

Alen Mehmedbegovic - 301476201

Assignment 4 - Math 343

Dec 1st 2024

1) T class where each node has # of children divisible by 3

node has 0, 3, 6, 9, ... children

$$T \approx z(u^{3n}), n \geq 0$$

$$T(z) = z \sum_{n=0}^{\infty} T(z)^{3n}$$

$$T(z) = \frac{z}{1-T(z)^3}, z = T(z)(1-T(z)^3)$$

Lagrange inversion: $\phi(u) = \frac{1}{1-u^3}, z = \frac{T(z)}{\phi(T(z))}$

$$\Rightarrow [z^n] T(z) = \frac{1}{n} [u^{n-1}] \phi(u)^n$$

$$= \frac{1}{n} [u^{n-1}] \frac{1}{(1-u^3)^n}, \text{ let } u^3 = t \rightarrow u^{3n} = t^n, u^{n-1} = t^{\frac{n}{3}-1}$$

$$= \frac{1}{n} \left[t^{\frac{n}{3}-1} \right] \frac{1}{(1-t)^n}, \text{ use negative binomial}$$

$$= \frac{1}{n} \binom{\frac{n}{3}-1+n-1}{n-1} = \frac{1}{n} \binom{\frac{4}{3}n-2}{n-1}$$

2) integer n with K summands

each summand at most r

$\Rightarrow 1 \leq k_i \leq r$ for each summand k_i

So $k_i = z + z^2 + \dots + z^r$ is OGF for # of summands

$$= z(1+z+\dots+z^{r-1})$$

$$(1+z+\dots+z^{r-1})(1-z) = (1-z^r)$$

$$= z \left(\frac{1-z^r}{1-z} \right)$$

$$\text{for } K \text{ summands} \Rightarrow \left(z \frac{1-z^r}{1-z} \right)^K$$

3a) Let's use RANKKSUBSET2(L, K, n)

UNRANKKSUBSET2(n, K, r) outputs $L = \{l_1, \dots, l_K\}$ s.t. $l_i \in \{1, \dots, n\}$
which has rank r in lex. order of K-subsets of $\{1, \dots, n\}$

RANKKSUBSET2(L, K, n)

$$r = 0$$

for i from 1 to K

$$r = r + \binom{n - L(i)}{K+1-i}$$

$$\text{return } \binom{n}{K} - 1 - r$$

\Rightarrow RANKSUBSET2(L, K, n)

$$r = 0$$

for i from 1 to K

$$r += \binom{n - L(i)}{K+1-i}$$

$$\text{return } \binom{n}{K} - 1 - r$$

UNRANKKSUBSET2(n, K, r)

let Rank = r

for i from 1 to K

$$\text{while } \binom{n - L(i)}{K+1-i} \leq \text{Rank}$$

$$\text{Rank} = \text{Rank} - \binom{n - L(i)}{K+1-i}$$

4) labeled trees n vertices, where each vertex has degree 1 or 3

in Prüfer sequence, degree 1 nodes don't appear
degree 3 nodes appear twice

Prüfer sequence length $n-2$

T has nodes $\{1, 2, \dots, n\}$

let $t_1 \in T$ s.t. $\deg(t_1) = 1$
let $t_3 \in T$ s.t. $\deg(t_3) = 3$

Prüfer(T) will have $n-2$ t_3 's, each appearing twice

If n is even

$$\# \text{ of } t_3 \text{'s} = \frac{n}{2} - 1$$

$$\# \text{ of } t_1 \text{'s} = n - \left\lceil \frac{n}{2} - 1 \right\rceil$$

$$\Rightarrow \# \text{ of all possible } t_1 \text{'s} \& t_3 \text{'s} = \left\lceil \frac{n}{2} - 1 \right\rceil \left\lceil n - \left\lceil \frac{n}{2} - 1 \right\rceil \right\rceil$$

If n is odd, round up

$$\# \text{ of } t_3 \text{'s} = \left\lceil \frac{n}{2} - 1 \right\rceil$$

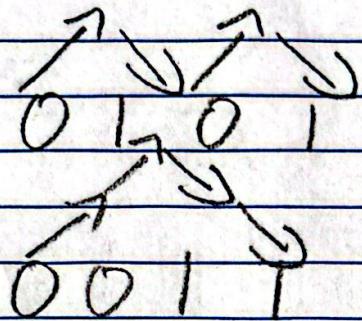
$$\# \text{ of } t_1 \text{'s} = \left\lceil n - \left\lceil \frac{n}{2} - 1 \right\rceil \right\rceil$$

$$\Rightarrow \left\lceil \frac{n}{2} - 1 \right\rceil \left\lceil n - \left\lceil \frac{n}{2} - 1 \right\rceil \right\rceil$$

5) Let's define minimal change as consecutive swap

let $\uparrow = \uparrow$
 $\downarrow = \downarrow$

example



middle two digits are swapped

$n=1$, length 2

$\uparrow \downarrow$ only one

$n=2$, length 4

0101
0011 two

$n=3$, length 6

010101
001101 four
001011
000111

$n=4$, length 8

01010101
00110101
00101101
00011101 seven
00001101
00010111
00001111

~~6a)~~ (2)

b) (1)

c) (3)

d) (1)