1. Which of the following is a combinatorial class? Give reasons.
   (a) $\mathcal{S} = \textsc{Seq}(\{0,1,2,3,\dots\})$, where $\textsc{Size}((s_1,s_2,\dots,s_k)) = \sum_{i=1}^k s_i$.

   (b) $\mathcal{T} = \textsc{Seq}(\{0,1,2,3\dots\})$, where $\textsc{Size}((s_1,s_2,\dots,s_k)) = k$.

   (c) $\mathcal{U} = \textsc{Seq}(\{0,1,2,3,\dots\})$, where $\textsc{Size}((s_1,s_2,\dots,s_k)) = k + \sum_{i=1}^k s_k$.

2. Suppose that the OGF of a combinatorial set $\mathcal{G}$ is
$$G(z) = \frac{1+3z}{1-6z+9z^2}$$
   Find G_n, for all $n \geq 0$.

3. Find the coefficient
$$[z^3](1-8z)^{\frac{1}{4}}$$

4. Suppose that the ordinary generating function of combinatorial class $\mathcal{C}$ is $C(z) = \dfrac{7z+1}{1-z-6z^2}$. Find the number C_n of objects having size n.

5. Let $\mathcal{W}$ be the set of binary words where every nonempty block of 1s has length 2, 3, or 5.
   (a) Describe the class $\mathcal{W}$ with a regular expression, using the symbols $0, 1, +, (\cdot)^*, \epsilon$.
   (b) Find the ordinary generating function $W(z)$.
   (c) Using a computer, we find the factorization
$$1 - z - z^3 - z^4 - z^6 = (1-az)(1-bz)(1-cz)(1-dz)(1-ez)$$

6. Let $\mathcal{H} = \textsc{Seq}(\{0,1,2\})$ the class of ternary sequences with $\textsc{Size}((h_1,h_2,\dots,h_k)) = k$.
   (a) Find the OGF $H(z)$.

   (b) Use $H(z)$ to find a formula for H_n.

7. For $n = 0,1,2\dots$, find the coefficient of z^n in the series expansion of $\dfrac{1+5z-2z^2}{1-2z}$.

8. Describe the first step in extracting the coefficient of z^n in the series of the following rational function? (Do **not** attempt to find the coefficient, just show and/or describe what must be done.)
$$G(z) = \frac{2z-3}{(1-3z)^3(1+z)(1-4z)}$$

9. How many objects of the combinatorial class $\mathcal{A}$ have size one, if its ordinary generating function is
$$A(z) = \frac{1 + 5z^2 - \sqrt{1 - 6z^2 + z^4}}{4z} \quad ?$$

10. Let $\mathcal{W}^{(k)}$ be the class of binary strings counted by length, which have no more than k consecutive 0s. Show that the generating function for $\mathcal{W}^{(k)}$ is
$$W^{(k)}(z) = \frac{1 - z^{k+1}}{1 - 2z + z^{k+2}}.$$