# Lab 5

## Problem 1: Component Analysis

An electrical engineer supervises the production of three types of electrical components. Three kinds of material -- metal, plastic, and rubber -- are required for production. The amounts needed to produce each component are:

| Component | Metal (g/component) | Plastic (g/component) | Rubber (g/component) |
|---|---|---|---|
| **1** | 15 | 0.30 | 1.0 |
| **2** | 17 | 0.40 | 1.2 |
| **3** | 19 | 0.55 | 1.5 |

Suppose that totals of 3.89, 0.095, and 0.282 kg of metal, plastic, and rubber, respectively, are used each day. We wish to find how many of each component has been produced per day.

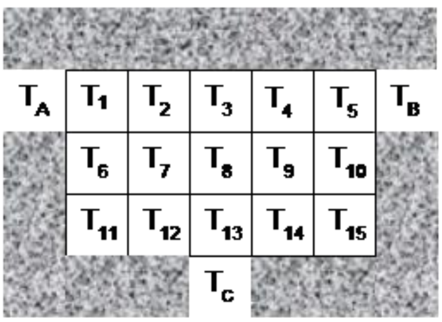
a. **Give the equations for this linear system**, making sure to identify what the unknowns are.
b. **Solve your system from part a using MATLAB** by creating the coefficient matrix and data vector and solving the system using the backslash operator.

## Problem 2: Heat Plate Linear System

Solve a linear system to determine the temperature at various locations across a heat plate. We have simplified the problem to focus on the general concepts used when modeling such a problem as a system of linear equations.

Imagine that you wish to know the temperature at all points on a plate that is insulated except for three positions on the edge where heating elements of known temperature are applied. At some point when the temperature at all positions on the plate have stabilized, what will the temperature at each point be?

To solve such a problem, we imagine the plate surface divided into equal sized "tiles" and restate the problem to find the temperature at the center of each tile. We know that the temperature of each tile depends upon its neighboring temperatures. In fact, it is a weighted average of those temperatures based on the proportion of our tile's edge that is shared with its neighbor. Here is a rectangular "plate" divided into 15 equal area tiles. Assume that each edge of each tile has identical length, so that the temperature of any one tile is a simple average of the temperatures of all neighboring tiles that share a side with the tile in question. This creates a system of 15 equations, one for each "tile" area.




$T_A$, $T_B$, and $T_C$ are the non-insulated positions along the edge where a heat source of known temperature will be applied. The temperatures at $T_1$ through $T_{15}$ are the unknowns that you wish to find.

a. **Define a function named `heatPlate` that accepts three temperature values `TA`, `TB`, and `TC`** (representing $T_A$, $T_B$, and $T_C$, respectively) and returns a single matrix result that is a 3x5 matrix of temperatures for the unknowns $T_1$ through $T_{15}$ in the order labeled above. If you can not figure out how to return the values as a 3x5 matrix, return them as a column vector for a minor (1/2 pt) deduction.

   To solve this problem, identify 15 equations. Each equation represents the temperature value for one tile based upon the temperature values of each of its neighboring tiles. For example, the temperature at $T_7$ is equal to the average of the temperatures: $T_2$, $T_6$, $T_8$, and $T_{12}$.

   *What about the corners?* Well, since the gray area is insulation, the only effect is from the non-insulating tiles. Thus, $T_{11}$ is equal to the average of only $T_6$ and $T_{12}$.

   *How do $T_A$, $T_B$, and $T_C$ factor into the equations?* They will be in the equations, but since their values will be known when the function is called, the terms with $T_A$, $T_B$, and $T_C$ should be on the right-hand side of the equations once the equations are put into general form.

   DO NOT SOLVE FOR ANY OF THE TEMPERATURES OF THIS PROBLEM BY HAND. Work out the equations for each tile and then reorder the terms in general form, with the unknown terms on the left-hand side and the known terms on the right-hand side of the equation. Once you have the equations, rewrite the linear system as a matrix equation and put the commands to solve this problem in your `heatPlate(TA,TB,TC)` function definition.

b. **Write code to call your `heatPlate` function to compute the values of `T1` through `T15` for each of the following conditions.** Do not suppress the output of your function calls.
   i. Find `T1` through `T15` when `TA` = 50, `TB` = 50, and `TC` = 50
   ii. Find `T1` through `T15` when `TA` = 20, `TB` = 20, and `TC` = 80
   iii. Find `T1` through `T15` when `TA` = 15, `TB` = 95, and `TC` = 40

## Problem 3: Interpolation

The table below lists values for dissolved oxygen concentration in water (with a chloride concentration of 20 g/L) as a function of temperature.

| ***T* (C)** | 0 | 5 | 10 | 15 | 20 | 25 | 30 |
|---|---|---|---|---|---|---|---|
| **Dissolved oxygen (mg/L)** | 11.4 | 10.3 | 8.96 | 8.08 | 7.35 | 6.73 | 6.20 |

We ultimately wish to estimate the dissolved oxygen concentration at a temperature of 2.5 C.

a. **Interpolate the data** in two ways:
   1. using a polynomial that goes through all the data points
   2. using a polynomial of the appropriate degree that goes through the first 3 data points

   Display the coefficients of each interpolating polynomial.

b. **Plot the data points and the interpolating polynomials on a single figure.** Use black circles to represent the data points. Plot the polynomial through all the data points over the range *T* = 0 to 30 C and as a magenta dashed line. Plot the polynomial through just the first 3 data points over the range *T* = 0 to 10 C and as a solid cyan line.
c. **Estimate the dissolved oxygen concentration at a temperature of 2.5 C in three ways:**
   i. using your polynomial interpolation through all the data points
   ii. using your polynomial interpolation through the first 3 data points
   iii. using a spline interpolation
d. **Which of your estimates from part c is the best? Which is the worst?** Briefly justify your answers.

## Problem 4: Approximation

For the data in the following table, the $y$ data obviously decays as $x$ increases:

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ***x* data** | 1.00 | 2.12 | 3.09 | 5.23 | 7.64 | 9.14 | 11.2 | 14.3 | 16.1 | 19.3 | 22.7 | 25.0 |
| ***y* data** | 50.6 | 44.1 | 40.3 | 31.2 | 28.7 | 24.8 | 23.2 | 18.3 | 16.2 | 14.8 | 12.9 | 12.6 |

a. **Fit this data to a curve of the form:**

$$y = \frac{1}{ln(ax^2 + bx + c)}$$

Make sure to display and identify the values you get for $a$, $b$, and $c$. (*Hint: transform the curve to a polynomial.*)

b. **Plot the data points and the approximating function** over the range $x$ = 1 to 25 on a single figure. Use blue pentagrams to represent the data points and a red dash-dot line for the approximating curve.

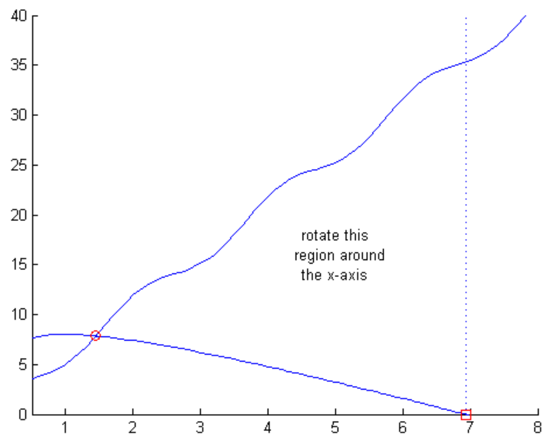
c. **Estimate the value of *y* at *x* = 8** using your approximating function.

## Problem 5: Volume of Revolution

This problem deals with the two curves:

$y = 5x + \cos(3x) + 1$
$y = 10 - 2x + \ln(x^2)$

These curves are displayed in the figure below:




a. **Determine the *x*- and *y*-coordinates of the point where the two curves intersect** (indicated with a red circle in the figure above). Make sure your script displays the coordinates of the intersection point.

b. **Determine where the second curve intersects the *x*-axis** (indicated with a red square in the figure above). Make sure your script displays this value.

**Determine the volume of the solid formed from rotating around the *x*-axis the region between the two curves** from where they intersect until where the second curve intersects the $x$-axis. Note that you will need to use your points from parts a and b.