universität freiburg

BAM **Winter semester 2025/2026**

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Tutorial 1

Exercise 1 (4 points)**.**
Show the following: Let Ω be a set, let I be an arbitrary index set, and let $A_i \subseteq \Omega$ for all $i \in I$. Then

(a)
$$\left(\bigcup_{i\in I} A_i\right)^c = \bigcap_{i\in I} A_i^c,$$

(b)
$$\left(\bigcap_{i\in I} A_i\right)^c = \bigcup_{i\in I} A_i^c.$$

Exercise 2 (4 points)**.**
Let X be Poisson distributed with parameter 1, i.e. $\mathbb{P}(X = k) = \frac{1}{k!}e^{-1}$ for all $k \in \{0,1,2,...\}$.

(a) Prove that the probability $\mathbb{P}(X \geq k)$ converges to zero for $k \to \infty$.

(b) Let $k_0 \in \mathbb{N}$. Prove that the conditional probability distribution

$$\mathbb{P}(X \in A|X \geq k_0) \quad \text{for } A \in \mathcal{P}(\mathbb{N}_{\geq k_0}).$$

satisfies the axioms of Kolmogorov (that is, conditions (2.3), (2.4), and (2.5) in the lecture notes).

(c) Prove that $\mathbb{P}(X = k|X \geq k)$ does not converge to zero for $k \to \infty$.

Exercise 3 (12 Points)**.**
Let X and Y be independent random variables taking values in the positive integers and having the same probability mass function

$$f(x) = 2^{-x}, \qquad x = 1,2,\ldots$$

Find:

(a) $\mathbb{P}(\min\{X,Y\} \leq x)$,

(b) $\mathbb{P}(Y > X)$,

(c) $\mathbb{P}(X = Y)$,

(d) $\mathbb{P}(X \geq kY)$, for a given positive integer k,

(e) $\mathbb{P}(X \,\text{divides}\, Y)$,

(f) $\mathbb{P}(X = rY)$, for a given positive rational number r.

Exercise 4 (2)**.**
Let X be a Poisson random variable with parameter λ, that is,

$$\mathbb{P}(X = n) = p_n(\lambda) = \frac{\lambda^n e^{-\lambda}}{n!}, \qquad n = 0,1,2,\ldots$$

Show that

$$\mathbb{P}(X \leq n) = 1 - \int_0^\lambda p_n(x)\,dx.$$

Exercise 5 (4 Points)**.**
Let Λ be a positive random variable with density function f and distribution function F, and let Y have a Poisson distribution with parameter Λ. Show that for $n = 0,1,2,\ldots$,

$$\mathbb{P}(Y \leq n) = \int_0^\infty p_n(\lambda)\,F(\lambda)\,d\lambda, \qquad \mathbb{P}(Y > n) = \int_0^\infty p_n(\lambda)\,[1 - F(\lambda)]\,d\lambda,$$

where

$$p_n(\lambda) = \frac{e^{-\lambda}\lambda^n}{n!}.$$