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ON BENCHMARKING FUNCTIONS FOR GENETIC ALGORITHMS

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This paper presents experimental results on the major benchmarking functions used for performance evaluation of Genetic Algorithms (GAs). Parameters considered include the effect of population size, crossover probability, mutation rate and pseudorandom generator. The general computational behavior of two basic GAs models, the Generational Replacement Model (GRM) and the Steady State Replacement Model (SSRM) is evaluated.

Keywords: Genetic algorithms; Benchmarking functions; Population size; Mutation rate; Pseudo-random number generation

C.R. Categories: I.2.8, F.2.2

1. INTRODUCTION

Genetic Algorithms (GAs) are a class of probabilistic algorithms that are loosely based on biological evolution. This paper presents experimental results on the major benchmarking functions used for performance evaluation of Genetic Algorithms (GAs). GAs rely heavily on random number generators. In addition, each of the basic genetic operators used in a simple GA (crossover, mutation) utilizes "random choice" to one extent or another. Many researchers of the field have frequently used various function groups in order to study the performance of GAs while a big part of the research involves the definition of control parameters and their potential

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adjustment. Up to now, no general conclusions are available concerning the optimum parameterization of operators.

In this paper we examine 14 different functions in order (a) to test specific parameter of GAs and (b) to evaluate the general computational behavior of GAs. The available theoretical analysis on genetic algorithms does not offer a tool which could help in a generalized adjustment of control parameters, leaving the choice of the proper operators, parameters and mechanisms to depend on the problem's demands, and the experience and preferences of the researcher.

The structure of this paper as follows: Section 2 describes the functions used in this paper. In Section 3 we set out the methodology used and then we present the experimental results of our experiments. Finally we draw some general conclusions.

2. SET OF BENCHMARKING FUNCTIONS

Taking the above under consideration and given the nature of our study, we concluded to a total of 14 optimization functions. Tables I–II summarizes some of the widely-used test functions. Next, we will try to make a short reference to these functions and to justify our selection of this specific set.

The first five test functions have been proposed by Dejong. All test functions reflect different degrees of complexity. Test functions F1–F4 are unimodal (*i.e.*, containing only one optimum), whereas the other test functions are multimodal (*i.e.*, containing many local optima, but only one global optimum).

Sphere [F1] is smooth, unimodal, strongly convex, symmetric and it does not have any of the problems we have discussed so far.

Rosenbrock [F2] is considered to be difficult, because it has a very narrow ridge. The tip of the ridge is very sharp, and it runs around a parabola. Algorithms that are not able to discover good directions underperfom in this problem.

Step [F3] is the representative of the problem of flat surfaces. Function F3 is piecewise continuous step function. Flat surfaces are obstacles for optimization algorithms, because they do not give any information as to which direction is favorable. Unless an algorithm has variable step sizes, it can get stuck on one of the flat plateaus. The background idea of the step function is to make the search more difficult by introducing small plateaus to the topology of an underlying continuous function.

Quartic [F4] is a simple unimodal function padded with noise. The gaussian noise makes sure that the algorithm never gets the same value on

TABLE I Unimodal and multimodal functions (F1 – F8)

Name	*Function*	*Limits*
F1	$f_1 = \sum_{i=1}^{2} x_i^2$	$-5.12 \le x_i \le 5.12$
F2	$f_2 = 100(x_1^2 - x_2)^2 + (1 - x_1)^2$	$-2{,}048 \le x_i \le 2{,}048$
F3	$f_3 = \sum_{i=1}^{5} \text{int.}(x_i)$	$-5{,}12 \le x_i \le 5{,}12$
F4	$f_4 = \sum_{i=1}^{30} (ix_i^4 + \text{Gauss}(0, 1))$	$-1{,}28 \le x_i \le 1{,}28$
F5	$f_5(x_i) = 0.002 + \sum_{j=1}^{25} \left(\frac{1}{j} + \sum_{i=1}^{2} (x_i - a_{ij})^6 \right)$	$-65536 \le x_i \le 65536$
F6	$f_6 = 10\,V + \sum_{i=1}^{10} (-x_i \sin(\sqrt{\lvert x_i \rvert}))$, $V = 4189.829101$	$-500 \le x_i \le 500$
F7	$f_7 = 20\,A + \sum_{i=1}^{20} (x_i^2 - 10\cos(2\pi x_i))$, $A = 10$	$-5{,}12 \le x_i \le 5{,}12$
F8	$f_8 = 1 + \sum_{i=1}^{10} \left(\frac{x_i^2}{4000} \right) - \prod_{i=1}^{10} \left(\cos\left(\frac{x_i}{\sqrt{i}} \right) \right)$	$-600 \le x_i \le 600$

the same point. Algorithms that do not do well on this test function will do poorly on noisy data.

Foxholes [F5] is an example of a function with many local optima. Many standard optimization algorithms get stuck in the first peak they find.

The Schwefel, Rastrigin, Griewangk [F6 – F8] functions are typical examples of non-linear multimodal functions. Rastrigin's function [F7] is a fairly difficult problem for genetic algorithms due to the large search space and large number of local minima. Rastrigin has a complexity of O(nln(n)), where n is the number of the function parameters. This function contains millions of local optima in the interval of consideration. The surface of the function is determined by the external variables A and ω, which control the amplitude and frequency of the modulation respectively. With $A = 10$ the selected domain is dominated by the modulation. The function is highly multimodal. The local minima are located at a rectangular grid with size 1. With increasing distance to the global minimum the fitness values of local minima become larger.

Schwefel's function is somewhat easier than Rastrigin's function, and is characterized by a second-best minimum which is far away from the global

TABLE II Non linear squares problems

Name	*Function definition*	*Dimensions*
F9	$f_i = \sum_{j=2}^{n}(j-1)x_j t_i^{(j-2)} - \sum_{j=1}^{n}(x_j t_i^{(j-1)})^2 - 1$ where $t_i = i/29$, $1 \le i \le 29$ $f_{30}(x) = x_1, f_{31}(x) = x_2 - x_1^2 - 1$ Standard starting point: $x_0 = (0, \ldots, 0)$	$2 \le n \le 31$, $m = 31$
F10	$f_{2i-1}(x) = 10(x_{2i} - x_{(i-1)}^2)$ $f_{2i}(x) = 1 - x_{(2i-1)}$ Standard starting point: $x_0 = (\xi_j)$ where $\xi_{2j-1} = -1.2, \quad \xi_{2j} = 1$	n variable but even $m = n$
F11	$f_1(x) = x_1 - 0.2$ $f_i(x) = \alpha^{(1/2)}\left(\exp\left[\frac{x_i}{10}\right] + \exp\left[\frac{x_{(i-1)}}{10}\right] - y_i\right), \quad 2 \le i \le n$ $f_i(x) = \alpha^{(1/2)}\left(\exp\left[\frac{x_{(i-n+1)}}{10}\right] - \exp\left[\frac{-1}{10}\right]\right), \quad n \le i \le 2n$ $f_{2n}(x) = \left(\sum_{j=1}^{n}(n-j+1)x_j^2\right) - 1$ where $\alpha = 10^5, y_i = \exp\left[\frac{i}{10}\right] + \exp\left[\frac{(i-1)}{10}\right]$ Standard starting point: $x_0 = \left(\frac{1}{2}, \ldots, \frac{1}{2}\right)$	n variable $m = 2n$
F12	$f_1(x) = 10^4 x_1 x_2 - 1$ $f_2(x) = \exp[-x_1] + \exp[-x_2] - 1.0001$ Standard starting point: $x_0 = (0, 1)$	$n = 2$, $m = 2$
F13	$f_1(x) = \exp\left[\frac{-\lvert y_i\ mix_2\rvert^{(x_1)}}{x_1}\right] - t_i$ $t_i = i/100$ $y_i = 25 + (-501\mathrm{n}(t_i))^{(2/3)}$ Standard starting point: $x_0 = (5, 2.5, 0.15)$	$n = 3$, $n \le m \le 100$
F14	$f_{(4i-3)}(x) = x_{(4i-3)} + 10x_{(4i-2)}$ $f_{(4i-2)}(x) = 5^{(1/2)}(x_{(4i-1)} - x_{4i})$ $f_{(4i-1)}(x) = (x_{(4i-2)} - 2x_{(4i-1)})^2$ $f_{(4i)}(x) = 10^{(1/2)}(x_{(4i-3)} - x_{(4i)})^2$ Standard starting point: $x_0 = (\xi_j)$ where $\xi_{4j-3} = 3, \quad \xi_{4j-2} = -1, \ \xi_{4j-1} = 0, \ \xi_{4j} = 1$	n variable but a multiple of 4 $m = n$

optimum. In Schwefel function, V is the negative of the global minimum, which is added to the function so as to move the global minimum to zero, for convenience. The exact value of V depends on system precision; for our experiments $V = 4179.829101$.

Griewangk has a complexity O(nln(n)), where n is the number of the function's parameters. The terms of the summation produce a parabola, while the local optima are above parabola level. The dimensions of the search range increase on the basis of the product, which results in the decrease of the local minimums. The more we increase the search range, the flatter the function. Generally speaking, this is a good function for testing GA performance mainly because the product creates sub-populations strongly codependent to parallel GAs models.

Most algorithms have difficulties to converge close to the minimum of such functions, because the probability of making progress decreases rapidly as the minimum is approached. The rest of the functions (Tab. II) of the GA performance testing set chosen, are drawn from the field of mathematical programming and primarily cover the area of non-linear programming problems [12, 9].

One of the reasons we chose this set of problems is that one of the major developments in mathematical programming has to do with algorithms, a kind of programming which deals with the possibility to define the problem's solution in its various forms.

The possibility to solve such a problem is determined by two factors. Firstly, the establishment of an automatic solution method must be feasible, *i.e.*, a method which solves all problems of the same category always following the same exact steps. Such a method, which also comprises genetic algorithms, is necessary in order for the method's programming and the problem's solving with the help of a computer to be possible. Further discussion of these test functions and their properties can be found in [15].

The standard functions of the available solving systems express in each case the optimization criterion. The dimensions of the systems are represented according to the constants or variables of the mathematical problem, while the limitations between the variables express the laws conditioning the system. The mathematical expression in many of our problems is such, that a big increase in the computational complexity of the mathematical system is entailed. The nature of each problem defines the feasibility and easiness with which the problem is solved from our computer. Genetic algorithms are also a systematic process, which is used for solving such problems, and it is thus used in our paper for a specific set of problems that tests their performance (see Tab. I – II).

3. METHODOLOGY

The main performance metric is the efficiency of the genetic algorithm (*i.e.*, the ability to reach an optimum is increased the number of populations we whether the number of GA iterations required to find a solution decreased). In order to facilitate an empirical comparison of the performance of GAs, a test environment for these algorithms must be provided in the form of several objective functions f.

We used the PGAPack [9] library and tested both population replacement models available in the field of serial GAs. The first, the generational replacement genetic algorithm (GRGA), replaces the entire population each generation and is the traditional genetic algorithm. The second variant is the steady state genetic algorithm (SSGA) in which only a few structures are generated in each iteration ($\lambda = 1$ or 2) and they are inserted in the current populations ($Q = P(t)$) The criterion we used for ending the algorithm stipulates the following:

"The executions stop only if after a certain number of repetitions (in this paper 100 repetitions) the best individual is not substantially mutated".

The population size affects both the overall performance and the efficiency of GAs. Two population replacement schemes are used. The first, the generational replacement model (GRM), replaces the entire population each generation and is the traditional genetic algorithm. The second the steady state replacement model (SSRM), typically replace as only a few strings each generation.

Two PNGs have been tested, *i.e.*, (a) PGARUniform and (b) ISAAC (Indirection, Shift, Accumulate, Add, and Count). Each time PGAPack is run, unique sequence of random numbers is used. In the package, the initial population of individuals is created by selecting each bit uniformly at random. For each bit position of each individual in the population, a call is made to a PNG to obtain a real number in the range [0, 1).

We obtained the ISAAC PNG from the World Wide Web [22]. It is intended to be a PNG worthy of use in cryptographic applications. Then, we found the online value for each PNG and test function, and then compared this value with the true optimal value for each function.

The crossover in our experiments is applied with six probability values [0.6, 0.95] with step 0.05 [4, 8, 17], and its application or non-application is defined by a random number generator. When it is not applied, the offspring is considered as an exact copy of one of the parents with equal probability.

The mutation in our experiments is applied with six probability values {0.001, 0.02, 0.04, 0.06, 0.08, 0.10, 0.18, 0.20} [4, 8, 17], and its application or

non-application is defined by a random number generator Mutation on the other hand is applied with a 1/21 probability where 1 is the alphanumerical string [2, 9].

4. EXPERIMENTAL RESULTS

Using the method described in the previous section we tried to determine the effect on the performance of a GA (a) of the population size, (b) of the population replacement model selection, (c) of the crossover probability of the parents and (d) of the pseudo-random generator and (e) of the mutation probability. For our experiments we used a Silicon – Graphics (Origin 200) [MIPS RISC R10000 64BIT, 128 MB memory] while the implementation of our programs was carried out with the help of the PGAPack library.

More specifically, the total of our parameters was:

(1) Population sizes: (50, 100, 150, 200, 250, 300, 350, 400).
(2) Population replacement models: (a) Generational Replacement Model, (b) Steady State Replacement.
(3) Crossover probability: [0.6 – 0.95] with step 0.05.
(4) Mutation probability: {0.00l, 0.02, 0.04, 0.06, 0.08, 0.10, 0.18, 0.20}
(5) Crossover mechanism: uniform crossover.
(6) Algorithm ending criteria: the executions stop only if after 100 repetitions the best individual has not substantially changed.
(7) Mutation mechanism: Gaussian.
(8) Selection: tournament selection.
(9) Fitness function: linear ranking.

We examined the average, best and worst values we took for each function in a total of 14336 executions of the GA ($2 \times 8 \times 8 \times 8 \times 14$). The main performance metric was the efficiency of the genetic algorithm model we used. This was studied by choosing a large set of test problems and trying to characterize on different problems profile how well the GA performed. Figures 1 – 14 shows that an SSGA performance improves when the population size is changed. A small size provides an insufficient sample, which makes them perform poorly. A large population size will undoubtedly raise the probability of the algorithm performing an informed and directed search. However, a large population requires more evaluations per iteration or generation possibly resulting in the method developing redundant controllers and becoming very slow especially when implemented serially. We found a populations of $200 \sim 250$ to be optimal for our task, but in

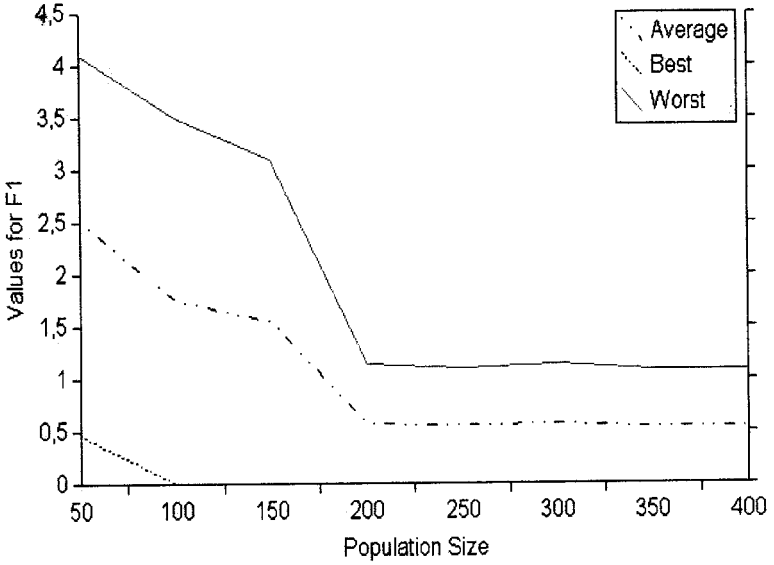

FIGURE 1 The effect of population size.

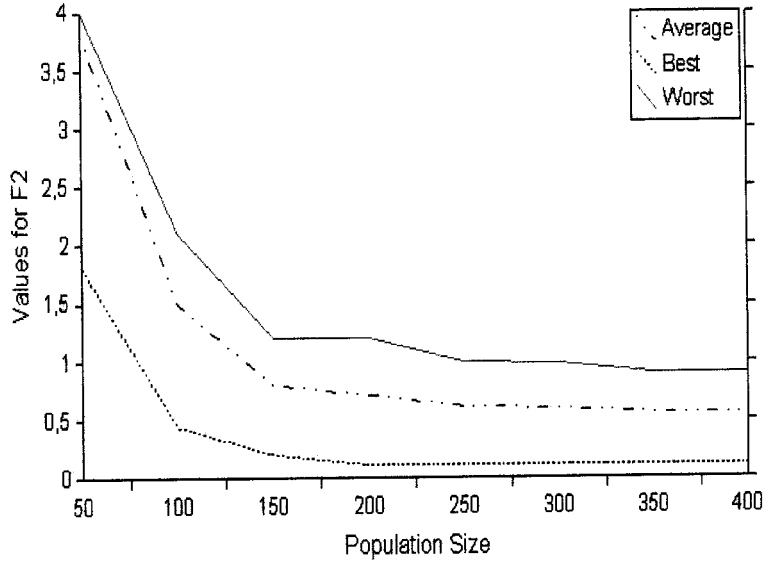

FIGURE 2 The effect of population size.

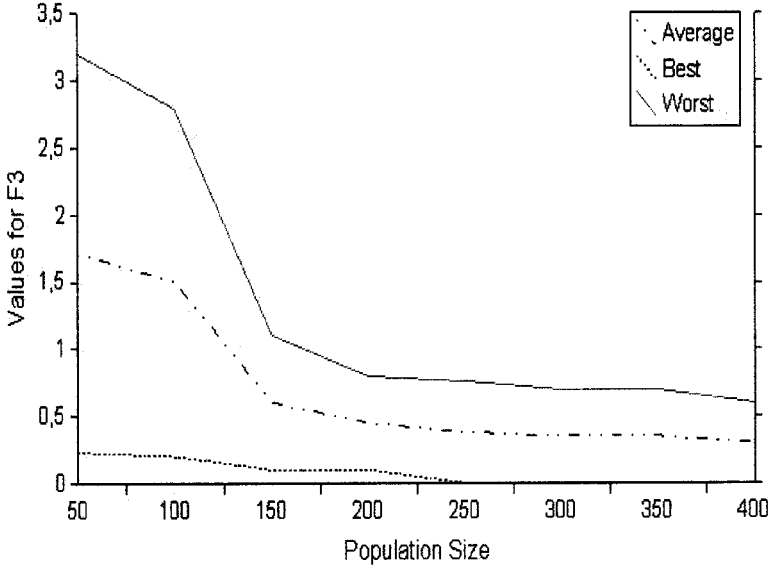

FIGURE 3 The effect of population size.

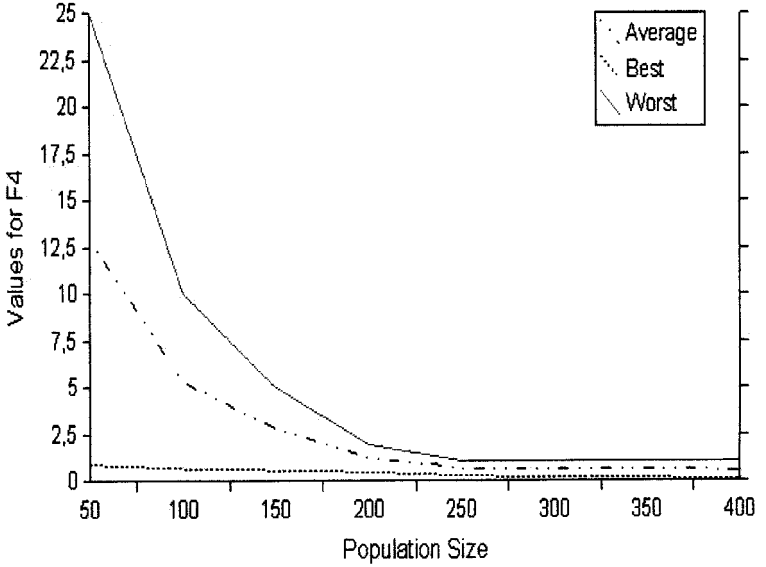

FIGURE 4 The effect of population size.

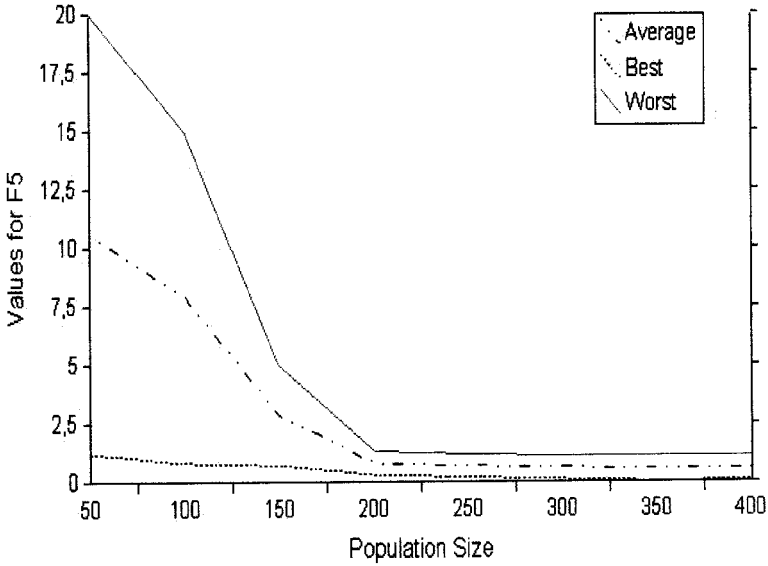

FIGURE 5 The effect of population size.

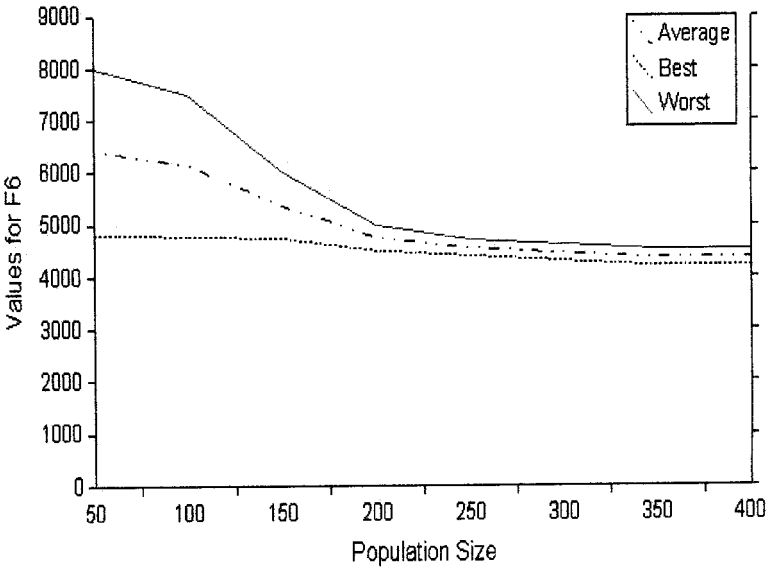

FIGURE 6 The effect of population size.

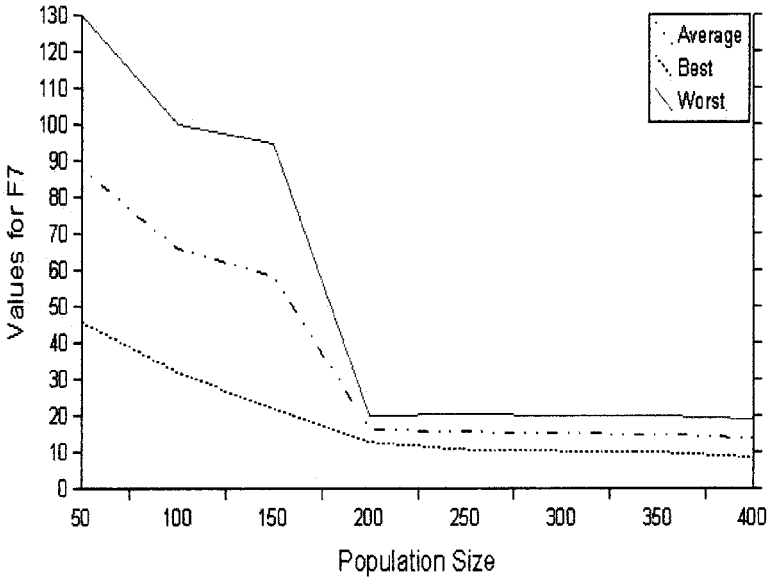

FIGURE 7 The effect of population size.

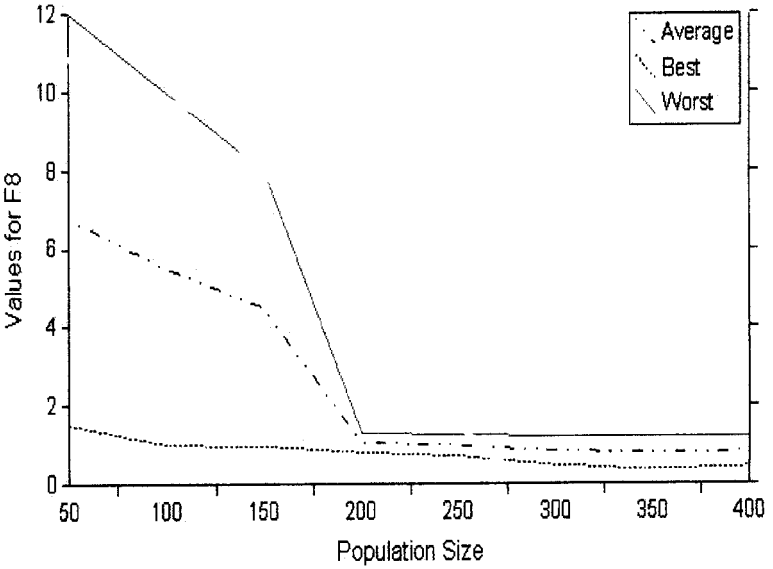

FIGURE 8 The effect of population size.

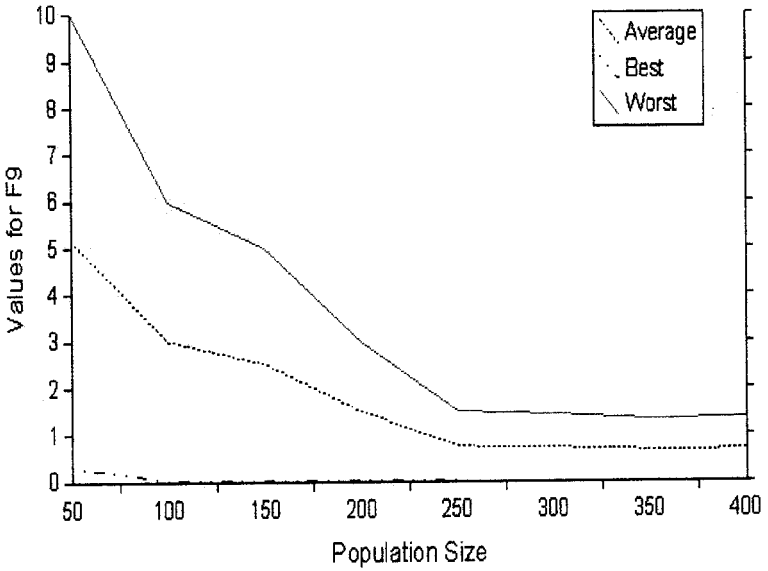

FIGURE 9 The effect of population size.

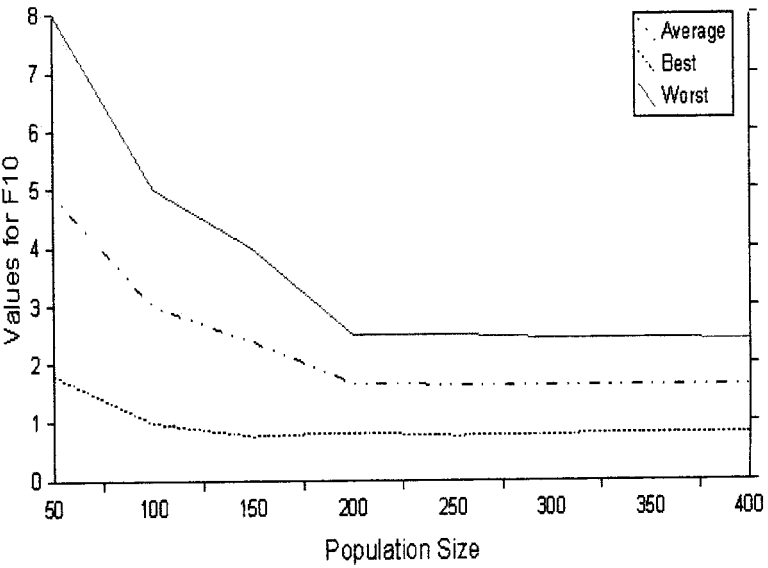

FIGURE 10 The effect of population size.

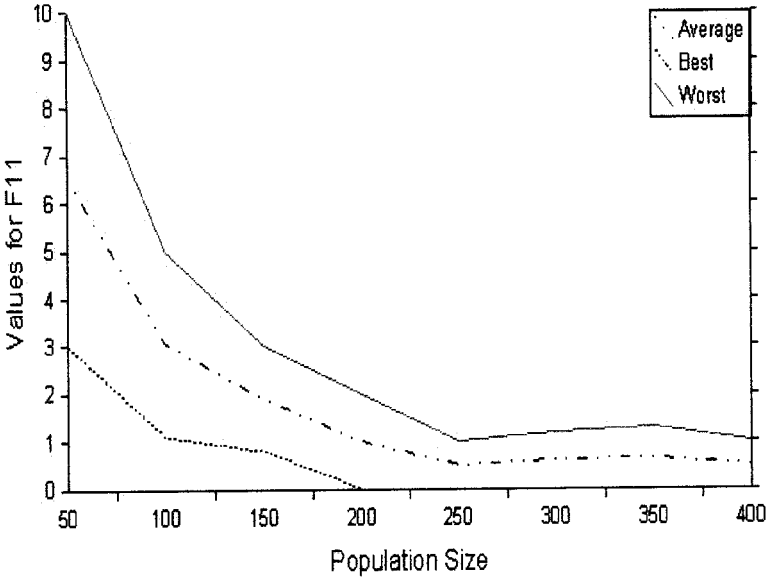

FIGURE 11 The effect of population size.

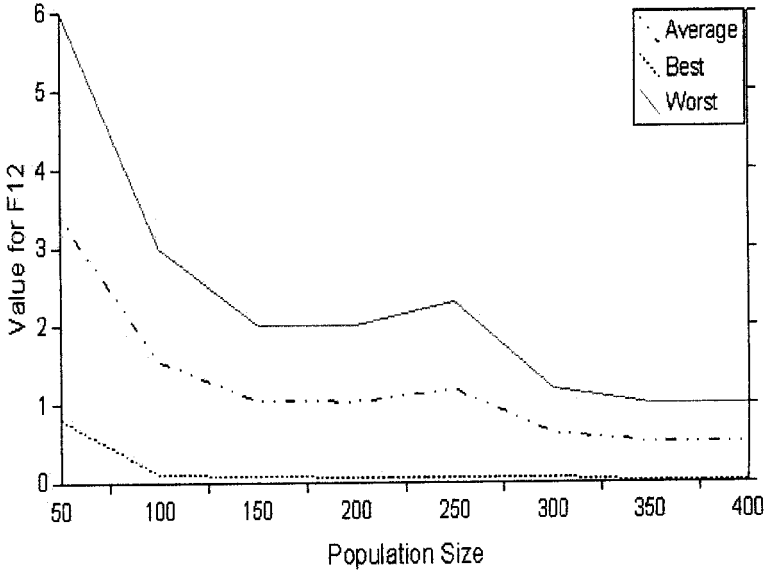

FIGURE 12 The effect of population size.

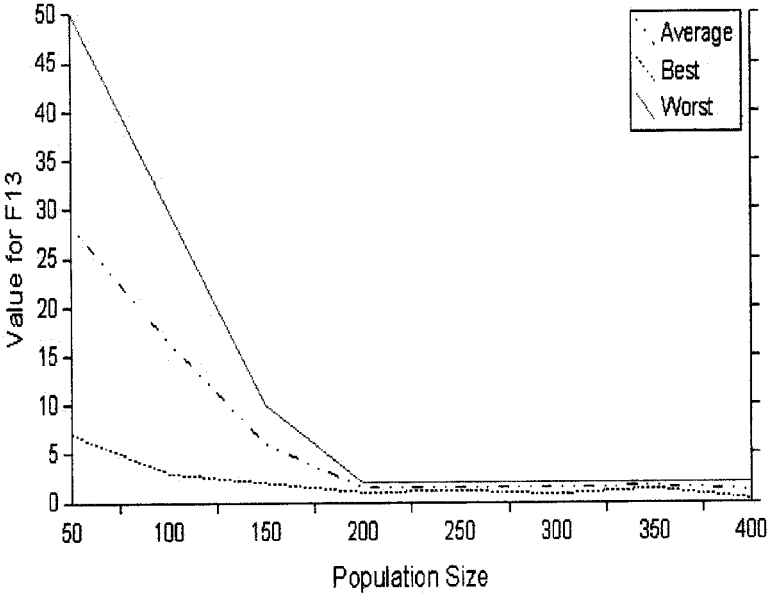

FIGURE 13 The effect of population size.

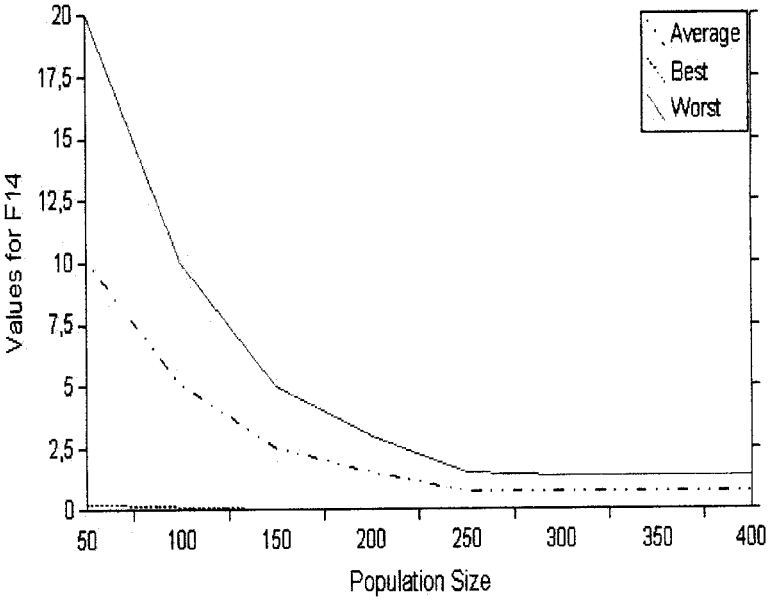

FIGURE 14 The effect of population size.

general we assert that the optimum population size depends on the complexity of the domain. When the population size is reduced to half, no significant performance different is observed as evident from the figures. This success with smaller population is because of the genetic variability which exists within each individual and in the population is sufficient to adapt in this environmental change. Of course, to achieve this level of performance the SSGA needed more memory space to keep redundant information as compared to the same size population.

Table IV shows the average number of generations taken be the GRM and SSRM on the occasions where it found the optimal value (within *0.001) for the test function in question. These results apply to the Generational Replacement model and Steady-State Replacement model. It should be noted that the absolute number of generation is important performance.

In [2] it is argued to that crossover alone is sufficient to find the global optimum of separable, multimodal functions within O(nlnn) time. Table V shows the results of experiments we did to compare eight values of crossover probability. Crossover rates suggest that a high rate, which disrupts many strings selected for reproduction is important in a small population. The same performance difference does not appear, however, between tests using different crossover probabilities. A higher crossover rate tends to disrupt the

TABLE IV The number of generations for convergence

Function	*F*1	*F*2	*F*3	*F*4	*F*5	*F*6	*F*7	*F*8	*F*9	*F*10	*F*11	*F*12	*F*13	*F*14
SSRM	25	232	25	223	20	35	450	100	350	300	310	172	430	456
GGRM	20	88	18	158	17	22	350	80	230	280	600	130	397	351

TABLE V Results for different crossover probabilities

Pc%	60	65	70	75	80	85	90	95
F1	1.25	0.850	0.600	0.610	0.644	0.727	0.737	0.733
F2	0.8	0.740	0.400	0.405	0.410	0.510	0.420	0432
F3	1.2	0.8	0.6	0.4	0.488	0.508	0.528	0.548
F4	2.35	1.21	1.2	0.8	0.7	0.71	0.72	0.73
F5	5.2	2.1	1.4	0.6	0.61	0.62	0.63	0.64
F6	5100	4500	4400	4380	4315	4317	4319	4321
F7	15	13.6	12.2	8.8	8.81	8.82	9.1	8.82
F8	1.8	1.2	2	0.7	0.71	0.72	0.73	0.74
F9	1.25	1.15	1.01	0.7	0.9	0.8	0.9	0.91
F10	2.3	2.1	1.8	1.1	1.12	1.14	1.16	1.18
F11	0.8	0.78	0.42	0.2	0.22	0.19	0.23	0.17
F12	3.2	1.86	0.85	0.6	0.51	0.48	0.6	0.63
F13	1.72	1.56	1.1	0.8	0.82	0.81	0.815	0.817
F14	1.2	1.1	0.5	0.56	0.54	0.52	0.51	0.5

structures selected for reproduction at a high rate, which is important in a small population, since high performance individuals are more likely to quickly dominate the population.

The focus of this paper is the GA's performance with respect to small mutation rates. These results has shown that a small mutation rate in the order of by applying mutation statistically to only one parameter per offspring, such GAs decompose the optimization task into n independent 1-dimensional tasks.

All mutation rate settings depended at least on 1, the length of the bit string. Thus, at the beginning, the mutation rate will allow for greater diversity at very little expense, since the growth rate will usually be high and will not be greatly affected by a slight mutation rate. A mutation rate such as $p = 0.001$ which is too small for the complete range of string lengths investigate here turns out to be a generally more reasonable choice by far than a mutation rate which is by the same factor larger than the optimal rate. In the functions discussed here, the minimum value of the population the GGGA with no mutation initially rises and then it falls as the population converges prematurely at the final values reached by the Gas.

Figures 15–29 shows that an GGGA performance improves when the mutation propability is changed. In Figures 15–29 we notice that there is a

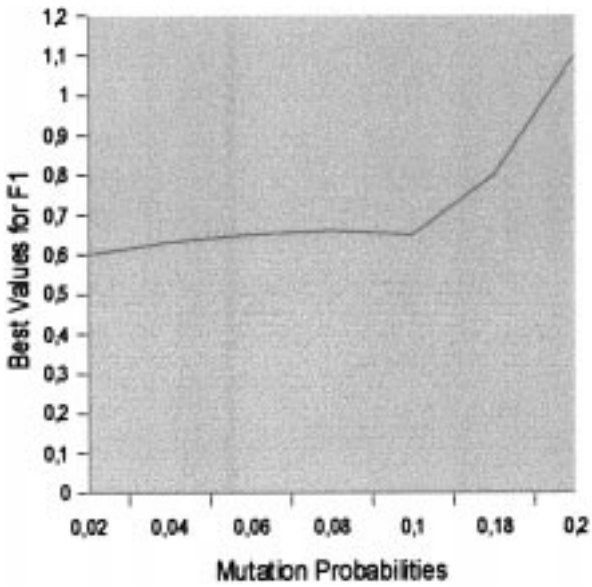

FIGURE 15 Results for different mutation probabilities.

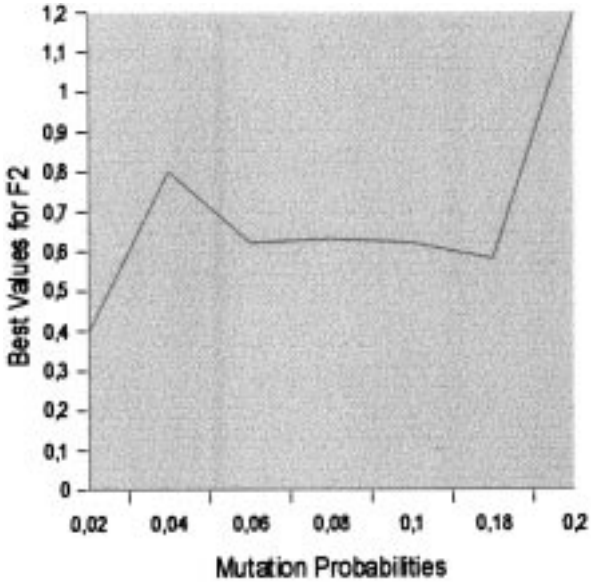

FIGURE 16 Results for different mutation probabilities.

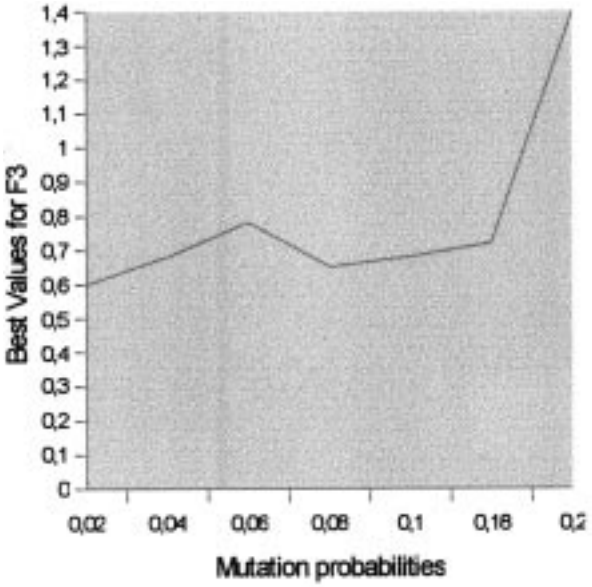

FIGURE 17 Results for different mutation probabilities.

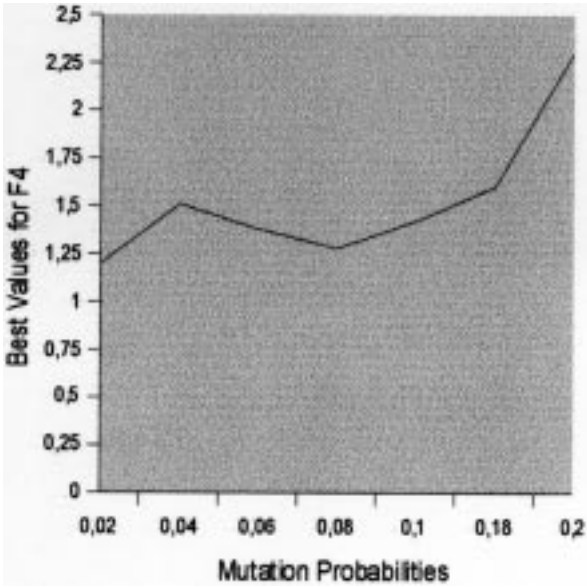

FIGURE 18 Results for different mutation probabilities.

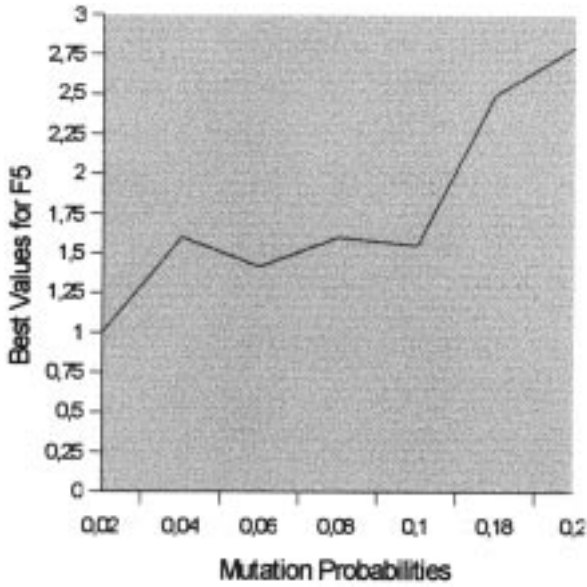

FIGURE 19 Results for different mutation probabilities.

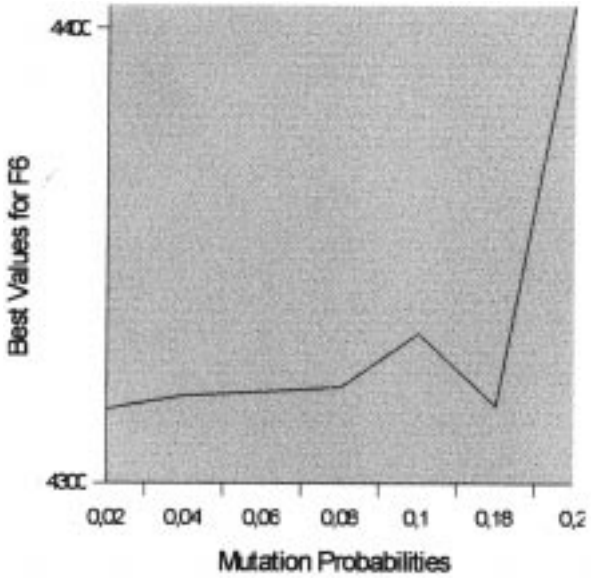

FIGURE 20 Results for different mutation probabilities.

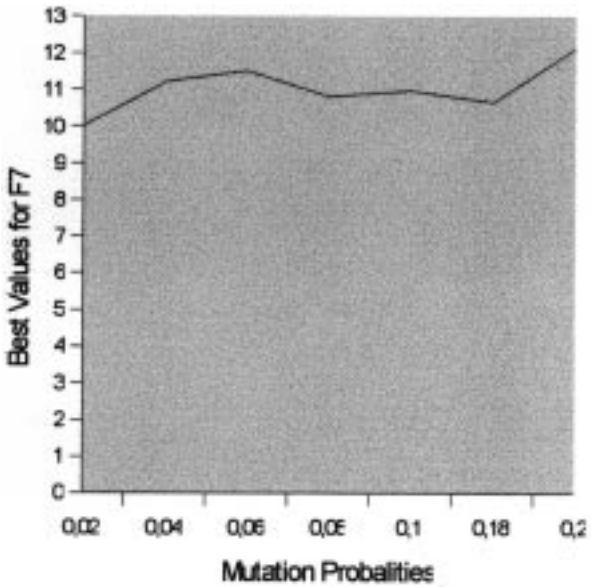

FIGURE 21 Results for different mutation probabilities.

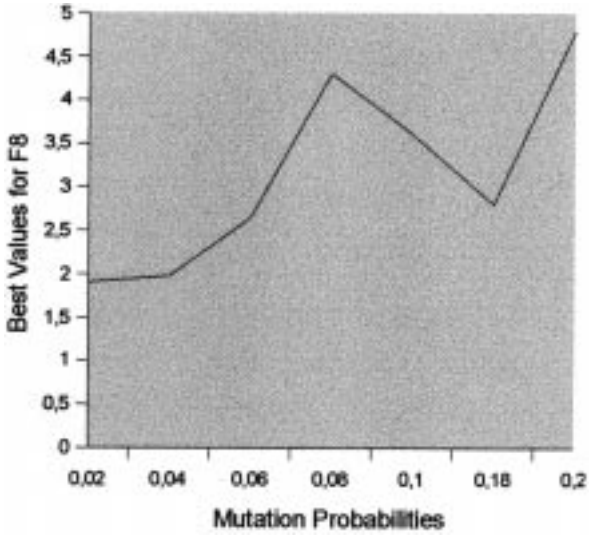

FIGURE 22 Results for different mutation probabilities.

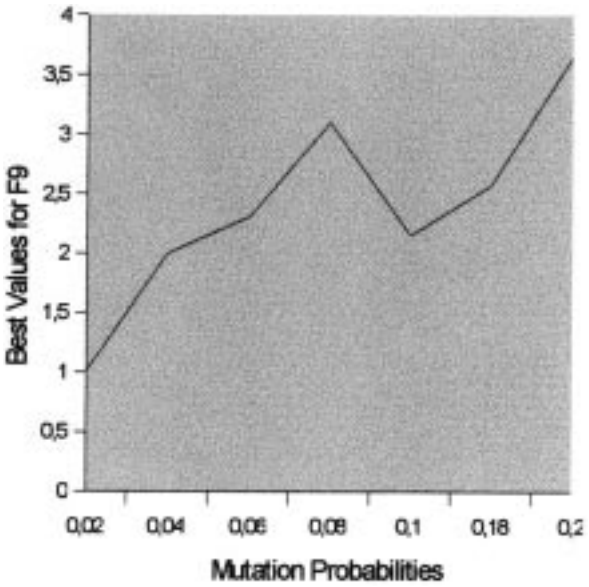

FIGURE 23 Results for different mutation probabilities.

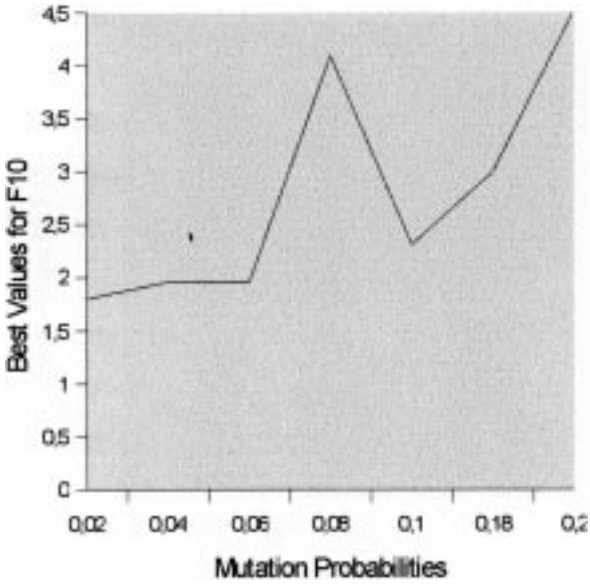

FIGURE 24 Results for different mutation probabilities.

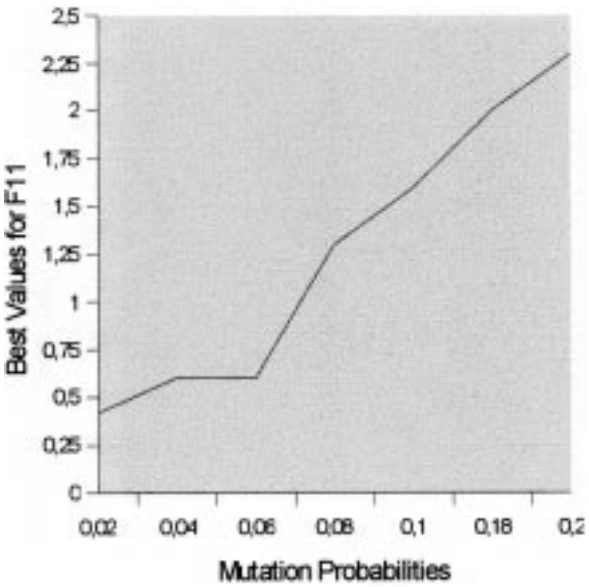

FIGURE 25 Results for different mutation probabilities.

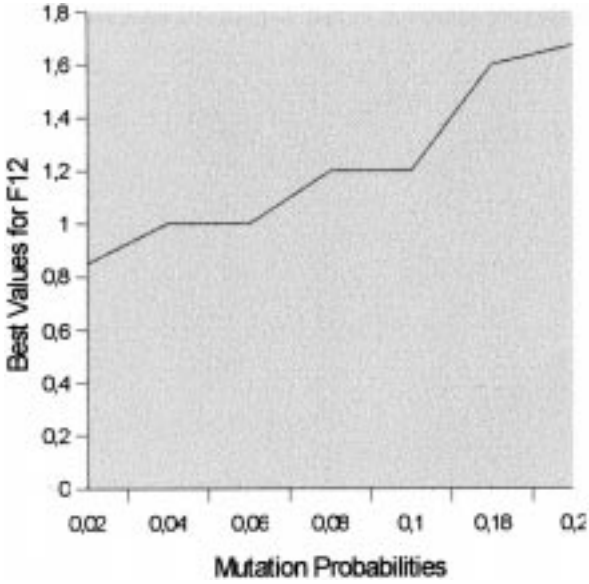

FIGURE 26 Results for different mutation probabilities.

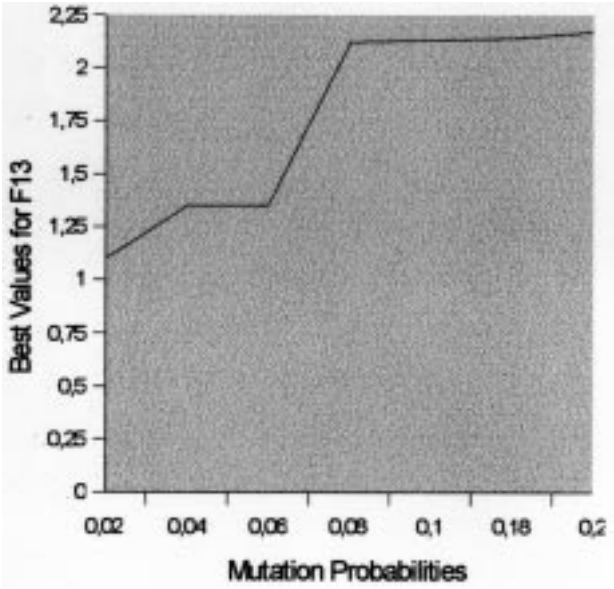

FIGURE 27 Results for different mutation probabilities.

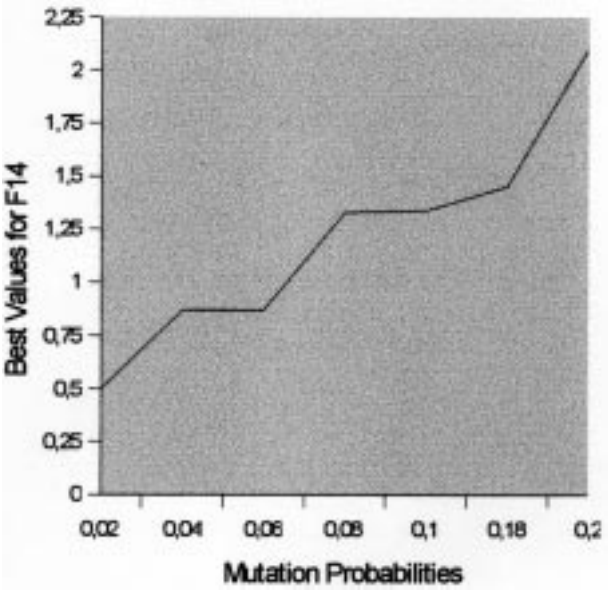

FIGURE 28 Results for different mutation probabilities.

FIGURE 29 Rules for different mutation probabilities.

significant difference in the result for mutation range from 0.001 to 0.02. This shows that the absence of mutation leads to poor results while the presence of a small mutation rate immediately improves our results.

The population size affects both the ultimate performance and the efficiency of GAs. A large population requires more evaluations per generation, possibly resulting in an unacceptably slow rate of convergence. This emfasizes that population size, if increased sufficiently, can offset differences in crossover probability. On the other hand, for small population size the choice of crossover operators plays a more dominant role than.

Using the method described in Section 4 we were able to determine to what extend the quality of PNG affects the performance of the SSRM and GRM. Figures 30, 31 show, in picture form, the similarity in performance

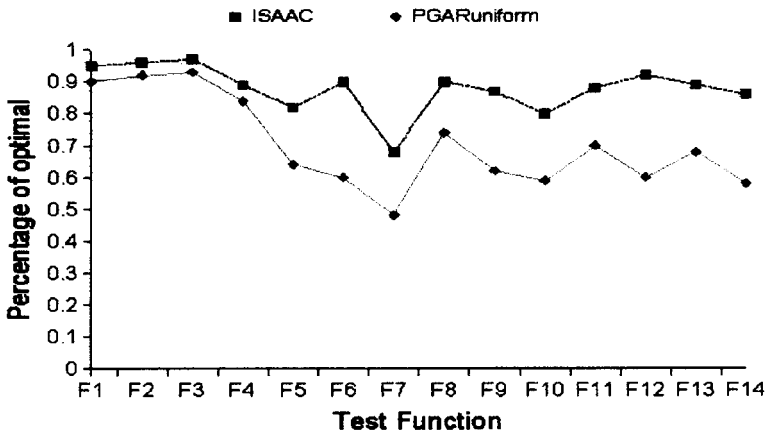

FIGURE 30 GA online percentage of optimal value, (GGRM model).

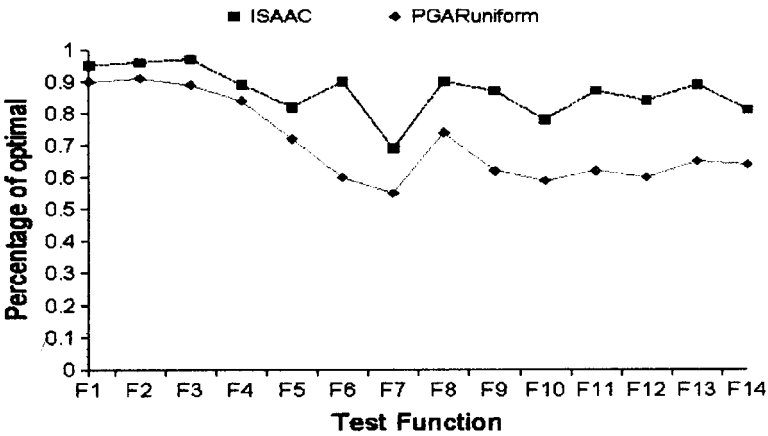

FIGURE 31 GA online percentage of optimal value, (SSRM model).

between the GA driven by the best PNG and the GA driven by one of the worst PNGs. Specifically, it compares the performance of the GA driven by the best PNG (ISAAC) *versus* the performance of the GA driven by the worst PNG (PGARUniform). It plots the online performance measure value found, expressed as a percentage of the optimal value, achieved by the GA across all fourteen of the test functions.

5. CONCLUSIONS

Genetic Algorithms, as have been discussed, provide a very good conceptional framework for optimization inspired by nature, but theoretical questions and algorithmic considerations discussed in this paper suggest that the set of functions on which GAs yield optimal performance converges to a set of benchmarking functions.

Experiments were performed to search for the optimal GAs for a given set of numerical optimization problems. Finding an appropriate and represented set of test problems is not easy task, since the particular combination of properties represented by any set of test functions does not allow for generalized performance statements. From the fourteen objective functions tested empirically and the results reported here we can give only a statement of limited generality about the efficiency of GA. Test functions that yield another ordering of the methods are likely to exist. The experimental data also suggest that, while it is possible to GA control parameters, very good performance can be obtained with a varying range of GA control parameter settings.

For a computationally bound GA properly sizing the population is important because both an oversized or undersized population reduces the final solution quality. Our results clearly show that the absence of crossover leads to poor results while the presence of even a small crossover rate immediately improves our results. Generally speaking, we can argue that a population size of $200 \sim 250$ and a crossover probability percentage of $70 \sim 75\%$ bring satisfactory results. The use of large population size values does not necessarily entail in all cases better results and consequently increases the computational time.

Results shows that the absence of mutation leads to poor results while the presence of a small mutation rate immediately improves our results. For all functions the differences in GA performance our results. For all functions the differences in GA performance, at the range of 0 to 0.02, are statistically significant. Experiments show the minimum values reached by GA using different values of mutation rate.

By using the Generational Replacement Model we can be more certain that the best selected parents of the current generation will participate in the reproduction of offsprings compared to the Steady-State model. This does not mean, however, that the Steady-State Replacement Model does not make proper use of the parent selection process, but simply that the possibility for a greater number of parents with good characteristics to be chosen is greater with the generational replacement model. This entails a computational cost since in all 14 functions of our paper we found a relatively good solution with a greater number of generations in the generational replacement model compared to the steady state replacement.

We do not expect that theses functions can always discriminate between mutation and crossover based searches: for example, a crossover-based search that separated the mutation and crossover strings into two independent population pools would easily find mutation optima if it is used a steady-state replacement scheme.

An issue we examine in this paper is to what degree the performance of the PNG employed effects the performance of the GA. For some test functions, ISAAC PNG drove the GA to its maximum value in fewer generations. In fact, when the GA did produce an individual with optimal value, there was no statistical difference between the number of generations required to produce the individual when the GA was driven by a good PNG (ISAAC) and when it was driven by PGARUniform. When optimizing multimodal functions, a PNG has even more severe consequences. Therefore, the experiments described above are important in that they identify approximately optimal parameter settings for the four performance measures considered.

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