# MATH 450
## Assignment 3

*Please submit using your Crowdmark link for this assignment by the due date.*

(1) Let $f : \mathbb{R} \to \mathbb{R}$ be a function. Show if $f$ is continuous in the analysis sense, then it is continuous in the topological sense.

(2) Let $Y$ be an ordered set which is given the order topology. Let $f, g : X \to Y$ be continuous.
  (a) Show that the set $\{x : f(x) \leq g(x)\}$ is closed in $X$.
  (b) Let $h : X \to Y$ be the function $h(x) = \min\{f(x), g(x)\}$. Show that $h$ is continuous. (Hint: use the pasting lemma.).

(3) Let $\mathbb{R}^\infty$ be the subset of $\mathbb{R}^\omega$ consisting of all sequences that are eventually zero, that is, all sequences $(x_1, x_2, \ldots)$ such that $x_i \neq 0$ for only finitely many values of $i$. What is the closure of $\mathbb{R}^\infty$ in $\mathbb{R}^\omega$ in the box and product topologies? Justify your answer.

(4) Given sequences $(a_1, a_2, \ldots)$ and $(b_1, b_2, \ldots)$ of real numbers with $a_i > 0$ for all $i$, define $h : \mathbb{R}^\omega \to \mathbb{R}^\omega$ by the equation

$$h((x_1, x_2, \ldots)) = (a_1 x_1 + b_1, a_2 x_2 + b_2, \ldots).$$

Show that if $\mathbb{R}^\omega$ is given the product topology, then $h$ is a homeomorphism of $\mathbb{R}^\omega$ to itself. What happens if $\mathbb{R}^\omega$ is given the box topology.

(5) (a) Let $p : X \to Y$ be a continuous map. Show that if there is a continuous map $f : Y \to X$ such that $p \circ f$ equals the identity map of $Y$, then $p$ is a quotient map.
  (b) If $A \subseteq X$, a retraction of $X$ onto $A$ is a continuous map $r : X \to A$ such that $r(a) = a$ for all $a \in A$. Show that a retraction is a quotient map.