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Assignment 4 - Math 343 Dec 1^{st}, 2024

1) τ class where each node has # of children divisble by 3

node has 0, 3, 6, 9, ... children

$\tau \cong z(\tau^{3n})$, $n \geq 0$

$$T(z) = z \sum_{n\geq 0}^{\infty} T(z)^{3n}$$

$$T(z) = \frac{z}{1-T(z)^3}, \quad z = T(z)(1-T(z)^3)$$

Lagrange inversion: $\phi(u) = \frac{1}{1-u^3}$, $z = \frac{T(z)}{\phi(T(z))}$

$$\Rightarrow [z^n]T(z) = \frac{1}{n}[u^{n-1}]\phi(u)^n$$

$$= \frac{1}{n}[u^{n-1}]\frac{1}{(1-u^3)^n}, \text{ let } u^3 = t \rightarrow u^{3n} = t^n, \; u^{n-1} = t^{\frac{n}{3}-1}$$

$$= \frac{1}{n}[t^{\frac{n}{3}-1}]\frac{1}{(1-t)^n}, \text{ use negative binomial}$$

$$= \frac{1}{n}\binom{\frac{n}{3}-1+n-1}{n-1} = \frac{1}{n}\binom{\frac{4}{3}n-2}{n-1}$$

2) integer n with K summands

each summand at most r

$\Rightarrow 1 \leq K_i \leq r$ for each summand K_i

So $K_i = z + z^2 + \dots + z^r$ is OGF for # of summands

$$= z(1 + z + \dots + z^{r-1})$$

$$, (1 + z + \dots + z^{r-1})(1-z) = (1-z^r)$$

$$= z\left(\frac{1-z^r}{1-z}\right)$$

for K summands $\Rightarrow \left(z\,\frac{1-z^r}{1-z}\right)^K$

3a) Let's use RANKKSUBSET2(L, K, n)

UNRANKSUBSET2(n, K, r) outputs $L=\{l_1,\ldots,l_K\}$ s.t. $l_1 < l_2 \ldots$ which has rank r in lex. order of K-subsets of $\{1,\ldots,n\}$

RANKKSUBSET2(L, K, n)

```
r=0
for i from 1 to K
    r = r+binom(n-L(i), K+1-i)
return binom(n, K)-1-r
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⇒ RANKSUBSET2(L, K, n)

$r=0$

for i from 1 to K

$r \mathrel{+}= \binom{n-L(i)}{K+1-i}$

return $\binom{n}{K}-1-r$

UNRANKSUBSET2(n, K, r)

```
let Rank=r
for i from 1 to K
    while binom(n-L(i), K+1-i) ≤ Rank
        Rank=Rank-binom(n-L(
```

4) labeled trees n vertices, where each vertex has degree 1 or 3

in Prüfer sequence, degree 1 nodes don't appear
degree 3 nodes appear twice

Prüfer sequence length $n-2$

T has nodes $\{1, 2, \ldots, n\}$

let $t_1 \in T$ s.t. $\deg(t_1) = 1$
let $t_3 \in T$ s.t. $\deg(t_3) = 3$

Prüfer(T) will have $n-2$ t_3's, each appearing twice

if n is even

$\# \text{ of } t_3\text{'s} = \frac{n}{2} - 1$

$\# \text{ of } t_1\text{'s} = n - (\frac{n}{2} - 1)$

$\Rightarrow \# \text{ of all possible } t_1\text{'s} \,\&\, t_3\text{'s} = (\frac{n}{2} - 1)(n - (\frac{n}{2} - 1))$

if n is odd, round up

$\# \text{ of } t_3\text{'s} = \lceil \frac{n}{2} - 1 \rceil$

$\# \text{ of } t_1\text{'s} = \lceil n - (\frac{n}{2} - 1) \rceil$

$\Rightarrow \lceil \frac{n}{2} - 1 \rceil \lceil n - (\frac{n}{2} - 1) \rceil$

5/ Lets define minimal change as consectutive swaps

let $0 = \nearrow$
$1 = \searrow$

example/ 0 1 0 1

0 0 1 1

middle two digits are swapped

n=1, length 2 0 1 only one

n=2, length 4
0 1 0 1
0 0 1 1
two

n=3, length 6
0 1 0 1 0 1
0 0 1 1 0 1
0 0 1 0 1 1
0 0 0 1 1 1
four

n=4, length 8
0 1 0 1 0 1 0 1
0 0 1 1 0 1 0 1
0 0 1 0 1 1 0 1
0 0 0 1 1 1 0 1
0 0 0 1 1 0 1 1
0 0 0 1 0 1 1 1
0 0 0 0 1 1 1 1
seven

6a) (2)

b) (1)

c) (3)

d) (1)