365 Assignment 1

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1 Assignment 1

1.1 1.5 – #5

5) Converting analog to digital isn't always necessary. If we have an analogy computer and we can read our format, we don't need a conversion if our task can be completed by an analog machine.
Analog $\rightarrow$ Digital
Pros:
- Digital is more generalized so can be transferred between digital machines easier
- More established protocols for conventions like text images
- Time independent for digital so computational speedup comes with faster and more efficient machines
- Digital has efficient compression methods

Cons:
- Digitization has round off errors from quantization
- For a specific analog signal, we need to know the intricacies of the DEs and functions that describe the signal (no standard)
- Takes significant time to run through analog signal & digitize it

1.2 3.3 – #2.3

2) 8 -bit grey scale can represent $2^8 = 256$ values
2 - bit grey scale can represent $2^2 = 4$ values
We can map equal ranges of values in 8-bit to each value in 2-bit

$$0 - 63 \rightarrow 0$$
$$64 - 127 \rightarrow 1$$
$$128 - 191 \rightarrow 2$$
$$192 - 255 \rightarrow 3$$

3) 5-bit grey scale to 7 -bit printer
For 2-level (1 bit output), we need a multiplier of 2 for each bit

$$\Rightarrow 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5 = 32$$

1.3 $4.4 - \#1, 8, 9$

1) wavelength $\iff$ hue
color level $\iff$ chrominance
brightness $\iff$ luminance
whiteness $\iff$ Saturation
8a) If no gamma correction is applied voltage to light level is too low. Meaning it will appear too dark, with less shades of distinguishable blacks
b) It will appear brighter with more distinguish able colours& appearance
9a) The levels will be lower than 256. It will reduce it to contain less representations as the ranges mapped values fall into overlap. (Many to one mapping)
b) Low end near 0:

$$0\text{: } 0 \rightarrow \text{int}\left(255\frac{0}{\sqrt{255}}\right) = 0$$

$$1\text{: } 1 \rightarrow \text{int}\left(255\frac{1}{\sqrt{255}}\right) = 1$$

Values 1-14 cannot be gamma corrected & displayed
High end near 255:

$$255\text{: } 255 \rightarrow \text{int}\left(255\frac{1}{\sqrt{255}}\right) = 255$$

$$254\text{: } 254 \rightarrow \text{int}\left(255\frac{254}{\sqrt{255}}\right) = 254$$

$$253\text{: } 253 \rightarrow \text{int}\left(255\frac{253}{\sqrt{255}}\right) = 253$$

b) High end is mapped with no values lost
Low end gets values 1 to 14 lost
Low end is more effected

1.4 $6.4 - \#1, 2, 3, 4, 5, 8$

1) If the lower frequency limit is f_L and the upper limit is f_U, the sampling rate should be at least 2*(f_U - f_L).
Meaning for 2-10 MHz, we should have a sampling rate of at least 2*(10-2) = 16 MHz

2a) Quantization levels
b) Peak SQNR for N bits is $20 * log_{10}((2^{N-1})/1/2)$ meaning for 8 bits we have $20 * log_{10}((2^{8-1})/1/2)$ which is approximately 48.16 dB

3) Each 10 dB is a multiplier by 10 so a 20dB increase is a ratio of intensities increase by a factor of 100

4) Each 10 dB is a multiplier by 10 so a 30dB reduction is a ratio of intensities reduction of 1000

5a) $20 * log_{10}(V_{signal}/1) = -3 \rightarrow log_{10}(V_{signal}) = -3/20 \rightarrow V \approx 0.708$
5b) 3 dB net change. $20 * log_{10}(V/1) = 3 \rightarrow V = 10^{3/20} \rightarrow V \approx 1.414$

8) Dynamic range gets mapped to voltage levels from $-2^{N-1}..2^{N-1} - 1$.
For 10 bits $-2^{9}..2^{9} - 1$, for 16 bits $-2^{15}..2^{15} - 1$.
Dynamic range is 60 dB so we have $20*log_{10}(V_{max}/V_{min}) = 60 \rightarrow V_{max}/V_{min} = 1000$.
Since we want the noise to be reduced to at least an order of magnitude below V_{min}, if the V_{max}/V_{noise} will be orders of magnitude above 1000. 10 bits just doesn't cut as it's V_{max}/V_{noise} is $V_{max}/2^{10} = V_{max}/1024$. Not an order of magnitude difference of 1000. However 16 bits is fine as V_{max}/V_{noise} is $V_{max}/2^{16} = V_{max}/65536$, plenty of factors.

1.5 Example question 1

1) In a very quiet room, a mic's output voltage is 0.5V. When this mic is moved to a busy street, what will be the output voltage?

What about moving to station with a train passing through? And two trains simultaneously?

In a very quiet room, 20dB has output 0.5V. Moving to a train passing has 90 dB.

$10 * log_{10}(r_{quietroom}) = 20 \rightarrow r_{quietroom} = 10^{2}$
$10 * log_{10}(r_{train}) = 90 \rightarrow r_{train} = 10^{9}$
$r_{train}/r_{quietroom} = 10^{9}/10^{2} = 10^{7}$ will give how much larger power is so sqrt it to get voltage multiplier
$0.5 * \sqrt{10^{7}} \approx 1581V$

Two trains simultaneously from one have power ratio of $2/1 = 2$. Meaning to get voltage scaling we multiply by sqrt(2). $\sqrt{2} * 1581 \approx 2236V$

1.6 Example question 2

Show the relationship between GBR and XYZ in matrix form for both directions.

$Y \to Y - 1/3 * X$.
$X \to X + 1/2 * Y \to X + 1/2 * (Y - 1/3 * X) \to 5/6 * X + 1/2 * Y$.
$Y \to Y - 1/2 * Z \to Y - 1/3 * X - 1/2 * Z$.
$Z \to Z + 1 * Y \to Z + 5/6 * X + 1/2 * Y$.

Final equations:

$G = 5/6 * X + 1/2 * Y$.
$B = -1/3 * X + Y - 1/2 * Z$.
$R = 5/6 * X + 1/2 * Y + Z$.

We can write this in matrix-vector form:
$$\begin{pmatrix} 5/6 & 1/2 & 0 \\ -1/3 & 1 & -1/2 \\ 5/6 & 1/2 & 1 \end{pmatrix} * \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} G \\ B \\ R \end{pmatrix}$$

To find the other direction, we need to find the inverse of the matrix so
$$\begin{pmatrix} 5/6 & 1/2 & 0 \\ -1/3 & 1 & -1/2 \\ 5/6 & 1/2 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 15/8 & -3/4 & -3/8 \\ -9/8 & 5/4 & 5/8 \\ -1 & 0 & 1 \end{pmatrix}$$
$$\begin{pmatrix} 15/8 & -3/4 & -3/8 \\ -9/8 & 5/4 & 5/8 \\ -1 & 0 & 1 \end{pmatrix} * \begin{pmatrix} G \\ B \\ R \end{pmatrix} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$