Alen Mehmedbegovic - Assignment 3

1) • $A^s = \begin{pmatrix} -1 & 4 & 1 & 0 & 1 & 0 \\ 4 & -1 & 0 & 1 & 0 & 1 \end{pmatrix}$, $c^s = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, $b = \begin{pmatrix} 18 \\ 18 \end{pmatrix}$

• Columns contain basis vectors $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ & $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $\Rightarrow \mathbb{R}^2 \Rightarrow$ rank 2

•

x_1	x_2	x_3	x_4	-1	
-1	4	1	0	18	$= -t_1$
4	-1	0	1	18	$= -t_2$
1	-1	0	0	0	$= f$

• Pivot on $(2,1) = 4$

t_2	x_2	x_3	x_4	-1	
$\frac{1}{4}$	$\frac{15}{4}$	1	$\frac{1}{4}$	$\frac{45}{2}$	$= -t_1$
$\frac{1}{4}$	$-\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{9}{2}$	$= -x_1$
$-\frac{1}{4}$	$-\frac{3}{4}$	0	$-\frac{1}{4}$	$-\frac{9}{2}$	$= f$

Basic sol: $t_2 = x_2 = x_3 = x_4 = 0 \Rightarrow t_1 = \frac{45}{2}, x_1 = \frac{9}{2} \Rightarrow f = \frac{9}{2}$

• Opt sol: $\frac{9}{2}$

2) We start with

$$\begin{array}{c|cccc|c|c}
 & x_1 & x_2 & \cdots & x_n & -1 & \\
\hline
 & a_{11} & a_{12} & \cdots & a_{1n} & b_1 & = -t_1 \\
 & a_{21} & a_{22} & \cdots & a_{2n} & b_2 & = -t_2 \\
 & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
 & a_{m1} & a_{m2} & \cdots & a_{mn} & b_m & = -t_m \\
\hline
 & c_1 & c_2 & \cdots & c_n & d & = f
\end{array}$$

$$\Rightarrow \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} & 1 & 0 & \cdots & 0 \\ a_{21} & a_{22} & \cdots & a_{2n} & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} & 0 & 0 & \cdots & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \\ t_1 \\ \vdots \\ t_m \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}$$

3) Given $a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \leq b_1$
$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \leq b_2$
$\vdots$
$a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{mn}x_n \leq b_m$

We can fourmulate an LP: Maximize $f = c_1x_1 + c_2x_2 + \ldots + c_nx_n$, $c_i \geq 0$
s.t. it follows the above constriants

Assume LP has optimal solution $\Rightarrow \max(f) = m$

m is acheived while following above constriants meaning for values of $x_1, \ldots, x_n$ plugged into constriants, system of lin inequalities has solutio

Conversely, assume system of lin. eq's. has solution $\Leftarrow$

$\exists \{x_1, \ldots, x_n\}$ s.t. inequalities are satisfied meaning we can plug into f, and get $\max(f) = m$ ✓

4) $f = x_1 + 2x_2 + x_3$
s.t.
$\Rightarrow$ $x_1 + 2x_2 + 2x_3 + t_1 = -4$
$2x_1 + 3x_3 + t_2 = 5$, $t_i \geq 0$
$2x_1 - x_2 + x_3 + t_3 = -4$
$x_1, x_2, x_3 \geq 0$

$$
\begin{array}{cccc|cl}
x_1 & x_2 & x_3 & -1 & & \\
\hline
1 & 2 & 2 & -4 & = -t_1 \\
2 & 0 & 3 & 5 & = -t_2 \\
2 & -1 & 1 & -4 & = -t_3 \\
\hline
1 & 2 & 1 & 0 & = f
\end{array}
$$

Pivot on $(3,2) = -1$

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4)

$$\begin{array}{cccc|cl} & x_1 & t_3 & x_3 & -1 & \\ \hline & -5 & 2 & -2 & -3 & = -t_1 \\ & -1 & 0 & -1 & -1 & = -t_2 \\ & -2 & -1 & -1 & 4 & = -x_2 \\ \hline & -5 & 2 & 3 & -8 & = f \end{array}$$

Pivot on (2,3) = -1

$$\begin{array}{ccc|cl} x_1 & t_3 & t_2 & -1 & \\ \hline -3 & 2 & -2 & -1 & = -t_1 \\ 1 & 0 & -1 & 1 & = -x_3 \\ -1 & -1 & -1 & 5 & = -x_2 \\ \hline 2 & 2 & 3 & -11 & = f \end{array}$$

Pivot on (1,3) = -2

$$\begin{array}{ccc|cl} x_1 & t_3 & t_1 & -1 & \\ \hline \frac{3}{2} & -1 & -\frac{1}{2} & \frac{1}{2} & = -t_2 \\ \frac{5}{2} & -1 & -\frac{1}{2} & \frac{3}{2} & = -x_3 \\ \frac{1}{2} & 0 & -\frac{1}{2} & \frac{11}{2} & = -x_2 \\ \hline -\frac{5}{2} & 5 & \frac{3}{2} & -\frac{19}{2} & = f \end{array}$$

Now MBFT

This column says it's unbounded

5)

$$\begin{array}{cc|c} x_1 & x_2 & -1 \\ \hline 1 & -1 & -1 \\ -1 & -1 & -3 \\ 2 & -1 & 2 \\ \hline 3 & 1 & 0 \end{array}$$

Pivot on (2,2) = -1

$$\begin{array}{cc|c} x_1 & x_2 & -1 \\ \hline 2 & -1 & 2 \\ 1 & -1 & 3 \\ 3 & -1 & 5 \\ \hline 2 & 1 & -3 \end{array}$$

Now MBFT

This column says it's unbounded

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6) two shifts from: 12am - 4am - 5 workers S_1
4am - 8am - 4 workers S_2
8am - 12pm - 15 workers S_3 shift # - S_i
12pm - 4pm - 10 workers S_4
4pm - 8pm - 20 workers S_5
8pm - 12am - 8 workers S_6

two shifts consecutive: \$25
two shifts nonconsecutive: \$32

Let x_{ij} = # of workers working shift i & j

Minimize $25(x_{12}+x_{23}+x_{34}+x_{45}+x_{56})+$
$32(x_{13}+x_{14}+x_{15}+x_{16}+x_{24}+x_{25}+x_{26}+x_{35}+x_{36}+x_{46})$

S.t. $x_{12}+x_{13}+x_{14}+x_{15}+x_{16} \geq 5$
$x_{23}+x_{24}+x_{25}+x_{26}+x_{12} \geq 4$
$x_{34}+x_{35}+x_{36}+x_{13}+x_{23} \geq 15$
$x_{45}+x_{46}+x_{14}+x_{24}+x_{34} \geq 10$
$x_{56}+x_{45}+x_{35}+x_{25}+x_{15} \geq 20$
$x_{16}+x_{26}+x_{36}+x_{46}+x_{56} \geq 8$