Red-black tree deletion steps
RB-DELETE

Pick one
- If the node we deleted has 2 NIL children, its replacement x is NIL.

- If the node we deleted has 1 NIL child and 1 non-NIL child, its replacement x is the non-NIL child.

- If the node we deleted has 2 non-NIL children, set x to the replacement's right child before the replacement is spliced out.

Pick one
- If the node we deleted is red and its replacement is red or NIL, we are done.

- If the node we deleted is red and its replacement is black and non-NIL, color the replacement red and proceed to the appropriate case.

- If the node we deleted is black and its replacement is red, color the replacement black. We are done.

- If the node we deleted is black, its replacement is black, and x is the root of the tree, we are done.

- If the node we deleted is black, its replacement is black, and x is not the root of the tree, proceed to the appropriate case.

RB-DELETE-FIXUP

Cases

0. Node x is red

1. Node x is black and its sibling w is red

2. Node x is black, its sibling w is black, and both of w's children are black

3. Node x is black, its sibling w is black, and

   - If x is the left child, w's left child is red and w's right child is black
   - If x is the right child, w's right child is red and w's left child is black

4. Node x is black, its sibling w is black, and

   - If x is the left child, w's right child is red
   - If x is the right child, w's left child is red

Case 0: Node x is red

  1. Color x black. We are done.

Case 1: Node x is black and its sibling w is red

  1. Color w black

  2. Color $x.p$ red

  3. Rotate $x.p$

    a. If x is the left child do a left rotation

    b. If x is the right child do a right rotation

  4. Change w

    a. If x is the left child set $w = x.p.right$

    b. If x is the right child set $w = x.p.left$

  5. With x and our new w, decide on case 2, 3, or 4 from here.

Case 2: Node x is black, its sibling w is black, and both of w's children are black

  1. Color w red

  2. Set $x = x.p$

    a. If our new x is red, proceed to case 0.

    b. If our new x is black and is the root of the tree, we are done.

    c. If our new x is black and is not the root of the tree, decide on case 1, 2, 3, or 4 from here. Note that we have a new w now.

Case 3: Node x is black, its sibling w is black, and

  • If x is the left child, w's left child is red and w's right child is black

  • If x is the right child, w's right child is red and w's left child is black

  1. Color w's child black

    a. If x is the left child, color $w.left$ black

b. If x is the right child, color $w.right$ black

2. Color w red

3. Rotate w

   a. If x is the left child do a right rotation

   b. If x is the right child do a left rotation

4. Change w

   a. If x is the left child set $w = x.p.right$

   b. If x is the right child set $w = x.p.left$

5. Proceed to case 4.

Case 4: Node x is black, its sibling w is black, and

- If x is the left child, w's right child is red

- If x is the right child, w's left child is red

1. Color w the same color as $x.p$

2. Color $x.p$ black

3. Color w's child black

   a. If x is the left child, color $w.right$ black

   b. If x is the right child, color $w.left$ black

4. Rotate $x.p$

   a. If x is the left child do a left rotation

   b. If x is the right child do a right rotation

5. We are done.