308- Assignment 4

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1) Scale 1-10 for grapes. 150,000 lbs of grade 9
200,000 lbs of grade 6

Average quality of grapes sold in bags must be at least 8
Average quality of grapes sold for juice must be at least 7

Mixing x lb's of grade A with y lb's of grade B grapes will give x+y lb of $(x \cdot A + y \cdot B)/x+y$ grade grapes

Each pound of grapes used for juice yields 2.55$ revenue with cost $1.05. => Profit: 1.5$

Each pound of grapes used for bags yields 0.65$ revenue with cost $0.15 => Profit: 0.5$

ulation

Let G_b be lb's of grapes for bags sold
Let G_j be lb's of grapes for juice sold

Maximize $f(G_b, G_j) = 0.5G_b + 1.5G_j$ s.t.

Let G_{6b} be lb's of grade 6 grapes used for bags, G_{6j} for juice
Let G_{9b} be lb's of grade 9 grapes used for bags, G_{9j} for juice

$G_{6b} + G_{6j} \leq 200000$, $G_{9b} + G_{9j} \leq 150000$. $G_{6b} + G_{9b} = G_b$, $G_{6j} + G_{9j} = G_j$.

mix G_6 & G_9 => $G_6 + G_9$ lb's of grade $\frac{(G_6 \cdot 6 + G_9 \cdot 9)}{15}$ grapes

meaning $\frac{6G_{6b} + 9G_{9b}}{15} \geq 8$, $\frac{6G_{6j} + 9G_{9j}}{15} \geq 7$.

Rewrite f using product specifics: $f(G_{6b}, G_{6j}, G_{9b}, G_{9j}) = 0.5(G_{6b} + G_{9b}) + 1.5(G_{6j} + G_{9j})$

2) Maximize: $f(x_1, x_2, x_3, x_4) = 2x_1 + x_2 + 3x_3 - x_4$ s.t.

s.t. $x_1 - x_2 + x_3 + 2x_4 \leq 6$
$x_1 + 2x_2 + 5x_3 - x_4 \leq 1$
$2x_1 + x_2 + x_4 = 12$, $x_1, x_2, x_3, x_4 \geq 0$

a) $\vec{x} = (10, 0, 0, 0) \Rightarrow$ Not optimal, infeasible

as $x_1 - x_2 + x_3 + 2x_4 \leq 6$
$10 - 0 + 0 + 2\cdot 0 \leq 6$
$10 \leq 6$ X

b) $\vec{x} = (5, \frac{11}{9}, -\frac{4}{3}, \frac{7}{9}) \Rightarrow 5 - \frac{11}{9} - \frac{4}{3} + 2\cdot\frac{7}{9} \leq 6$
$5 - 1 \leq 6$ ✓, $4 \leq 6$

$5 + 2\cdot\frac{11}{9} + 5\cdot -\frac{4}{3} - \frac{7}{9} \leq 1$
$5 + \frac{22}{9} - \frac{60}{9} - \frac{7}{9} \leq 1$
$5 - 5 \leq 1$ ✓, $0 \leq 1$

$2(5) + \frac{11}{9} + \frac{7}{9} = 12$ ✓ Feasible

$\frac{4}{3} + \frac{1}{5} = -\frac{20}{15} + \frac{3}{15} = \frac{17}{15}$

Although this is feasible, it is not optimal as look at (1) constriants we have $4 \leq 6$, we could make $x_3 = \frac{-17}{15}$ instead of $-\frac{4}{3}$ and we would get $\frac{21}{15} \leq 6$ for (1) & $1 \leq 1$ for (2) & (3) would be okay

Increasing the value of f by $\frac{3}{5}$ but still being in constriants

c) $\vec{x} = (6, 0, -1, 0) \Rightarrow 6 - 1 \leq 6$ ✓, $5 \leq 6$
$6 - 5 \leq 1$ ✓ $1 \leq 1$
$2\cdot 6 = 12$ ✓ $12 = 12$

This is optimal as changing any would violate others.

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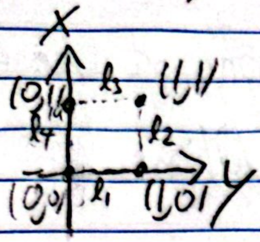
3) $f(x,y) = 3x + 19y$



Constraints: $0 \leq x \leq 1$, $0 \leq y \leq 1$ by line

$l_1: y = 0,\ 0 \leq x \leq 1$

$l_2: x = 1,\ 0 \leq y \leq 1$

$l_3: y = 1,\ 0 \leq x \leq 1$

$l_4: x = 0,\ 0 \leq y \leq 1$

Non-negativity: $y, x \geq 0 \Rightarrow x \leq 1, y \leq 1$

Based off constriants, $\max(x,y) = (1,1)$

$\Rightarrow$ $(1,1)$ is an opt. sol as coefficients of x & y in f are positive & x, y are maxxed

4) Minimize

Q – # of people sent from Quadrilaterals
R – # of people sent from Rhomboids
S – # of people sent from Spheres
T – # of people sent from Tessellations s.t.

The names of the people in each research group is 1 if they go, 0 if not

5) (P) canonical max problem.

feasible set for (P) is bounded, none of the extreme points

$\Rightarrow$ (P) has infinitely many optimal solutions

$\Rightarrow \exists$ infinitely many points in which (P) is max

Complementary slackness: Any pair of feasible sols, x^*, y^* of the canonic dual LP problem for which

$$\begin{aligned} &\text{i)}\ x_j \neq 0 \Rightarrow s_j = 0,\ j = 1, \dots, n\ \& \\ &\text{ii)}\ y_i \neq 0 \Rightarrow t_i = 0,\ i = 1, \dots, m \end{aligned}$$

$\Downarrow$

Say x_j for max (P) has corresponding dual problem with $s_j = 0$

$\Rightarrow$ All basic variables are $0 \Rightarrow$ degenerate solution

6) Minimize: $g(x_1, x_2, x_3) = -5x_1 + x_2 - 2x_3$

$$\begin{aligned} \text{s.t.}\quad & 2x_1 + x_3 = 0 \Rightarrow 2x_1 + x_3 \geq 0,\ 2x_1 + x_3 \leq 0 \\ & x_1 - x_2 \geq 1 \\ & 3x_1 - x_2 + x_3 \leq 3 \end{aligned}$$

$$\begin{aligned} \text{s.t.}\quad & 2x_1 + x_3 \geq 0 \\ & -2x_1 - x_3 \geq 0 \\ & x_1 - x_2 \geq 1 \\ & -3x_1 + x_2 - x_3 \geq 3 \end{aligned}$$

$$\Rightarrow \begin{array}{c|ccc|c} x_1 & 2 & -2 & -3 & -5 \\ x_2 & 0 & 0 & 1^* & 1 \\ x_3 & 1 & -1 & -1 & -2 \\ \hline -1 & 0 = s_1 & 0 = s_2 & 3 = s_3 & 0 = g \end{array} \rightarrow \begin{array}{c|ccc|c} x_1 & 2 & -2 & 3 & -2 \\ s_3 & 0 & 0 & 1 & 1 \\ x_3 & 1 & -1 & 1 & -1 \\ \hline -1 & 0 & -2 & -3 & \\ & = s_1 & = s_2 & = x_2 & = g \end{array}$$

BFT $\xrightarrow{\text{now}}$ Second row says it's unbounded

7) Maximize $f(x_1, x_2, x_3) = 3x_1 - 2x_2 + 3x_3$

St. (1) $x_1 - 2x_2 + 2x_3 = 6 \rightarrow$

$x_1 - 2x_2 + 2x_3 \le 6$

$-x_1 + 2x_2 - 2x_3 \ge 6$

$x_1 + 2x_3 \le 8$

$-x_1 - 2x_3 \ge 8$

$-x_2 - 2x_3 \le 2$, $x_2, x_3 \ge 0$ x_1 is unconstrained

$$\Rightarrow \begin{array}{cccc|cl} x_1 & x_2 & x_3 & & -1 & \\ \hline 1^* & -2 & 2 & & 6 & = -t_1 \\ 1 & 0 & 2 & & 8 & = -t_2 \\ 0 & -1 & 2 & & 2 & = -t_3 \\ \hline 3 & -2 & 3 & & 0 & = f \end{array}$$

$$\rightarrow \begin{array}{cccc|cl} t_1 & x_2 & x_3 & & -1 & \\ \hline \cancel{1} & \cancel{-2} & \cancel{2} & & \cancel{6} & = -x \\ 1 & 0 & 2 & & 8 & = -t_2 \\ 0 & -1 & -2 & & 2 & = -t_3 \\ \hline 3 & -2 & 3 & & 0 & = f \end{array}$$

$x_1 = 6 - 2x_3 + 2x_2 - t_1$

$$\rightarrow \begin{array}{cccc|cl} t_1 & x_2 & x_3 & & -1 & \\ \hline 1^* & 0 & 2 & & 8 & = -t_2 \\ 0 & -1 & -2 & & 2 & = -t_3 \\ \hline 3 & -2 & 3 & & 0 & = f \end{array}$$

MFBT

$$\Rightarrow \begin{array}{cccc|cl} t_2 & x_2 & x_3 & & -1 & \\ \hline 1 & 0 & 2 & & 8 & = -t_1 \\ 0 & -1 & -2 & & 2 & = -t_3 \\ \hline -3 & -2 & -3 & & -24 & = f \end{array}$$

$\Rightarrow$ Opt sol, $f = 24$ with $t_1 = 8$, $t_3 = 2$

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