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Exercise 1 (10 points)

Let's consider a 2-class problem in a one-dimensional feature space bounded in [0,1], i.e., $x \in [0,1]$. The class-conditional densities are: $p(x|\omega_1) = 2 - 2x$, and $p(x|\omega_2) = 2x$, both defined in [0,1]. Assume that the prior probabilities of the two data classes are $P_1 = P_2$, and that the cost of errors of class ω_2 is 1.5 times that of class ω_1, that is, $\lambda_{12} = 1.5\lambda_{21}$.

- (5 points) Compute the minimum-risk decision regions, and the Bayesian decision regions, and compute the classification error in both cases.
- (3 points) Plot the joint distributions $P_k p(x|\omega_k)$ for k=1,2 on the one-dimensional feature space, along with the minimum-risk and the Bayesian decision regions.
- (2 points) In the plot, highlight the area(s) corresponding to the <u>additional error</u> incurred when using the minimum-risk decision. One class shows a higher error. *Which one? Why?*

Exercise 2 (10 points)

Let us consider a 3-class problem in R^2 (two-dimensional feature space), where the likelihood of each class is Gaussian and given as $p(x|\omega_i) = N(\boldsymbol{\mu_i}, \Sigma_i)$, with

$\Sigma_i = \sigma^2\mathbf{I}$; $\mu 1 = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$; $\mu 2 = \begin{pmatrix} +1 \\ -1 \end{pmatrix}$; $\mu 3 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$;

and prior probabilities $P_1 = P_2 = P_3$. Compute the decision boundaries and plot them.

Exercise 3 (10 points)

Given the two-dimensional training points $\mathbf{x}_{\mathrm{tr}}$, along with their labels $\mathbf{y}_{\mathrm{tr}}$, and a set of test examples $\mathbf{x}_{\mathrm{ts}}$, with their labels $\mathbf{y}_{\mathrm{ts}}$: $\mathbf{x}_{\mathrm{tr}} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ 2 & 2 \\ 1 & 0 \\ 3 & 3 \end{bmatrix}$, $\mathbf{y}_{\mathrm{tr}} = \begin{bmatrix} 2 \\ 2 \\ 0 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{x}_{\mathrm{ts}} = \begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 0 & 0 \\ 1 & 2 \end{bmatrix}$, $\mathbf{y}_{\mathrm{ts}} = \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \end{bmatrix}$, classify the points in $\mathbf{x}_{\mathrm{ts}}$ with a k-NN algorithm with k=1, using the l2 distance as the distance metric. The distance matrix computed by comparing $\mathbf{x}_{\mathrm{ts}}$ against $\mathbf{x}_{\mathrm{tr}}$ is given below:

[[2.24 2.24 1.41 1.00 2.83]
[3.61 3.61 0.00 2.24 1.41]
[1.00 1.00 2.83 1.00 4.24]
[2.83 3.16 1.00 2.00 2.24]]

- (5 points) Compute the classification error. In case of equal (minimum) distances between a given test sample and a subset of the training points, assign the test sample to the class of the first point of the training set (from left to right in the distance matrix).
- (5 points) Plot the decision function of the given k-NN classifier.