MATH 308 - Assignment 1

1. Find necessary and sufficient conditions for the numbers s and t to make the LP problem

$$\begin{array}{ll} \text{Maximize:} & f(x_1, x_2) = x_1 + x_2 \\ \text{subject to} & sx_1 + tx_2 \leq 1 \\ & x_1, x_2 \geq 0 \end{array}$$

a) have an optimal solution.

b) be infeasible.

c) be unbounded.

Prove your answers.

2. Prove or disprove: If a canonical LP problem

$$\begin{array}{ll} \text{Maximize:} & f(x_1, x_2, \ldots, x_n) = c_1x_1 + c_2x_2 + \cdots + c_nx_n \\ \text{subject to} & \sum_{j=1}^{n} a_{ij}x_j \leq b_i \ (i = 1, 2, \ldots, m) \\ & x_1, x_2, \ldots, x_n \geq 0 \end{array}$$

is unbounded, then there is a subscript k so that the LP problem

$$\begin{array}{ll} \text{Maximize:} & f(x_1, x_2, \ldots, x_n) = x_k \\ \text{subject to} & \sum_{j=1}^{n} a_{ij}x_j \leq b_i \ (i = 1, 2, \ldots, m) \\ & x_1, x_2, \ldots, x_n \geq 0 \end{array}$$

is unbounded.

3. Show that the LP problem

$$\begin{array}{ll} \text{Minimize:} & g(x_1, x_2) = 2x_1 - 5x_2 \\ \text{subject to} & x_1 + x_2 \geq 2 \\ & x_1 - 2x_2 \leq 0 \\ & x_2 - 2x_1 \leq 1 \\ & x_1, x_2 \geq 0 \end{array}$$

is unbounded.

4. Solve each of the LP problems below by sketching the constraint set and applying Theorem 22 and 23.

a)
$$\begin{array}{ll} \text{Maximize:} & f(x,y) = 5x + 2y \\ \text{subject to} & x + 3y \leq 14 \\ & 2x + y \leq 8 \\ & x, y \geq 0 \end{array}$$

b)
$$\begin{array}{ll} \text{Minimize:} & g(x,y) = 5x + 2y \\ \text{subject to} & x + 3y \geq 14 \\ & 2x + y \geq 8 \\ & x, y \geq 0 \end{array}$$

5. A drug company sells three different formulations of vitamin complex and mineral complex. The first formulation consists entirely of vitamin complex and sells for \$1 per unit. The second formulation consists of $3/4$ of a unit of vitamin complex and $1/4$ of a unit of mineral complex and sells for \$2 per unit. The third formulation consists of $1/2$ of a unit of each of the complexes and sells for \$3 per unit. If the company has 100 units of vitamin complex and 75 units of mineral complex available, how many units of each formulation should the company produce as to maximize profit? Write down the corresponding LP, and solve it by using Theorem 22 and 23.

6. Prove that there are infinitely many optimal solutions for the problem in Exercise 5 above. First prove that there are two solutions at extreme points of the constraint set. Then consider the line segment between these solutions/points.

Strayer, Chapter 1., p.23–26 Ex. 5, Ex. 8, Ex. 10, Ex. 12