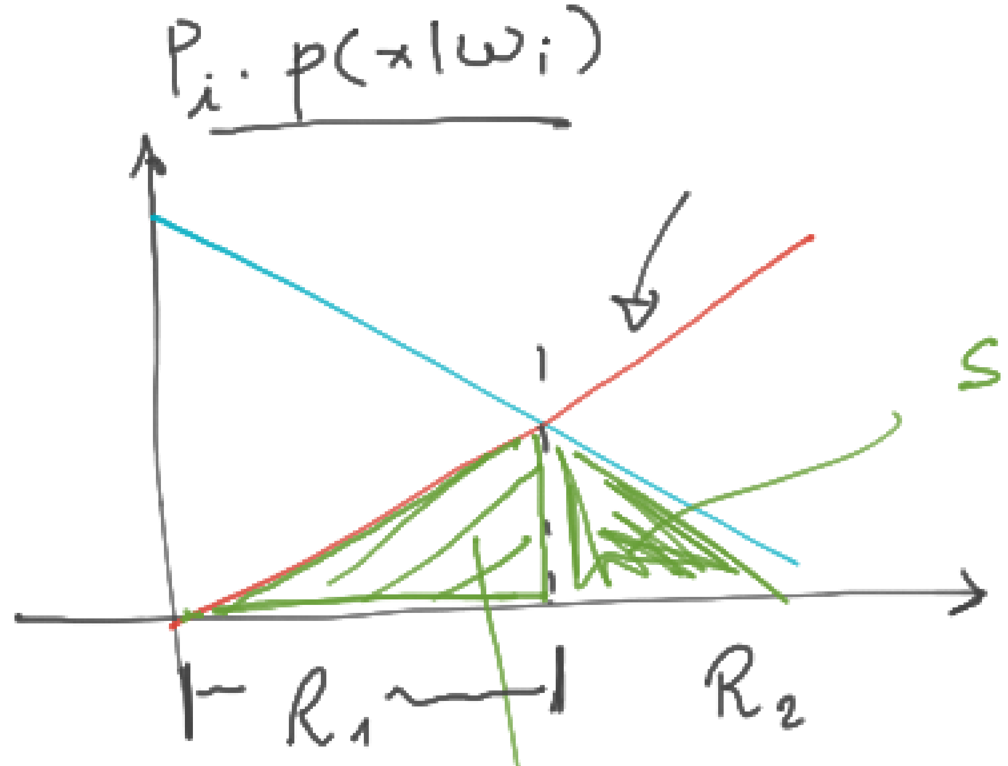
$$P_1 \cdot (2 - 2x) = (1 - P_1)(2x)$$

$$2P_1 - \cancel{2xP_1} = 2x - \cancel{2xP_1} \rightarrow \boxed{P_1 = x^*} \rightarrow \boxed{\begin{array}{ll} R_1 & x \in [0, P_1] \\ R_2 & x \in [P_1, 1] \end{array}}$$



$P_1 = 0 \rightarrow x^* = 0$

$P_1 = 1 \rightarrow x^* = 1$



samples from class ω_1 assigned to R_2

samples from class ω_2 assigned to R_1

$$P(\text{Error}) = \int_{R_1} P_2 \cdot 2x \, dx + \int_{R_2} P_1 \cdot (2-2x) \, dx =$$

$P_2 = (1-P_1)$

$$= (1-P_1) \cdot \int_0^{P_1} 2x \, dx + P_1 \int_{P_1}^{1} (2-2x) \, dx$$

$P_2 = (1-P_1)$

$$= (1-P_1)\cdot\int_0^{P_1} 2x\,dx + P_1 \int_{P_1}^{1}(2-2x)\,dx =$$

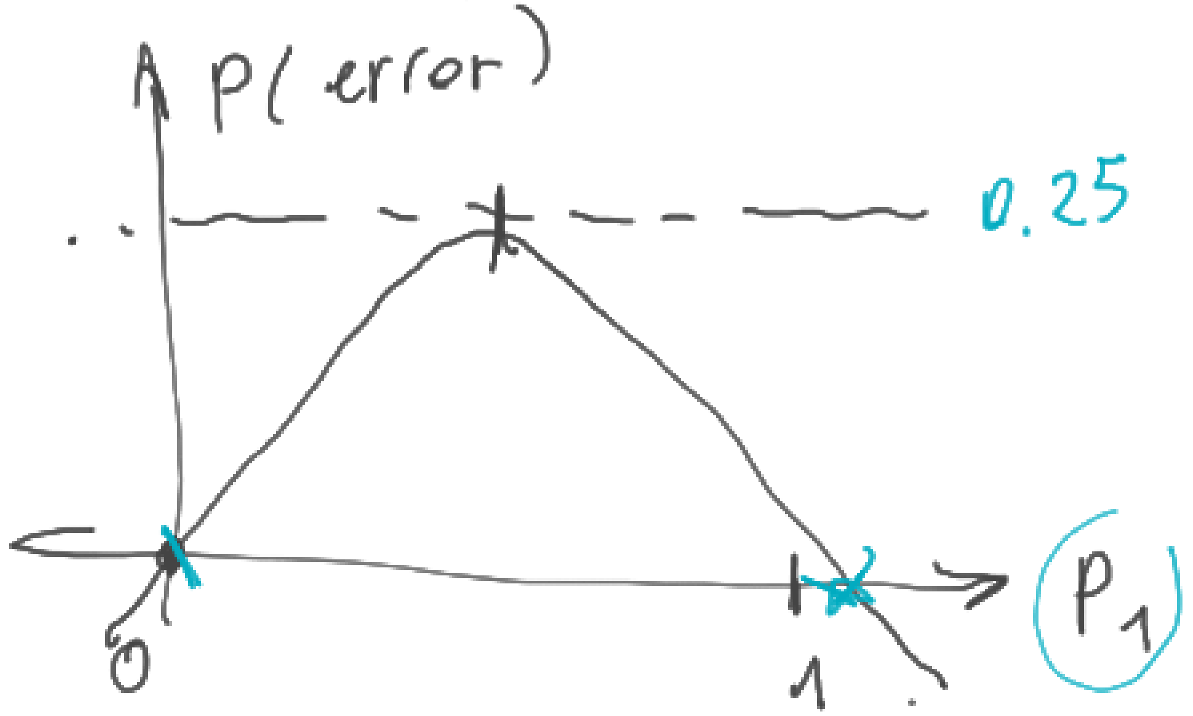
$$= (1-P_1)\left[x^2\right]_0^{P_1} + P_1\left[2x - x^2\right]_{P_1}^{1} =$$

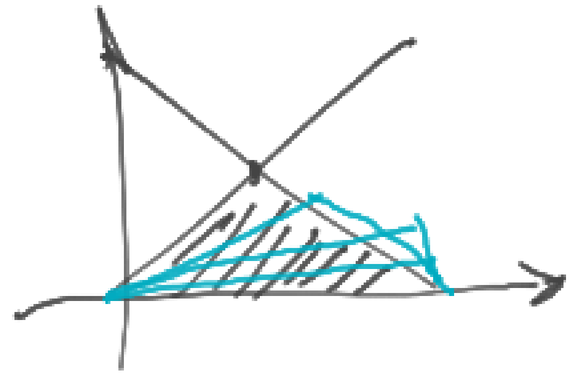
$P_1 = 0.5$

$P_1 = 0$ $\qquad$ $P_1 = 1$

$$= (1-P_1)(+P_1^2) + P_1(\overbrace{2-1}^{1} - 2P_1 + P_1^2) =$$

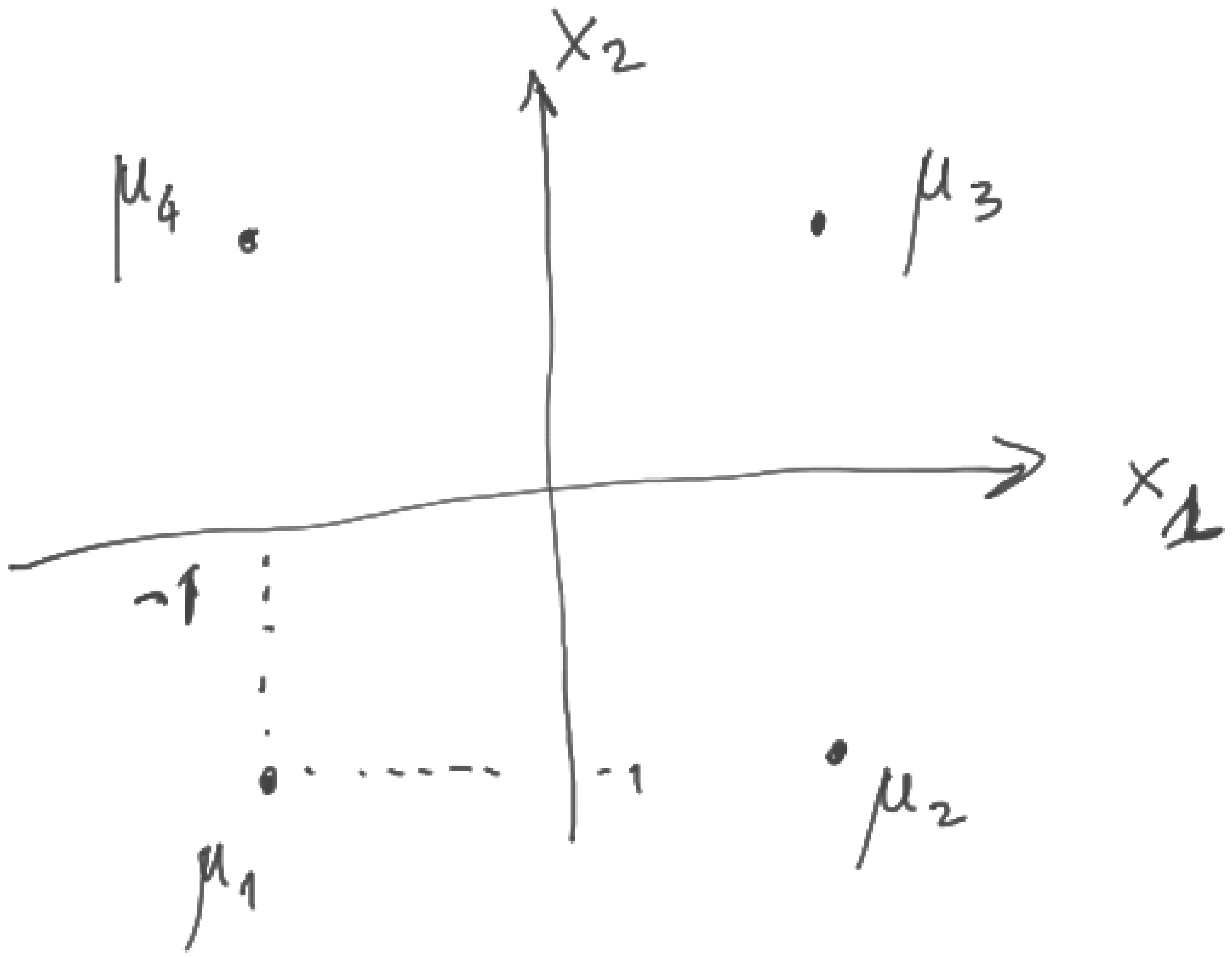
$$= P_1^2 - \cancel{P_1^3} + P_1 - 2P_1^2 + \cancel{P_1^3} = -P_1^2 + P_1 = \underbrace{P_1(1-P_1)}$$

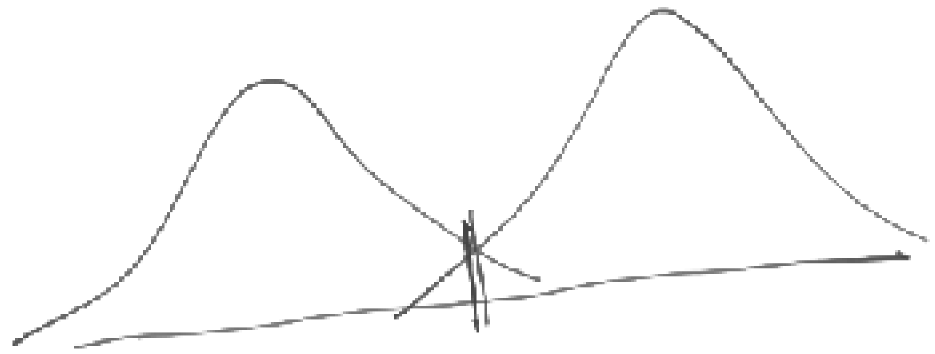


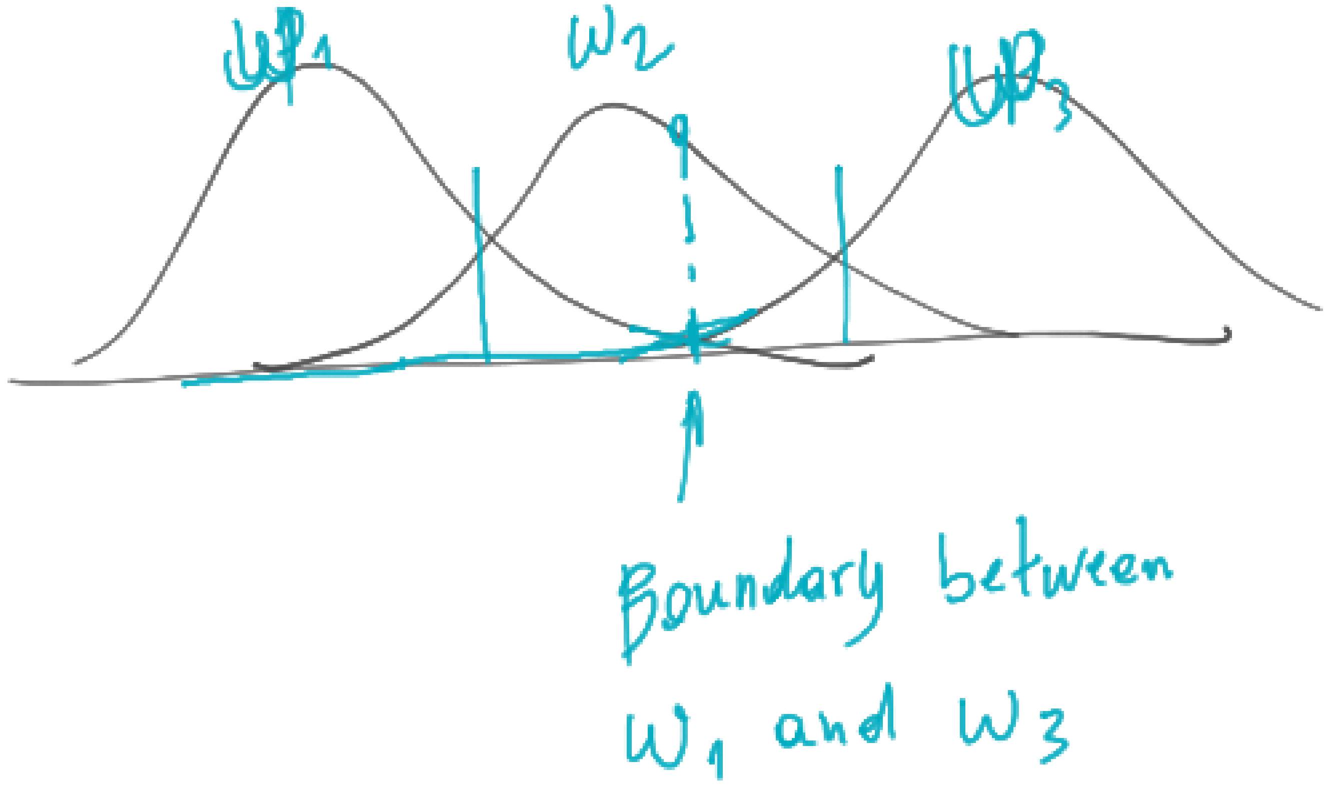


$$\Sigma_i = \sigma^2 I \qquad \mu_1 = \begin{pmatrix} -1 \\ -1 \end{pmatrix} \quad \mu_2 = \begin{pmatrix} +1 \\ -1 \end{pmatrix} \quad \mu_3 = \begin{pmatrix} +1 \\ +1 \end{pmatrix} \quad \mu_4 = \begin{pmatrix} -1 \\ +1 \end{pmatrix}$$

$$P_1 = P_2 = P_3 = P_4 = 1$$







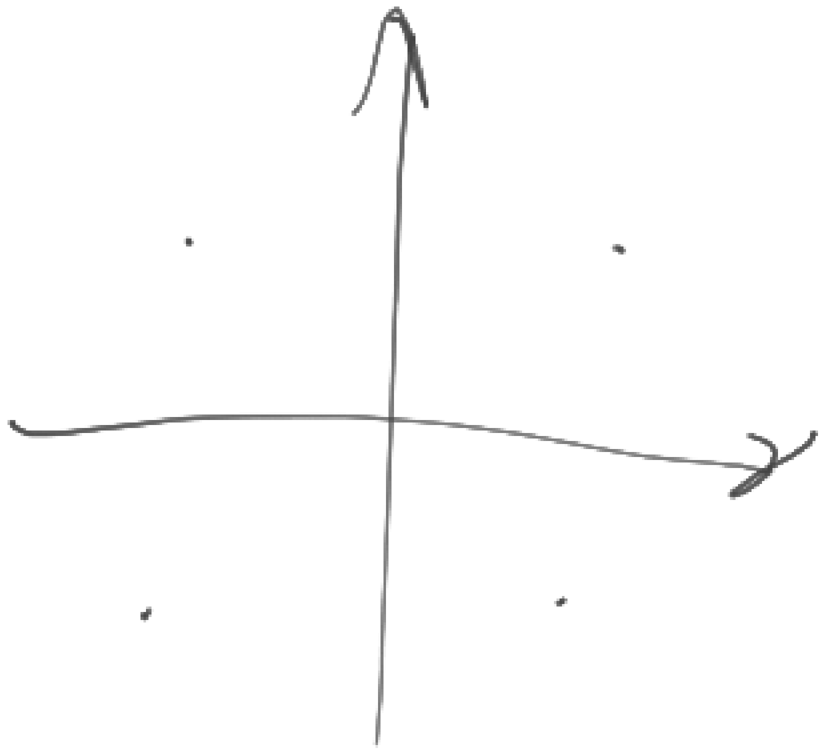
$$\Sigma_i = \sigma^2 I \qquad \mu_1 = \begin{pmatrix}-1\\-1\end{pmatrix} \quad \mu_2 = \begin{pmatrix}+1\\-1\end{pmatrix} \quad \mu_3 = \begin{pmatrix}+1\\+1\end{pmatrix} \quad \mu_4 = \begin{pmatrix}-1\\+1\end{pmatrix}$$

$$P_1 = P_2 = P_3 = P_4 = 1$$

$$g(x) = \boxed{-\frac{1}{2} x^T \Sigma^{-1} x} + \mu^T \cancel{\Sigma}^{-1} x - \frac{1}{2} \mu^T \cancel{\Sigma}^{-1} \mu + \boxed{\ln p(\omega)} \boxed{-\frac{1}{2} \ln |\Sigma|}$$

$$\mu^T \mu \longrightarrow \begin{pmatrix}-1\\-1\end{pmatrix}\begin{pmatrix}-1 & -1\end{pmatrix} = 1 + 1$$

$$\begin{pmatrix}-1\\+1\end{pmatrix}\begin{pmatrix}-1 & +1\end{pmatrix} = 1 + 1$$



$$g(x) = \mu^T x \qquad x = \begin{pmatrix}x_1\\x_2\end{pmatrix}$$

$$\Sigma_i = \sigma^2 I \qquad \mu_1 = \begin{pmatrix}-1\\-1\end{pmatrix} \quad \mu_2 = \begin{pmatrix}+1\\-1\end{pmatrix} \quad \mu_3 = \begin{pmatrix}+1\\+1\end{pmatrix} \quad \mu_4 = \begin{pmatrix}-1\\+1\end{pmatrix}$$

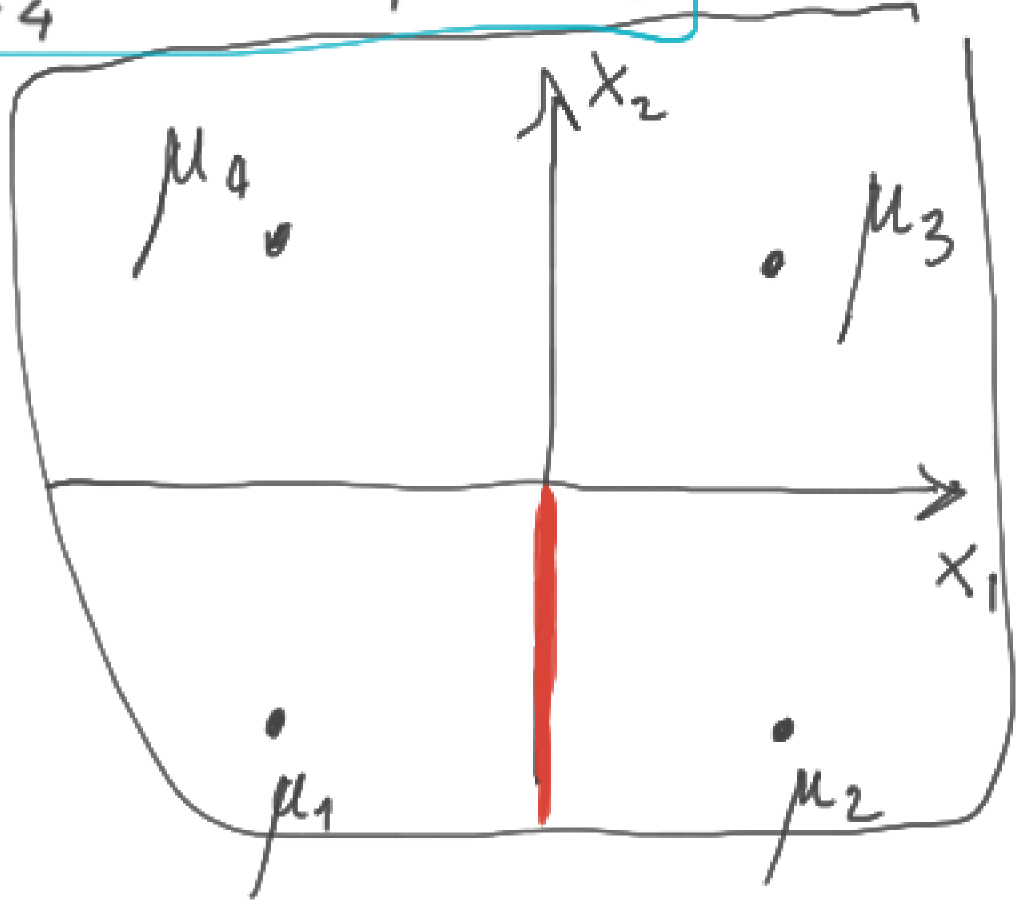
$$P_1 = P_2 = P_3 = P_4 = 1$$

$$\boxed{g_1(x) = \begin{pmatrix}-1 & -1\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix} = -x_1 - x_2}$$

$$\boxed{g_2(x) = x_1 - x_2}$$

$$\boxed{g_3(x) = x_1 + x_2}$$

$$\boxed{g_4(x) = -x_1 + x_2}$$



$$g_1 = g_2 \to \begin{cases} g_1 > g_3 \\ g_1 > g_4 \end{cases}$$

$$g_1 = g_2 \to -x_1 - \cancel{x_2} = x_1 - \cancel{x_2} \qquad \underline{x_1 = 0}$$

$$g_1 > g_3 \to -\cancel{x_1} - x_2 > \cancel{x_1} + x_2 \to x_2 < 0$$

$$g_1 > g_4 \to -\cancel{x_1} - x_2 > -\cancel{x_1} + x_2 \to x_2 < 0$$

AND F

$$\Sigma_i = \sigma^2 I \qquad \mu_1 = \begin{pmatrix} -1 \\ -1 \end{pmatrix} \quad \mu_2 = \begin{pmatrix} +1 \\ -1 \end{pmatrix} \quad \mu_3 = \begin{pmatrix} +1 \\ +1 \end{pmatrix} \quad \mu_4 = \begin{pmatrix} -1 \\ +1 \end{pmatrix}$$

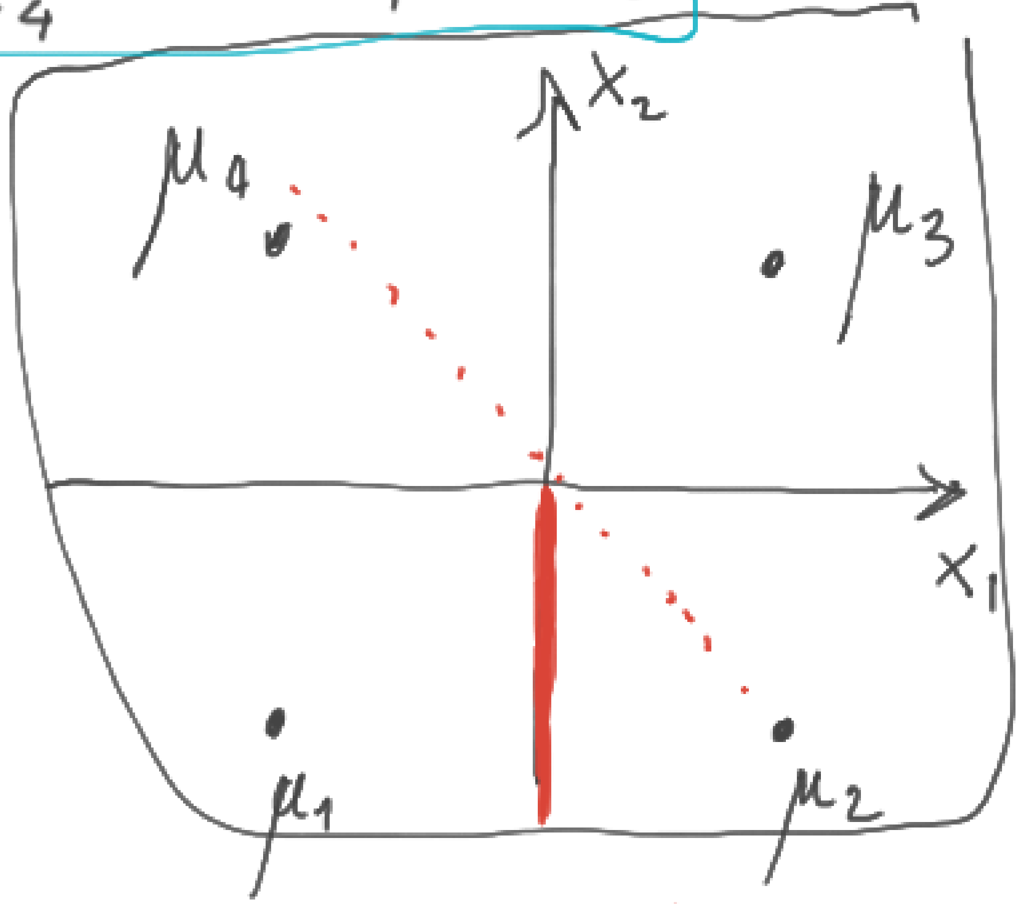
$$P_1 = P_2 = P_3 = P_4 = 1$$

$$\boxed{g_1(x) = \begin{pmatrix} -1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = -x_1 - x_2}$$

$$\boxed{g_2(x) = x_1 - x_2}$$

$$\boxed{g_3(x) = x_1 + x_2}$$

$$\boxed{g_4(x) = -x_1 + x_2}$$



$g_1 = g_3 \rightarrow$ ACTIVE WHEN $g_1 = g_3 > g_2$

$g_1 = g_3 > g_4$

$$-x_1 - x_2 = x_1 + x_2 \rightarrow x_1 + x_2 = 0 \qquad x_1 = -x_2$$

$$-x_1 - x_2 > x_1 - x_2$$

$$\cancel{+x_2} - \cancel{x_2} > -x_2 - x_2 \rightarrow x_2 > 0 \quad \text{AND}$$

$$-x_1 - x_2 > -x_1 + x_2 \rightarrow \cancel{x_1} - \cancel{x_2} > x_2 + x_2 \rightarrow x_2 < 0$$

..... = INACTIVE

$$\Sigma_i = \sigma^2 I \qquad \mu_1 = \begin{pmatrix}-1\\-1\end{pmatrix} \quad \mu_2 = \begin{pmatrix}+1\\-1\end{pmatrix} \quad \mu_3 = \begin{pmatrix}+1\\+1\end{pmatrix} \quad \mu_4 = \begin{pmatrix}-1\\+1\end{pmatrix}$$

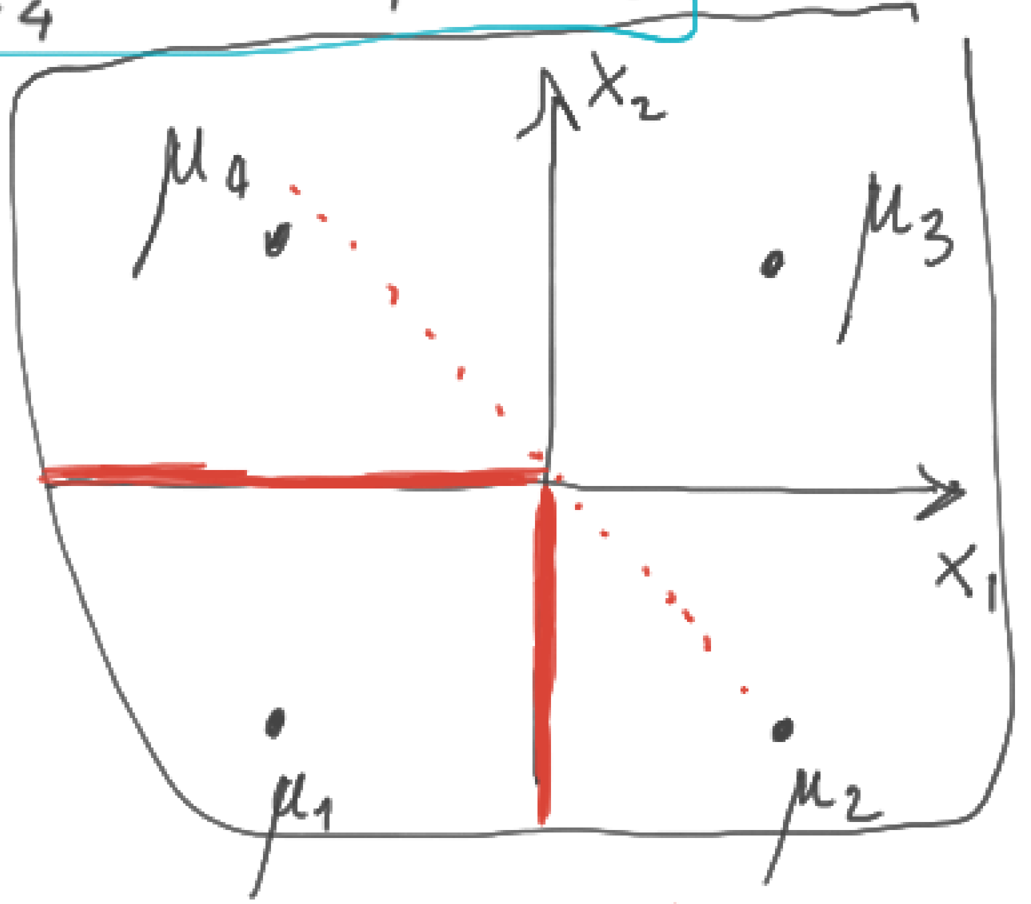
$$P_1 = P_2 = P_3 = P_4 = 1$$

$$\boxed{g_1(x) = \begin{pmatrix}-1 & -1\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix} = -x_1 - x_2}$$

$$\boxed{g_2(x) = x_1 - x_2}$$

$$\boxed{g_3(x) = x_1 + x_2}$$

$$\boxed{g_4(x) = -x_1 + x_2}$$



$$g_1 = g_4 \rightarrow -\cancel{x_1} - x_2 = -\cancel{x_1} + x_2 \qquad x_2 = 0$$

$g_1 > g_2$ AND $g_1 > g_3$

$$-x_1 - \cancel{x_2} > x_1 - \cancel{x_2} \rightarrow x_1 < 0$$

AND

$$g_1 > g_3 \qquad -x_1 - \cancel{x_2} > x_1 + \cancel{x_2} \qquad x_1 < 0$$

...... = INACTIVE

$$\Sigma_i = \sigma^2 I \qquad \mu_1 = \begin{pmatrix} -1 \\ -1 \end{pmatrix} \quad \mu_2 = \begin{pmatrix} +1 \\ -1 \end{pmatrix} \quad \mu_3 = \begin{pmatrix} +1 \\ +1 \end{pmatrix} \quad \mu_4 = \begin{pmatrix} -1 \\ +1 \end{pmatrix}$$

$$P_1 = P_2 = P_3 = P_4 = 1$$

$$\boxed{g_1(x) = \begin{pmatrix} -1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = -x_1 - x_2}$$

$$\boxed{g_2(x) = x_1 - x_2}$$

$$\boxed{g_3(x) = x_1 + x_2}$$

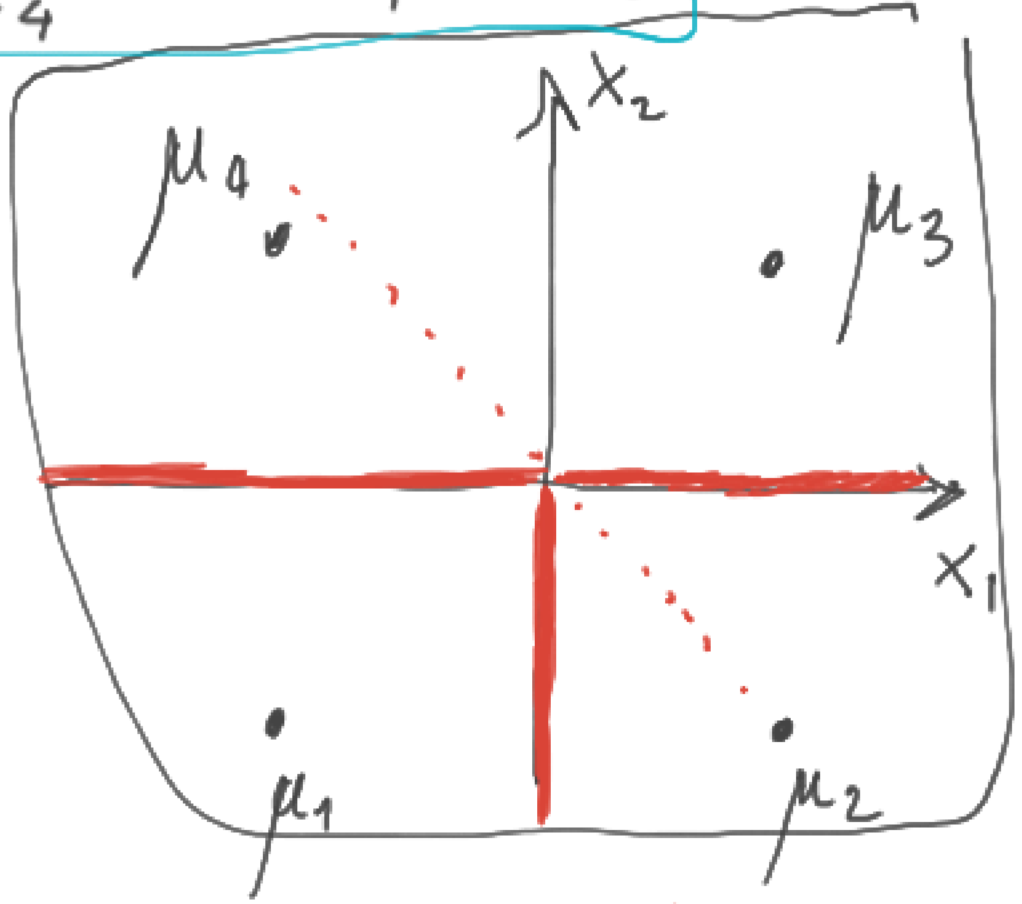
$$\boxed{g_4(x) = -x_1 + x_2}$$

$$g_2 = g_3 \longrightarrow \cancel{x_1} - x_2 = \cancel{x_1} + x_2 \qquad \underline{x_2 = 0}$$

$$g_2 > g_1 \longrightarrow x_1 > -x_1 \longrightarrow x_1 > 0$$

AND

$$g_2 > g_4 \longrightarrow x_1 > -x_1 \longrightarrow x_1 > 0$$



...... = INACTIVE

$$\Sigma_i = \underline{\sigma^2 I} \qquad \underline{\mu_1} = \begin{pmatrix} -1 \\ -1 \end{pmatrix} \quad \mu_2 = \begin{pmatrix} +1 \\ -1 \end{pmatrix} \quad \mu_3 = \begin{pmatrix} +1 \\ +1 \end{pmatrix} \quad \mu_4 = \begin{pmatrix} -1 \\ +1 \end{pmatrix}$$

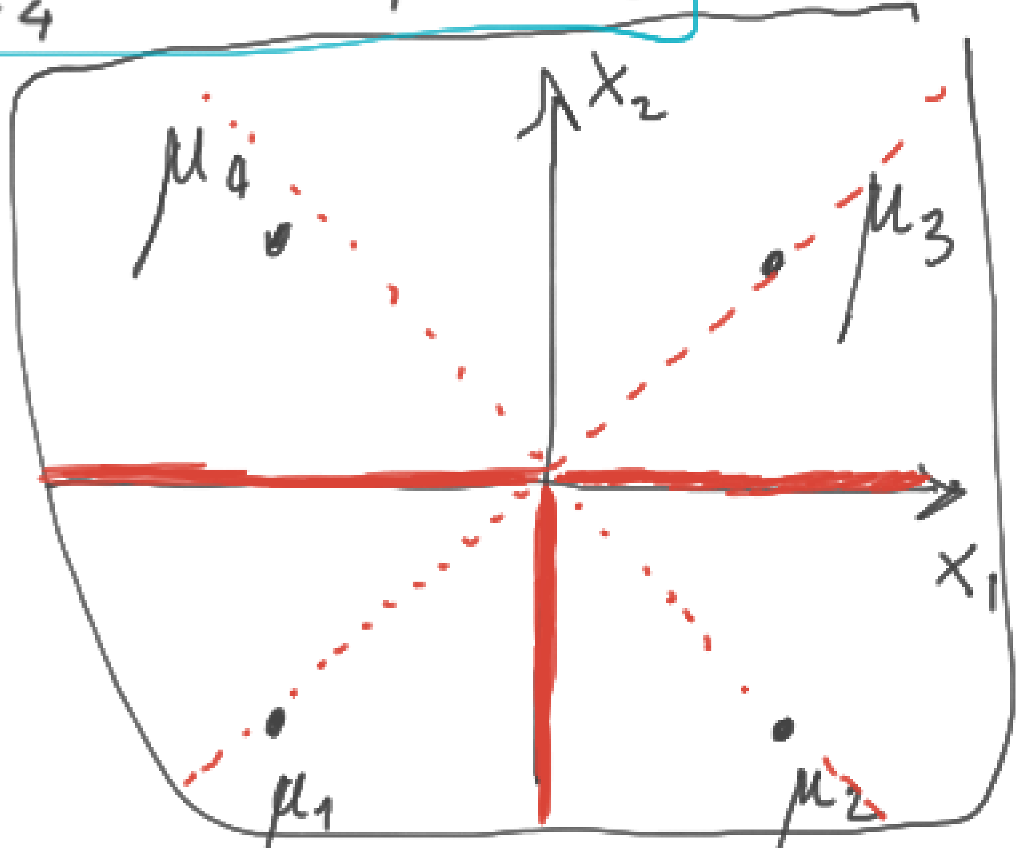
$$P_1 = P_2 = P_3 = P_4 = 1$$

$$\boxed{g_1(x) = \begin{pmatrix} -1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = -x_1 - x_2}$$

$$\boxed{g_2(x) = x_1 - x_2}$$

$$\boxed{g_3(x) = x_1 + x_2}$$

$$\boxed{g_4(x) = -x_1 + x_2}$$



..... = INACTIVE

$g_2 = g_4$ ACTIVE WEN $g_2 > g_1$ AND $g_2 > g_3$

$$x_1 - x_2 = -x_1 + x_2 \rightarrow x_1 - x_2 = 0 \qquad \underline{x_1 = x_2}$$

$$x_1 - x_2 > -x_1 - x_2 \qquad \cancel{x_1} - \cancel{x_1} > -x_1 - x_1 \qquad x_1 > 0$$

$$x_1 - x_2 > x_1 + x_2 \rightarrow x_1 \; x_1 > x_1 + x_1 \qquad x_1 < 0$$

AND X

$$\Sigma_i = \sigma^2 I \qquad \mu_1 = \begin{pmatrix} -1 \\ -1 \end{pmatrix} \quad \mu_2 = \begin{pmatrix} +1 \\ -1 \end{pmatrix} \quad \mu_3 = \begin{pmatrix} +1 \\ +1 \end{pmatrix} \quad \mu_4 = \begin{pmatrix} -1 \\ +1 \end{pmatrix}$$

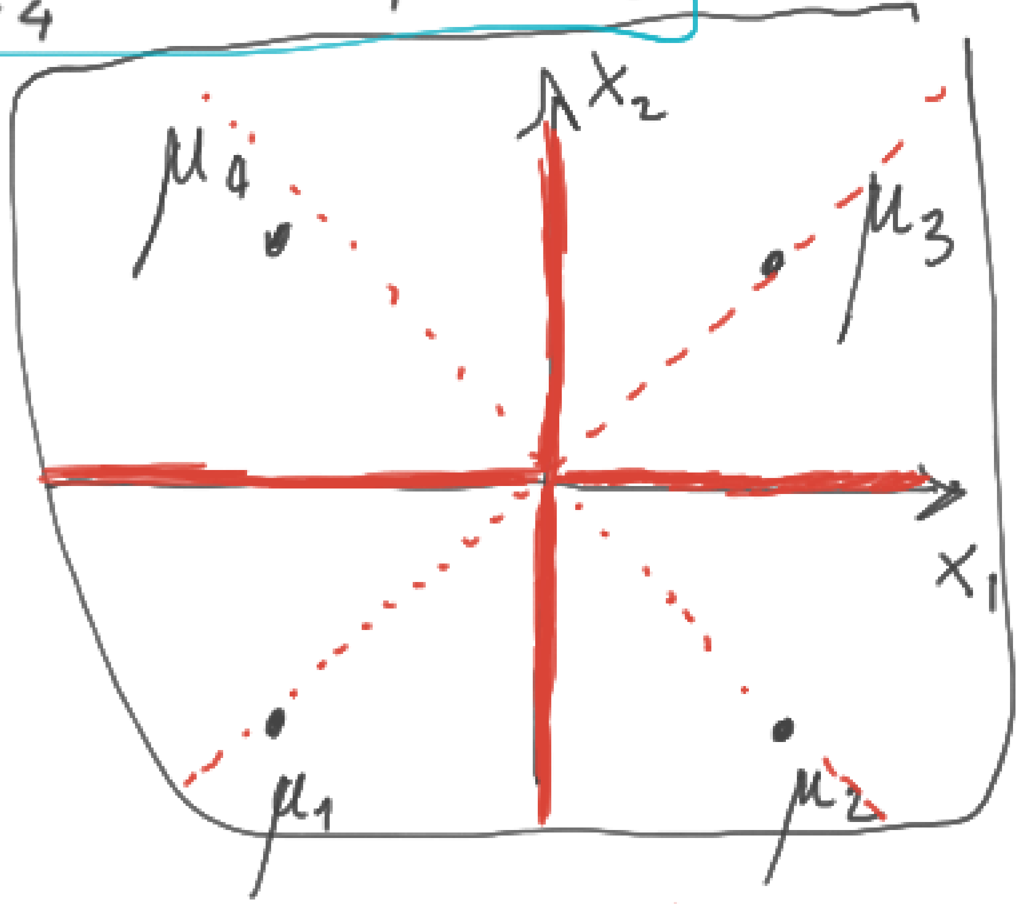
$$P_1 = P_2 = P_3 = P_4 = 1$$

$$\boxed{g_1(x) = \begin{pmatrix} -1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = -x_1 - x_2}$$

$$\boxed{g_2(x) = x_1 - x_2}$$

$$\boxed{g_3(x) = x_1 + x_2}$$

$$\boxed{g_4(x) = -x_1 + x_2}$$



..... = INACTIVE

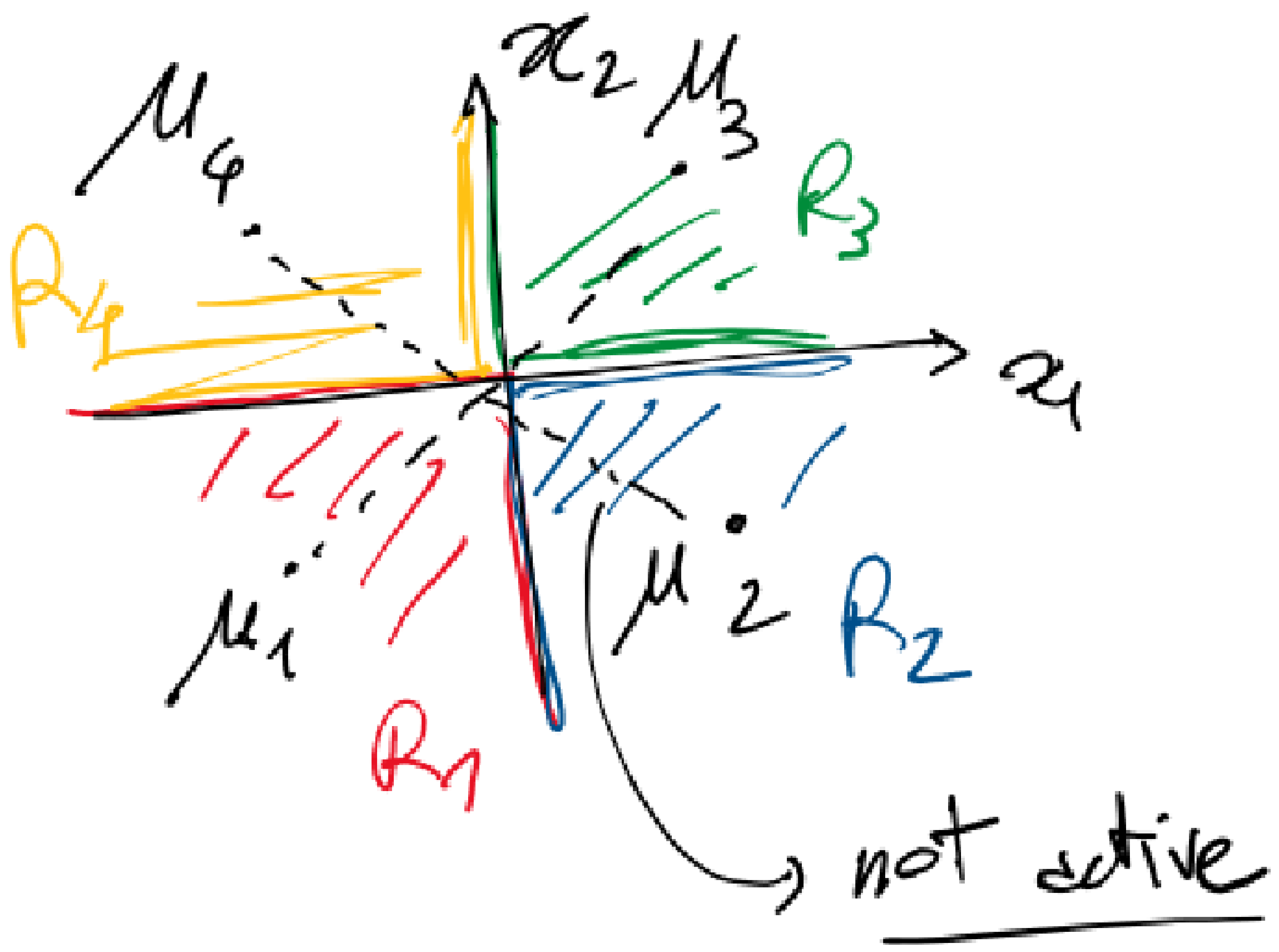
$$\underline{g_3 = g_4} \to g_3 > g_1 \text{ AND } g_3 > g_2$$

$$x_1 + \cancel{x_2} = -x_1 + \cancel{x_2} \to \underline{x_1 = 0}$$

$$\cancel{x_1} + x_2 > -\cancel{x_1} - x_2 \qquad x_2 > 0$$

AND

$$\cancel{x_1} + x_2 > \cancel{x_1} - x_2 \qquad x_2 > 0$$



x_2
x_1
μ_1
μ_2
μ_3
μ_4
R_1
R_2
R_3
R_4
not active

$$\mathbf{x}_{tr} = \begin{bmatrix} -1 & 1 \\ 1 & 1 \\ 1 & -1 \end{bmatrix}, \quad \mathbf{y}_{tr} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{x}_{ts} = \begin{bmatrix} -0.5 & 1.5 \\ 0.1 & 0.5 \\ -1 & -1.5 \\ 1 & -0.5 \end{bmatrix}, \quad \mathbf{y}_{ts} = \begin{bmatrix} 2 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

training data

Testing data

```
[[ 0.50   2.50   8.50]
 [ 1.46   1.06   3.06]
 [ 6.25  10.25   4.25]
 [ 6.25   2.25   0.25]]
```

y_{tr} [2 1 0]

$$d_{ij} = d(x_{ts_i}, x_{tr_j})$$

row column

$\hat{y}_{ts_1} = 2$

$\hat{y}_{ts_2} = 1$

$y_{ts_3} = 0$

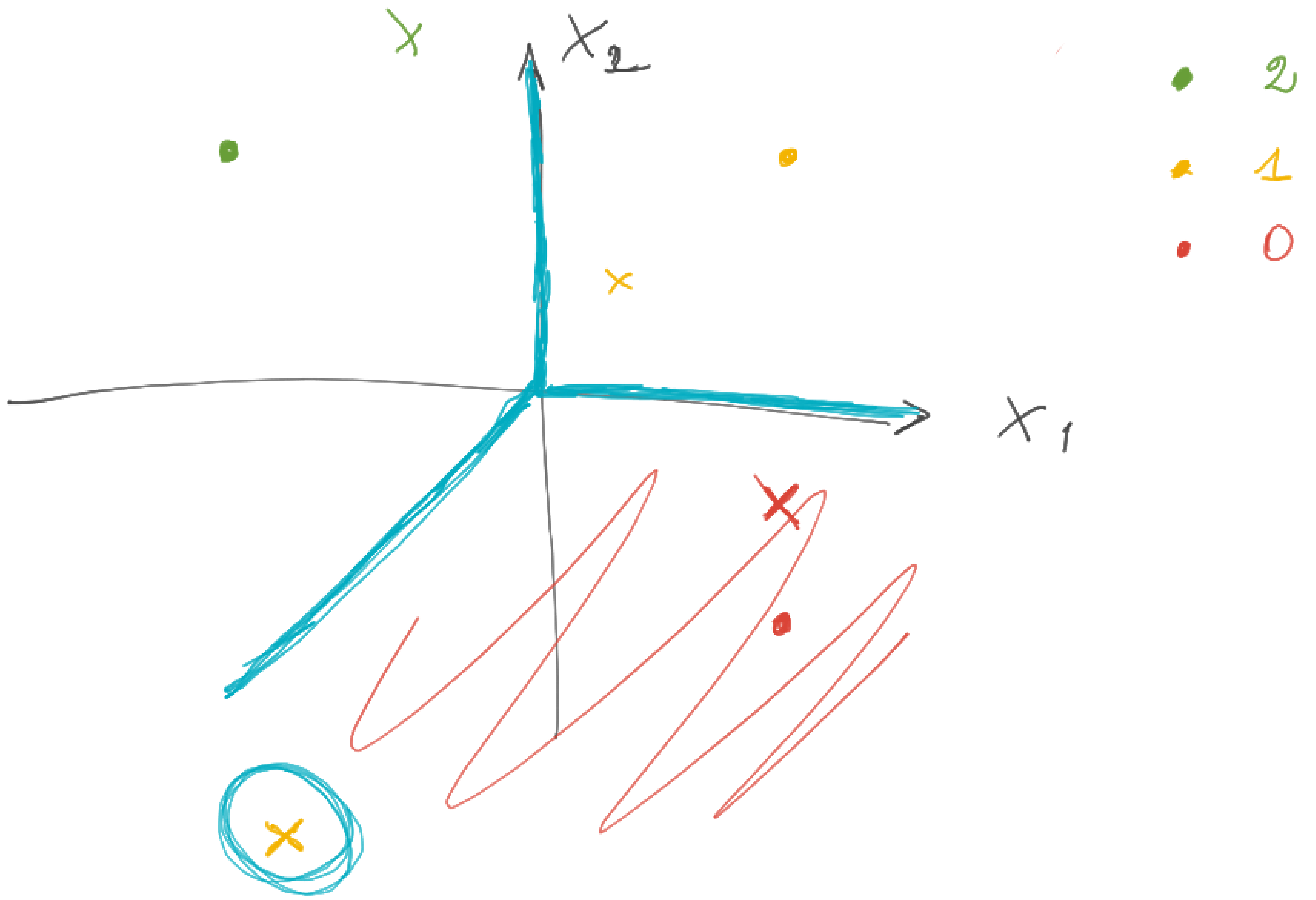
$y_{ts_4} = 0$

$$y_{ts} = \begin{bmatrix} 2 \\ 1 \\ 1 \\ 0 \end{bmatrix} \begin{matrix} \checkmark \\ \checkmark \\ \times \\ \checkmark \end{matrix}$$

classification error

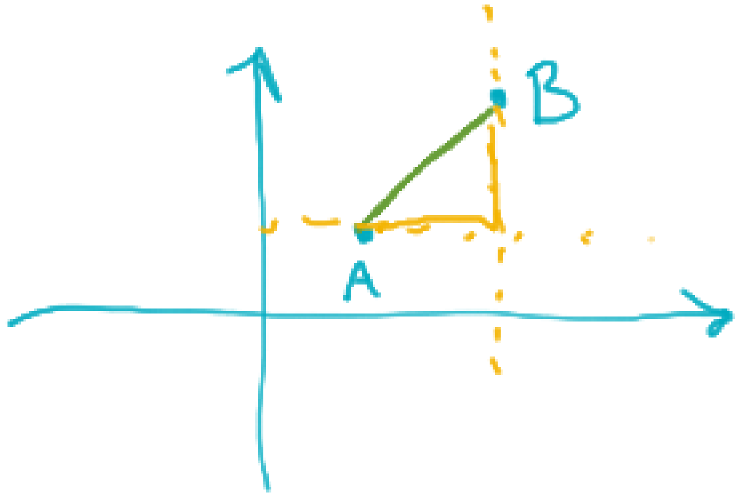
$$\frac{1}{4} = 0.25 = 25\%$$

$$\mathbf{x}_{\text{tr}} = \begin{bmatrix} -1 & 1 \\ 1 & 1 \\ 1 & -1 \end{bmatrix}, \quad \mathbf{y}_{\text{tr}} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{x}_{\text{ts}} = \begin{bmatrix} -0.5 & 1.5 \\ 0.1 & 0.5 \\ -1 & -1.5 \\ 1 & -0.5 \end{bmatrix}, \quad \mathbf{y}_{\text{ts}} = \begin{bmatrix} 2 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$



$$\mathbf{x} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \\ -2 & -1 \\ -1 & 1 \\ 1 & 0 \\ -2 & -1 \end{bmatrix} \qquad v_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \qquad v_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\mathbf{v} = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \qquad D \in \mathbb{R}^{6 \times 2}$$



2 ~ number of features

$$d(x_1, v_1) = \sum_{i=1}^{2} |x_{1_i} - v_{1_i}| = |-2-(-1)| + |0-1| = 1+1 = 2$$

$$d(x_1, v_2) = |-2-1| + |0-(-1)| = +3+1 = 4$$

$$d(x_2, v_1) = |0-(-1)| + |-2-1| = 1+3 = 4$$

$$d(x_2, v_2) = |0-1| + |-2-(-1)| = 1+1 = 2$$

$$d(x_3, v_1) = |-2-(-1)| + |-1-(+1)| = 1+2 = 3$$

$$d(x_3, v_2) = |-2-(1)| + |-1-(-1)| = 3+0 = 3$$

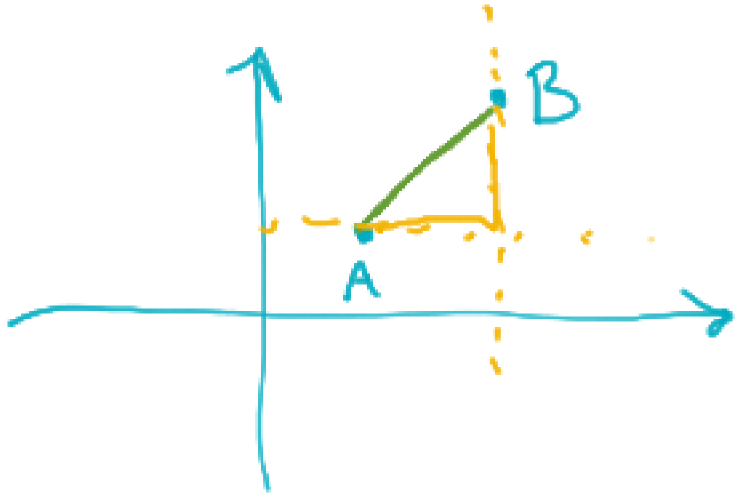
$$D = \begin{array}{c} \begin{array}{cc} v_1 & v_2 \end{array} \\ \begin{bmatrix} 2 & 4 \\ 4 & 2 \\ 3 & 3 \\ & \\ & \end{bmatrix} \end{array}$$

$$\mathbf{x} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \\ -2 & -1 \\ -1 & 1 \\ 1 & 0 \\ -2 & -1 \end{bmatrix}$$

$v_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ $\quad v_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\mathbf{v} = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$

$D \in \mathbb{R}^{6 \times 2}$



$d(x_4, v_1) = |-1-(-1)| + |1-1| = 0$

$d(x_4, v_2) = \times$

$d(x_5, v_1) = |1-(-1)| + |0-1| = 2+1 = 3$

$d(x_5, v_2) = |1-1| + |0-(-1)| = 0+1 = 1$

$d(x_6, v_1) = |-2-(-1)| + |-1-(+1)| = 1+2 = 3$

$d(x_6, v_2) = |-2-1| + |-1-(-1)| = 3+0 = 3$

$$D = \begin{array}{cc} v_1 & v_2 \\ \end{array}$$
$$D = \begin{bmatrix} 2 & 4 \\ 4 & 2 \\ 3 & 3 \\ 0 & \times \\ 3 & 1 \\ 3 & 3 \end{bmatrix}$$

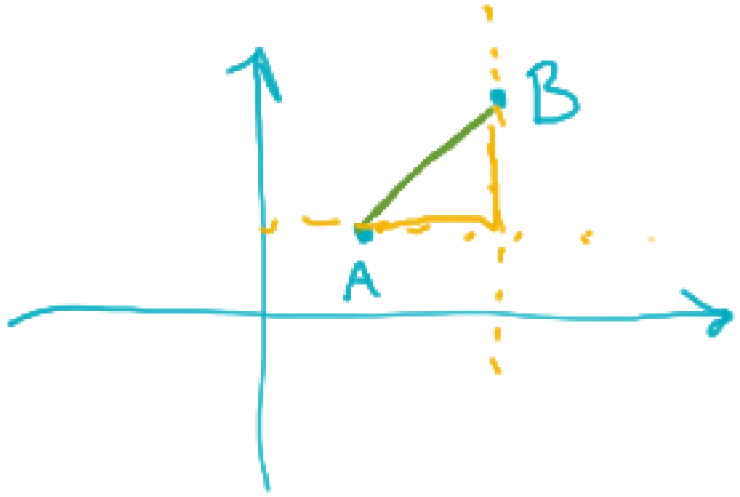
$L = 2+2+3+0+1+3 = 11$

$$\mathbf{x} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \\ -2 & -1 \\ -1 & 1 \\ 1 & 0 \\ -2 & -1 \end{bmatrix}$$

$$V_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \qquad V_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\mathbf{v} = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$D \in \mathbb{R}^{6 \times 2}$$



$$V_1 = \frac{1}{4}\left[x_1 + x_3 + x_4 + x_6\right] =$$

$$= \frac{1}{4}\left[\begin{bmatrix} -2 \\ 0 \end{bmatrix} + \begin{bmatrix} -2 \\ -1 \end{bmatrix} + \begin{bmatrix} -1 \\ 1 \end{bmatrix} + \begin{bmatrix} -2 \\ -1 \end{bmatrix}\right] = \begin{bmatrix} -\frac{7}{4} \\ -\frac{1}{4} \end{bmatrix} = \begin{bmatrix} -1.75 \\ -0.25 \end{bmatrix}$$

$$V_2 = \frac{1}{2}\left[\begin{bmatrix} 0 \\ -2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}\right] = \begin{bmatrix} \frac{1}{2} \\ -1 \end{bmatrix} = \begin{bmatrix} 0.5 \\ -1 \end{bmatrix}$$

$$D = \begin{array}{cc} v_1 & v_2 \\ \end{array}$$
$$D = \begin{bmatrix} 2 & 4 \\ 4 & 2 \\ 3 & 3 \\ 0 & X \\ 3 & 1 \\ 3 & 3 \end{bmatrix}$$

$$\mathbf{x} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \\ -2 & -1 \\ -1 & 1 \\ 1 & 0 \\ -2 & -1 \end{bmatrix} \qquad V_1 = \begin{bmatrix} -1.75 \\ -0.25 \end{bmatrix} \qquad V_2 = \begin{bmatrix} 0.5 \\ -1 \end{bmatrix} \qquad D = \begin{bmatrix} 0.5 & 3.5 \\ 3.5 & 1.5 \\ 1 & 2.5 \end{bmatrix}$$

$$d(x_1, V_1) = |-2-(-1.75)| + |0-(-0.25)| = |0.25| + |0.25| = 0.5$$

$$d(x_1, V_2) = |-2-(0.5)| + |0-(-1)| = 2.5 + 1 = 3.5$$

$$d(x_2, V_1) = |0-(-1.75)| + |-2-(-0.25)| = 1.75 + 1.75 = 3.5$$

$$d(x_2, V_2) = |0-(0.5)| + |-2-(-1)| = 0.5 + 1 = 1.5$$

$$d(x_3, V_1) = |-2-(-1.75)| + |-1-(-0.25)| = 0.25 + 0.75 = 1$$

$$d(x_3, V_2) = |-2-(+0.5)| + |-1-(-1)| = 2.5$$

$$\mathbf{x} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \\ -2 & -1 \\ -1 & 1 \\ 1 & 0 \\ -2 & -1 \end{bmatrix} \qquad V_1 = \begin{bmatrix} -1.75 \\ -0.25 \end{bmatrix} \qquad V_2 = \begin{bmatrix} 0.5 \\ -1 \end{bmatrix} \qquad D = \begin{bmatrix} 0.5 & 3.5 \\ 3.5 & 1.5 \\ 1 & 2.5 \\ 2 & 3.5 \\ 3 & 1.5 \\ 1 & 2.5 \end{bmatrix}$$

$$d(x_4, v_1) = |-1 - (-1.75)| + |1 - (-0.25)| = 0.75 + 1.25 = 2$$

$$d(x_4, v_2) = |-1 - 0.5| + |1 - (-1)| = 1.5 + 2 = 3.5$$

$$d(x_5, V_1) = |1 - (-1.75)| + |0 - (-0.25)| = 2.75 + 0.25 = 3$$

$$d(x_5, V_2) = |1 - (0.5)| + |0 - (-1)| = 0.5 + 1 = 1.5$$

$$d(x_6, V_1) = |-2 - (-1.75)| + |-1 - (-0.25)| = 0.25 + 0.75 = 1$$

$$d(x_6, V_2) = |-2 - 0.5| + |-1 - (-1)| = 2.5 + 0 = 2.5$$

PREVIOUS ITER.

$$L = 11 \qquad D = \begin{bmatrix} 2 & 4 \\ 4 & 2 \\ 3 & 3 \\ 0 & \times \\ 3 & 1 \\ 3 & 3 \end{bmatrix}$$

CURRENT ITER.

$$D = \begin{bmatrix} 0.5 & 3.5 \\ 3.5 & 1.5 \\ 1 & 2.5 \\ 2 & 3.5 \\ 3 & 1.5 \\ 1 & 2.5 \end{bmatrix}$$

$$L = 0.5 + 1.5 + 1 + 2 + 1.5 + 1 = 7.5$$