Due on **Wednesday September 18, 9:00pm** via Crowdmark [Total points: 6+6+8+8 =28].

- Solutions must be **typeset** or **neatly written** on paper
- Begin each solution by writing the question number at the **top of an empty page**.
- **Scan** your answer sheets using a scanner or a scanning app.
- **Upload** each PDF file to the appropriate answer box in the Crowdmark site for Math 343.
- Late homework submissions are **not accepted**.

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1. [One point each] Consider each of the following sets:
   - $\mathcal{A}$ is the set of all finite unlabeled undirected simple graphs. (A graph is *simple* if it has no loops or multiple edges).
   - $\mathcal{B}$ is the set of all finite *connected* unlabeled undirected loopless graphs (multiple edges are allowed).
   - $\mathcal{C}$ is the set of all finite unlabeled undirected loopless graphs (multiple edges are allowed).
   - $\mathcal{W}$ is the set of finite binary words.

   For each of the following, decide whether it is a combinatorial class. If the answer is "no", then give a reason for this.

   (a) $\mathcal{A}$, where the size of a graph is the number of vertices.
   (b) $\mathcal{A}$, where the size of a graph is the number of edges.
   (c) $\mathcal{B}$, where the size of a graph is the number of edges.
   (d) $\mathcal{C}$, where the size of a graph is the number of vertices.
   (e) $\mathcal{C}$, where the size of a graph is the number of edges.
   (f) $\mathcal{W}$, where the size of a word is its length plus the number of zeros in the word.

2. (a) [4 points] Consider the formal power series $A(x) = 1 + x^2$. Compute the inverse power series $A(x)^{-1}$ by solving a triangular system of equations as in Section 2.3 of Lecture Notes 2.

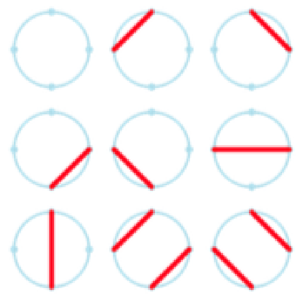
   (b) [2 points] Evaluate the geometric series $\sum_{n\geq 0} r^n$, where $r = -x^2$. Why is your answer not surprising?

3. (a) [5 points] We define the derivative of a formal power series $A(z) = \sum_{n\geq 0} a_n z^n$ to be the power series
$$\frac{d}{dz}A(z) := \sum_{n\geq 0}(n+1)a_{n+1}z^n.$$
   Using the definitions for the sum and the product of two formal power series as given in Section 2.2 of Lecture Notes 2 prove that the "product" rule holds for formal power series:
$$\frac{d}{dz}\left(A(z)B(z)\right) = \left(\frac{d}{dz}A(z)\right)B(z) + A(z)\left(\frac{d}{dz}B(z)\right).$$

   (b) [3 points] Define the formal power series
$$\exp(z) = \sum_{n\geq 0}\frac{z^n}{n!}.$$
   Show that $\frac{d}{dz}\exp(z) = \exp(z)$.

4. We have several tools at our disposal to help understand and play with counting sequences. This exercise is to help you gain familiarity with some of these tools. Let $\mathcal{M}$ be the following class of combinatorial objects: An object of size n is a circle on n (labeled) points, with some collection of chords between the vertices under the condition that no two chords are intersecting. Here are all 9 objects of size 4:

$\mathcal{M}_4$:



   (a) [2 points] Prove that $\mathcal{M}$ satisfies the definition of combinatorial class.

   (b) [2 points] Determine by hand the first five elements of the counting sequence $(M_n)_{n\geq 0}$.

   (c) [1 point] **Fact:** The generating function for this sequence is

$$M(z) = \frac{1 - z - \sqrt{1 - 2z - 3z^2}}{2z^2}.$$

   Determine the first 10 terms of the counting sequence for $\mathcal{M}$. To do this, use command `series` or `taylor` in Maple®, or the command `series` an the prompt at the website http://www.wolframalpha.com/. Submit a printout or screen capture of your work.

   (d) [3 points] Look up this sequence on the On-line Encyclopedia of Integer Sequences (http://oeis.org). What is its sequence number? List two other combinatorial classes also counted by these numbers. Choose one, and draw all the elements of size less than or equal to 5.