$$\mathbf{x} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \\ -1 & -1 \\ 1 & 2 \\ 1 & 1 \\ 2 & 0 \end{bmatrix}, \quad w = [0.2, \ -1]^T$$

$f(x) \longrightarrow L = \ldots$

$$L(\mathbf{w}, b) = \frac{1}{n}\sum_{i=1}^{n}\left[\max\left(0, 1 - \underbrace{y_i f(\boldsymbol{x}_i)}\right) - \max\left(0, -y_i f(\boldsymbol{x}_i)\right)\right] + \frac{\lambda}{2}\mathbf{w}^T\mathbf{w}$$

$f(x) = w^T x + b$

$$\begin{bmatrix} \boxed{-2 \quad 0} \\ 0 \quad -2 \\ -1 \quad -1 \\ 1 \quad 2 \\ 1 \quad 1 \\ 2 \quad 0 \end{bmatrix} \begin{bmatrix} 0.2 \\ -1 \end{bmatrix} = \begin{bmatrix} -0.4 \\ +2 \\ 0.8 \\ -1.8 \\ -0.8 \\ 0.4 \end{bmatrix}$$

$$\boxed{y_i \circ f(x) = \begin{bmatrix} -0.4 \\ +2 \\ 0.8 \\ -1.8 \\ -0.8 \\ 0.4 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \\ -1 \\ -1 \end{bmatrix} = \begin{bmatrix} -0.4 \\ 2 \\ 0.8 \\ 1.8 \\ 0.8 \\ -0.4 \end{bmatrix}}$$

$L = \frac{1}{6}\Big[\Big($

$$\mathbf{x} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \\ -1 & -1 \\ 1 & 2 \\ 1 & 1 \\ 2 & 0 \end{bmatrix}, \quad w = [0.2, \; -1]^T$$

$f(x) \longrightarrow L = \ldots$

$$0.25 \cdot \begin{bmatrix} 0.2 & -1 \end{bmatrix} \begin{bmatrix} 0.2 \\ -1 \end{bmatrix} = 0.25(1.4)$$

$$f(x) = w^T x + b$$

$$L(\mathbf{w}, b) = \frac{1}{n} \sum_{i=1}^{n} [\max(0, 1 - y_i f(\boldsymbol{x}_i)) - \max(0, -y_i f(\boldsymbol{x}_i))] + \frac{\lambda}{2} \mathbf{w}^T \mathbf{w},$$

$$y_i \circ f(x) = \begin{bmatrix} -0.4 \\ +2 \\ 0.8 \\ -1.8 \\ -0.8 \\ 0.4 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \\ -1 \\ -1 \end{bmatrix} = \begin{bmatrix} -0.4 \\ 2 \\ 0.8 \\ 1.8 \\ 0.8 \\ -0.4 \end{bmatrix}$$

$$[1 - y \circ f(x)] = \begin{bmatrix} 1.4 \\ -1 \;\; 0 \\ 0.2 \\ -0.8 \;\; 0 \\ 0.2 \\ 1.4 \end{bmatrix} \qquad -y \cdot f(x) = \begin{bmatrix} 0.4 \\ -2 \\ -0.8 \\ -1.8 \\ -0.8 \\ 0.4 \end{bmatrix}$$

$$L = \frac{1}{6}\left[\left(1.4 + \cancel{0} + 0.2 + \cancel{0} + 0.2 + 1.4\right) - \left(0.4 + \cancel{0} + \cancel{0} + \cancel{0} + \cancel{0} + 0.4\right)\right] + 0.25(1.4)$$

$$\mathbf{x} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \\ -1 & -1 \\ 1 & 2 \\ 1 & 1 \\ 2 & 0 \end{bmatrix}, \quad w = [0.2, \ -1]^T$$

$$0.25 \cdot \begin{bmatrix} 0.2 & -1 \end{bmatrix} \begin{bmatrix} 0.2 \\ -1 \end{bmatrix} = 0.25 (1.4)$$

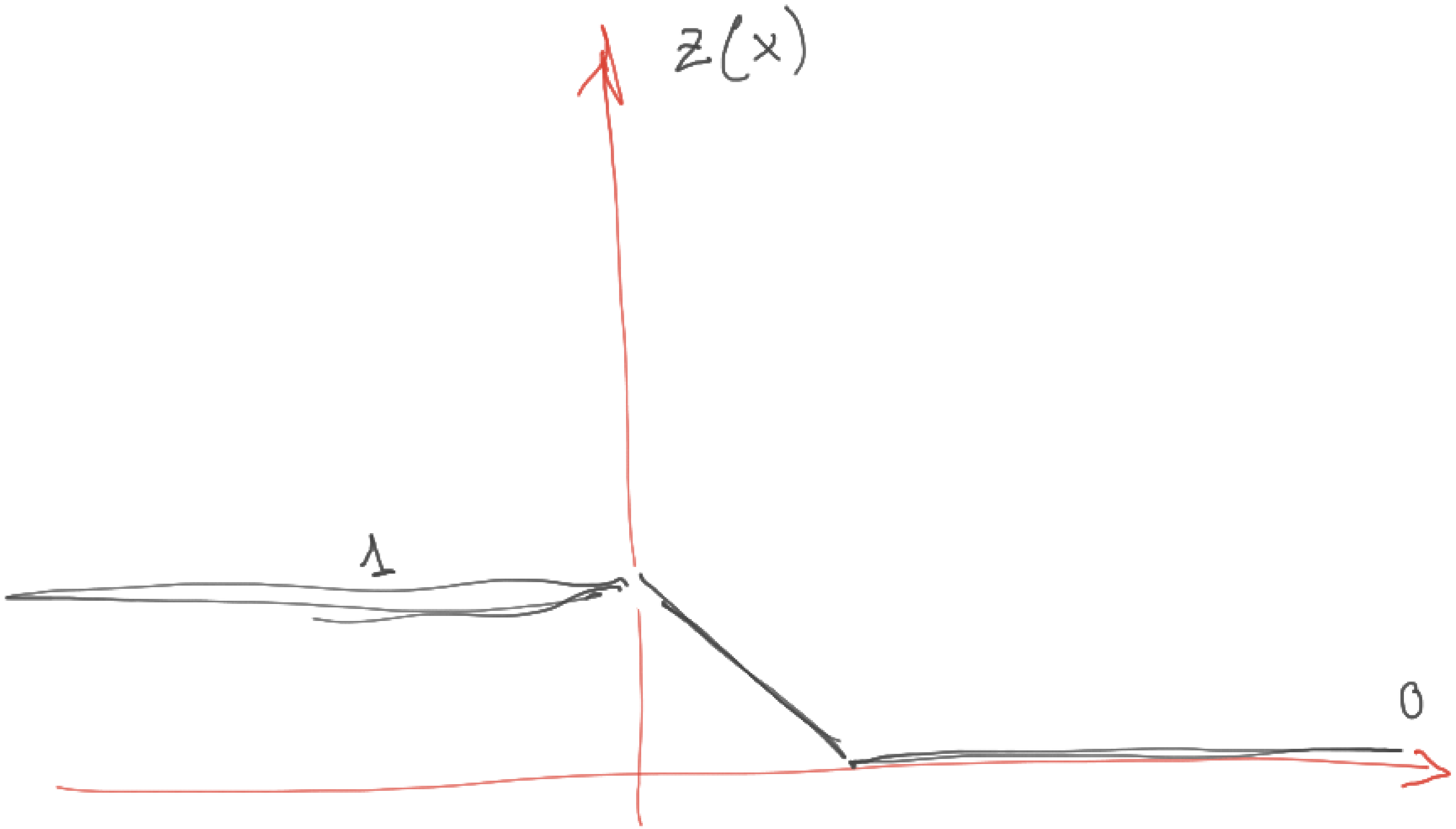
$$f(x) \rightarrow L = \ldots$$

$$L(\mathbf{w}, b) = \frac{1}{n} \sum_{i=1}^{n} [\max(0, 1 - y_i f(\mathbf{x}_i)) - \max(0, -y_i f(\mathbf{x}_i))] + \frac{\lambda}{2} \mathbf{w}^T \mathbf{w},$$

$$f(x) = w^T x + b$$

$$L = \frac{1}{6} \left[\left(\boxed{1.4} + \cancel{0} + 0.2 + \cancel{0} + 0.2 + \boxed{1.4} \right) - \left(\boxed{0.4} + \cancel{0} + \cancel{0} + \cancel{0} + \cancel{0} + \boxed{0.4} \right) \right] + 0.25 (1.4)$$

$$L = \frac{1}{6} \left[1 + 0 + 0.2 + 0 + 0.2 + 1 \right] + 0.35 = 0.75$$


z(x)
1
0

$$\nabla_w L = \begin{cases} \frac{1}{n}\sum(-y_i \vec{x}_i) & \text{if } (y_i f(x_i) \text{ in } (0,1)) \\ 0 & \text{otherwise} \end{cases} + \lambda \vec{w}$$

$$\mathbf{x} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \\ -1 & -1 \\ 1 & 2 \\ 1 & 1 \\ 2 & 0 \end{bmatrix}, \quad \mathbf{w} = [0.2, \; -1]^T, \quad \mathbf{y} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \\ -1 \\ -1 \end{bmatrix}$$

$$\nabla_b L = \begin{cases} \frac{1}{n}\sum -y_i & \text{if } f(x_i) \cdot y_i \text{ is in } (0,1) \\ 0 & \text{otherwise} \end{cases} + 0$$

$$y_i \circ f(x) = \begin{bmatrix} -0.4 \\ +2 \\ 0.8 \\ -1.8 \\ -0.8 \\ 0.4 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \\ -1 \\ -1 \end{bmatrix} = \begin{array}{c} \begin{bmatrix} -0.4 \\ 2 \\ \boxed{0.8} \\ 1.8 \\ \boxed{0.8} \\ -0.4 \end{bmatrix} \end{array} \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{array}$$

$$\nabla_w L = \frac{1}{6} \sum_{i=\{3,5\}} (-y_i x_i) + \lambda w$$

$$= \frac{1}{6}\left[-\underbrace{(1)}_{y_3} \underbrace{\begin{bmatrix} -1 \\ -1 \end{bmatrix}}_{x_3} - \underbrace{(-1)}_{y_5} \underbrace{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}_{x_5} \right] + \underbrace{0.5}_{\lambda} \underbrace{\begin{bmatrix} 0.2 \\ -1 \end{bmatrix}}_{w} =$$

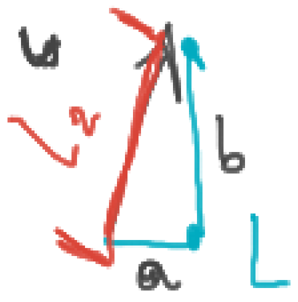
$$= \frac{1}{6}\begin{bmatrix} 2 \\ 2 \end{bmatrix} + \begin{bmatrix} 0.1 \\ -0.5 \end{bmatrix} = \begin{bmatrix} 0.43 \\ -0.17 \end{bmatrix}$$

$$w \leftarrow w - \eta \nabla_w L = \begin{bmatrix} 0.2 \\ -1 \end{bmatrix} - 0.5 \begin{bmatrix} 0.43 \\ -0.17 \end{bmatrix} = \begin{bmatrix} -0.02 \\ -0.92 \end{bmatrix}$$

$$\nabla_b L = \frac{1}{6} \underbrace{(1 - 1)}_{y_3 + y_5} = 0$$

$$b \leftarrow b - \eta 0 = b$$

$$\eta\left(\|\nabla_w L\| + \|\nabla_b L\|\right) < \theta$$



L_1

$c^2 = \sqrt{a^2 + b^2}$

$|a| + |b|$

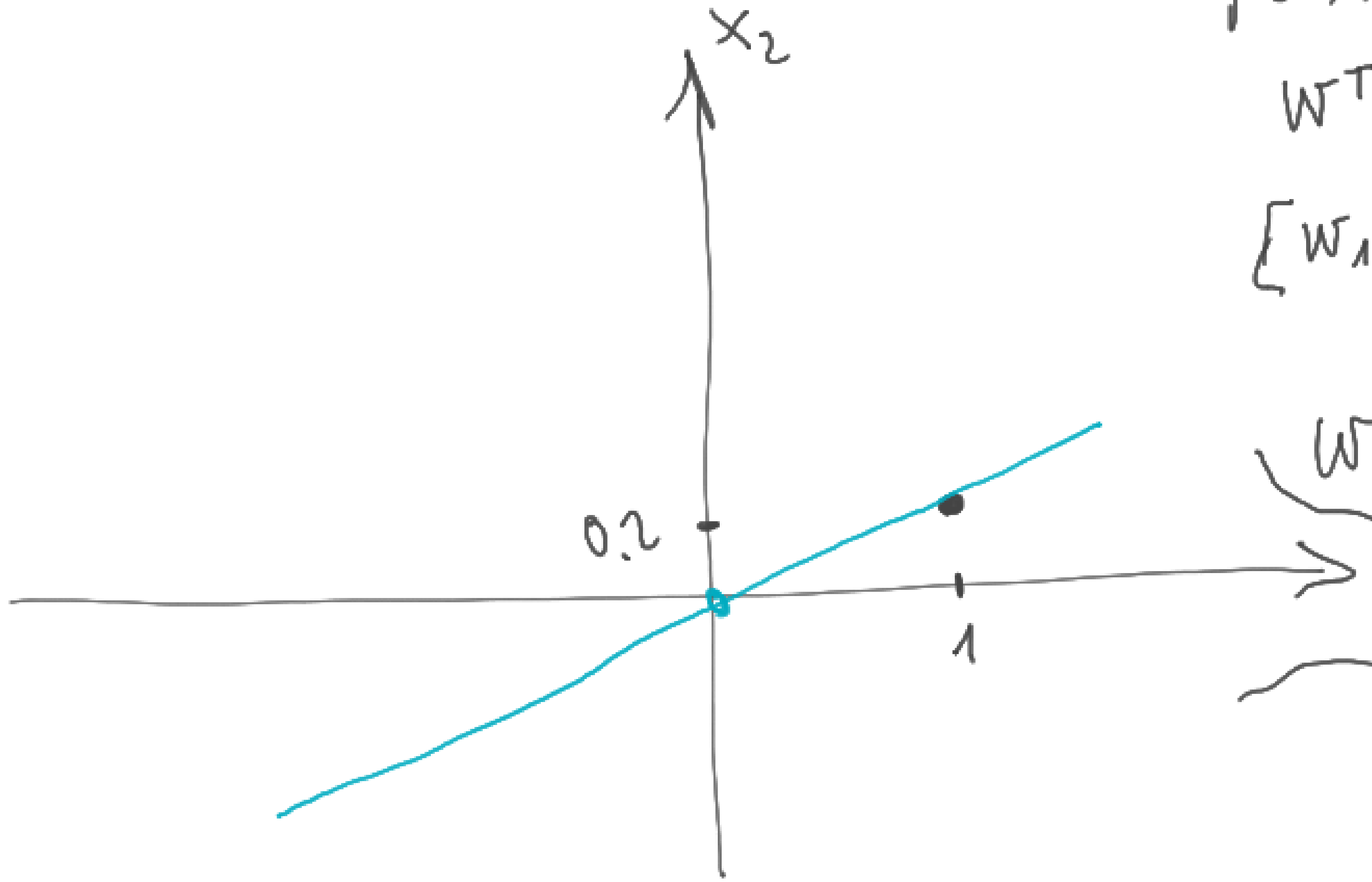
$\nabla_w = \begin{bmatrix} 0.43 \\ -0.17 \end{bmatrix}$ $\quad \nabla_b L = 0 \quad \eta = 0.5 \quad \theta = 0.7$

$$0.5\left[|0.43| + |-0.17| + 0\right] < 0.7$$

$$0.3 < 0.7$$

Plot of initial decision boundary

$$w = \begin{bmatrix} 0.2 \\ -1 \end{bmatrix} \qquad b = 0$$



$$f(x) = 0$$

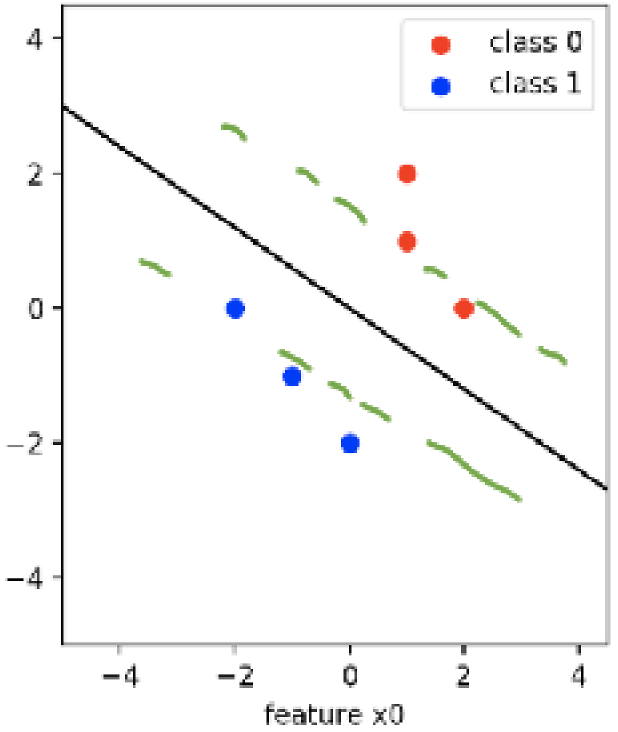
$$w^T x + b = 0$$

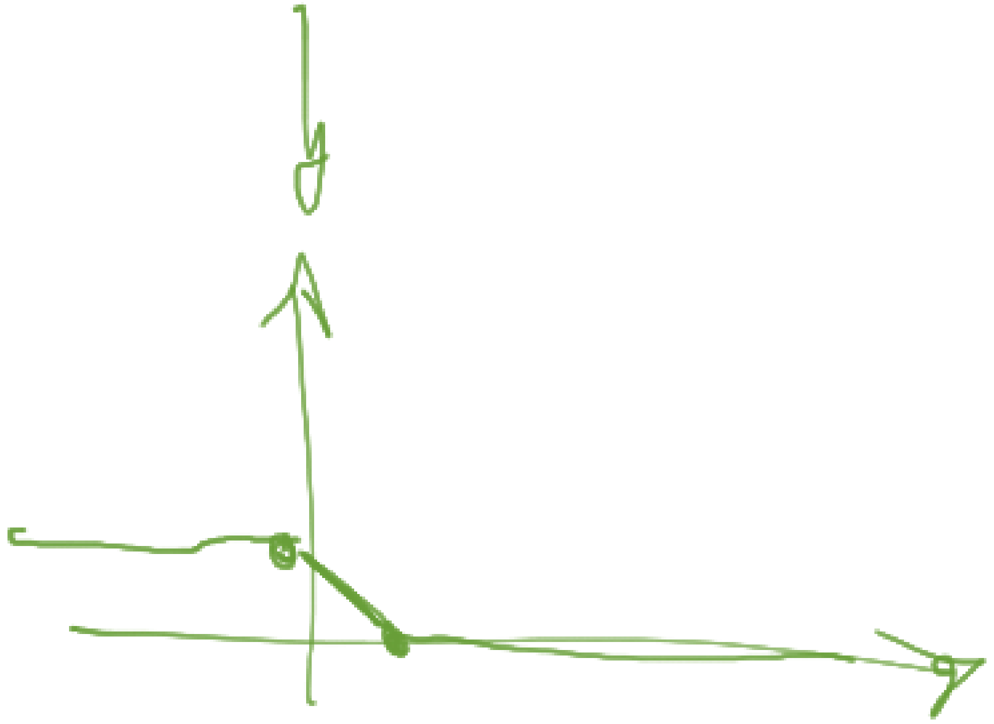
$$[w_1 \;\; w_2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \cancel{b} = 0$$

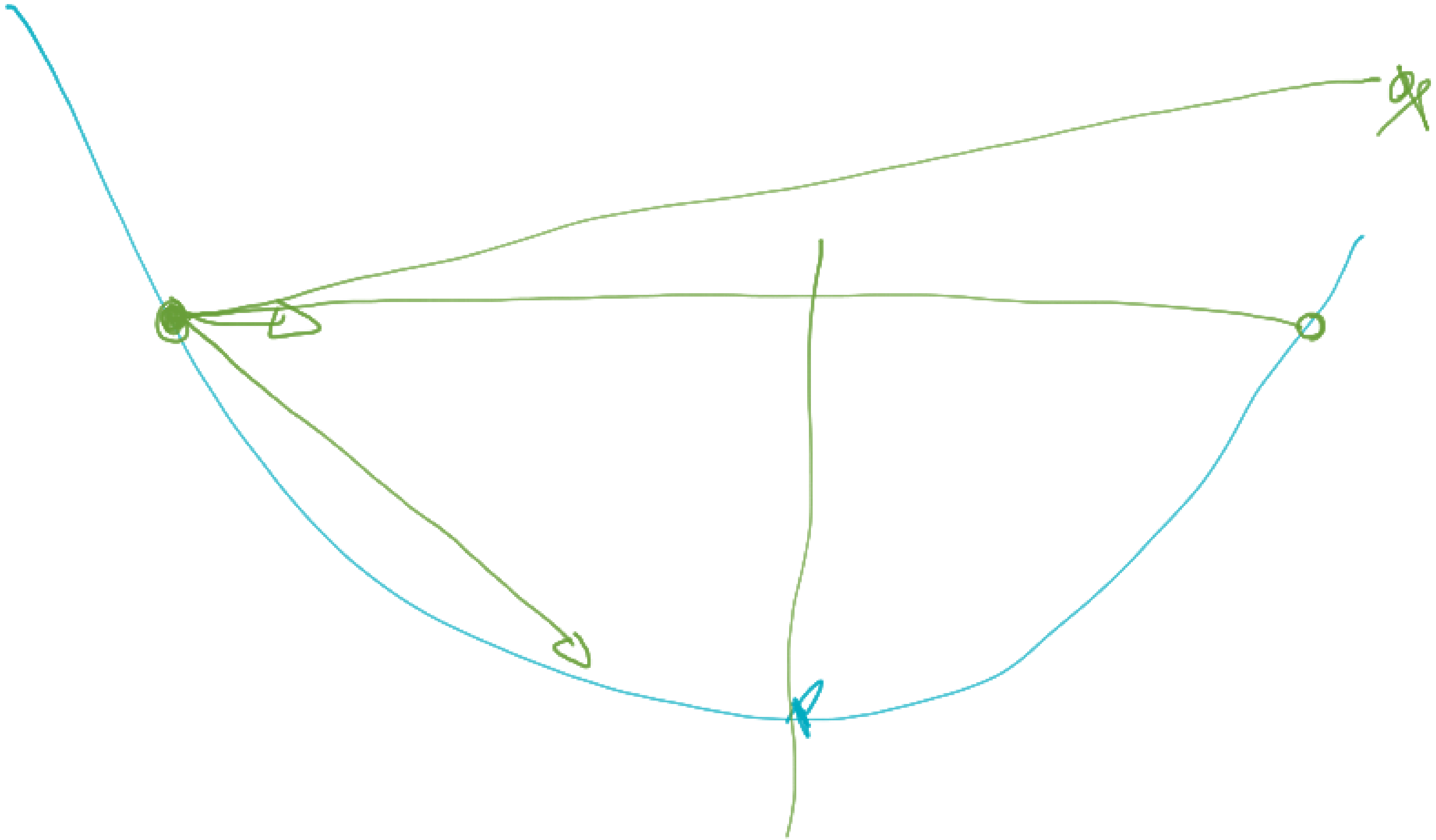
$$w_1 x_1 + w_2 x_2 = 0$$

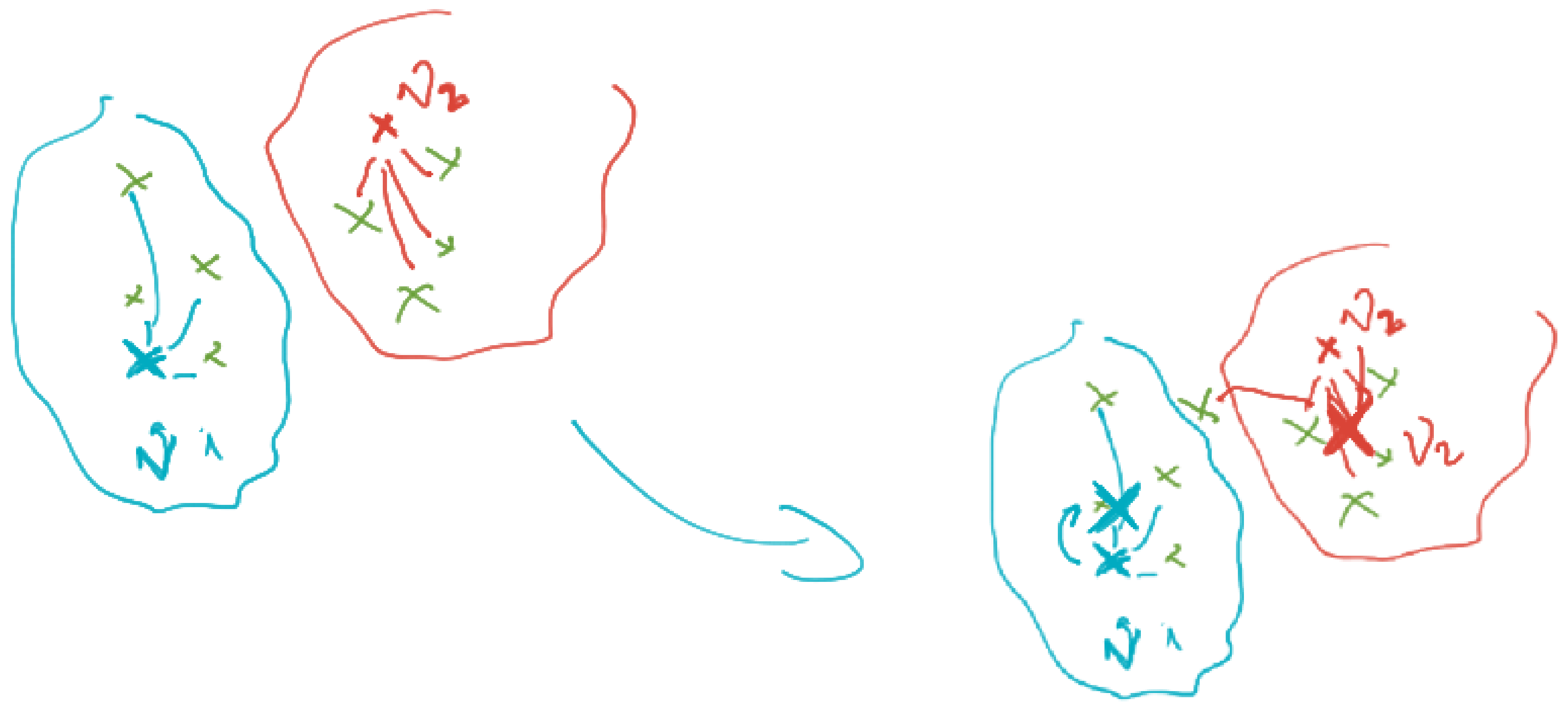
$$x_2 = \frac{-w_1 x_1}{w_2}$$

$$x_1 = 1 \rightarrow x_2 = -\frac{w_1}{w_2} = 0.2$$


class 0
class 1
feature x0







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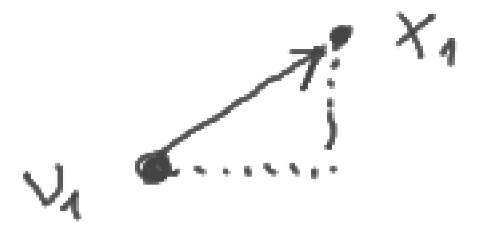


$v_1 - x_1$

$$\mathbf{x} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \\ -2 & -1 \\ 1 & 1 \\ 1 & 0 \\ 2 & 1 \end{bmatrix} \qquad \mathbf{v} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$v_1 \to y = 1$

$v_2 \to y = 2$

$$d(x_1, v_1) = |v_{1,1} - x_{1,1}| + |v_{1,2} - x_{1,2}| =$$

$$= |1 - (-2)| + |1 - 0| = 3 + 1 = \underline{4} \qquad y_1 = 1$$

$$d(x_1, v_2) = |1 - (-2)| + |-1 - 0| = \underline{4}$$

$$d(x_2, v_1) = 1 + 3 = 4$$

$$d(x_2, v_2) = 1 + 1 = 2 \qquad y_2 = 2$$

$$d(x_3, v_1) = 3 + 2 = 5$$

$$d(x_3, v_2) = 3 + 0 = 3 \qquad y_3 = 2$$

$$d(x_4, v_1) = 0 \to y_4 = 1$$

$$d(x_4, v_2) = 2$$

$$d(x_5, v_1) = 1 - y_5 = 1$$

$$d(x_5, v_2) = 1$$

$$d(x_6, v_1) = 1 \qquad y_6 = 1$$

$$d(x_6, v_2) = 3$$

$$\mathbf{x} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \\ -2 & -1 \\ 1 & 1 \\ 1 & 0 \\ 2 & 1 \end{bmatrix} \qquad \mathbf{v} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$V_1 \to y = 1$

$V_2 \to y = 2$

$$\sum_{i=1}^{n} \|\boldsymbol{x}_i - \boldsymbol{v}_k\|_1 = 11$$

$$V_1 = \frac{1}{4}\left[\begin{bmatrix} -2 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix}\right] = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$$

$$V_2 = \frac{1}{2}\left[\begin{bmatrix} 0 \\ -2 \end{bmatrix} + \begin{bmatrix} -2 \\ -1 \end{bmatrix}\right] = \begin{bmatrix} -0.5 \\ -1.5 \end{bmatrix}$$

$d(x_1, V_1) = 4 \quad y_1 = 1$

$d(x_2, V_2) = 1 + 1 = 2 \quad y_2 = 2$

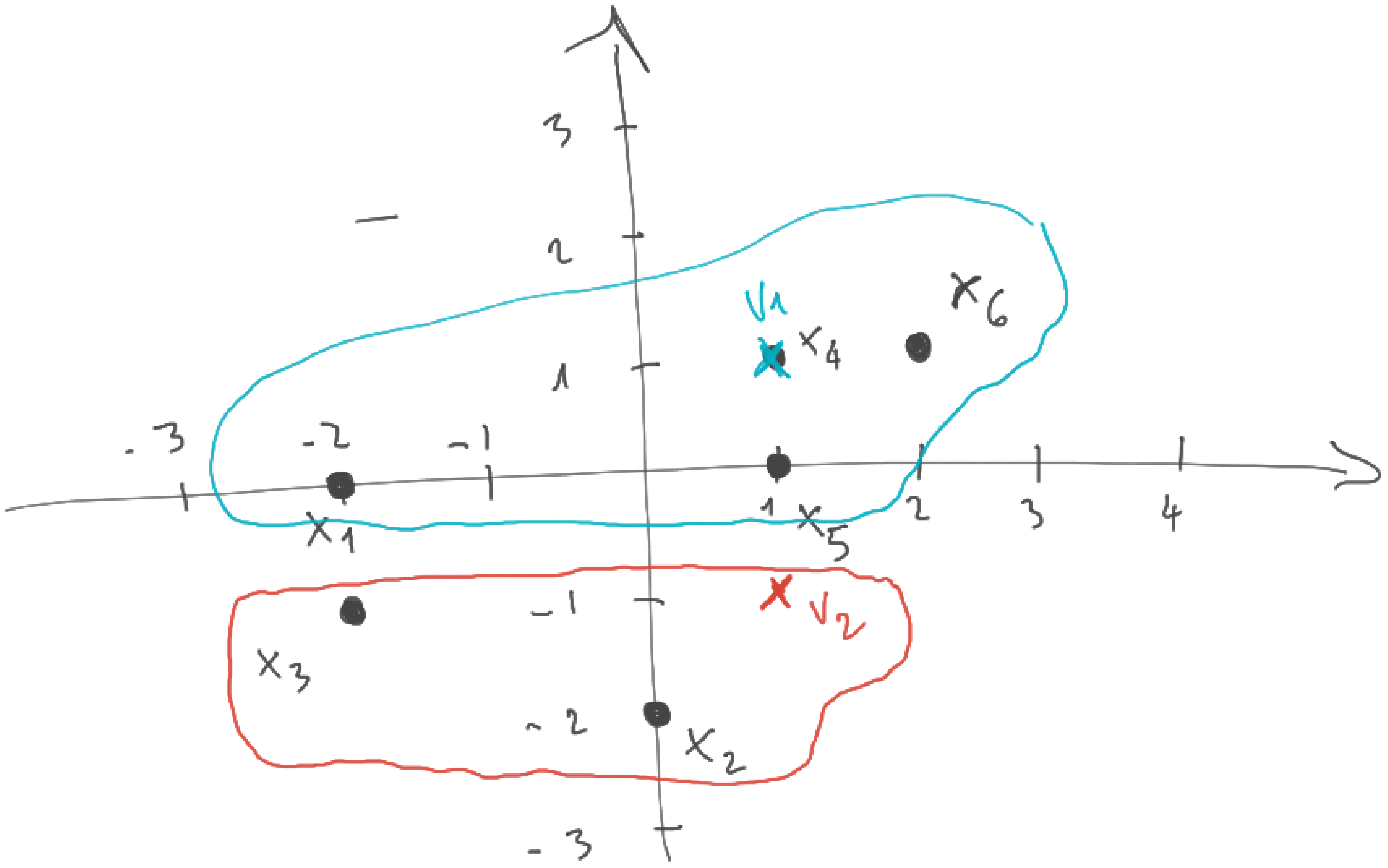
$d(x_3, V_2) = 3 + 0 = 3 \quad y_3 = 2$

$d(x_4, V_1) = 0 \to y_4 = 1$

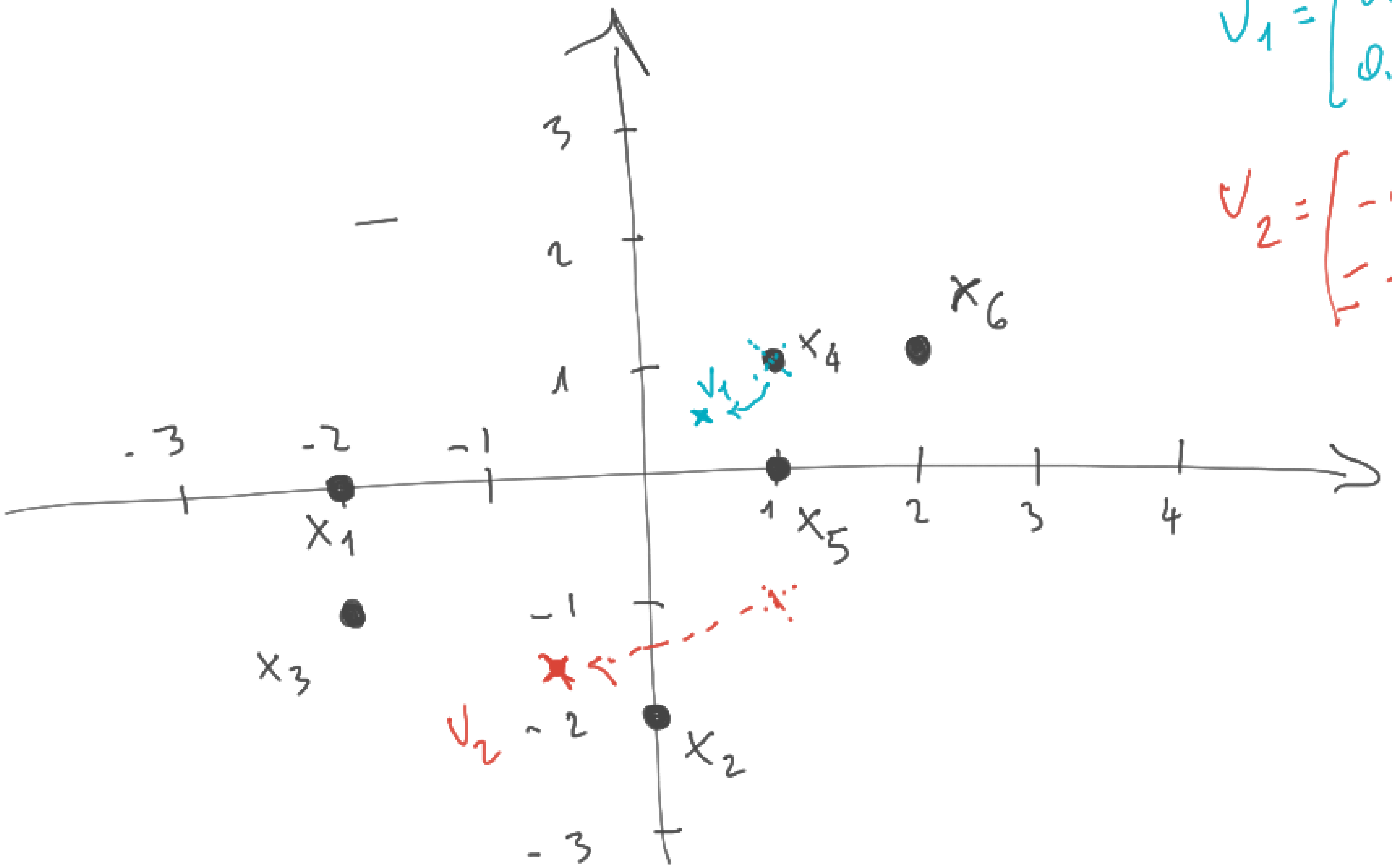
$d(x_5, V_1) = 1 - y_5 = 1$

$d(x_6, V_1) = 1 \quad y_6 = 1$

4 samples



3
2
1
-1
-2
-3
-3
-2
-1
1
2
3
4
v_1
x_4
x_6
x_1
x_5
x_3
x_2
v_2



$$v_1 = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$$

$$v_2 = \begin{bmatrix} -0.5 \\ -1.5 \end{bmatrix}$$

$$p(\mathbf{x}) = \frac{1}{(2\pi)^{d/2}|\Sigma|^{1/2}} \exp\left[-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^t \Sigma^{-1}(\mathbf{x}-\boldsymbol{\mu})\right]$$

$x^T x$ same for all

$P(\omega_i)$ same for all

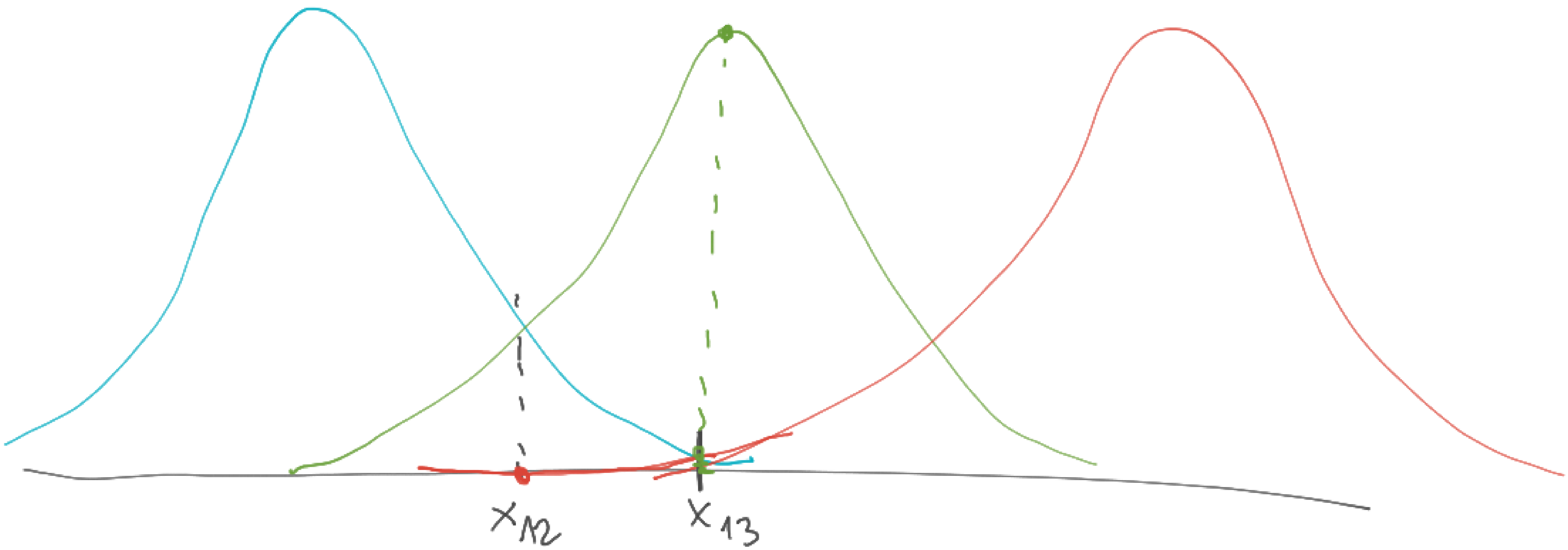
$$g(x) = -\frac{1}{2}x^T \Sigma^{-1} x + \mu^T \Sigma^{-1} x - \frac{1}{2}\mu^T \Sigma^{-1}\mu + \ln p(\omega_i) - \frac{1}{2}\ln|\Sigma|$$

$$P(\omega_i|x) = P(\omega_i)\,p(x|\omega_i)$$

$$g_1(x) = g_2(x)$$

$$g_1(x) = \mu^T x - \frac{1}{2}\mu^T\mu \qquad \mu_1 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$\begin{bmatrix} -1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - \frac{1}{2}\begin{bmatrix} -1 & 0 \end{bmatrix}\begin{bmatrix} -1 \\ 0 \end{bmatrix} = -x_1 - \frac{1}{2}$$



$g_1(x) = g_2(x) > g_3(x)$ if this is valid, then x_{12} exists

$g_1(x) = g_3(x) > g_2(x)$ if this is valid, then x_{13} exists