365 Written Assignment 2

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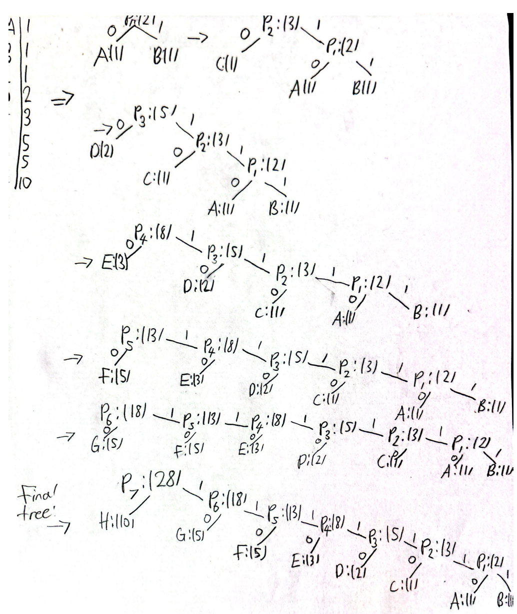
1 7.8 Q1

50/50 chance for black or white.
prob(white) = 1/2, prob(black) = 1/2.

$$\sum_{i=1}^{2} -\frac{1}{2} * log_2 * P_i = 2 * -\frac{1}{2} * log_2 \frac{1}{2} = 2 * \frac{1}{2} * log_2 2 = 2 * \frac{1}{2} = 1$$

2 7.8 Q2



3 7.8 Q3

3a)
This is an 8x8 grid meaning 64 total entries.
8 99's, 8 20's, 16 50's, 32 0's.
pr(99) = 1/8, pr(20) = 1/8, pr(50) = 1/4, pr(0) = 1/2.

$$Entropy = -\frac{1}{8} * log_2\frac{1}{8} - \frac{1}{8} * log_2\frac{1}{8} - \frac{1}{4} * log_2\frac{1}{4} - \frac{1}{2} * log_2\frac{1}{2}$$

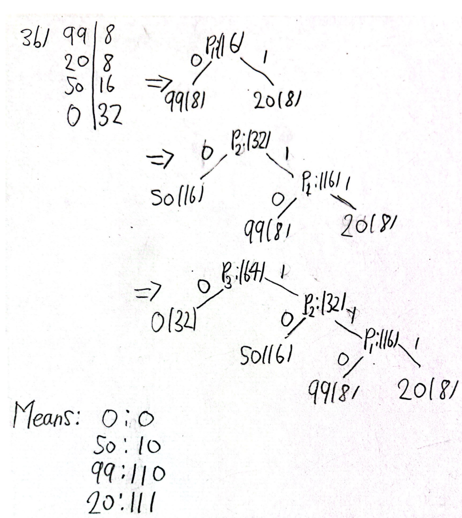
$$Entropy = \frac{1}{8} * log_2 8 + \frac{1}{8} * log_2 8 + \frac{1}{4} * log_2 4 + \frac{1}{2} * log_2 2$$

$$Entropy = \frac{1}{8} * 3 + \frac{1}{8} * 3 + \frac{1}{4} * 2 + \frac{1}{2} * 1$$

$$Entropy = \frac{3}{8} + \frac{3}{8} + \frac{2}{4} + \frac{1}{2}$$

$$Entropy = \frac{7}{4}$$

3b)



3c)

$$Average = \frac{1}{8} * 3 + \frac{1}{8} * 3 + \frac{1}{4} * 2 + \frac{1}{2} * 1 = 7/4$$

4 7.8 Q7

7a)
Pros of arithmetic: Minimum codeword length is 1 bit in Huffman coding, while in arithmetic it can be less than 1.
Cons of arithmetic: Huffman coding is more efficient than arithmetic as there's minimum redunacy, with the unique prefix property. This property will ensure no huffman code is a prefix of another code, which prevents ambiguity when decoding.

7b) (change ii to "If Arithmetic coding is used, what are the lower and higher bounds of the interval after encoding BBB and ABC, respectively")

Original bounds from 0.0 to 1.0 are A: [0,0.5), B: [0.5,0.9), C: [0.9,1)

BBB:
First B changes bounds to 0.5-0.9. Now the range is A: [0.5,0.7), B: [0.7,0.86), C: [0.86,0.90).
Second B changes bounds to 0.7-0.86. Now the range is A: [0.7,0.78), B: [0.78,0.844), C: [0.844,0.86).
Last B changes bounds to 0.78-0.844. As this is the last symbol and the question isn't asking the specific symbol ranges within the bound, we can answer with lowerbound: 0.78, higher bound: 0.844

ABC:
A changes bounds to 0-0.5. Now the range is A: [0,0.25), B: [0.25,0.45), C: [0.45,0.5).
B changes bounds to 0.25-0.45. Now the range is A: [0.25,0.35), B: [0.35,0.43), C: [0.43,0.45).
C changes bounds to 0.43-0.45. As this is the last symbol and the question isn't asking the specific symbol ranges within the bound, we can answer with lowerbound: 0.43, higher bound: 0.45

5 EX 1

Given sequence 00100011001011011101001110111011

1) String length: 32. Number of zeroes: 14, ones: 18. Pr(0) = 7/16, Pr(1) = 9/16.

$$Entropy = -\frac{7}{16} * log_2\frac{7}{16} - \frac{9}{16} * log_2\frac{9}{16}$$

2) Pr(0,0) = 49/256, Pr(0,1) = 63/256 , Pr(1,0) = 63/256, Pr(1,1) = 81/256

$$Secondorderentropy = -\frac{49}{256}*log_2\frac{49}{256} - \frac{63}{256}*log_2\frac{63}{256} - \frac{63}{256}*log_2\frac{63}{256} - \frac{81}{256}*log_2\frac{81}{256}$$

3) 2 bits needed per symbol. Occurrence of 00: 4, 01: 2, 10: 4, 11: 6. With 16 total symbols meaning
Pr(00) = 1/4, Pr(01) = 1/8, Pr(10) = 1/4, Pr(11) = 3/8.
Which leads us to

$$L = \frac{1}{4} * 2 + \frac{1}{8} * 2 + \frac{1}{4} * 2 + \frac{3}{8} * 2 = 2$$

6 Example 2

Given input sequence XZZXYYXXYZYY, and assume that the initial dictionary is: 0 X, 1 Y, 2 Z

Output: 0 2 2 0 1 1 0 6 2 7

Code:
3 XZ
4 ZZ
5 ZX
6 XY
7 YY
8 YX
9 ZY