Red-black tree deletion steps

RB-DELETE

Pick one

- \bullet If the node we deleted has 2 NIL children, its replacement x is NIL.
- ullet If the node we deleted has 1 NIL child and 1 non-NIL child, its replacement x is the non-NIL child.
- \bullet If the node we deleted has 2 non-NIL children, set x to the replacement's right child before the replacement is spliced out.
- If the node we deleted is red and its replacement is red or NIL, we are done.
- If the node we deleted is red and its replacement is black and non-NIL, color the replacement red and proceed to the appropriate case.
- Pick one
- If the node we deleted is black and its replacement is red, color the replacement black. We are done.
- ullet If the node we deleted is black, its replacement is black, and x is the root of the tree, we are done.
- If the node we deleted is black, its replacement is black, and x is not the root of the tree, proceed to the appropriate case.

RB-DELETE-FIXUP

Cases

- 0. Node x is red
- 1. Node x is black and its sibling w is red
- 2. Node x is black, its sibling w is black, and both of w's children are black
- 3. Node x is black, its sibling w is black, and
 - If x is the left child, w's left child is red and w's right child is black
 - If x is the right child, w's right child is red and w's left child is black
- 4. Node x is black, its sibling w is black, and
 - If x is the left child, w's right child is red
 - If x is the right child, w's left child is red

Case 0: Node x is red

1. Color x black. We are done.

Case 1: Node x is black and its sibling w is red

- 1. Color w black
- 2. Color x.p red
- 3. Rotate x.p
 - a. If x is the left child do a left rotation
 - b. If x is the right child do a right rotation
- 4. Change w
 - a. If x is the left child set w = x.p.right
 - b. If x is the right child set w = x.p.left
- 5. With x and our new w, decide on case 2, 3, or 4 from here.

Case 2: Node x is black, its sibling w is black, and both of w's children are black

- 1. Color w red
- 2. Set x = x.p
 - a. If our new x is red, color x black. We are done.
 - b. If our new x is black and is the root of the tree, we are done.
 - c. If our new x is black and is not the root of the tree, decide on case 1, 2, 3, or 4 from here. Note that we have a new w now.

Case 3: Node x is black, its sibling w is black, and

- If x is the left child, w's left child is red and w's right child is black
- If x is the right child, w's right child is red and w's left child is black
- 1. Color w's child black
 - a. If x is the left child, color w.left black

- b. If x is the right child, color w.right black
- 2. Color w red
- 3. Rotate w
 - a. If x is the left child do a right rotation
 - b. If x is the right child do a left rotation
- 4. Change w
 - a. If x is the left child set w = x.p.right
 - b. If x is the right child set w = x.p.left
- 5. Proceed to case 4.

Case 4: Node x is black, its sibling w is black, and

- If x is the left child, w's right child is red
- If x is the right child, w's left child is red
- 1. Color w the same color as x.p
- 2. Color x.p black
- 3. Color w's child black
 - a. If x is the left child, color w.right black
 - b. If x is the right child, color w.left black
- 4. Rotate x.p
 - a. If x is the left child do a left rotation
 - b. If x is the right child do a right rotation
- 5. We are done.