# MATH 450
## Assignment 2

*Please submit using your Crowdmark link for this assignment by the due date.*

(1) Determine the closures of the following subsets of the ordered square.
   (a) $A = \{\frac{1}{n} \times 0 : n \in \mathbb{N}\}$
   (b) $B = \{\frac{1}{n} \times a : n \in \mathbb{N}, a \in \mathbb{Q}\}$
   (c) $C = \{x \times 0 : 0 < x < 1\}$
   (d) $D = \{\frac{y}{2} \times y : 0 < y < 1\}$.

(2) Let $A \subseteq X$, $B \subseteq Y$ where $X$ and $Y$ are topological spaces. Show that in the space $X \times Y$, we have that
$$\overline{A \times B} = \bar{A} \times \bar{B}.$$

(3) Show that a topological space $X$ is Hausdorff if and only if the diagonal $\Delta = \{x \times x : x \in X\}$ is closed in $X \times X$.

(4) Show that the $T_1$ axiom is equivalent to the condition that for each pair of points of a topological space $X$, each has a neighbourhood not containing the other.

(5) If $A \subseteq X$ a topological space, we define the boundary of $A$ by
$$\operatorname{Bd} A = \bar{A} \cap \overline{X - A}.$$
   (a) Show that $x \in \operatorname{Bd} A$ if and only if every neighbourhood of $x$ contains a point in $A$ and a point in $X - A$.
   (b) Show that $\operatorname{Int} A$ and $\operatorname{Bd} A$ are disjoint, and $\bar{A} = \operatorname{Int} A \cup \operatorname{Bd} A$.
   (c) Show that $\operatorname{Bd} A = \varnothing$ if and only if $A$ is both open and closed.
   (d) Show that $U$ is open if and only if $\operatorname{Bd} U = \bar{U} - U$.
   (e) If $U$ is open, is it true that $U = \operatorname{Int} \bar{U}$? Justify your answer.

(6) Find the boundary and the interior of each of the following subsets of $\mathbb{R}^2$.
   (a) $A = \{x \times y : y = 0, x \in \mathbb{R}\}$
   (b) $B = \{x \times y : x > 0, y \neq 0\}$
   (c) $C = A \cup B$
   (d) $D = \{x \times y : x \in \mathbb{Q}\}$