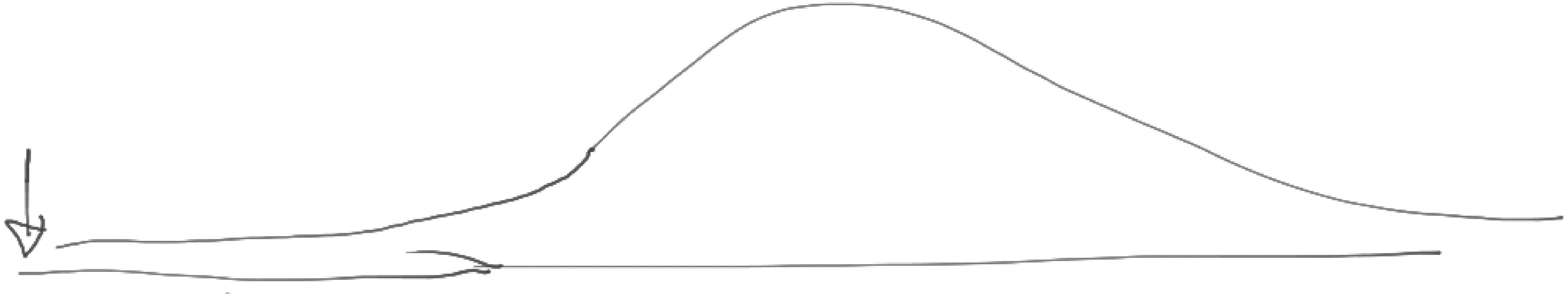
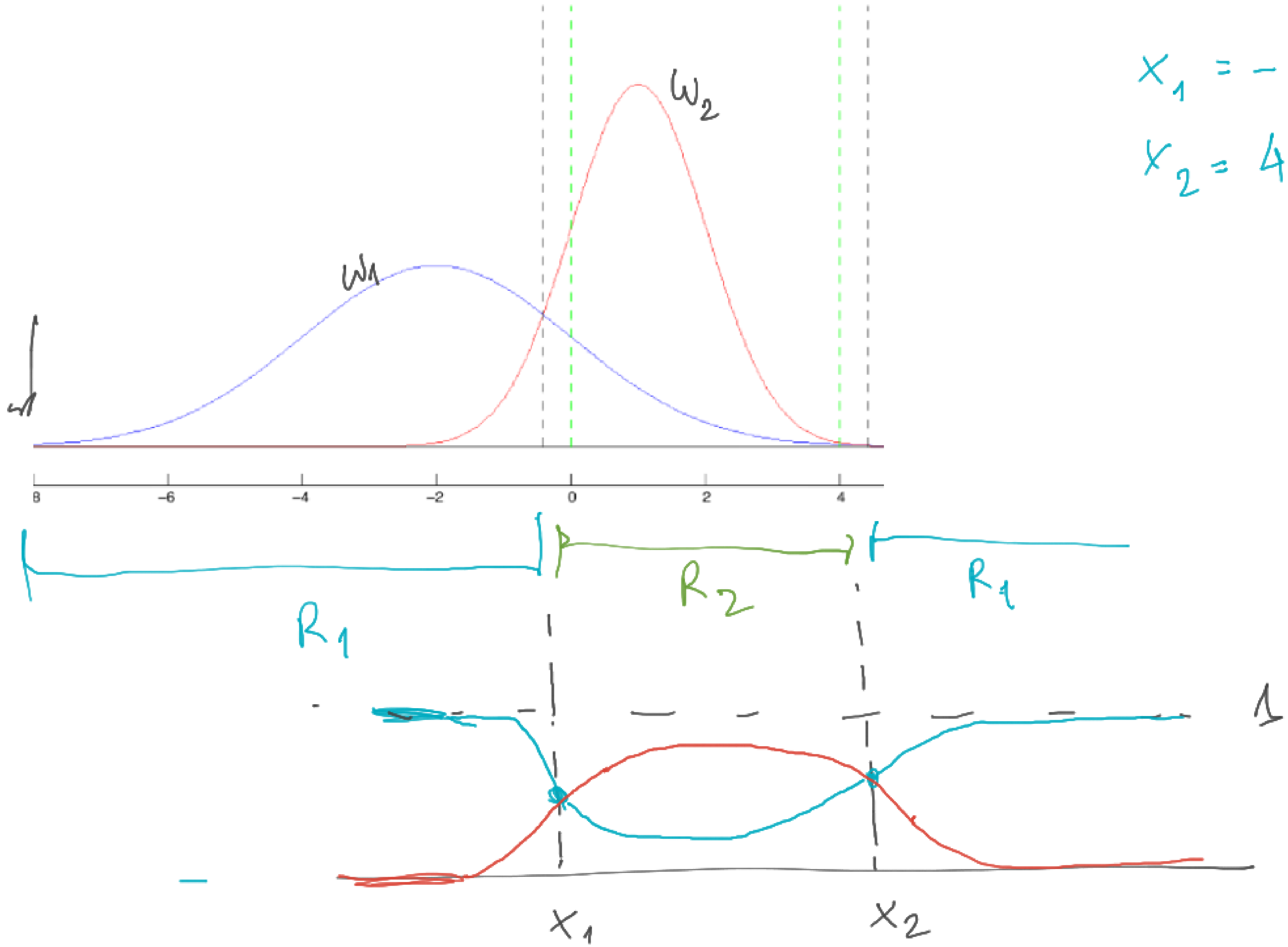
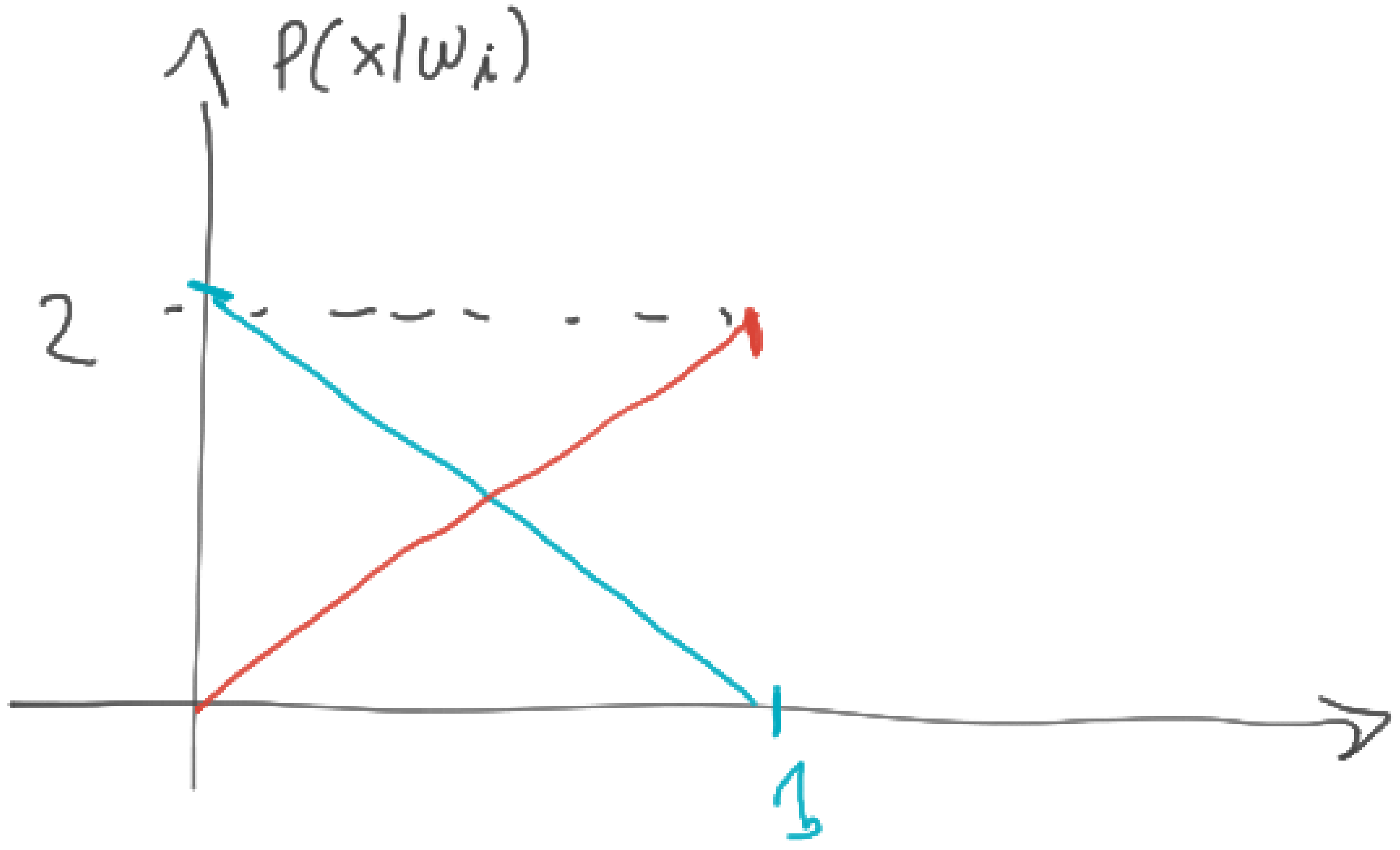
$$P(w_i|x) = \frac{P(w_i)\,P(x|w_i)}{P(x)}$$

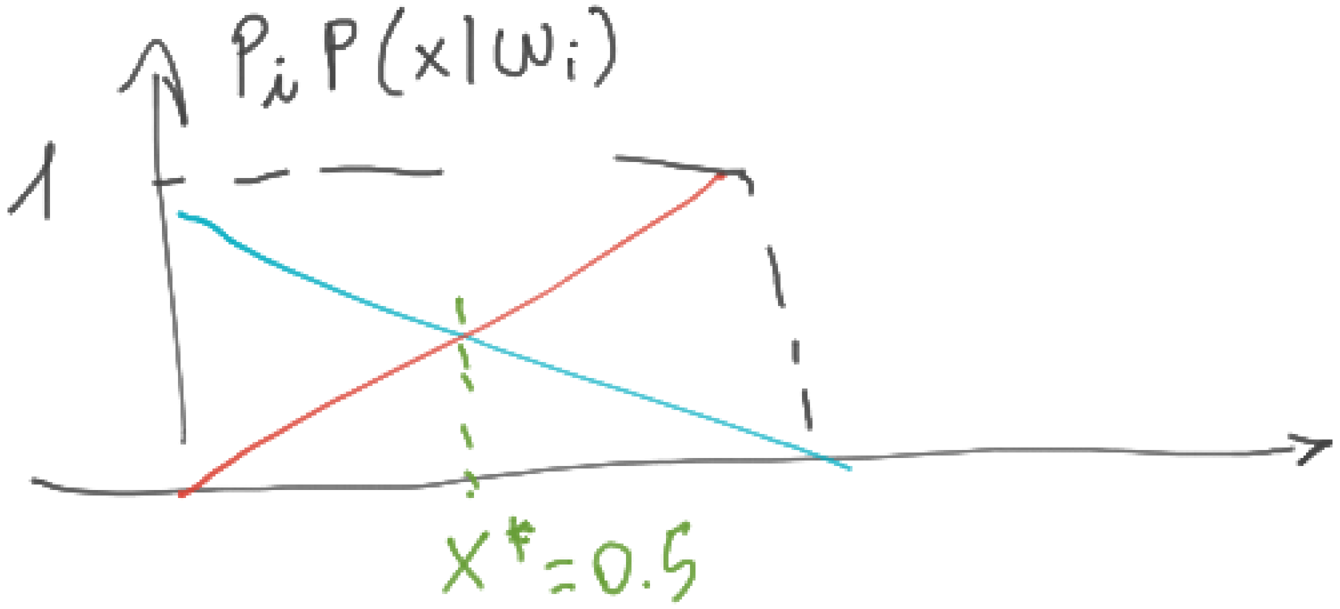




$x_1 = -0{,}4$

$x_2 = 4$





$P(x|\omega_1) = 2 - 2x$

$P(x|\omega_2) = 2x$

$P_1 = \frac{1}{2} \qquad P_2 = \frac{1}{2}$

$x \in [0,1]$

$$P_1\, p(x|\omega_1) = P_2\, p(x|\omega_2)$$

$$\cancel{\frac{1}{2}} \cdot (2-2x) = \cancel{\frac{1}{2}} (2x) \qquad 4x = 2 \rightarrow x^* = \frac{1}{2}$$

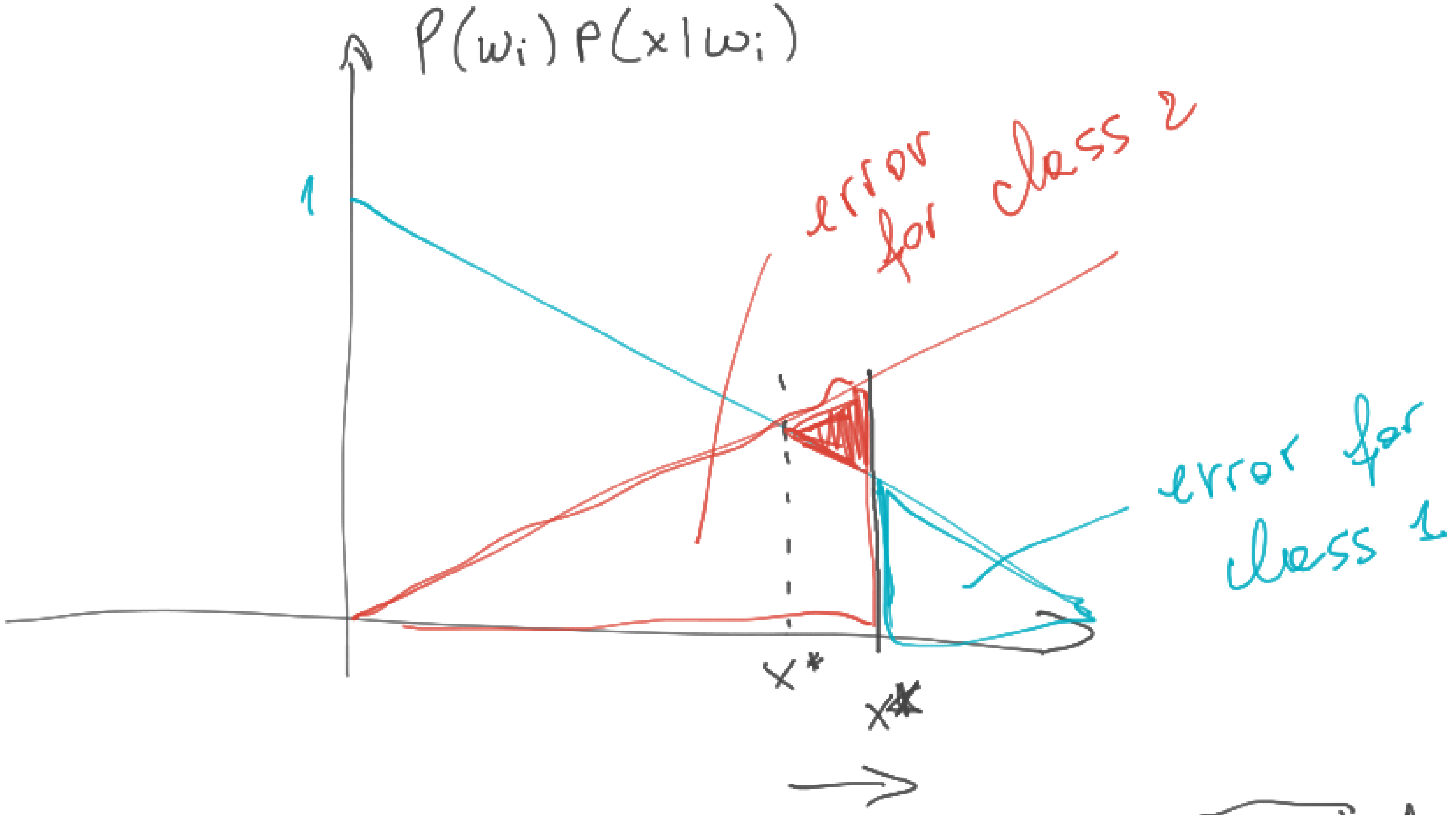
$$R_1 : x \in [0, 0.5]$$

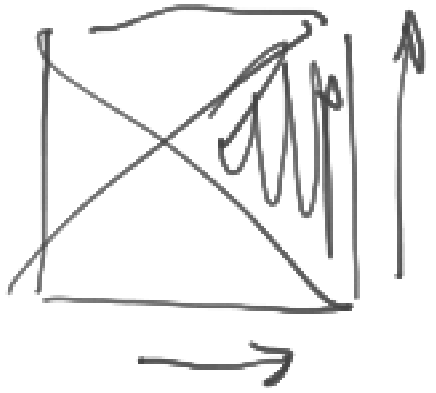
$$R_2 : x \in [0.5, 1]$$

$$P(\text{error}) = P_1 \int_{R_2} (2-2x)\,dx + P_2 \int_{R_1} 2x \, dx =$$

$$= \frac{1}{2}\left[2x - x^2\right]_{0.5}^{1} + \frac{1}{2}\left[x^2\right]_{0}^{0.5} = \frac{1}{2}\left\{\left[\cancel{2} - \cancel{1} - (\cancel{1} - 0.25)\right] + \left[0.25\right]\right\} =$$

$$= 0.25$$

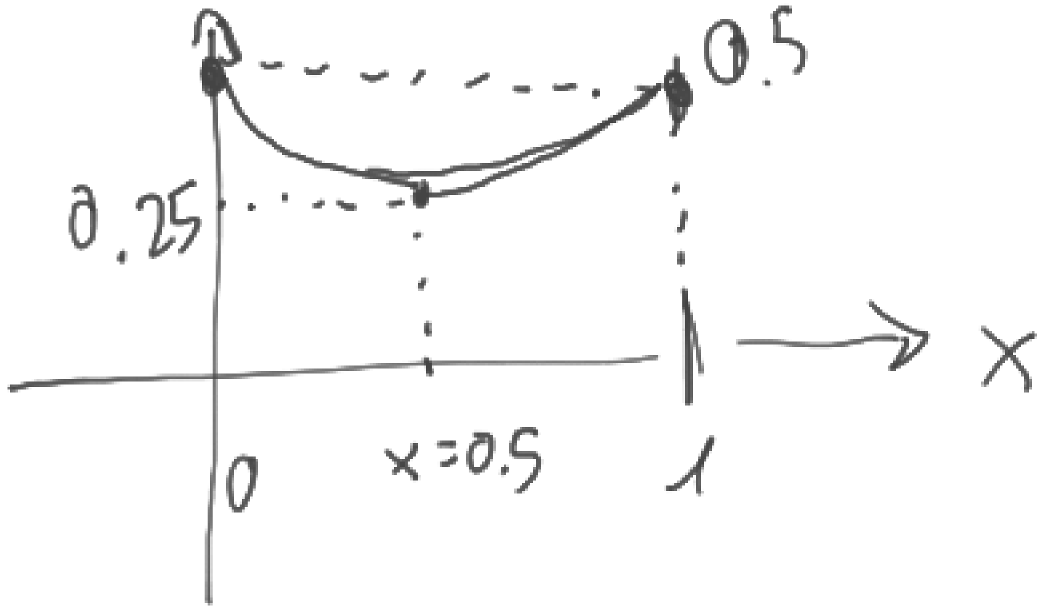

P(wi) P(x|wi)
1
error for class 2
error for class 1
x*
x*



$$P(\text{error}) = P_1 \int_{R_2} (2-2x)\,dx + P_2 \int_{R_1} 2x\,dx =$$

$$= \frac{1}{2}\left[2x - x^2\right]_{x^*}^{1} + \frac{1}{2}\left[x^2\right]_0^{x^*} = \frac{1}{2}\left\{2-1-\left(2x^* - {x^*}^2\right) + {x^*}^2\right\}$$

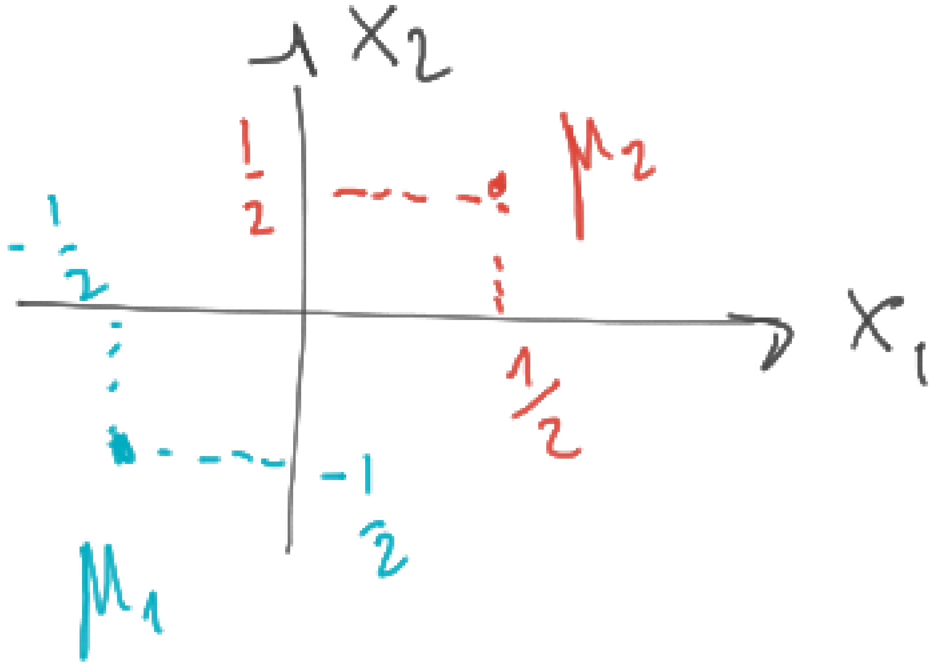
$$= \frac{1}{2}\left(2{x^*}^2 - 2x^* + 1\right) = {x^*}^2 - x^* + \frac{1}{2}$$



$$P(x|\omega_i) = N(\mu_i, \Sigma_i)$$

$$\Sigma = \sigma^2 I \qquad \mu_1 = \begin{pmatrix} -\frac{1}{2} \\ -\frac{1}{2} \end{pmatrix} \qquad \mu_2 = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$$

$$P_1 = \frac{3}{4} \quad P_2 = \frac{1}{4}$$



$$g(x) = \cancel{-\frac{1}{2} x^T \Sigma^{-1} x} + \mu^T \Sigma^{-1} x - \underbrace{\frac{1}{2} \mu^T \Sigma^{-1} \mu} + \ln p(\omega_i) - \cancel{\frac{1}{2} \ln|\Sigma|}$$

the same for both classes

$P(x|\omega_i) = N(\mu_i, \Sigma_i)$

$\Sigma = \sigma^2 I \qquad \mu_1 = \begin{pmatrix} -\frac{1}{2} \\ -\frac{1}{2} \end{pmatrix} \qquad \mu_2 = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$

$P_1 = \frac{3}{4} \quad P_2 = \frac{1}{4}$

x_2

$$\sigma^2 I = \begin{bmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{bmatrix}$$

$$\Sigma^{-1} = \begin{bmatrix} \frac{1}{\sigma^2} & 0 \\ 0 & \frac{1}{\sigma^2} \end{bmatrix}$$

$$x \Sigma^{-1} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \begin{bmatrix} \frac{1}{\sigma^2} & 0 \\ 0 & \frac{1}{\sigma^2} \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{x_1}{\sigma^2} \\ \frac{x_2}{\sigma^2} \end{bmatrix}$$

$$\mu^T \Sigma^{-1} \mu = \frac{1}{\sigma^2} \mu^T \mu$$

$$\mu_1^T \mu = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

$$\mu_2^T \mu = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

$$g_i(x) = \frac{1}{\sigma^2} \mu_i^T x + \ln(p_i)$$

$$g_i(x) = \mu_i^T x + \sigma^2 \ln(p_i)$$

$$g_i(x) = \mu_i^T x + \sigma^2 \ln(p_i)$$

$$\begin{bmatrix} -\frac{1}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \sigma^2 \ln\left(\frac{3}{4}\right) = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \sigma^2 \ln\left(\frac{1}{4}\right)$$

$$-\frac{1}{2}x_1 - \frac{1}{2}x_2 + \sigma^2 \ln\left(\frac{3}{4}\right) = \frac{1}{2}x_1 + \frac{1}{2}x_2 + \sigma^2 \ln\left(\frac{1}{4}\right)$$

$$x_1 + x_2 + \sigma^2 \ln\left(\frac{1}{4}\right) - \sigma^2 \ln\left(\frac{3}{4}\right) = 0$$

$$\frac{1}{\cancel{4}} \cdot \frac{\cancel{4}}{3}$$

$$3$$

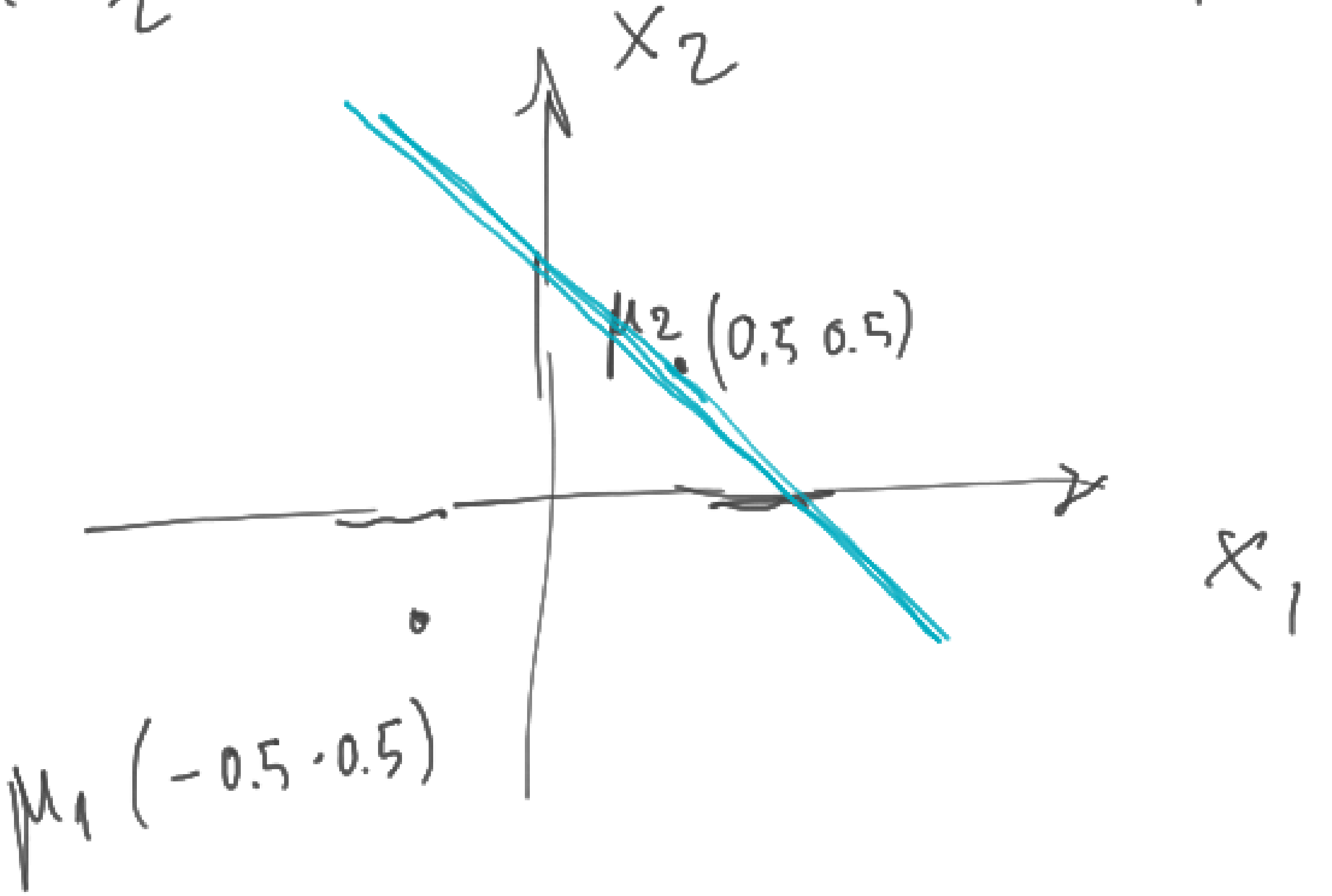
$$\begin{cases} x_1 + x_2 - \underline{\sigma^2 \ln 3} = 0 \\ x_1 + x_2 = 1 \end{cases}$$

$$x_1 + x_2 = \sigma^2 \ln 3$$

$$\uparrow$$

$$\sigma^2 = \frac{1}{\ln 3}$$

$x_1 + x_2 = 1$ $\qquad$ $x_1 = 1 - x_2$



$$\begin{array}{c c} & \overbrace{\hphantom{[[2. \ 2. \ 5. \ 10. \ 10.]]}}^{\text{Distances } x_{tr}} \\ x_{ts} & \begin{bmatrix} 2. & 2. & 5. & 10. & 10. \\ 17. & 13. & 2. & 5. & 9. \\ 1. & 5. & 4. & 5. & 13. \\ 5. & 1. & 2. & 9. & 5. \end{bmatrix} \\ & \begin{array}{ccccc} 2 & 2 & 1 & 1 & 0 \end{array} \end{array}$$

$$\hat{y}_{ts} = \begin{bmatrix} 2 \\ 1 \\ 2 \\ 2 \end{bmatrix}$$

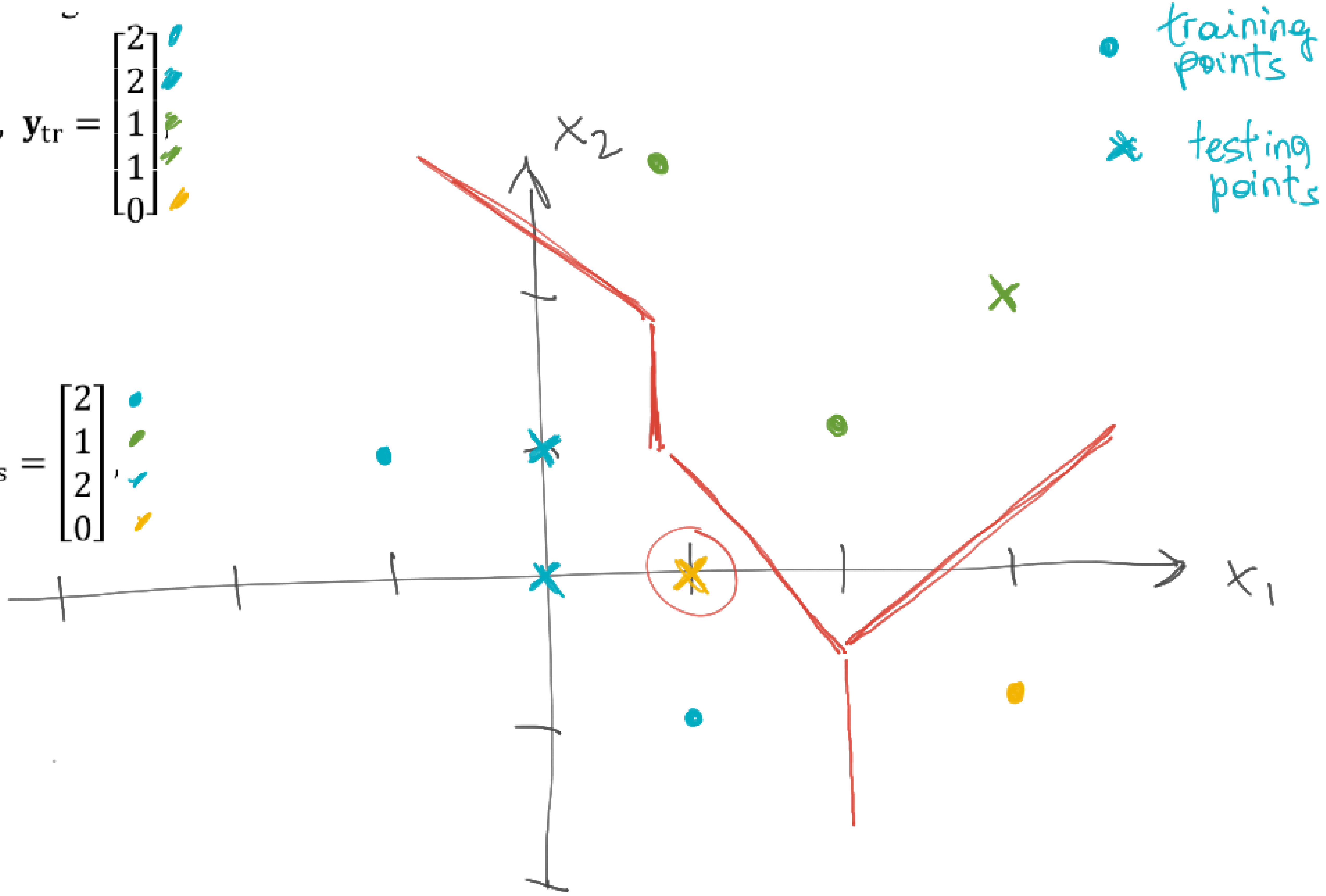
$$\mathbf{x}_{tr} = \begin{bmatrix} -1 & 1 \\ 1 & -1 \\ 2 & 1 \\ 1 & 3 \\ 3 & -1 \end{bmatrix}, \quad \mathbf{y}_{tr} = \begin{bmatrix} 2 \\ 2 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{x}_{ts} = \begin{bmatrix} 0 & 0 \\ 3 & 2 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \mathbf{y}_{ts} = \begin{bmatrix} 2 \\ 1 \\ 2 \\ 0 \end{bmatrix},$$

classification error $= \frac{1}{4}$

$$\hat{y}_{ts} = \begin{bmatrix} 2 \\ 1 \\ 2 \\ 2 \end{bmatrix} \qquad y_{ts} = \begin{bmatrix} 2 \\ 1 \\ 2 \\ \cancel{0} \end{bmatrix}$$

$$\mathbf{x}_{\mathrm{tr}} = \begin{bmatrix} -1 & 1 \\ 1 & -1 \\ 2 & 1 \\ 1 & 3 \\ 3 & -1 \end{bmatrix}, \quad \mathbf{y}_{\mathrm{tr}} = \begin{bmatrix} 2 \\ 2 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\mathbf{x}_{\mathrm{ts}} = \begin{bmatrix} 0 & 0 \\ 3 & 2 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \mathbf{y}_{\mathrm{ts}} = \begin{bmatrix} 2 \\ 1 \\ 2 \\ 0 \end{bmatrix},$$



$$\mathbf{x} = \begin{bmatrix} -2 & 0 \\ 1 & 1 \\ 1 & 2 \\ -1 & 1 \\ -3 & -1 \\ 2 & 1 \end{bmatrix} \qquad \mathbf{v} = \begin{bmatrix} -1 & 0 \\ 1 & 1 \end{bmatrix}:$$

$$v_1 = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \qquad v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & 4 \\ 3 & 0 \\ 4 & 1 \\ 1 & 2 \\ 3 & 6 \\ 4 & 1 \end{bmatrix} \rightarrow d(x_i, v_k)$$

$$d(x_1, v_1) = |-2-(-1)| + |0-0| = 1$$

$$d(x_1, v_2) = |-2-1| + |0-1| = 4$$

$$d(x_2, v_1) = |2| + |1| = 3$$

$$d(x_2, v_2) = 0$$

$$d(x_3, v_1) = |2| + |2| = 4$$

$$d(x_3, v_2) = |0| + |1| = 1$$

$$d(x_4, v_1) = |0| + |1| = 1$$

$$d(x_4, v_2) = |-2| + |0| = 2$$

$$d(x_5, v_1) = |-2| + |-1| = 3$$

$$d(x_5, v_2) = |-4| + |-2| = 6$$

$$d(x_6, v_1) = |3| + |1| = 4$$

$$d(x_6, v_2) = |1| + |0| = 1$$

$$y = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 1 \\ 1 \\ 2 \end{bmatrix}$$

$$L = 1+0+1+1+3+1 = 7$$

$$\mathbf{x} = \begin{bmatrix} -2 & 0 \\ 1 & 1 \\ 1 & 2 \\ -1 & 1 \\ -3 & -1 \\ 2 & 1 \end{bmatrix} \quad y = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 1 \\ 1 \\ 2 \end{bmatrix}$$

$$V_1 = \frac{1}{3}\left[\left[x_1\right] + \left[x_4\right] + \left[x_5\right]\right] = \frac{1}{3}\left[\begin{bmatrix} -2 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 \\ -1 \end{bmatrix} + \begin{bmatrix} -3 \\ -1 \end{bmatrix}\right] = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$$

$$V_2 = \frac{1}{3}\left[\left[x_2\right] + \left[x_3\right] + \left[x_6\right]\right] = \frac{1}{3}\left[\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix}\right] = \begin{bmatrix} 4/3 \\ 4/3 \end{bmatrix}$$

$$\mathbf{x} = \begin{bmatrix} -2 & 0 \\ 1 & 1 \\ 1 & 2 \\ -1 & 1 \\ -3 & -1 \\ 2 & 1 \end{bmatrix}$$

$$v_1 = \begin{bmatrix} -2 \\ 0 \end{bmatrix} \qquad v_2 = \begin{bmatrix} 4/3 \\ 4/3 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & \left|-\frac{10}{3}\right| + \left|\frac{4}{3}\right| \\ |3|+|1| & \left|\frac{1}{3}\right| + \left|\frac{1}{3}\right| \\ |3|+|2| & \left|\frac{1}{3}\right| + \left|\frac{2}{3}\right| \\ |1|+|1| & \left|-\frac{7}{3}\right| + \left|\frac{1}{3}\right| \\ |-1|+|-1| & \left|-\frac{13}{3}\right| + \left|-\frac{7}{3}\right| \\ |4|+|1| & \left|\frac{2}{3}\right| + \left|\frac{1}{3}\right| \end{bmatrix} = \begin{bmatrix} 0 & \frac{14}{3} \\ 4 & \frac{2}{3} \\ 5 & 1 \\ 2 & \frac{8}{3} \\ 2 & \frac{20}{3} \\ 5 & 1 \end{bmatrix}$$

$$y = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 1 \\ 1 \\ 2 \end{bmatrix}$$

same assignments

$$L = 0 + \frac{2}{3} + 1 + 2 + 2 + 1 = 6 + \frac{2}{3}$$

$$\mathbf{x} = \begin{bmatrix} -2 & 0 \\ 1 & 1 \\ 1 & 2 \\ -1 & 1 \\ -3 & -1 \\ 2 & 1 \end{bmatrix} \quad y = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 1 \\ 1 \\ 2 \end{bmatrix}$$

$$V_1 = \begin{bmatrix} -2 \\ 0 \end{bmatrix} \qquad V_2 = \begin{bmatrix} 4/3 \\ 4/3 \end{bmatrix}$$