MATH 308 - Assignment 4

1. A company sells bags of grapes and cartons of grape juice. The company grades grapes on the scale from 1 (poor) to 10 (excellent). At present, the company has on hand 150,000 lb of grade 9 grapes and 200,000 lb of grade 6 grapes. The average quality of grapes sold in bags must be at least 8, and the average quality of grapes used to produce the juice must be at least 7 (we assume that different grades of quality level blend linearly, i.e., mixing 2 lb of grade 6 grapes and 1 lb of grade 9 grades will produces 3 lb of grade 7 grapes). Each pound of grapes used for the juice yields a revenue of \$2.55 and incurs a cost (consisting of labor cost, packing cost, overhead cost, inventory cost,...) of \$1.05. Each pound of grapes sold in bags yields a revenue of \$0.65 and incurs a cost of \$0.15.

Formulate as an LP problem to help the company maximize the profit. **You do not need to solve the problem!**

2. Consider the following LP problem:

$$\begin{aligned} \text{Maximize} \quad & f(x_1, x_2, x_3, x_4) = 2x_1 + x_2 + 3x_3 - x_4 \\ \text{subject to} \quad & x_1 - x_2 + x_3 + 2x_4 \leqslant 6 \\ & x_1 + 2x_2 + 5x_3 - x_4 \leqslant 1 \\ & 2x_1 + x_2 + x_4 = 12 \\ & x_1, x_2, x_3, x_4 \geqslant 0 \end{aligned}$$

For each point $\boldsymbol{x}$ below determine whether or not it is an optimal solution of this problem. You may try to repeat everything with the assumption that x_3 is unconstrained variable. You could use complementary slackness for non-canonical LPs. This second part does not need to be completed for assignment.

(a) $\boldsymbol{x} = (10, 0, 0, 0)$

(b) $\boldsymbol{x} = (5, \frac{11}{9}, -\frac{4}{3}, \frac{7}{9})$

(c) $\boldsymbol{x} = (6, 0, -1, 0)$

3. Suppose the following information is known about an LP:
The extreme points of the feasible set are $(0,0), (1,0), (0,1), (1,1)$. The objective is to maximise $f(x, y) = 3x + 19y$. Prove that $(x, y) = (1, 1)$ is an optimal solution. (Hint: This isn't as obvious as it looks! Determine the constraint set.)

4. A mathematics department has four geometry research groups: **Q**uadrilaterals, **R**homboids, **S**pheres, and **T**essellations. There are seven geometers in the department, some of whom work in more than one geometry research group. Specifically, the groups consist of the following members:

Q: Anne, Beatrice

R: Beatrice, Charles, David, Etta

S: David, Etta, Francois

T: Etta, Francois, Gerhard

The department wishes to organize a meeting between these research groups. Each group must be represented by at least one member, and to save on coffee expenses, the attendance at the meeting must be as small as possible. A researcher who is sent to the meeting is counted as a representative for all the groups of which he or she is a member. For example, one feasible solution is {Anne, David, Gerhard}.

(a) Formulate this problem as an LP.

(b) State the dual of the problem.

(c) What is the meaning of the dual? That is, what situation does it model?

(d) One of the optimal solutions to the primal is {Beatrice, Etta}. From this information, what does complementary slackness predict about any optimal solution to the dual?

5. Let (P) be a canonical maximization problem. Suppose that the feasible set for (P) is bounded, and that none of the extreme points are degenerate. Use complementary slackness to prove that if (P) has infinitely many optimal solutions, then its dual (D) has a degenerate optimal solution.

6. Solve the following LP problem using Dual SA.

$$
\begin{array}{ll}
\text{Minimize:} & g(x_1, x_2, x_3) = -5x_1 + x_2 - 2x_3 \\
\text{subject to} & 2x_1 + x_3 = 0 \\
 & x_1 - x_2 \geq 1 \\
 & 3x_1 - x_2 + x_3 \leq 3
\end{array}
$$

7. Solve the following LP problem using SA.

$$\begin{aligned} \text{Maximize:} \quad & f(x_1, x_2, x_3) = 3x_1 - 2x_2 + 3x_3 \\ \text{subject to} \quad & x_1 - x_2 + 2x_3 = 6 \\ & x_1 + 2x_3 = 8 \\ & x_2 + 2x_3 \geq 2 \\ & x_2, x_3 \geq 0 \end{aligned}$$

Strayer, Chapter 4, Exercise 5d, 5e (p. 111), Strayer, Chapter 4, Exercise 8 (p. 111), Strayer, Chapter 4, Exercise 12 (p. 113), Strayer, Chapter 4, Exercise 13a, 13b (p. 113)