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Assignment 3- Feb. 15 30147620

1) $|\psi\rangle \in \mathbb{C}^d$, $\| |\psi\rangle \| = 1$. $\| |\psi^\perp\rangle \| = 1$, $\langle \psi^\perp | \psi \rangle = \langle \psi | \psi^\perp \rangle = 0$

1i) Let $|\psi\rangle = \begin{pmatrix} a_1 \\ \vdots \\ a_d \end{pmatrix}$ s.t. $\| |\psi\rangle \| = 1$

$$P = |\psi\rangle\langle\psi| = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_d \end{pmatrix} \begin{pmatrix} a_1^* & a_2^* & \cdots & a_d^* \end{pmatrix}$$

$$= \begin{pmatrix} a_1 a_1^* & a_1 a_2^* & \cdots & a_1 a_d^* \\ a_2 a_1^* & a_2 a_2^* & \cdots & a_2 a_d^* \\ \vdots & \vdots & \ddots & \vdots \\ a_d a_1^* & a_d a_2^* & \cdots & a_d a_d^* \end{pmatrix}$$

$$(I - 2P)|\psi\rangle = \left(\begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix} - 2 \begin{pmatrix} a_1 a_1^* & a_1 a_2^* & \cdots & a_1 a_d^* \\ a_2 a_1^* & a_2 a_2^* & \cdots & a_2 a_d^* \\ \vdots & & \ddots & \vdots \\ a_d a_1^* & a_d a_2^* & \cdots & a_d a_d^* \end{pmatrix} \right) \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_d \end{pmatrix}$$

$$= \begin{pmatrix} 1 - 2a_1 a_1^* & -2a_1 a_2^* & \cdots & -2a_1 a_d^* \\ -2a_2 a_1^* & 1 - a_2 a_2^* & \cdots & -2a_2 a_d^* \\ \vdots & \vdots & \ddots & \vdots \\ -2a_d a_1^* & -2a_d a_2^* & \cdots & 1 - 2a_d a_d^* \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_d \end{pmatrix}$$

$$= \begin{pmatrix} (1 - 2a_1 a_1^*) a_1 - 2a_1 a_2^* a_2 - \cdots - 2a_1 a_d^* a_d \\ -2a_2 a_1^* a_1 + (1 - a_2 a_2^*) a_2 - \cdots - 2a_2 a_d^* a_d \\ \vdots \\ -2a_d a_1^* a_1 - 2a_d a_2^* a_2 - \cdots + (1 - 2a_d a_d^*) a_d \end{pmatrix}$$

1ii)

$$\begin{pmatrix} a_1 - 2a_1 a_1^* a_1 - 2a_1 a_2^* a_2 - \dots - 2a_1 a_d^* a_d \\ -2a_2 a_1^* a_1 + a_2 - a_2 a_2^* a_2 - \dots - 2a_2 a_d^* a_d \\ \vdots \\ -2a_d a_1^* a_1 - 2a_d a_2^* a_2 - \dots a_d - 2a_d a_d^* a_d \end{pmatrix}$$

$$\begin{pmatrix} a_1 - (2a_1 \langle \psi | \psi \rangle \\ a_2 - (2a_2 \langle \psi | \psi \rangle \\ \vdots \\ a_d - (2a_d \langle \psi | \psi \rangle \end{pmatrix}$$ Unit vector dotted with itself equal

$$\begin{pmatrix} a_1 - 2a_1 \\ a_2 - 2a_2 \\ \vdots \\ a_d - 2a_d \end{pmatrix} = -|\psi\rangle$$

=

ii) $(I-2|\psi\rangle\langle\psi|) = \begin{pmatrix} 1-2a_1a_1^* & -2a_1a_2^* & \cdots & -2a_1a_d^* \\ -2a_2a_1^* & 1-a_2a_2^* & \cdots & -2a_2a_d^* \\ \vdots & \vdots & \ddots & \vdots \\ -2a_da_1^* & -2a_da_2^* & \cdots & 1-2a_da_d^* \end{pmatrix} = P$

$P^{\perp} = \begin{pmatrix} 1-2a_1^*a_1 & -2a_2^*a_1 & \cdots & -2a_d^*a_1 \\ -2a_1^*a_2 & 1-a_2^*a_2 & \cdots & -2a_d^*a_2 \\ \vdots & \vdots & \ddots & \vdots \\ -2a_1^*a_d & -2a_2^*a_d & \cdots & 1-2a_d^*a_d \end{pmatrix}, \; P = P^{\perp}$

$$\begin{pmatrix} \vdots \\ a_2 - (2a_2\langle\psi|\psi\rangle) \\ \vdots \\ a_d - (2a_d\langle\psi|\psi\rangle) \end{pmatrix}$$

$$\begin{pmatrix} a_1 - 2a_1 \\ a_2 - 2a_2 \\ \vdots \\ a_d - 2a_d \end{pmatrix} = -|\psi\rangle$$

ii) Let $|\psi^\perp\rangle = \begin{pmatrix} a_1^\perp \\ a_2^\perp \\ \vdots \\ a_d^\perp \end{pmatrix}$ s.t. $\||\psi^\perp\rangle\| = 1$ & $\langle\psi|\psi^\perp\rangle = \langle\psi^\perp|\psi\rangle = 0$

$$(I - 2P)|\psi^\perp\rangle = \begin{pmatrix} 1-2a_1a_1^* & -2a_1a_2^* & \cdots & -2a_1a_d^* \\ -2a_2a_1^* & 1-2a_2a_2^* & \cdots & -2a_2a_d^* \\ \vdots & \vdots & \ddots & \vdots \\ -2a_da_1^* & -2a_da_2^* & \cdots & 1-2a_da_d^* \end{pmatrix} \begin{pmatrix} a_1^\perp \\ a_2^\perp \\ \vdots \\ a_d^\perp \end{pmatrix}$$

$$= \begin{pmatrix} (1-2a_1a_1^*)a_1^\perp - 2a_1a_2^*a_2^\perp - \cdots - 2a_1a_d^*a_d^\perp \\ -2a_2a_1^*a_1^\perp + (1-2a_2a_2^*)a_2^\perp - \cdots - 2a_2a_d^*a_d^\perp \\ \vdots \\ -2a_da_1^*a_1^\perp - 2a_da_2^*a_2^\perp - \cdots + (1-2a_da_d^*)a_d^\perp \end{pmatrix}$$

$$= \begin{pmatrix} a_1^\perp - 2a_1a_1^*a_1^\perp - 2a_1a_2^*a_2^\perp - \cdots - 2a_1a_d^*a_d^\perp \\ -2a_2a_1^*a_1^\perp + a_2^\perp - 2a_2a_2^*a_2^\perp - \cdots - 2a_2a_d^*a_d^\perp \\ \vdots \\ -2a_da_1^*a_1^\perp - 2a_da_2^*a_2^\perp - \cdots - a_d^\perp - 2a_da_d^*a_d^\perp \end{pmatrix}$$

ii) $\begin{vmatrix} a_1^{\perp} - 2a_1 \langle \psi | \psi^{\perp} \rangle \\ a_2^{\perp} - 2a_2 \langle \psi | \psi^{\perp} \rangle \\ \vdots \\ a_d^{\perp} - 2a_d \langle \psi | \psi^{\perp} \rangle \end{vmatrix}$, $\langle \psi | \psi^{\perp} \rangle = 0$ by defn

$= |\psi^{\perp}\rangle$

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ii) Let's look at $PP^{\dagger}_{1,1}$. Just the first element

$$PP^{\dagger}_{1,1} = |1-2a_1a_1^*||1-2a_1^*a_1| + |4a_1a_1^*a_2a_2^*| + \ldots + |4a_1a_1^*a_da_d^*|$$

$$= 1 - 2a_1^*a_1 - 2a_1a_1^* + 4a_1|a_1^*|^2 + 4a_1a_1^*a_2a_2^* + \ldots + 4a_1a_1^*a_da_d^*$$

$$= 1 - 4a_1^*a_1 + 4a_1^2(a_1^*)^2 + 4a_1a_1^*a_2a_2^* + \ldots + 4a_1a_1^*a_da_d^*$$

$$= 1 - 4a_1^*a_1 + 4a_1a_1^*(a_1a_1^*) + 4a_1a_1(a_2a_2^*) + \ldots + 4a_1a_1^*a_da_d^*$$

$$= 1 - 4a_1^*a_1 + 4a_1a_1^*(a_1a_1^* + a_2a_2^* + \ldots + a_da_d^*)$$

$$= 1 - 4a_1^*a_1 + 4a_1a_1^* \langle\psi|\psi^{\perp}\rangle$$

$$= 1 - 4a_1^*a_1 + 4a_1a_1^*$$

$$= 1$$

Infact, any diagonal element will be 1. Let's prove this.

Assume $X \neq 1$

$$PP^{\perp}_{x,x} = 4a_1a_1^*a_xa_x^* + \ldots + |1-2a_x^*a_x||1-2a_xa_x^*| + \ldots + 4a_xa_x^*a_da_d^*$$

$$= 4a_1a_1^*a_xa_x^* + \ldots + 1 - 2a_x^*a_x - 2a_xa_x^* + 4a_xa_x^*a_xa_x^* + \ldots + 4a_xa_x^*$$

$$= 4a_1a_1^*a_xa_x^* + \ldots + 1 - 4a_xa_x^* + 4a_xa_x^*a_xa_x^* + \ldots + 4a_xa_x^*a_da_d^*$$

$$= 1 - 4a_xa_x^* + 4a_xa_x^*(a_1a_1^* + \ldots + a_xa_x^* + \ldots + a_da_d^*)$$

$$= 1 - 4a_xa_x^* + 4a_xa_x^* \langle\psi|\psi^{\perp}\rangle$$

$$= 1 - 4a_xa_x^* + 4a_xa_x^*$$

$$= 1$$

So we know all of the diagonal entries are 1, what about all of the other entries?

Let's look at $PP^{\dagger}_{1,2}$

Included 3rd term for clarification

$$PP^{\dagger}_{1,2} = (1-2a_1^*a_1)(-2a_1a_2^*) + (-2a_2^*a_1)(1-2a_2^*a_2) + (-2a_1a_3^*)(-2a_2^*a_3) \dots + (-2a_1a_d^*)(-2a_2^*a_d)$$

$$= -2a_1a_2^* + 4a_1a_1^*a_1a_2^* - 2a_2^*a_1 + 4a_2^*a_1a_2^*a_2 + 4a_1a_3^*a_2^*a_3 \dots + 4a_1a_d^*a_2^*a_d$$

$$= -4a_1a_2^* + 4a_1a_2^*a_1^*a_1 + 4a_1a_2^*a_2a_2^* + 4a_1a_2^*a_3a_3^* + \dots + 4a_1a_2^*a_d^*a_d$$

$$= -4a_1a_2^* + 4a_1a_2^*(a_1^*a_1 + a_2a_2^* + a_3a_3^* + \dots + a_da_d^*)$$

$$= -4a_1a_2^* + 4a_1a_2^*\langle\psi|\psi^{\dagger}\rangle$$

$$= -4a_1a_2^* + 4a_1a_2^* = 0$$

In fact, all non-diagonal elements are zero. Let's prove this

Assume $x \neq y,\ x \neq 1,\ y \neq 2$

$$PP^{\dagger}_{x,y} = P_{row\,x} P_{col\,y} = (-2a_xa_1^* \quad -2a_xa_2^* \dots 1-a_xa_x^* \dots -2a_xa_d^*)\begin{pmatrix}-2a_y^*a_1\\ -2a_y^*a_2\\ \vdots\\ 1-a_y^*a_y\\ \vdots\\ -2a_y^*a_d\end{pmatrix}$$

$$= 4a_xa_y^*a_1a_1^* + 4a_xa_y^*a_2a_2^* \dots (1-a_xa_x^*)(-2a_y^*a_x) + \dots + (-2a_xa_y^*)(1-a_ya_y^*) + \dots 4a_xa_y^*a_da_d^*$$

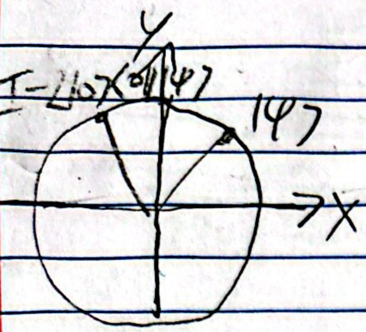
$$= 4a_xa_y^*a_1a_1^* + 4a_xa_y^*a_2a_2^* + \dots - 2a_xa_y^* + 2a_xa_y^*a_xa_x^* + \dots - 2a_xa_y^* + 2a_xa_y^*a_ya_y^* + \dots + 4a_xa_y^*a_da_d^*$$

$$= -4a_xa_y^* + 4a_xa_y^*\langle\psi|\psi^{\dagger}\rangle = -4a_xa_y^* + 4a_xa_y^* = 0$$

Now we know $PP^{\dagger} = I$. Since $P = P^{\dagger}$, $PP^{\dagger} = P^{\dagger}P = I$.

$\Rightarrow P$ is unitary when $\||\psi\rangle\| = 1$ as $\langle\psi|\psi^{\dagger}\rangle = 1$ was used in all these eq'ns

3/ $I-2|0\rangle\langle 0|\psi\rangle$



$$I-2|0\rangle\langle 0| = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} - 2\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix} = \begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}$$

for $|\psi\rangle = \begin{pmatrix}a\\ b\end{pmatrix}$ $\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}a\\ b\end{pmatrix} = \begin{pmatrix}-a\\ b\end{pmatrix}$

negates/flips x value & keeps y value

4/ Yes it does but it's under a rotation as we're in 3D
It would be a rotation to the $-x$ value of the state along an x axis of symettry

2i/ $|ab\rangle \Rightarrow a \oplus b$
$|ab\rangle \Rightarrow |0/1\rangle|0/1\rangle \Rightarrow 0/1 \oplus 0/1$
measure in comp basis

First if the qubits are in a different basis than comp basis, we would either have to express each of the qubits as a lin. combo of $|0\rangle, |1\rangle$ vecto and take parity or set our own convention for parity values (like in Ass.

Secondly the comp basis measurement will collapse the qubits based on their probabilities of $|0\rangle, |1\rangle$. We can possibly get a wrong result becaus we didn't do the composite representation described above

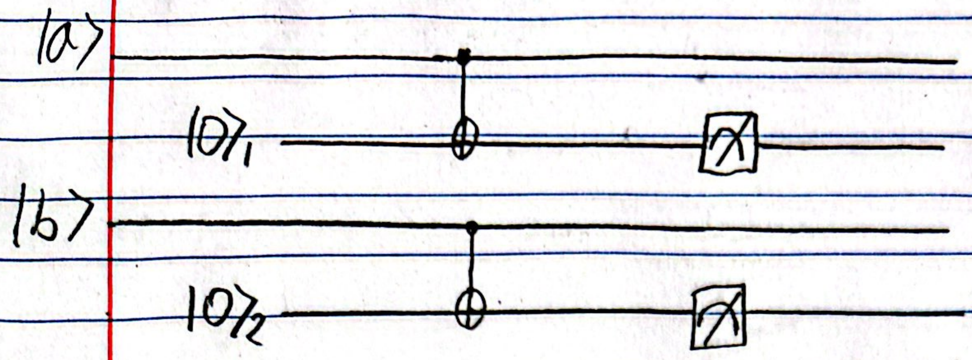
2ii/ Use ancilla on $|ab\rangle \Rightarrow |a\rangle|b\rangle$
We can transform two ancilla's on each to play with CNOT & get value of $|a\rangle$ & $|b\rangle$ by not measuring them

$\Rightarrow |a\rangle|0\rangle|b\rangle|0\rangle$

Apply CNOT on $|a\rangle|0\rangle$ & $|b\rangle|0\rangle$ & measure ancillas in comp basis.

If $|a\rangle$ or $|b\rangle$ was $|1\rangle$, we would get ancilla being $|1\rangle$
If $|a\rangle$ or $|b\rangle$ was $|0\rangle$, we would get ancilla being $|0\rangle$
We can then use this to compute parity

ii) Let's do this formally now



Add two distinct ancillas

CNOT

Comp basis measure

Observe results of ancillas to get values of $|a\rangle$ & $|b\rangle$

$\Rightarrow$ Compute parity of $|a\rangle|b\rangle$ without measuring $|a\rangle$ or $|b\rangle$

3/

$$dm = \sum_i p_i |\psi_i\rangle\langle\psi_i|$$

i/

$$1\cdot\left(\frac{1}{\sqrt2}|0\rangle + \frac{1}{\sqrt2}|+\rangle\right)\left(\frac{1}{\sqrt2}\langle 0| + \frac{1}{\sqrt2}\langle +|\right)$$

$$\frac{1}{2}\left(|0\rangle\langle 0| + |+\rangle\langle 0| + |0\rangle\langle +| + |+\rangle\langle +|\right)$$

$$\frac{1}{2}\left(\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} + \begin{pmatrix}0 & 0\\ 0 & \frac{1}{\sqrt2}\end{pmatrix} + \begin{pmatrix}\frac{1}{\sqrt2} & 0\\ 0 & 0\end{pmatrix} + \begin{pmatrix}\frac{1}{2} & \frac{1}{2}\\ \frac{1}{2} & \frac{1}{2}\end{pmatrix}\right)$$

$$\frac{1}{2}\begin{pmatrix}1+\frac{1}{\sqrt2}+\frac{1}{2} & \frac{1}{2}\\ \frac{1}{2} & 1+\frac{1}{\sqrt2}+\frac{1}{2}\end{pmatrix} = \begin{pmatrix}\frac{1}{2}+\frac{1}{2\sqrt2}+\frac{1}{4} & \frac{1}{4}\\ \frac{1}{4} & \frac{1}{2}+\frac{1}{2\sqrt2}+\frac{1}{4}\end{pmatrix}$$

ii/

$$\frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|+\rangle\langle +|$$

$$\begin{pmatrix}\frac{1}{2} & 0\\ 0 & 0\end{pmatrix} + \begin{pmatrix}\frac{1}{4} & \frac{1}{4}\\ \frac{1}{4} & \frac{1}{4}\end{pmatrix} = \begin{pmatrix}\frac{3}{4} & \frac{1}{4}\\ \frac{1}{4} & \frac{1}{4}\end{pmatrix}$$

iii/

$$\frac{1}{2}|00\rangle\langle 00| + \frac{1}{4}|01\rangle\langle 01| + \frac{1}{4}|10\rangle\langle 10|$$

$$\frac{1}{2}\begin{pmatrix}1&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix} + \frac{1}{4}\begin{pmatrix}0&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix} + \frac{1}{4}\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&1&0\\0&0&0&0\end{pmatrix} = \begin{pmatrix}\frac{1}{2}&0&0\\0&\frac{1}{4}&0\\0&0&\frac{1}{4}\\0&0&0\end{pmatrix}$$

4) $\mathrm{Tr}_2\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix} = \begin{pmatrix} a_{11}+a_{22} & a_{13}+a_{24} \\ a_{31}+a_{42} & a_{33}+a_{44} \end{pmatrix}$

$\mathrm{Tr}_1\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix} = \begin{pmatrix} a_{11}+a_{33} & a_{12}+a_{34} \\ a_{21}+a_{43} & a_{22}+a_{44} \end{pmatrix}$

$\Rightarrow \mathrm{Tr}_A\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0+\frac{1}{2} & 0+0 \\ 0+0 & \frac{1}{2}+0 \end{pmatrix} = \frac{1}{2} I$

5) A is positive-semidefinite if $\langle v|A|v\rangle \in \mathbb{R}$ & ≥ 0

Consider the density matrix of an ensemble of pure states

$\rho = \sum_i p_i |\phi_i\rangle\langle\phi_i|$ & let $|V\rangle$ be any vector size d where d is max i (same number as # of states)

WTS: $\langle V|\left(\sum_i p_i |\phi_i\rangle\langle\phi_i|\right)|V\rangle \in \mathbb{R}^+$

$\Rightarrow \sum_i p_i \langle V|\phi_i\rangle\langle\phi_i|V\rangle$

$\sum_i p_i \langle V|\phi_i\rangle\langle V|\phi_i\rangle^*$

$\sum_i p_i |\langle V|\phi_i\rangle|^2 \Rightarrow p_i \in \mathbb{R}^+$ & any square $\in \mathbb{R}^+$

$\Rightarrow \langle V|\rho|V\rangle \in \mathbb{R}^+ \Rightarrow$ semi-definite operator

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6) $\rho \in H_A \otimes H_B$

Measurement of A in $\{|e_i\rangle\}$ basis $\Leftrightarrow \{P_i = |e_i\rangle\langle e_i| \otimes I\}$, $\rho \to \sum_i P_i \rho P_i$

Bob's basis H_B has $\{|f_j\rangle\}$ $\Rightarrow$ Bob's density matrix $DM_B = \sum_j P_j |f_j\rangle\langle f_j|$

Bob's reduced density matrix is a partial trace over Alice's

$$\Rightarrow Tr_A \rho = \sum_{ij} P_{ij} |e_i\rangle\langle e_i| Tr(|f_j\rangle\langle f_j|)$$

Measurement of A: $|e_i\rangle\langle e_i| \left(\sum_{ij} P_{ij} |e_i\rangle\langle e_i| Tr(|f_j\rangle\langle f_j|)\right) |e_i\rangle\langle e_i|$

$$\sum_{ij} P_{ij} |e_i\rangle\langle e_i|e_i\rangle\langle e_i|e_i\rangle\langle e_i| Tr(|f_j\rangle\langle f_j|)$$

As we are in orthonormal basis, any basis vector dotted with itself is its magnitude squared $\Rightarrow 1^2 = 1$

$$\Rightarrow \sum_{ij} P_{ij} |e_i\rangle 1 \cdot 1 \langle e_i| Tr(|f_j\rangle\langle f_j|).$$

$$\sum_{ij} P_{ij} |e_i\rangle\langle e_i| Tr(|f_j\rangle\langle f_j|)$$

Concluding that

$$|e_i\rangle\langle e_i| \left(\sum_{ij} P_{ij} |e_i\rangle\langle e_i| Tr(|f_j\rangle\langle f_j|)\right) |e_i\rangle\langle e_i| = \sum_{ij} P_{ij} |e_i\rangle\langle e_i| Tr(|f_j\rangle\langle f_j|)$$

$$\Rightarrow \quad P_i \, Tr_A \rho \, P_i = Tr_A \rho$$

$\Rightarrow$ Bob's reduced density matrix isn't effected by Alice's measurement in any basis

7) Alice $|\psi\rangle$, Bob $|\phi\rangle$

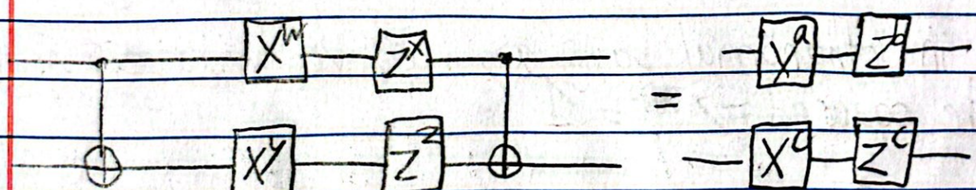
- Alice & Bob have classical communication
- Alice & Bob have access to unlimited entangled qubits
- Alice & Bob do NOT have a quantum communication channel

i) Start with an entangled state with entangled qubits $|\psi\rangle|\phi\rangle$ between the both of them

Bob applies CNOT with Alice as control bit as only Bob's qubit will change with CNOT

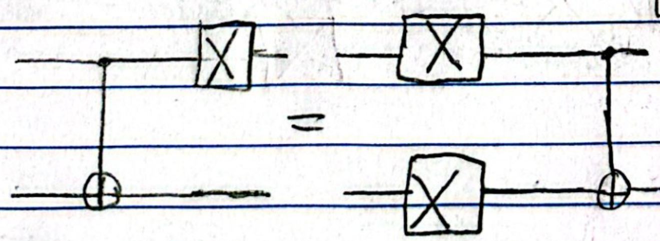
$\Rightarrow$ CNOT is applied

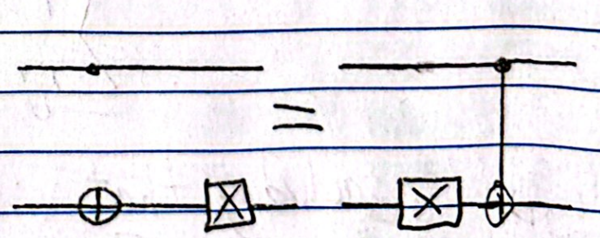
ii) Find values of $a, b, c, d \in \{0,1\}$ as functions of w, x, y, z

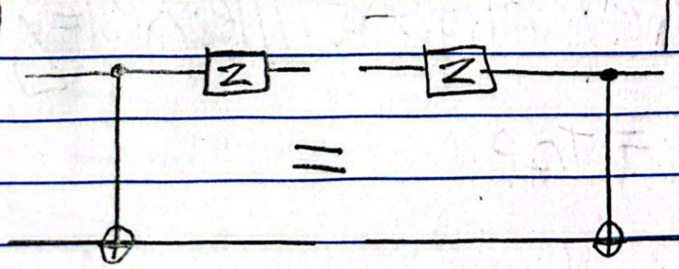


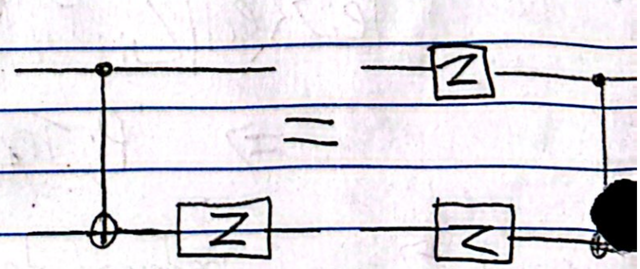
is there a typo? where is d in this circu I just went with the two c's here

Circuit equalities:

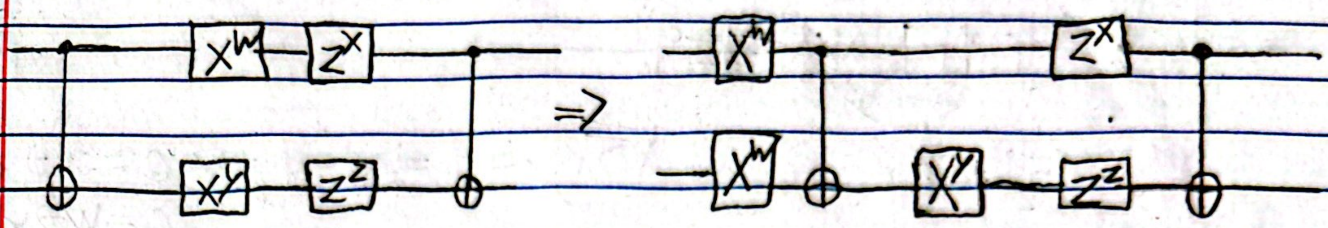
(1) 

(2) 

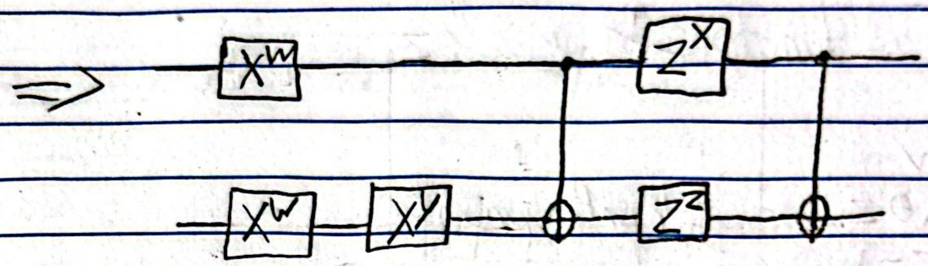
(3) 

(4) 

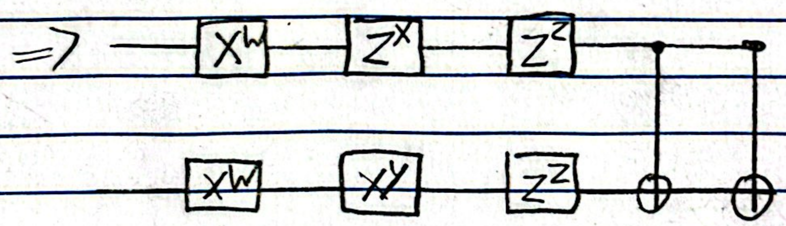
ii) Using (1) we can move X^w to rhs



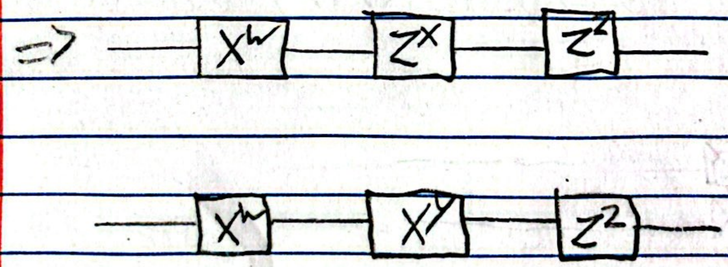
(2) move X^y to rhs too



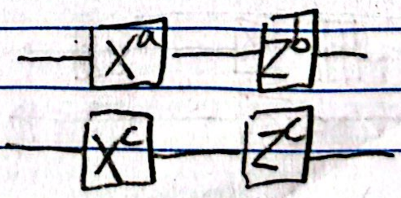
(3) & (4) to bring CNOT's together



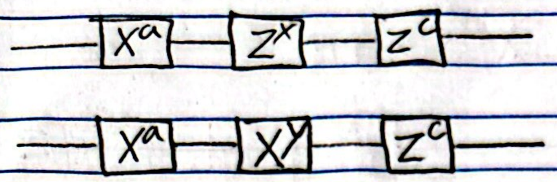
Double CNOT cancels them out $(CNOT(CNOT(|11\rangle)) = CNOT(|10\rangle) =$
$(CNOT(CNOT(|10\rangle)) = CNOT(|11\rangle) =$



Transforming this into



$\Rightarrow a = w$ as it's the only X at top
$c = z$ as it's the only Z at bottom

ii) 

$\Rightarrow$

$a = y = 1 \Rightarrow c = 0.\quad a \text{ XOR } y = 1 \Rightarrow c = 1 \rightarrow c = a \oplus y$

$c = w \oplus y$

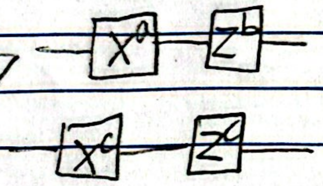
$x = c = 1 \Rightarrow b = 0.\quad x \text{ XOR } c = 1 \Rightarrow b = 1 \rightarrow b = x \oplus c$

$b = x \oplus z$

full list as functions of w, x, y, z

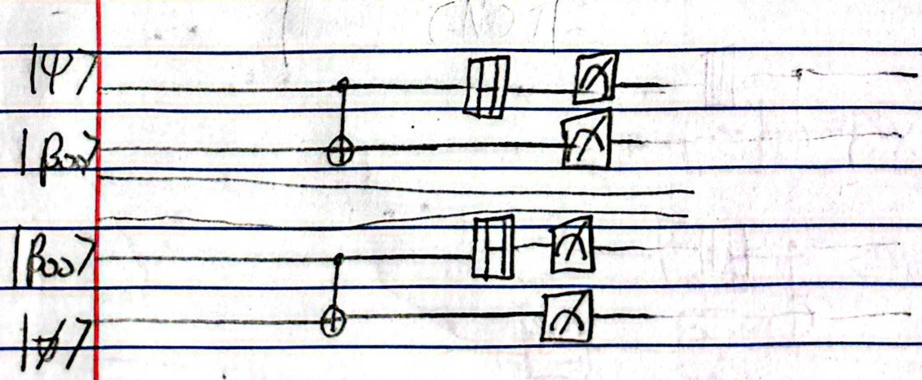
$\Rightarrow a = w$
$b = x \oplus z$
$c = z = w \oplus y$

, d isn't present

$\Rightarrow$ 

iii) On the state $|\psi\rangle|\phi\rangle$

Bell measurement of $|\psi\rangle|\beta_{00}\rangle$ & $|\beta_{00}\rangle|\phi\rangle$ can be implemented



Look at them individually

iii) $|\psi\rangle|\beta_{00}\rangle|\beta_{00}\rangle|\phi\rangle \Rightarrow |\psi\rangle|\beta_{00}\rangle \;\&\; |\beta_{00}\rangle|\phi\rangle$

Assume $|\psi\rangle = \alpha_A|0\rangle + \beta_A|1\rangle$

$$|\psi\rangle|\beta_{00}\rangle = (\alpha_A|0\rangle + \beta_A|1\rangle)\otimes\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)$$

$$\Rightarrow \frac{1}{\sqrt{2}}(\alpha_A|000\rangle + \alpha_A|011\rangle + \beta_A|100\rangle + \beta_A|111\rangle)$$

Apply CNOT on first two qubits

$$\Rightarrow \frac{1}{\sqrt{2}}(\alpha_A|000\rangle + \alpha_A|011\rangle + \beta_A|110\rangle + \beta_A|101\rangle)$$

Apply H on first qubit

$$\Rightarrow \frac{1}{2}(\alpha_A|000\rangle + \alpha_A|100\rangle + \alpha_A|011\rangle + \alpha_A|111\rangle$$
$$+ \beta_A|010\rangle - \beta_A|110\rangle + \beta_A|001\rangle - \beta_A|101\rangle)$$

Try to get $|\psi\rangle$ back $\Rightarrow \frac{1}{2}(\alpha_A|0\rangle(|00\rangle+|11\rangle) + (-\beta_A|1\rangle(|10\rangle+|01$

Assume $|\phi\rangle = \alpha_B|0\rangle + \beta_B|1\rangle$

$$|\beta_{00}\rangle|\phi\rangle = (\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)\otimes(\alpha_B|0\rangle+\beta_B|1\rangle)$$
$$= \frac{1}{\sqrt{2}}(\alpha_B|000\rangle + \alpha_B|110\rangle + \beta_B|001\rangle + \beta_B|111\rangle)$$

Apply CNOT on last two qubits

$$\Rightarrow \frac{1}{\sqrt{2}}(\alpha_B|000\rangle + \alpha_B|111\rangle + \beta_B|001\rangle + \beta_B|110\rangle)$$

Apply H on second qubit

$$\frac{1}{2}(\alpha_B|000\rangle + \alpha_B|010\rangle + \alpha_B|101\rangle - \alpha_B|111\rangle + \beta_B|001\rangle + \beta_B|011\rangle + \beta_B|101\rangle - \beta_B|11$$

Try to get $|\phi\rangle$ back $\Rightarrow \frac{1}{2}(\alpha_B|0\rangle\otimes(|00\rangle+|10\rangle) - \beta_B|1\rangle$