Due Wednesday, Oct 2, 9:00pm.. [Total points: 4+4+9+6+4 =27].

1. Let

$$F(z) = \frac{z(1-z)}{(1-3z)^3}.$$

   (a) [2 points] Compute the coefficient $[z^n]F(z)$ by using partial fractions.

   (b) [2 points] Compute the coefficient $[z^n]F(z)$ by using only the rules in Section 3 of Lecture 2 Notes, together with the binomial formula

$$(1-w)^{-3} = \sum_{n \geq 0} \binom{n+3-1}{3-1} w^n.$$

2. [4 points] Use partial fractions to compute the coefficient

$$[z^n]\frac{10z^2 - 4z + 4}{1 - 3z + 4z^3}.$$

   Hint: To factor the denominator, substitute small integer values for z to find the unknown values.

3. Let $\mathcal{T}$ be the class of binary rooted trees, **not** including the empty tree, where *every node in the tree has either 0 or 2 children.* The size of a tree is its number of nodes.

   (a) [1 point] Every tree $\tau \in \mathcal{T}$ is either the (atomic) tree t_1 which consists of one single node, or τ consists of the atomic tree t_1 (its root) together with a left and a right subtree, each selected from $\mathcal{T}$. Find an equation which involves the combinatorial classes $\mathcal{T}$ and $\mathcal{Z} = \{t_1\}$, and the operations *cross product* "$\times$" and *set addition* "$+$", which expresses the decomposition of trees in $\mathcal{T}$ described in the previous sentence.

   (b) [1 point] Use part (a) to find an equation which the generating function $T(z)$ must satisfy.

   (c) [3 points] Solve the equation from part (b) for the generating function $T(z)$. You will have to reject one of two possible solutions for $T(z)$ by using a fact such as $[z^{-1}]T(z) = 0$ or $[z^1]T(z) \geq 0$.

   (d) [3 points] Extract the nth coefficient $T_n = [z^n]T(z)$ from the formula for $T(z)$ that you found in part (c). You computation should be similar to finding the formula for the nth Catalan number. You might find a shortcut by reviewing the derivation of $(W_{odd})_n$ at the end of Lecture 4.

   (e) [1 point] How does your formula for T_n compare to that of the the class of all binary rooted trees $\mathcal{B}$ described in a footnote of Lecture 5?

4. (a) [3 points] Let $A = \{00, 1, 01\}$. Are the words of $\mathrm{Seq}(A)$ uniquely generated? Either give a word that can be generated in more than one way, or give a proof that each word in the class can be built in only one way.

   (b) [3 points] Let $B = \{00, 101, 11\}$. Are the words of $\mathrm{Seq}(B)$ uniquely generated? Either give a word that can be generated in more than one way, or give a proof that each word in the class can be built in only one way.

5. Both parts of this question regard the rational expression

$$C(x) = \frac{80000\,x^8 - 84000\,x^7 + 20200\,x^6 + 10890\,x^5 - 7207\,x^4 + 1640\,x^3 - 218\,x^2 + 26\,x - 2}{40000\,x^7 - 62000\,x^6 + 41100\,x^5 - 15105\,x^4 + 3324\,x^3 - 438\,x^2 + 32\,x - 1}.$$

   Submit a screenshot or printout of the calculation.

   (a) [2 points] Use Maple, or another program, to convert $C(x)$ into partial fractions.

   (b) [2 points] Use Maple, or another program, to compute the first 5 terms of the series expansion for $C(x)$.