

# Osnove

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## Programska orodja za statistiko

- splošni programski jeziki, low-level (nizko-nivojski), hitrejši (Fortran, C/C++, Java, Python,...)
- bolj matematično statistični, uporabniku prijaznejši (Mathematica, Matlab/Octave, R, Julia)

## R

- uporablja vektorje, matrike in delo z njimi
- open-source
- najbolj uporabljan jezik v statistiki
- uporabniki dograjujejo njegovo funkcionalnost s programskimi paketi
- instalacija z osnovnim grafičnim vmesnikom
  - RStudio za dodatne funkcionalnosti
  - IDE (integrated development environment)
- dokumentacija dosegljiva prek RStudia, na spletu, forum *stackexchange* (omejite se z R)

## RStudio, Rmarkdown

cheatsheet **rstudio-IDE**

- ukazna vrstica, R Markdown izpis
- help, knjižnice, pregled grafov, pregled datotek v mapi
- globalno okolje (Global Environment), zgodovina ukazov
- datoteke (bližnjice <Ctrl> + <Enter>, <Ctrl> + <Shift> + <Enter>)

cheatsheet: **rmarkdown**

- .Rmd datoteka
- reproducible research (ponovljive raziskave)
- koda v R med tremi enojnimi narekovaji in crko **r**: “`{r }`”
- R koda znotraj besedila med enima enojnima narekovajema in crko **r**: `{r <code>}`
- enačbe kot v LaTeX-u (cheatsheet **LaTeX**)
- za kreiranje PDF dokumentov: `install.packages('tinytex');` `tinytex::install_tinytex()`

## Naloge

- Naredite nov .Rmd dokument kot je tukajle.

## Osnovni gradniki v R

# - znak pred komentarjem

? - znak za izpis pomoči (ali pa uporaba funkcije `help()`)

cheatsheet: **base-r**

## Vrste stavkov

### Izrazi

```
2+4
```

### Prirejanja

```
a1 = 1+6
a2 <- "vaje"
a3 = a1 == a2
```

a1 je objekt, ki je shranjen v globalnem okolju.

Logične primerjave: `==`, `!=`, `<`, `>`, `is.na()`, `is.null()`

Operacije: `+`, `-`, `/`, `*`, `%%`, `%/%` (zadnja dva: deljenje po modulu in celoštevilsko deljenje)

## (Osnovne) vrste objektov

### Skalar

```
as1 = 1 # numeric
as2 = "Uporabna statistika" # character
as3 = TRUE # logical
str(as3)
class(as3)
is.numeric(a1)
as.numeric(a3)
ls() # list of objects
```

Datume bomo obravnavali kasneje.

## Naloge

- Kakšna je numerična vrednost `as3`?
- Kako izbrišemo vse, kar je trenutno v globalnem okolju? Poglejte v `help` funkcije `rm()`.

```

# setting global seed for reproducibility
set.seed(8)

# solutions

# as3 = TRUE
as.numeric(as3)

## [1] 1

# equal to
as.numeric(TRUE)

## [1] 1

# a way to remove all global variables (found in ?rm)
rm(list=ls())

```

Posebne vrednosti:

- NA - not available
- pi -  $\pi$
- NaN - not a number
- Inf - infinite value
- NULL - brez vrednosti, prazno
- TRUE in FALSE (okrajšavi T in F)

## Vektor

```

av1 = c(1,2,3,4,5)
av2 = vector(mode="character",length=4)
av2
class(av2)
av2[1] = "\u017Div\u00E9" # Žive
av2[2] = "naj"
av2[4] = "narodi"
av2
av2[-1]
av3 = 1:10
length(av3)

```

## Naloge

- Naredite vektor **av4**, v katerem so števila in znaki. Izpišite ga na zaslon. Kakšnega tipa je?
- Vektor **av3** skrajšajte:
  - na prve tri znake,
  - na zadnja dva znaka,
  - izberite le vsak drugi znak,
  - vektor naj vsebuje samo lihe številke
- Vektor **av2** podaljšate za naslednjo vrstico Zdravljuje.
- Kaj se zgodi, če seštejemo **av1** in **av3[1:2]**?

Seštevamo lahko tudi vektorje neenakih dolžin, vendar moramo biti pri tem **zelo pazljivi!** (npr. prištevamo skalar)

```

# solutions

```

```

# av4 creation
av4 = c(8, 22, 9, "Alen", "Kahteran")
# printing av4
print(av4)

## [1] "8"          "22"          "9"          "Alen"       "Kahteran"

# type of av4
typeof(av4)

## [1] "character"

# shortening av3 to first 3 elements
av3[1:3]

## [1] 1 2 3

# shortening av3 to last 2 elements
av3[(length(av3) - 1):length(av3)]

## [1] 9 10

# only selecting every second element in av3
av3[seq(2, 10, 2)]

## [1] 2 4 6 8 10

# only selecting odd numbers in av3
av3[av3 %% 2 == 1]

## [1] 1 3 5 7 9

# imputing 3rd element of av2 as it's missing
av2[3] <- "vsi"

# adding the next row of Zdravljica
next_row <- c("ki", "hrepené", "dočakat", "dan")

for (i in 5:8) {
  av2[i] <- next_row[i-4]
}

print(av2)

## [1] "Živé"      "naj"      "vsi"      "narodi"   "ki"      "hrepené" "dočakat"
## [8] "dan"

# what happens when summing av1 and av3[1:2]
print(sum(av1, av3[1:2])) # we get the sum of all elements

## [1] 18

# as I assume the above wasn't the answer that was expected
av1 + av3[1:2]

## [1] 2 4 4 6 6

# this above returns a warning message that we can't sum objects with different
# lengths which are not a multiple of eachother.
# what it does it expands the second vector to match the length and then sums.
# the warning is raised as it has to slice the second vector at the end.

```

```
# this would work (without warning) as only 1 element of av3 was added to each element
# of av1 (objects match by multiple length)
av1 + av3[1]
```

```
## [1] 2 3 4 5 6
```

```
# this could work as well
```

```
av1 + av3[1:5]
```

```
## [1] 2 4 6 8 10
```

```
# this sums av1 and av3[1:5] element-wise (first with first, second with second, etc.)
```

```
# in cases where some object lengths are a multiple of eachother R automatically
# multiplies the shorter one by the multiplier so the lengths match.
```

## Matrika

```
am1 = matrix(c(1:6),nrow=2)
am1
am1[2,3] = 10
am1
am1[2,] # second row
is.matrix(am1)
dim(am1)
# dimensions can be added to the vector
dim(av3) = c(1,10)
# matrix as binded vectors
am2 = cbind(av1,av1)
class(am2)
str(am2)
colnames(am2)
am3 = rbind(av1,av1,rev(av1))
am1[1,]
am1[1,,drop=FALSE] # preserve dimensions
```

## Naloge

- Seštejte dve matriki am1.
- Zmnožite am1z 2.
- Kaj dobite z naslednjim izrazom: `am1*c(1,2)` ? Zakaj?
- Kaj dobite z naslednjim izrazom: `am1*c(1,2,3)` ? Zakaj?
- Kaj dobite, če po vrsticah skupaj združite av1 in vektor 1:8? Zakaj?
- Kaj dobite, če želite po stolpcih združiti av1 in av2? Zakaj?
- Kaj omogoča parameter `byrow` v funkciji `matrix`? Zapišite primer.

Vektorsko množenje in množenje matrik (`%*%`):

```
# solutions
```

```
# sums element-wise
```

```
am1 + am1
```

```
##      [,1] [,2] [,3]
## [1,]    2    6   10
## [2,]    4    8   20
```

```
# multiplies 2 with every element  
am1 * 2
```

```
##      [,1] [,2] [,3]  
## [1,]    2    6   10  
## [2,]    4    8   20
```

```
# multiplies 1st row with 1 and 2nd row with 2  
am1*c(1, 2)
```

```
##      [,1] [,2] [,3]  
## [1,]    1    3    5  
## [2,]    4    8   20
```

```
# multiplies 1st column with 1, 2nd column with 2, 3rd column with 3  
am1*c(1, 2, 3)
```

```
##      [,1] [,2] [,3]  
## [1,]    1    9   10  
## [2,]    4    4   30
```

```
# why? I assume R checks the lengths, as you can't multiply column-wise  
# in this particular example if the vector we're multiplying with is shorter  
# than the matrix size.
```

```
# returns a matrix of size 2x8, as av1 vector is length 5, it expands it  
# so the lengths match (adds first three elements to the end).  
# why? it's somewhat senseless to bind if the lengths don't match.  
# also, if the lengths are a multiple of each other, it does the same but doesn't  
# raise a warning.  
rbind(av1, 1)
```

```
##      [,1] [,2] [,3] [,4] [,5]  
## av1    1    2    3    4    5  
##      1    1    1    1    1
```

```
# this does similarly as above, but in this case as the types don't match  
# it converts the numeric type to character type as the data in matrix can only  
# be of one type. (also expands the first vector as it is shorter)  
cbind(av1, av2)
```

```
##      av1 av2  
## [1,] "1"  "Živé"  
## [2,] "2"  "naj"  
## [3,] "3"  "vsi"  
## [4,] "4"  "narodi"  
## [5,] "5"  "ki"  
## [6,] "1"  "hrepené"  
## [7,] "2"  "dočakat"  
## [8,] "3"  "dan"
```

```
# returns a matrix of size 2x3 where the selected method of filling is column-wise  
# which means first fills the values sequentially in first column, then second, etc.  
matrix(c(1, 1, 1, 2, 2, 2), nrow = 2)
```

```
##      [,1] [,2] [,3]  
## [1,]    1    1    2  
## [2,]    1    2    2
```

```
# returns a matrix of size 2x3 (byrow=FALSE by default)
# similarly as above, byrow setting just changes the filling method to rows.
matrix(c(1, 1, 1, 2, 2, 2), nrow = 2, byrow=TRUE)
```

```
##      [,1] [,2] [,3]
## [1,]    1    1    1
## [2,]    2    2    2
```

```
am1 %>% t(am1) # transpose
av1 %>% t(av1)
as.matrix(av1)
av1 %>% av1
t(av1) %>% av1
```

## Naloge

- Izračunajte naslednji produkt (preverite, ali je rezultat na desni pravilen)

$$\begin{matrix} \text{Matrix A} & & \text{Matrix B} & & \text{Product} \\ \begin{bmatrix} 1 & 4 & 6 & 10 \\ 2 & 7 & 5 & 3 \end{bmatrix} & \cdot & \begin{bmatrix} 1 & 4 & 6 \\ 2 & 7 & 5 \\ 9 & 0 & 11 \\ 3 & 1 & 0 \end{bmatrix} & = & \begin{bmatrix} 93 & 42 & 92 \\ 70 & 60 & 102 \end{bmatrix} \end{matrix}$$

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+ Preverite v help-u, kaj naredi funkcija `solve`. (`?solve`) + Uporabite funkcijo `solve`, da boste dobili kvadratno matriko X dimenzije 4x4, za katero velja, da  $Y \%*\% X = Y$ . Matrika Y naj bo katerakoli matrika dimenzije 4x4. Zakaj dobite tak rezultat?

```
# solutions
```

```
# defining matrix A and matrix B
matr_a <- matrix(c(1, 4, 6, 10,
                  2, 7, 5, 3),
                nrow=2,
                byrow=TRUE)
```

```
print(matr_a)
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    4    6   10
## [2,]    2    7    5    3
```

```
matr_b <- matrix(c(1, 4, 6,
                  2, 7, 5,
                  9, 0, 11,
                  3, 1, 0),
                nrow=4,
                byrow=TRUE)
```

```
print(matr_b)
```

```

##      [,1] [,2] [,3]
## [1,]    1    4    6
## [2,]    2    7    5
## [3,]    9    0   11
## [4,]    3    1    0

# product
prod_matr <- matr_a %*% matr_b

print(prod_matr)

##      [,1] [,2] [,3]
## [1,]   93   42   92
## [2,]   70   60  102

# result from image
result <- matrix(c(93, 42, 92,
                   70, 60, 102),
                 nrow=2,
                 byrow=TRUE)

# do all values match?
all(prod_matr == result)

## [1] TRUE

# solve() solves the equation a %*% x = b for x, where b can be either a vector or a matrix.
# (from ?solve)
matr_y <- matrix(c(1, 2, 3, 4,
                   5, 6, 7, 8,
                   9, 10, 11, 12,
                   13, 14, 15, 16),
                 nrow=4,
                 byrow=TRUE)

print(matr_y)

##      [,1] [,2] [,3] [,4]
## [1,]    1    2    3    4
## [2,]    5    6    7    8
## [3,]    9   10   11   12
## [4,]   13   14   15   16

# this case can't be solved as the matrix is not invertible
# solve(matr_y, matr_y)
# commented due to otherwise .rmd not compiling.

# as this inverts one of the matrixes to solve the problem the matrix must be invertible
# one of the cases when a matrix is not invertible is when the determinant is equal to 0.
det(matr_y)

## [1] 4.733165e-30

# 4.733165e-30 - which is basically 0, within machine precision (floating point error).

# example of a matrix with non-zero determinant
matr_y2 <- matrix(c(1, 0, 2, 0,

```



```

-4, -3, 1, 5,
-6, 2, 4, 1,
0, 1, 3, 0),
nrow=4,
byrow=TRUE)

```

```
print(matr_y2)
```

```
##      [,1] [,2] [,3] [,4]
## [1,]  1   0   2   0
## [2,] -4  -3   1   5
## [3,] -6   2   4   1
## [4,]  0   1   3   0

```

```

# determinant
det(matr_y2)

```

```
## [1] -32
```

```
# usage of solve again.
```

```
ret_matrix <- solve(matr_y2, matr_y2)
```

```

# returns an identity matrix, as this is the only possible solution anyway.
# This is the only matrix which if any matrix is multiplied by the identity matrix
# it returns the "original" matrix

```

```
print(ret_matrix)
```

```
##      [,1] [,2] [,3] [,4]
## [1,]  1   0   0   0
## [2,]  0   1   0   0
## [3,]  0   0   1   0
## [4,]  0   0   0   1

```

## Seznam

```

al1 = list(ime=c("Anton", "Janez"), priimek=c("Novak", "Trilar"), starost=c(67, 58, 34))
al1[1]
al1$priimek
al1$starost[3]
al1[[3]]
al1[[3]][1]
names(al1)

```

Seznami so uporabni za združevanje več (različnih tipov) objektov v skupni objekt. Rezultati funkcij so ponavadi sezname različnih objektov.

## Podatkovni okvir (osnovna statistična tabela)

```

#df1 = data.frame(ime=c("Anton", "Janez"), priimek=c("Novak", "Trilar"), starost=c(67, 58, 34))
df1 = data.frame(ime=c("Anton", "Janez"), priimek=c("Novak", "Trilar"), starost=c(67, 58))
df1
df1$starost
df1[, "priimek"]
df1[[2]]

```

```
dim(df1)
names(df1)
rownames(df1) = c("oseba1", "oseba2")
df1
str(df1)
class(df1)
class(df1$ime)
```

Faktor je poseben tip podatka. Gre za opis kategorialnih podatkov, ki jim lahko priredimo opis posameznih kategorij in povemo, ali so urejeni.

```
af1 = c(0,0,0,0,1,1,1,1) # 4 men, 4 women, numeric
af2 = as.factor(af1)
af2
as.numeric(af2) # factor starts with 1, levels sorted
af3 = factor(af1, levels = c(0,1), labels=c("M","F")) # ordered=TRUE for ordered factors
af3
```

## Naloge

- Iz `af3` izbrišite vse moške in shranite rezultat v `af4`. Kakšen je izpis? Katere vrednosti ima lahko faktor?
- Ali lahko faktor dodamo novo kategorijo? V `af3` poskusite dodati kategorijo 0 - otrok. Kaj se zgodi? Kako boste dodali otroka, da bo to tudi nova kategorija?
- Poženite ukaz `data()`.
- Za podatkovje `USArrests` ugotovite, kaj so statistične enote in preverite, kakšne spremenljivke imamo na voljo (katerega tipa, kaj pomenijo).
- Prikažite le imena držav.
- Izberite iz podatkov vse države, ki imajo vsaj 70% populacije urbane. Koliko jih je?
- Iz podatkov izbrišite spremenljivko `Rape`.
- Kaj se zgodi, če `USArrests` spremenimo v matriko? Zakaj?

```
# solutions
```

```
# remove all "M" values.
af4 <- af3[af3 != "M"]
```

```
print(af4)
```

```
## [1] F F F F
## Levels: M F
```

```
# output returns only "F" values. Factor can only hold specific values which are usually
# predetermined. In this case af4 can hold "M" and "F".
```

```
# can we add a new factor category? yes
# labels and levels needed to be changed as they now have different values.
factor(af3,
      levels=c("M", "F", "0"),
      labels=c("M", "F", "0")
)
```

```
## [1] M M M M F F F F
## Levels: M F 0
```

```
# if we wanted to do the same from af1, and number 2 represent a child ("0")
factor(af1,
```

```

    levels=c(0, 1, 2),
    labels=c("M", "F", "O")
  )

## [1] M M M M F F F F
## Levels: M F O

# data() returns all data sets within R.

### USArrests

# the statistical units are US states.

# Murder represents Murder arrests per 100.000 people.
# Assault represents Assault arrests per 100.000 people.
# UrbanPop represents percent of urban population.
# Rape represents Rape arrests per 100.000 people.

# all of the values are numeric (double or integer)
typeof(USArrests$Murder)

## [1] "double"

typeof(USArrests$Assault)

## [1] "integer"

typeof(USArrests$UrbanPop)

## [1] "integer"

typeof(USArrests$Rape)

## [1] "double"

# displaying US state names.
rownames(USArrests)

## [1] "Alabama"      "Alaska"      "Arizona"      "Arkansas"
## [5] "California"   "Colorado"    "Connecticut"  "Delaware"
## [9] "Florida"      "Georgia"     "Hawaii"       "Idaho"
## [13] "Illinois"     "Indiana"     "Iowa"         "Kansas"
## [17] "Kentucky"     "Louisiana"   "Maine"        "Maryland"
## [21] "Massachusetts" "Michigan"    "Minnesota"    "Mississippi"
## [25] "Missouri"     "Montana"     "Nebraska"     "Nevada"
## [29] "New Hampshire" "New Jersey"  "New Mexico"   "New York"
## [33] "North Carolina" "North Dakota" "Ohio"         "Oklahoma"
## [37] "Oregon"       "Pennsylvania" "Rhode Island" "South Carolina"
## [41] "South Dakota" "Tennessee"   "Texas"        "Utah"
## [45] "Vermont"      "Virginia"    "Washington"   "West Virginia"
## [49] "Wisconsin"    "Wyoming"

# selecting US states with at least 70% urban population
USArrests[USArrests$UrbanPop >= 70, ]

##           Murder Assault UrbanPop Rape
## Arizona      8.1    294      80 31.0
## California   9.0    276      91 40.6

```

```
## Colorado      7.9      204      78 38.7
## Connecticut   3.3      110      77 11.1
## Delaware      5.9      238      72 15.8
## Florida       15.4     335      80 31.9
## Hawaii        5.3       46      83 20.2
## Illinois      10.4     249      83 24.0
## Massachusetts 4.4      149      85 16.3
## Michigan      12.1     255      74 35.1
## Missouri       9.0      178      70 28.2
## Nevada        12.2     252      81 46.0
## New Jersey     7.4      159      89 18.8
## New Mexico     11.4     285      70 32.1
## New York       11.1     254      86 26.1
## Ohio          7.3      120      75 21.4
## Pennsylvania   6.3      106      72 14.9
## Rhode Island   3.4      174      87  8.3
## Texas          12.7     201      80 25.5
## Utah           3.2      120      80 22.9
## Washington     4.0      145      73 26.2
```

```
# how many?
```

```
nrow(USArrests[USArrests$UrbanPop >= 70, ])
```

```
## [1] 21
```

```
# remove Rape from data.frame
```

```
USArrests$Rape <- NULL
```

```
print(USArrests)
```

```
##           Murder  Assault  UrbanPop
## Alabama      13.2     236      58
## Alaska       10.0     263      48
## Arizona       8.1     294      80
## Arkansas      8.8     190      50
## California    9.0     276      91
## Colorado      7.9     204      78
## Connecticut   3.3     110      77
## Delaware      5.9     238      72
## Florida       15.4     335      80
## Georgia       17.4     211      60
## Hawaii        5.3      46      83
## Idaho         2.6     120      54
## Illinois      10.4     249      83
## Indiana       7.2     113      65
## Iowa          2.2      56      57
## Kansas        6.0     115      66
## Kentucky      9.7     109      52
## Louisiana     15.4     249      66
## Maine         2.1      83      51
## Maryland      11.3     300      67
## Massachusetts 4.4     149      85
## Michigan      12.1     255      74
## Minnesota     2.7      72      66
## Mississippi   16.1     259      44
## Missouri      9.0     178      70
```

```
## Montana      6.0      109      53
## Nebraska     4.3      102      62
## Nevada       12.2     252      81
## New Hampshire 2.1       57      56
## New Jersey    7.4     159      89
## New Mexico    11.4    285      70
## New York      11.1    254      86
## North Carolina 13.0    337      45
## North Dakota  0.8       45      44
## Ohio          7.3     120      75
## Oklahoma      6.6     151      68
## Oregon        4.9     159      67
## Pennsylvania  6.3     106      72
## Rhode Island  3.4     174      87
## South Carolina 14.4    279      48
## South Dakota  3.8       86      45
## Tennessee     13.2    188      59
## Texas         12.7    201      80
## Utah          3.2     120      80
## Vermont       2.2       48      32
## Virginia      8.5     156      63
## Washington    4.0     145      73
## West Virginia 5.7       81      39
## Wisconsin     2.6       53      66
## Wyoming       6.8     161      60
```

*# what happens when converting `USArrests` into a matrix, why?*

```
matrix(USArrests)
```

```
##      [,1]
## [1,] Numeric,50
## [2,] Integer,50
## [3,] Integer,50
```

*# it uses each column as an object which is then placed sequentially  
 # Why? as data.frame is basically a list of lists (or list of vectors)  
 # in the background. So basically matrix() function is trying to convert a list  
 # of 3 objects into a matrix which it succeeds. It is usually not what we want*

## Array (večdimenzionalna tabela)

```
aa = array(dim=c(3,5,2))
aa[1,,2] = "M"
aa[, ,1] = 3
dimnames(aa) = list(c("oseba1","oseba2","oseba3"),c("cas1","cas2","cas3","cas4","cas5"),c("treatment","treatment2"))
```