Alen Mehmedbegovic- Assignment 2

1) $B = \{|A\rangle, |B\rangle\}$ rotated by θ to be $\{\cos\theta|A\rangle + \sin\theta|B\rangle,$
$-\sin\theta|A\rangle + \cos\theta|B\rangle\}$

i) Rotating by 2θ: $\cos 2\theta|A\rangle + \sin 2\theta|B\rangle, -\sin 2\theta|A\rangle + \cos 2\theta|B\rangle$

$(\cos^2\theta - \sin^2\theta)|A\rangle + 2\sin\theta\cos\theta|B\rangle, -2\sin\theta\cos\theta|A\rangle + (\cos^2\theta - \sin^2\theta)|B\rangle$

Rotating by θ once: $\cos\theta|A\rangle + \sin\theta|B\rangle, -\sin\theta|A\rangle + \cos\theta|B\rangle$

Rotating by θ a second time
$\cos\theta(\cos\theta|A\rangle + \sin\theta|B\rangle) + \sin\theta(-\sin\theta|A\rangle + \cos\theta|B\rangle),$
$-\sin\theta(\cos\theta|A\rangle + \sin\theta|B\rangle) + \cos\theta(-\sin\theta|A\rangle + \cos\theta|B\rangle$

$(\cos^2\theta - \sin^2\theta)|A\rangle + 2\sin\theta\cos\theta|B\rangle, -2\sin\theta\cos\theta|A\rangle + (\cos^2\theta - \sin^2\theta)|B\rangle$

They are equivalent ✓

ii) $B = \{|A\rangle, |B\rangle\}$ rotated by θ to be $\{\cos\theta|A\rangle + \sin\theta|B\rangle, -\sin\theta|A\rangle + \cos\theta|B\rangle\}$

B can be any orthonormal basis so let B be computational basis

$B = \{|0\rangle, |1\rangle\}$ rotated to $\{\cos\theta|0\rangle + \sin\theta|1\rangle, -\sin\theta|0\rangle + \cos\theta|1\rangle\}$

$\Rightarrow |0'\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle \ , \ |1'\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$

Measuring $|0\rangle$ gives $|0'\rangle$ with prob $\cos^2\theta$ & $|1'\rangle$ with prob $\sin^2\theta$

global phase- Assuming θ is small, rotate basis again to get another map with same prob

2)

$$P(\theta) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix}$$

$$P(0) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I, \quad P(\tfrac{\pi}{4}) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{4}} \end{pmatrix}$$

Given a state $|\psi\rangle$ where $|\psi\rangle = \frac{1}{\sqrt{2}}(e^{i\frac{\pi}{4}}|0\rangle + |1\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix} e^{i\frac{\pi}{4}} \\ 1 \end{pmatrix}$

OR $|\phi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + e^{i\frac{3\pi}{4}}|1\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ e^{i\frac{3\pi}{4}} \end{pmatrix}$

Protocol: Apply $P(\frac{\pi}{4})$, H, measure in comp basis.
If result is $|1\rangle$, we had $|\phi\rangle$. If result is $|0\rangle$, we had $|\psi\rangle$

Proof: As if $|\psi\rangle = |\phi\rangle$, $P(\frac{\pi}{4})|\phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{4}} \end{pmatrix}\begin{pmatrix} 1 \\ e^{i\frac{3\pi}{4}} \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ e^{i\pi} \end{pmatrix}$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$H \cdot \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |1\rangle,$$ prob of measuring $|1\rangle$ is 100%

if $|\psi\rangle = |\psi\rangle$, $P(\frac{\pi}{4})|\psi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{4}} \end{pmatrix}\begin{pmatrix} e^{i\frac{\pi}{4}} \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} e^{i\frac{\pi}{4}} \\ e^{i\frac{\pi}{4}} \end{pmatrix}$

$$H \cdot \frac{1}{\sqrt{2}}\begin{pmatrix} e^{i\frac{\pi}{4}} \\ e^{i\frac{\pi}{4}} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} e^{i\frac{\pi}{4}} \\ e^{i\frac{\pi}{4}} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 2 \cdot e^{i\frac{\pi}{4}} \\ 0 \end{pmatrix} = \begin{pmatrix} e^{i\frac{\pi}{4}} \\ 0 \end{pmatrix}$$

prob of measuring $|0\rangle$: $|e^{i\frac{\pi}{4}}|^2 = \left|\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right|^2$

$$= \left(\frac{1}{\sqrt{2}}\right)^2 + \left(-\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2} + \frac{1}{2} = 1$$

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3i) $X \otimes Z = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$, $Z \otimes X = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$

ii) ${}_3C_2 = 3$ possible pairs

$X \& Y \Rightarrow XY = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$

$-YX = -\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$, $XY = -YX$ ✓

$X \& Z \Rightarrow XZ = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

$-ZX = -\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$, $XZ = -ZX$ ✓

$Y \& Z \Rightarrow YZ = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$

$-ZY = -\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$, $YZ = -ZY$ ✓

iii) Does $\exists\ a \cdot I + b \cdot X + c \cdot Y + d \cdot Z = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ where $a, b, c, d \neq 0$

$a\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + b\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + c\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} + d\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

Look at 1,2 entry: $a \cdot 0 + b \cdot 1 + c \cdot -i + d \cdot 0 = 0$

$b - i \cdot c = 0$

Look at 2,1 entry: $a \cdot 0 + b \cdot 1 + c \cdot i + d \cdot 0 = 0$

$b + i c = 0$

3.iii) $\nexists$ a linear combo that satisfies both $b-ic=0$ & $b+ic=0$

That isn't $b=c=0$ $\Rightarrow$ You cannot get zero matrix from a linear combo of Pauli matrices

$\Rightarrow$ Pauli matrices are linearly independant.

iv) iii) showed their linear independance ✓

Show every 2x2 complex matrix can be written as a linear combo of the Pauli matrices

Let C be a generic complex matrix: $C = \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix}, c_{ij} \in \mathbb{C}$

$C = a \cdot I + b \cdot X + c \cdot Y + d \cdot Z, \quad a, b, c, d \in \mathbb{C}$

$$\begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix} = a \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + b \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + c \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} + d \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\Rightarrow \begin{aligned} c_{11} &= a + d \\ c_{12} &= b - i \cdot c \\ c_{21} &= b + i \cdot c \\ c_{22} &= a - d \end{aligned}$$

Since $a, b, c, d \in \mathbb{C}$, we can get any complex number c_{ij} as $a, b, c,$ and d are all any complex number themselves ✓

$\Rightarrow$ Pauli matrices form a basis for the space of 2x2 complex-valued matrices

4)

$$CZ = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad |\psi\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}, \quad |\phi\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$$|\psi\rangle \otimes |\phi\rangle = \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$$

$$CZ|\psi\rangle \otimes |\phi\rangle = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$$

For contradiction assume $CZ|\psi\rangle \otimes |\phi\rangle$ can be written as a joint state product where $\begin{pmatrix} \alpha \\ \beta \end{pmatrix} \otimes \begin{pmatrix} \gamma \\ \delta \end{pmatrix} = \begin{pmatrix} \alpha\gamma \\ \alpha\delta \\ \beta\gamma \\ \beta\delta \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$

$\Rightarrow \alpha\gamma = \beta\gamma = \beta\delta = \frac{1}{2}, \quad \alpha\delta = -\frac{1}{2}$

$\to$ one of α or δ has to be negative, not both

But if α negative, then δ is positive. $\alpha\gamma = \frac{1}{2}$ implies γ negative, so $\beta\gamma = \frac{1}{2}$ means β negative too $\Rightarrow\Leftarrow$ as $\beta\delta$ is a positive result

if δ negative, then α is positive. $\beta\delta = \frac{1}{2}$ implies β negative, so $\beta\gamma = \frac{1}{2}$ means γ negative too $\Rightarrow\Leftarrow$ as $\alpha\gamma$ is a positive result

$\Rightarrow\Leftarrow$ $CZ|\psi\rangle \otimes |\phi\rangle$ cannot be written as a joint state

$\to CZ|\psi\rangle \otimes |\phi\rangle$ is entangled

5) Let $|\psi\rangle = \frac{i\sqrt{2}}{\sqrt{3}}|00\rangle + \frac{1}{\sqrt{6}}|01\rangle + \frac{1}{\sqrt{6}}|10\rangle$

prob of measuring zero in first qubit: $\left|\frac{i\sqrt{2}}{\sqrt{3}}\right|^2 + \left|\frac{1}{\sqrt{6}}\right|^2$

$$\frac{2}{3} + \frac{1}{6} = \frac{5}{6}$$

Resulting state: $\frac{i\sqrt{2}}{\sqrt{3}}|00\rangle + \frac{1}{\sqrt{6}}|01\rangle$

normalize $\Rightarrow$ $\frac{1}{\sqrt{\frac{5}{6}}}\left(\frac{i\sqrt{2}}{\sqrt{3}}|00\rangle + \frac{1}{\sqrt{6}}|01\rangle\right)$

$$\frac{\sqrt{6}}{\sqrt{5}}\left(\frac{i\sqrt{2}}{\sqrt{3}}|00\rangle + \frac{1}{\sqrt{6}}|01\rangle\right)$$

$$\frac{i\sqrt{12}}{\sqrt{15}}|00\rangle + \frac{1}{\sqrt{5}}|01\rangle = \frac{2i}{\sqrt{5}}|00\rangle + \frac{1}{\sqrt{5}}|01\rangle$$

prob of measuring one in first qubit: $\left|\frac{1}{\sqrt{6}}\right|^2 = \frac{1}{6}$

Resulting state: $\frac{1}{\sqrt{6}}|10\rangle$

normalize $\Rightarrow$ $\frac{1}{\sqrt{\frac{1}{6}}}\left(\frac{1}{\sqrt{6}}|10\rangle\right)$

$$\sqrt{6}\left(\frac{1}{\sqrt{6}}|10\rangle\right)$$

$$|10\rangle$$

6) $|\psi\rangle = \frac{1}{2}(|000\rangle - |110\rangle - |011\rangle - |101\rangle)$

i) $U \cdot \left(\frac{1}{\sqrt{2}}|000\rangle - \frac{1}{\sqrt{2}}|111\rangle\right) = |\psi\rangle$

$$\Rightarrow \frac{1}{\sqrt{2}} U(|0\rangle\otimes|0\rangle\otimes|0\rangle - |1\rangle\otimes|1\rangle\otimes|1\rangle) = \frac{1}{2}(|0\rangle\otimes|0\rangle\otimes|0\rangle - |1\rangle\otimes|1\rangle\otimes|0\rangle - |0\rangle\otimes|1\rangle\otimes|1\rangle - |1\rangle\otimes|0\rangle\otimes|1\rangle)$$

Use shorthand: $|\phi\rangle\otimes|\phi\rangle = |\phi\rangle|\phi\rangle$

Apply H on 0 qubit

$$\frac{1}{\sqrt{2}}(|0\rangle|0\rangle|0\rangle - |1\rangle|1\rangle|1\rangle)$$

$$\Rightarrow \frac{1}{2}(|0\rangle|0\rangle|0\rangle + |1\rangle|0\rangle|0\rangle + |1\rangle|1\rangle|1\rangle - |0\rangle|1\rangle|1\rangle)$$

Apply $-X$ on 1 qubit

$$\Rightarrow \frac{1}{2}(-|0\rangle|1\rangle|0\rangle - |1\rangle|1\rangle|0\rangle - |1\rangle|0\rangle|1\rangle + |0\rangle|0\rangle|1\rangle)$$

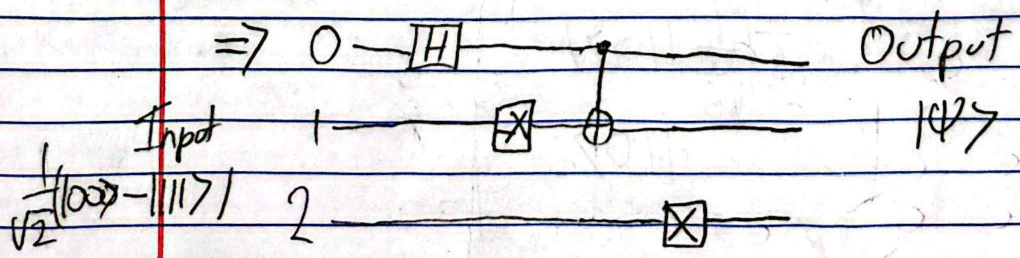
Apply CNOT on 0 & 1 qubit

$$\Rightarrow \frac{1}{2}(-|0\rangle|1\rangle|0\rangle - |1\rangle|0\rangle|0\rangle - |1\rangle|1\rangle|1\rangle + |0\rangle|0\rangle|1\rangle)$$

Apply X on 2 qubit & rearrange

$$\frac{1}{2}(|0\rangle|0\rangle|0\rangle - |1\rangle|1\rangle|0\rangle - |0\rangle|1\rangle|1\rangle - |1\rangle|0\rangle|1\rangle) = |\psi\rangle$$

$\Rightarrow$



6.ii) $|\psi\rangle = \frac{1}{2}\left(|000\rangle - |110\rangle - |011\rangle - |101\rangle\right)$

Measure 0^{th} zero: $\left|\frac{1}{2}\right|^2 + \left|-\frac{1}{2}\right|^2 = \frac{1}{2}$ prob

Resulting state normalized: $\frac{\frac{1}{2}}{\sqrt{\left|\frac{1}{2}\right|}}\left(|00\rangle - |11\rangle\right)$

$$\frac{1}{\sqrt{2}}|00\rangle - \frac{1}{\sqrt{2}}|11\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\-1\end{pmatrix}$$

For contradiction, assume $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\-1\end{pmatrix}$ can be expressed as a tensor product

$$\Rightarrow \begin{pmatrix}\alpha\\\beta\end{pmatrix} \otimes \begin{pmatrix}\gamma\\\delta\end{pmatrix} = \begin{pmatrix}\alpha\gamma\\\alpha\delta\\\beta\gamma\\\beta\delta\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\-1\end{pmatrix}, \quad \alpha\gamma = \frac{1}{\sqrt{2}}, \alpha\delta = 0, \beta\gamma = 0, \beta\delta = -\frac{1}{\sqrt{2}}$$

$\Rightarrow \Leftarrow$ as $\alpha\delta = 0$ means at least one of α or δ are 0, but $\alpha\gamma = \frac{1}{\sqrt{2}}$ & $\beta\delta = \frac{1}{\sqrt{2}}$ $\Rightarrow\Leftarrow$

$\rightarrow$ Assumption incorrect, state cannot be expressed as a tensor product

$\Rightarrow$ entangled state

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6ii) $|\psi\rangle = \frac{1}{2}(|000\rangle - |110\rangle - |011\rangle - |101\rangle)$

Measure 0^{th} one: prob $\left|-\frac{1}{2}\right|^2 + \left|-\frac{1}{2}\right|^2 = \frac{1}{2}$

Resulting normalized state: $\frac{1}{\sqrt{\left(\frac{1}{2}\right)}} \cdot \frac{1}{2}(-|10\rangle - |01\rangle)$

$$-\frac{1}{\sqrt{2}}|01\rangle - \frac{1}{\sqrt{2}}|10\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ -1 \\ -1 \\ 0 \end{pmatrix}$$

Assume for contradiction it can be expressed as a tensor product

$$\Rightarrow \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \otimes \begin{pmatrix} \gamma \\ \delta \end{pmatrix} = \begin{pmatrix} \alpha\gamma \\ \alpha\delta \\ \beta\gamma \\ \beta\delta \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ -1 \\ -1 \\ 0 \end{pmatrix} \Rightarrow \alpha\gamma = 0,\ \alpha\delta = -\frac{1}{\sqrt{2}},\ \beta\gamma = -\frac{1}{\sqrt{2}},\ \beta\delta = 0$$

$\Rightarrow\Leftarrow$ As $\beta\delta = 0$ implies at least one of β or δ is 0

This can't be as $\beta\gamma = \frac{1}{\sqrt{2}}$ & $\alpha\delta = \frac{1}{\sqrt{2}}$ $\Rightarrow\Leftarrow$

Our assumption is wrong so our state cannot be expressed as a tensor prod.

$\Rightarrow$ entangled state

Measure 1^{st} zero: prob $\left|\frac{1}{2}\right|^2 + \left|-\frac{1}{2}\right|^2 = \frac{1}{2}$

Resulting normalized state: $\frac{1}{\sqrt{\frac{1}{2}}} \cdot \frac{1}{2}(|00\rangle - |11\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}$

Assume $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \otimes \begin{pmatrix} \gamma \\ \delta \end{pmatrix} = \begin{pmatrix} \alpha\gamma \\ \alpha\delta \\ \beta\gamma \\ \beta\delta \end{pmatrix}$, $\alpha\gamma = \frac{1}{\sqrt{2}}$, $\alpha\delta = \beta\gamma = 0$, $\beta\delta = -\frac{1}{\sqrt{2}}$

$\Rightarrow\Leftarrow$ As $\alpha\delta = 0$, α or δ is 0 but $\alpha\gamma = \frac{1}{\sqrt{2}}$ & $\beta\delta = \frac{1}{\sqrt{2}}$ $\Rightarrow\Leftarrow$

State cannot be expressed as tensor product $\Rightarrow$ entangled

6ii) Measure 1^{st} one: prob $\left|-\frac{1}{2}\right|^2 + \left|-\frac{1}{2}\right|^2 = \frac{1}{2}$

Resulting normalized state: $\frac{1}{\sqrt{\frac{1}{2}}} \cdot \frac{1}{2}\left(-|10\rangle - |01\rangle\right) = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\-1\\-1\\0\end{pmatrix}$

Assume $\frac{1}{\sqrt{2}}\begin{pmatrix}0\\-1\\-1\\0\end{pmatrix} = \begin{pmatrix}\alpha\\\beta\end{pmatrix} \otimes \begin{pmatrix}\gamma\\\delta\end{pmatrix} = \begin{pmatrix}\alpha\gamma\\\alpha\delta\\\beta\gamma\\\beta\delta\end{pmatrix}$, $\alpha\gamma = \beta\delta = 0$, $\alpha\delta = \beta\gamma = -\frac{1}{\sqrt{2}}$

$\Rightarrow\Leftarrow$ As $\beta\delta = 0$, β or δ are 0 but $\beta\gamma = \frac{1}{\sqrt{2}}$ & $\alpha\delta = 0$ $\Rightarrow\Leftarrow$

$\rightarrow$ State cannot be expressed as tensor product $\Rightarrow$ entangled

Measure 2^{nd} zero: prob $\left|\frac{1}{2}\right|^2 + \left|-\frac{1}{2}\right|^2 = \frac{1}{2}$

Resulting normalized state: $\frac{1}{\sqrt{\frac{1}{2}}} \cdot \frac{1}{2}\left(|00\rangle - |11\rangle\right) = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\-1\end{pmatrix}$

Assume $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\-1\end{pmatrix} = \begin{pmatrix}\alpha\\\beta\end{pmatrix} \otimes \begin{pmatrix}\gamma\\\delta\end{pmatrix} = \begin{pmatrix}\alpha\gamma\\\alpha\delta\\\beta\gamma\\\beta\delta\end{pmatrix}$, $\alpha\delta = \beta\gamma = 0$, $\alpha\gamma = \frac{1}{\sqrt{2}}$, $\beta\delta = -\frac{1}{\sqrt{2}}$

$\Rightarrow\Leftarrow$ As $\alpha\delta = 0$, α or δ are 0 but $\alpha\gamma = \frac{1}{\sqrt{2}}$ & $\beta\delta = \frac{1}{\sqrt{2}}$ $\Rightarrow\Leftarrow$

$\rightarrow$ State cannot be expressed as tensor product $\rightarrow$ entangled

Measure 2^{nd} one: prob $\left|-\frac{1}{2}\right|^2 + \left|-\frac{1}{2}\right|^2 = \frac{1}{2}$

Resulting normalized state: $\frac{1}{\sqrt{\frac{1}{2}}} \cdot \frac{1}{2}\left(-|01\rangle - |10\rangle\right) = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\-1\\-1\\0\end{pmatrix}$

Assume $\frac{1}{\sqrt{2}}\begin{pmatrix}0\\-1\\-1\\0\end{pmatrix} = \begin{pmatrix}\alpha\\\beta\end{pmatrix} \otimes \begin{pmatrix}\gamma\\\delta\end{pmatrix} = \begin{pmatrix}\alpha\gamma\\\alpha\delta\\\beta\gamma\\\beta\delta\end{pmatrix}$, $\alpha\gamma = \beta\delta = 0$, $\alpha\delta = \beta\gamma = -\frac{1}{\sqrt{2}}$

$\Rightarrow\Leftarrow$ As $\beta\delta = 0$, β or δ are 0 but $\beta\gamma = \frac{1}{\sqrt{2}}$ & $\alpha\delta = \frac{1}{\sqrt{2}}$ $\Rightarrow\Leftarrow$

$\rightarrow$ State cannot be expressed as tensor product $\Rightarrow$ entangled

6iii) $a \oplus b \oplus c = a + b + c \mod 2$

a) $|000\rangle - |110\rangle - |011\rangle - |101\rangle$

$0 \oplus 0 \oplus 0 \quad 1 \oplus 1 \oplus 0 \quad 0 \oplus 1 \oplus 1 \quad 1 \oplus 0 \oplus 1$

$0 \quad 0 \quad 0 \quad 0$, all 0's

b) $|0\rangle|0\rangle|0\rangle \Rightarrow \frac{1}{2}(|+\rangle + |-\rangle)(|+\rangle + |-\rangle)|0\rangle$

$\frac{1}{2}(|+\rangle|+\rangle + |-\rangle|+\rangle + |+\rangle|-\rangle + |-\rangle|-\rangle)|0\rangle$

$\frac{1}{2}(|+\rangle|+\rangle|0\rangle + |-\rangle|+\rangle|0\rangle + |+\rangle|-\rangle|0\rangle + |-\rangle|-\rangle|0\rangle)$

$0 \oplus 0 \oplus 0 \quad 1 \oplus 0 \oplus 0 \quad 0 \oplus 1 \oplus 0 \quad 1 \oplus 1 \oplus 0$

$0 \quad 1 \quad 1 \quad 0$,

Add all together $2 \mod 2 = 0$

$|1\rangle|1\rangle|0\rangle \Rightarrow \frac{1}{2}(|+\rangle - |-\rangle)(|+\rangle - |-\rangle)(|0\rangle)$

$\frac{1}{2}(|+\rangle|+\rangle - |-\rangle|+\rangle - |+\rangle|-\rangle + |-\rangle|-\rangle)|0\rangle$

$0 \oplus 0 \oplus 0 \quad 1 \oplus 0 \oplus 0 \quad 0 \oplus 1 \oplus 0 \quad 1 \oplus 1 \oplus 0$

$0 \quad 1 \quad 1 \quad 0$

Add all together $2 \mod 2 = 0$

b) $|0\rangle|1\rangle|1\rangle \Rightarrow \frac{1}{2}(|+\rangle+|-\rangle)(|+\rangle-|-\rangle)|1\rangle$

$\frac{1}{2}(|+\rangle|+\rangle+|-\rangle|+\rangle-|+\rangle|-\rangle-|-\rangle|-\rangle)|1\rangle$

$0\oplus0\oplus1 \quad 1\oplus0\oplus1 \quad 0\oplus1\oplus1 \quad 1\oplus1\oplus1$

$1 \quad 0 \quad 0 \quad 1$

Add all together $2 \bmod 2 = 0$

$|1\rangle|0\rangle|1\rangle \Rightarrow \frac{1}{2}(|+\rangle-|-\rangle)(|+\rangle+|-\rangle)|1\rangle$

$\frac{1}{2}(|+\rangle|+\rangle-|-\rangle|+\rangle+|+\rangle|-\rangle-|-\rangle|-\rangle)|1\rangle$

$0\oplus0\oplus1 \quad 1\oplus0\oplus1 \quad 0\oplus1\oplus1 \quad 1\oplus1\oplus1$

$1 \quad 0 \quad 0 \quad 1$

All add together, $2 \bmod 2 = 0$

c) $|0\rangle|0\rangle|0\rangle \Rightarrow \frac{1}{2}(|+\rangle+|-\rangle)|0\rangle(|+\rangle+|-\rangle)$

$\frac{1}{2}(|+\rangle|+\rangle+|-\rangle|+\rangle+|+\rangle|-\rangle+|-\rangle|-\rangle)|0\rangle$

$0\oplus0\oplus0 \quad 1\oplus0\oplus0 \quad 0\oplus1\oplus0 \quad 1\oplus1\oplus0$

$0 \quad 1 \quad 1 \quad 0$

Sum $2 \bmod 2 = 0$

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d) $|1\rangle|1\rangle|0\rangle \Rightarrow \frac{1}{2}(|+\rangle - |-\rangle)|1\rangle(|+\rangle + |-\rangle)$

$\frac{1}{2}(|+\rangle|+\rangle - |-\rangle|+\rangle + |+\rangle|-\rangle - |-\rangle|-\rangle)|1\rangle$

$0\oplus0\oplus1 \quad 1\oplus0\oplus1 \quad 0\oplus1\oplus1 \quad 1\oplus1\oplus1$

$1 \quad 0 \quad 0 \quad 1$

Add all together 2mod2=0

$|0\rangle|1\rangle|1\rangle \Rightarrow \frac{1}{2}(|+\rangle + |-\rangle)|1\rangle(|+\rangle - |-\rangle)$

$\frac{1}{2}(|+\rangle|+\rangle + |-\rangle|+\rangle - |+\rangle|-\rangle - |-\rangle|-\rangle)|1\rangle$

$0\oplus0\oplus1 \quad 1\oplus0\oplus1 \quad 0\oplus1\oplus1 \quad 1\oplus1\oplus1$

$1 \quad 0 \quad 0 \quad 1$

sum, 2mod2=0

$|1\rangle|0\rangle|1\rangle \Rightarrow \frac{1}{2}(|+\rangle - |-\rangle)|0\rangle(|+\rangle - |-\rangle)$

$\frac{1}{2}(|+\rangle|+\rangle - |-\rangle|+\rangle - |+\rangle|-\rangle - |-\rangle|-\rangle)|0\rangle$

$0\oplus0\oplus0 \quad 1\oplus0\oplus0 \quad 0\oplus1\oplus0 \quad 1\oplus0\oplus0$

$0 \quad 1 \quad 1 \quad 0$

sum, 2mod2=0

d) $|0\rangle|0\rangle|0\rangle \Rightarrow |0\rangle\frac{1}{2}(|+\rangle+|-\rangle)(|+\rangle+|-\rangle)$

$|0\rangle\frac{1}{2}(|+\rangle|+\rangle+|-\rangle|+\rangle+|+\rangle|-\rangle+|-\rangle|-\rangle)$

$0\oplus0\oplus0 \quad 0\oplus1\oplus0 \quad 0\oplus0\oplus1 \quad 0\oplus1\oplus1$

$0 \quad 1 \quad 1 \quad 0$

Sum $2 \bmod 2 = 0$

$|1\rangle|1\rangle|0\rangle \Rightarrow |1\rangle\frac{1}{2}(|+\rangle-|-\rangle)(|+\rangle+|-\rangle)$

$|1\rangle\frac{1}{2}(|+\rangle|+\rangle-|-\rangle|+\rangle+|+\rangle|-\rangle-|-\rangle|-\rangle)$

$1\oplus0\oplus0 \quad 1\oplus1\oplus0 \quad 1\oplus0\oplus1 \quad 1\oplus1\oplus1$

$1 \quad 0 \quad 0 \quad 1$

Sum $2 \bmod 2 = 0$

$|0\rangle|1\rangle|1\rangle \Rightarrow |0\rangle\frac{1}{2}(|+\rangle-|-\rangle)(|+\rangle-|-\rangle)$

$|0\rangle\frac{1}{2}(|+\rangle|+\rangle-|-\rangle|+\rangle-|+\rangle|-\rangle+|-\rangle|-\rangle)$

$0\oplus0\oplus0 \quad 0\oplus1\oplus0 \quad 0\oplus0\oplus1 \quad 0\oplus1\oplus1$

$0 \quad 1 \quad 1 \quad 0$

Sum $2 \bmod 2 = 0$

d) $|1\rangle|0\rangle|1\rangle \Rightarrow |1\rangle\frac{1}{2}(|+\rangle+|-\rangle)(|+\rangle-|-\rangle)$

$|1\rangle\frac{1}{2}(|+\rangle|+\rangle+|-\rangle|+\rangle-|+\rangle|-\rangle-|-\rangle|-\rangle)$

$1\oplus0\oplus0 \quad 1\oplus1\oplus0 \quad 1\oplus0\oplus1 \quad 1\oplus1\oplus1$

$1 \quad 0 \quad 0 \quad 1$

Sum $2 \bmod 2 = 0$

In all parts partity is all zero

6iv) While it is possible to have dependant qubits through measurement with entanglement, not every joint qubit system is entangled.
We cannot have a pre-determined value independant of the basis.

For example look at $\frac{1}{\sqrt{2}}(|000\rangle - |111\rangle)$

Measure 2 qubit to get $\frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$ remaining.

This is an entangled state as shown in 6ii).
Now that we have an entangled state, we know that measuring 1 qubit affects the other, which is the closest we'll get to a "pre-determined value"

But the result of measuring a_i or b_i on the 0 or 1 qubit isn't predetermined, it's dependant on the possibilities & basis

v) Let Alice, Bob & Charile share $|\psi\rangle = \frac{1}{2}(|000\rangle - |110\rangle - |011\rangle - |101\rangle)$

Alice - x, Bob - y, Charile - z. Assume $x \oplus y \oplus z = 0$

Meaning either $x = y = z = 0$ or 2 of the 3 are 0, the other is 1

$\Rightarrow$ 4 cases:
1) $x = y = z = 0$
2) $x = y = 0, z = 1$
3) $x = z = 0, y = 1$
4) $y = z = 0, x = 1$

1) $|\psi\rangle$ collapses to $|000\rangle \Rightarrow$ Measure all in comp basis

& get $a = b = c = 0$. $0 \oplus 0 \oplus 0 = 0 \vee 0 \vee 0 \vee 0$ ✓
$0 = 0$

2) $x=y=0$, $z=1$ Not possible in $|\psi\rangle$

3) $x=z=0$, $y=1$ Not possible in $|\psi\rangle$

4) $y=z=0$, $x=1$ Not possible in $|\psi\rangle$