The Same

$d(x_1, x_2)$

12 x_5 $d(x_1, x_{12})$

A x_1

12

$d(x_2, x_1)$

$$d_{2,1} = \sqrt{(x_{21} - x_{12})^2 + (\dots)}$$

$$\sum_{j=1}^{d} \sqrt{(x_j - y_j)^2}$$

$$f(\vec{x}) = \vec{W}^{-T} \vec{x} + b$$

$$L = \frac{1}{2} \sum_{i=1}^{N} \left(W^T x_i + b - y_i \right)^2$$

$$W = [0.1, 0.1]^T \quad b = 0.1 \quad \eta = 0.1 \quad \theta = 0.06$$
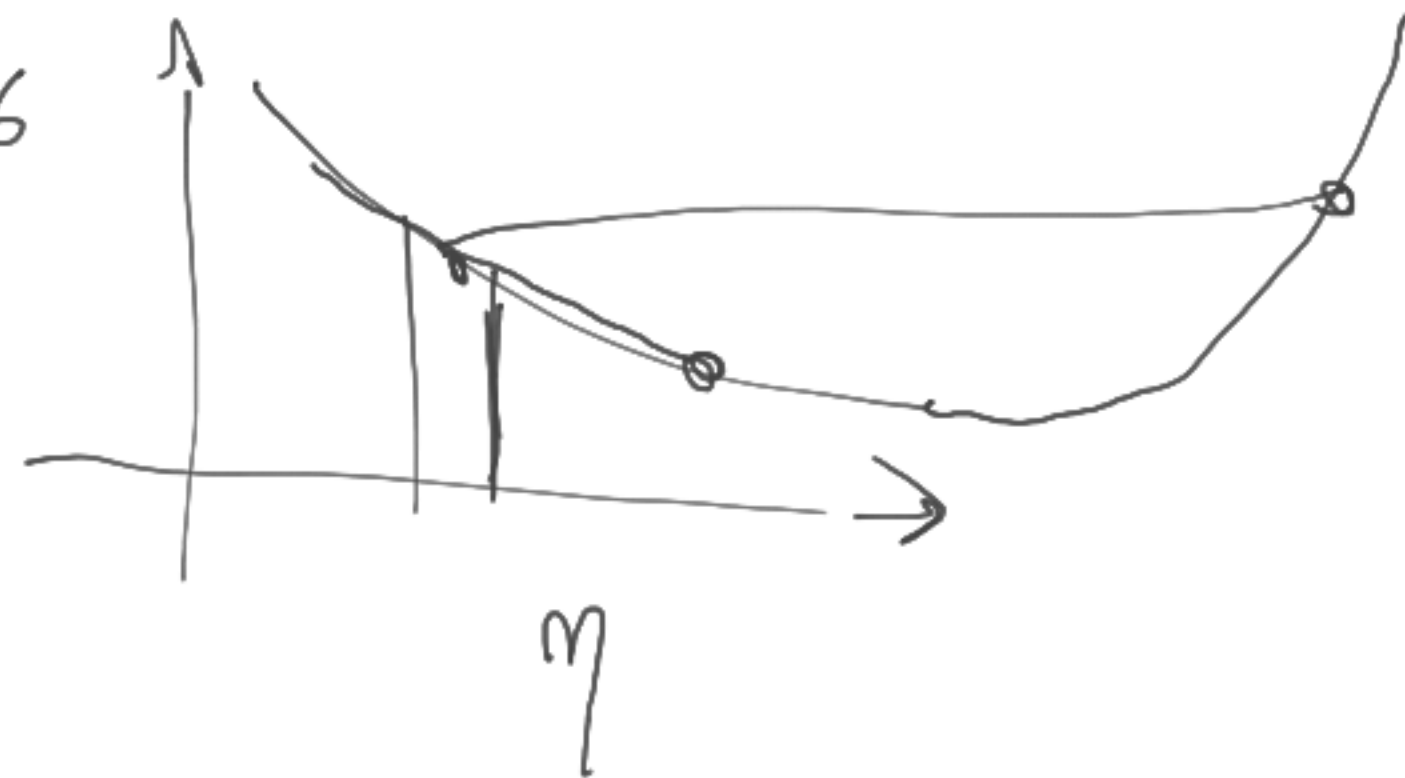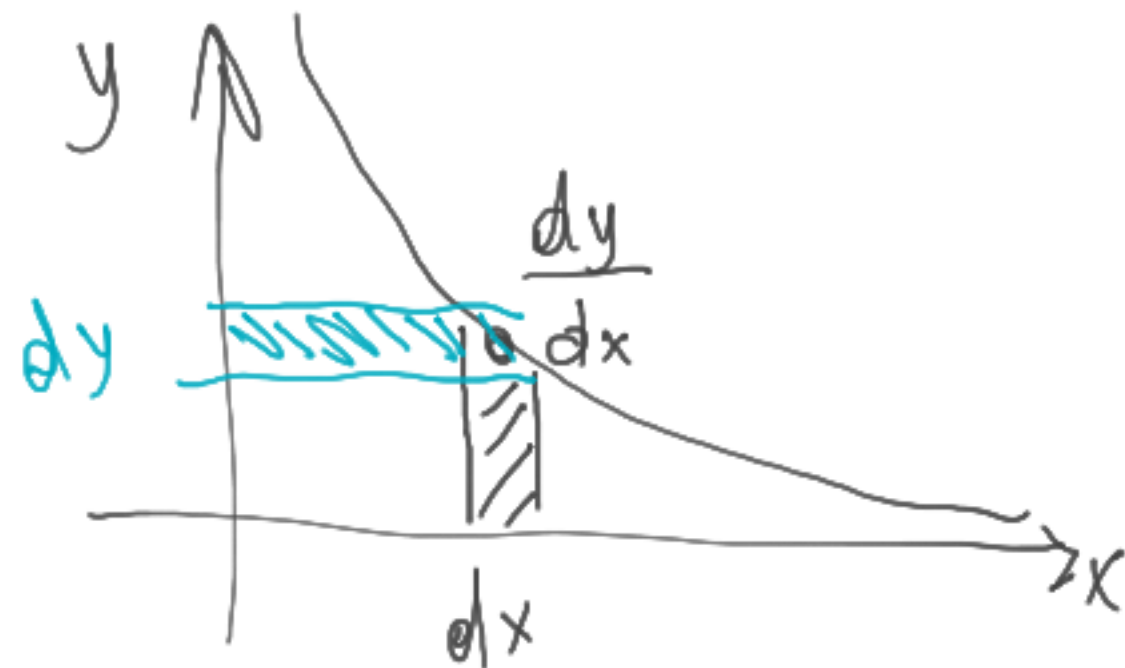
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$$W = [0.1, 0.1]^T \quad b = 0.1 \quad \eta = 0.1 \quad \theta = 0.06$$

$$X_i = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad W = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$$

$$W^T x = \overset{1 \times 2}{[\quad]} \overset{2 \times 1}{\begin{bmatrix} \\ \\ \end{bmatrix}} \rightarrow \overset{1 \times 1}{[\quad]}$$

$$b = \overset{1 \times 1}{[\quad]}$$

$$f(x) = \overset{1 \times 1}{[\quad]} + \overset{1 \times 1}{[\quad]} = \overset{1 \times 1}{[\quad]}$$

$$\mathbf{X} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ 0 & 0 \\ 0 & 2 \\ 1 & 2 \\ 2 & 1 \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \\ 0.1 \\ 0.1 \\ 0.1 \\ 0.1 \end{bmatrix},$$

$$w = [0.1, 0.1]^T \qquad b = 0.1 \qquad \eta = 0.1 \qquad \theta = 0.06$$

$$f(\vec{x}) = \vec{w}^T \vec{x} + b \qquad X_i = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad w = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$$

$$L = \frac{1}{2} \sum_{i=1}^{N} \underbrace{\left(w^T x_i + b - y_i \right)^2}_{\longrightarrow \; e(x)}$$

$$\overset{w^T x \qquad\qquad b}{f(x_1) = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \end{bmatrix} + 0.1 = -0.1 + 0 + 0.1 = 0}$$

$$f(x_2) = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \end{bmatrix} + 0.1 = 0 - 0.1 + 0.1 = 0$$

$$f(x_3) = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} + 0.1 = 0.1$$

$$f(x_4) = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \end{bmatrix} + 0.1 = 0.3$$

$$f(x_5) = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} + 0.1 = 0.4$$

$$f(x_6) = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix} \begin{bmatrix} ? \\ 1 \end{bmatrix} + 0.1 = 0.4$$

$$e(x_1) = f(x_1) - y_1) = 0 - 0.1 = -0.1$$

$$e(x_2) = 0 - 0.1 = -0.1$$

$$e(x_3) = 0.1 - 0.1 = 0$$

$$e(x_4) = 0.3 - 0.1 = 0.2$$

$$e(x_5) = 0.4 - 0.1 = 0.3$$

$$e(x_6) = 0.4 - 0.1 = 0.3$$

$$L = \frac{1}{2} \sum_{i=1}^{N} (e(x)_i)^2 \frac{1}{2} \left[(-0.1)^2 + (-0.1)^2 + (0)^2 + (0.2)^2 + \cdots \right]$$

$$\mathbf{X} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ 0 & 0 \\ 0 & 2 \\ 1 & 2 \\ 2 & 1 \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \\ 0.1 \\ 0.1 \\ 0.1 \end{bmatrix},$$

$$w = [0.1, 0.1]^T \quad b = 0.1 \quad \eta = 0.1 \quad \theta = 0.06$$

$$f(\vec{x}) = \vec{w}^T \vec{x} + b \qquad X_i = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad w = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$$

$$L = \frac{1}{2} \sum_{i=1}^{N} \underbrace{(w^T x_i + b - y_i)}_{\longrightarrow \ e(x)}{}^2$$

$$e(x_1) = f(x_1) - y_1) = 0 - 0.1 = -0.1$$

$$e(x_2) = 0 - 0.1 = -0.1$$

$$e(x_3) = 0.1 - 0.1 = 0$$

$$e(x_4) = 0.3 - 0.1 = 0.2$$

$$e(x_5) = 0.4 - 0.1 = 0.3$$

$$e(x_6) = 0.4 - 0.1 = 0.3$$

$$f(x_1) = 0$$

$$f(x_2) = 0$$

$$f(x_3) = 0.1$$

$$f(x_4) = 0.3$$

$$f(x_5) = 0.4$$

$$f(x_6) = 0.4$$

$$L = \frac{1}{2} \sum_{i=1}^{N} (e(x)_i)^2 = \frac{1}{2}\left[(-0.1)^2 + (-0.1)^2 + (0)^2 + (0.2)^2 + (0.3)^2 + (0.3)^2 \right] = 0.5 \begin{bmatrix} 0.01 + 0.01 + 0.04 \\ + 0.09 + 0.09 \end{bmatrix}$$
$$= 0.12$$

$$L = \frac{1}{2} \sum_{i=1}^{N} \left(\vec{w}^T \vec{x}_i + b - y_i \right)^2 \longrightarrow \frac{1}{2} \cdot \sum \left(\boxed{w_1 x_{i1}} + w_2 x_{i2} + b - y_i \right)^2$$

$$\alpha$$

$$\nabla L_{\vec{w}} = \begin{bmatrix} \frac{\partial L}{\partial w_1} \\ \frac{\partial L}{\partial w_2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \sum \left[2 \left(w_1 x_{i1} + w_2 x_{i2} + b - y_i \right) x_{i1} \right] \\ \frac{1}{2} \sum \left[2 \left(w_1 x_{i1} + w_2 x_{i2} - y_i \right) x_{i2} \right] \end{bmatrix} = \begin{bmatrix} \left(w^T x_i + b - y_i \right) x_{i1} \\ \left(\quad \right) x_{i2} \end{bmatrix} =$$

↑
same shape
of $\vec{w}$

GRADIENTS

$$\nabla L_{\vec{w}} = \sum \left(w^T x + b - y_i \right) \cdot \vec{x}_i$$

$$\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot \boxed{g'(x)}$$

$$g(x)$$

$$\nabla L_b = \sum \left(w^T x + b - y_i \right)$$

$$f \to \alpha^2 \qquad \frac{df}{d\alpha} = 2\alpha$$

$$\nabla L_{\vec{w}} \cong \sum \underbrace{\left(w^T x + b - y_i\right)}_{e(x_i)} \cdot \vec{x_i}$$

$$\nabla L_b \cong \sum \left(w^T x + b - y_i\right)$$

$$\nabla L_{\vec{w}} = e(x_1) \cdot x_1 + \ldots = -0.1 \begin{bmatrix} -1 \\ 0 \end{bmatrix} + (-0.1)\begin{bmatrix} 0 \\ -1 \end{bmatrix} + 0 \cdot \begin{bmatrix} 0 \\ 0 \end{bmatrix} +$$

$$+ 0.2 \begin{bmatrix} 0 \\ 2 \end{bmatrix} + 0.3 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + 0.3 \begin{bmatrix} 2 \\ 1 \end{bmatrix} =$$
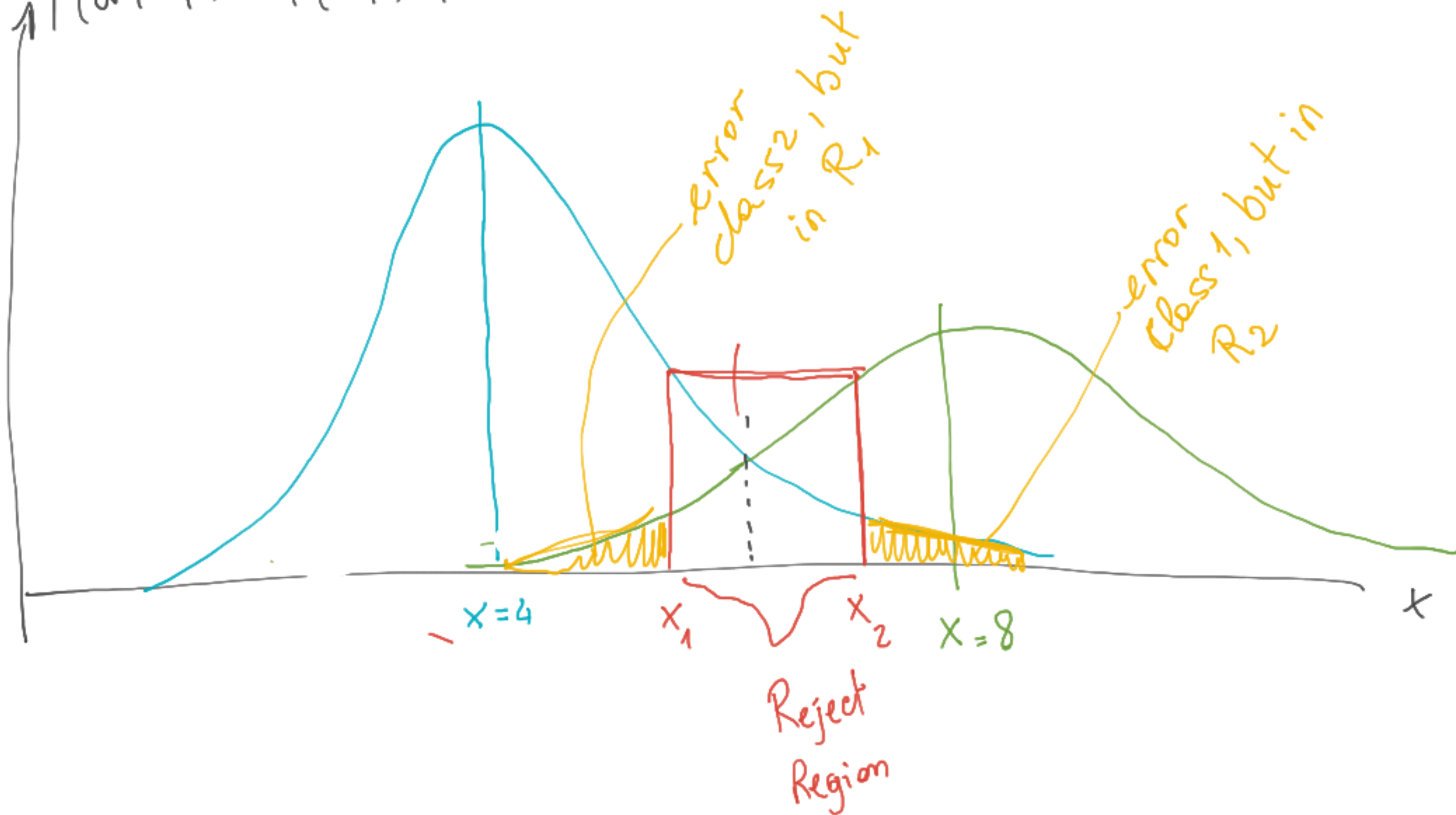
$$= \begin{bmatrix} 0.1 + 0 + 0 + 0 + 0.3 + 0.6 \\ 0 + 0.1 + 0 + 0.4 + 0.6 + 0.3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1.4 \end{bmatrix}$$
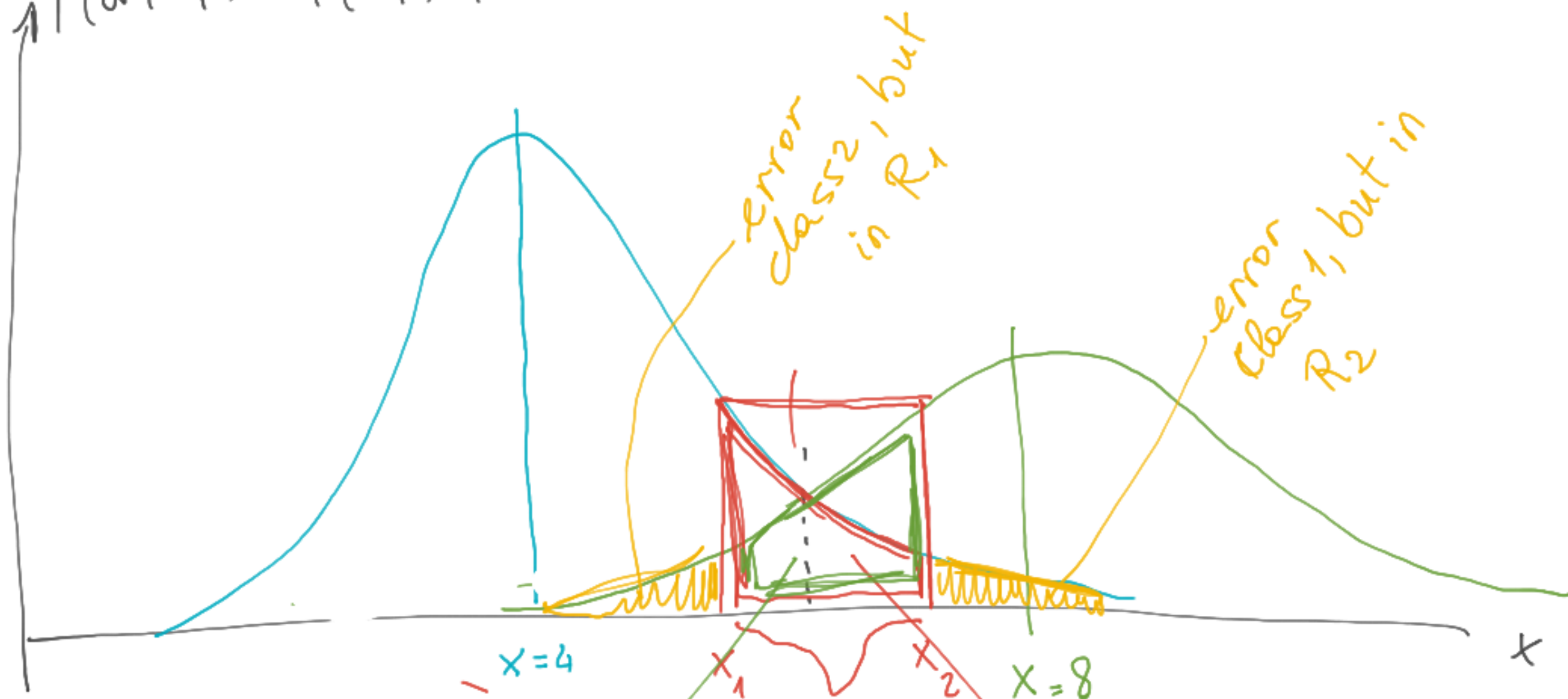
$$b = -0.1 - 0.1 + 0 + 0.2 + 0.3 + 0.3 = 0.6$$

$$W \leftarrow W - \eta \cdot \nabla L_W = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix} - 0.1 \begin{bmatrix} 1 \\ 1.4 \end{bmatrix} = \begin{bmatrix} 0.1 - 0.1 \\ 0.1 - 0.14 \end{bmatrix} = \begin{bmatrix} 0 \\ -0.04 \end{bmatrix}$$

$$b \leftarrow b - \eta \cdot \nabla L_b = 0.1 - (0.1 \cdot 0.6) = 0.04$$
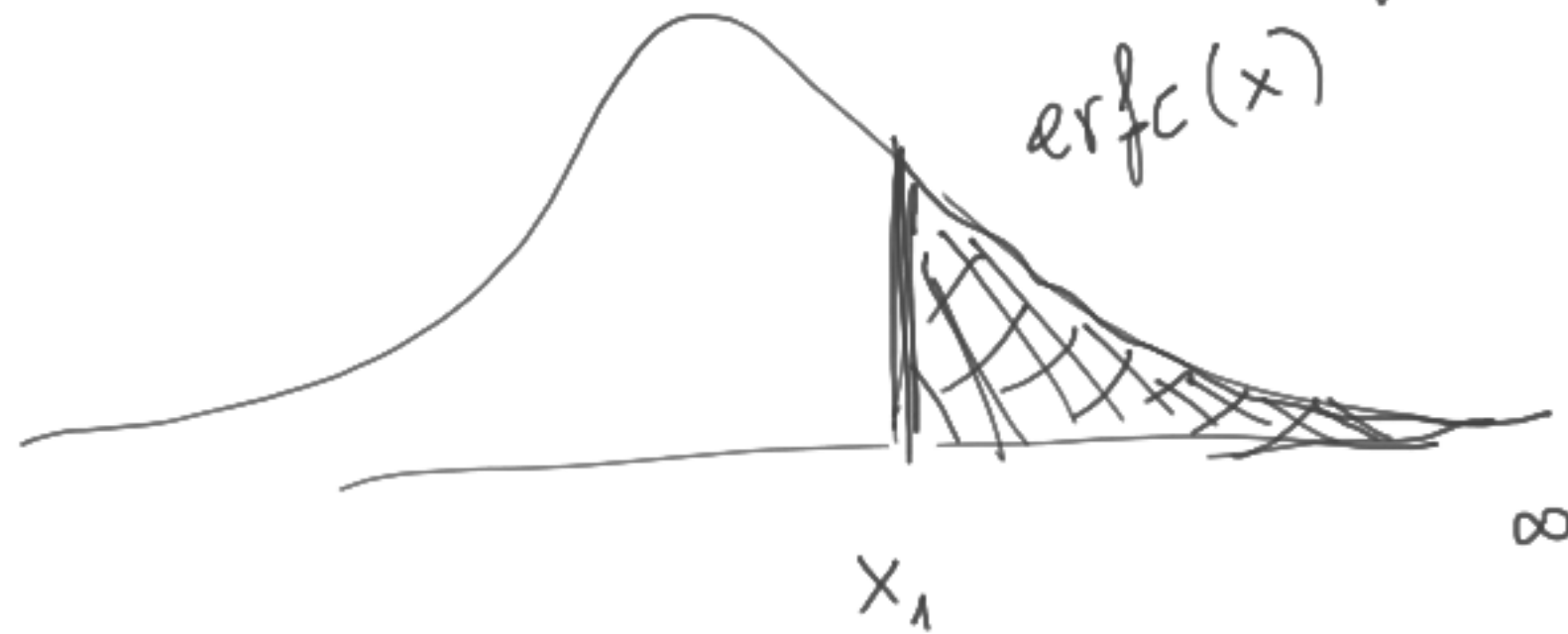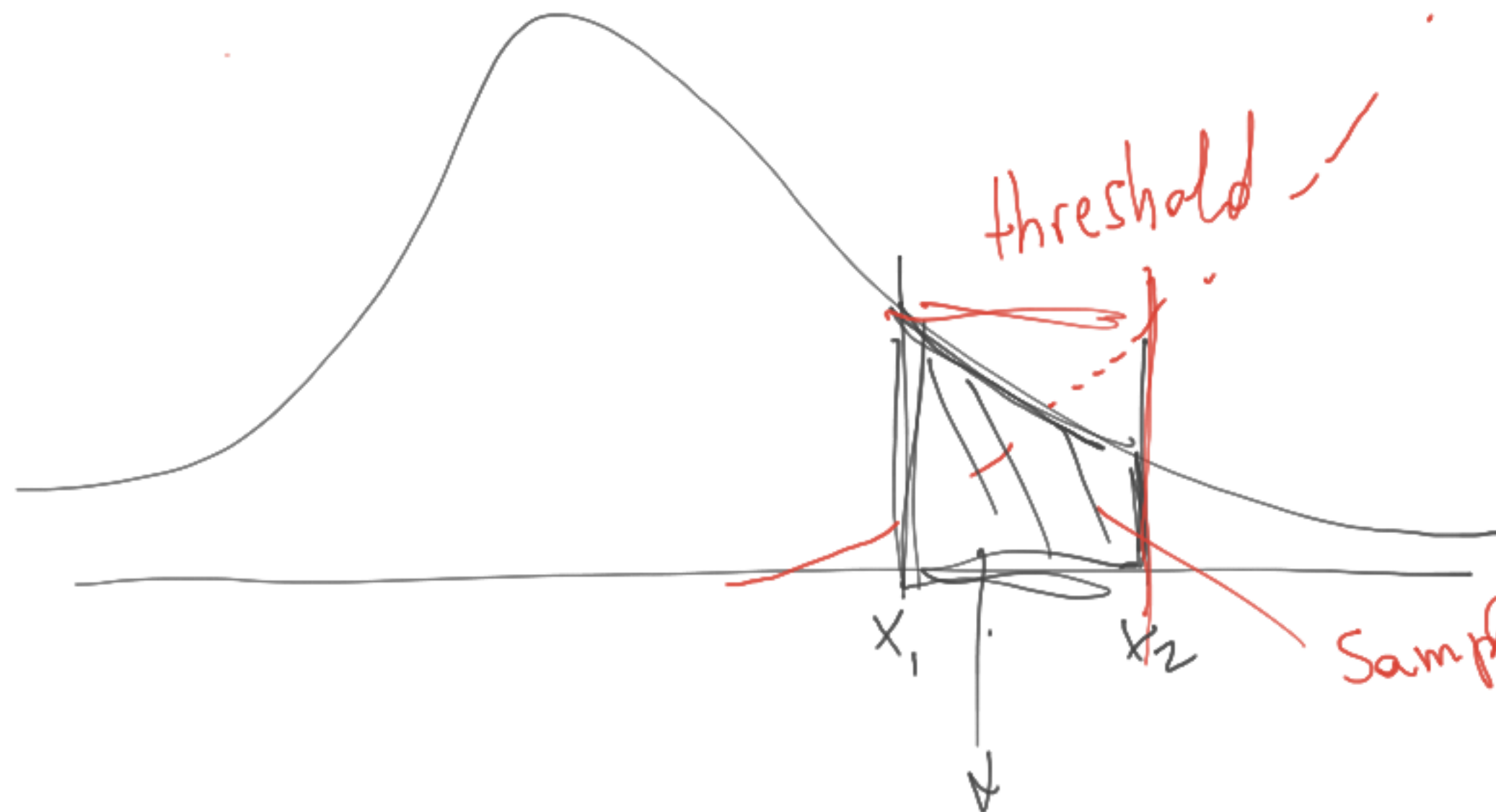
$P(\omega|x_i) = P(\omega_i) \cdot p(x|\omega_i)$

error class2, but in R_1

error class1, but in R_2

$x = 4$

x_1

x_2

$x = 8$

Reject Region

threshold

Samples rejected for class 1

x_1 x_2

$erfc(x)$

∞

x_1

error

x_2

2

$erf(x)$

x_1 x_2

x_2

x_1