Alen Mehmedbegovic Assignment 4 301476201

1) $E(U,V) = ||U-V|| = \max_{|\psi\rangle} ||(U-V)|\psi\rangle||$

1i) $E(U_2U_1, V_2V_1) = ||U_2U_1 - V_2V_1||$

$= ||U_2U_1 - V_2V_1 - V_2U_1 + V_2U_1||$

$= ||U_2U_1 - V_2U_1 + V_2U_1 - V_2V_1||$

$= ||U_1(U_2 - V_2) + V_2(U_1 - V_1)||$

$\leq ||U_1(U_2 - V_2)|| + ||V_2(U_1 - V_1)||$

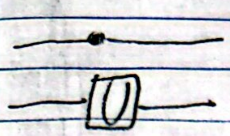
$\leq ||U_2 - V_2|| + ||U_1 - V_1||$

$\leq E(U_2, V_2) + E(U_1, V_1)$

$\Rightarrow E(U_2U_1, V_2V_1) \leq E(U_2, V_2) + E(U_1, V_1)$ ✓

ii) $\frac{\epsilon}{K}$

Scanned with CamScanner

2| 

2i| $C\text{-}U \Rightarrow$ Want to apply U only when control is $|1\rangle$ i.e. 1| $C\text{-}U|0\rangle|\psi\rangle = |0\rangle|\psi\rangle$

2| $C\text{-}U|1\rangle|\psi\rangle = |1\rangle U|\psi\rangle$

Verify that $|0\rangle\langle 0|\otimes I + |1\rangle\langle 1|\otimes U = C\text{-}U$

$\boxed{1} \Rightarrow (|0\rangle\langle 0|\otimes I + |1\rangle\langle 1|\otimes U)|0\rangle|\psi\rangle$

$$(|0\rangle\langle 0|\otimes I)|0\rangle|\psi\rangle + (|1\rangle\langle 1|\otimes U)|0\rangle|\psi\rangle$$

$$|0\rangle\overset{1}{\langle 0|0\rangle}\otimes I|\psi\rangle + |1\rangle\overset{\text{zero}}{\cancel{\langle 1|0\rangle}}\otimes U|\psi\rangle$$

$$|0\rangle\otimes I|\psi\rangle = |0\rangle|\psi\rangle \checkmark$$

$\boxed{2} \Rightarrow (|0\rangle\langle 0|\otimes I + |1\rangle\langle 1|\otimes U)|1\rangle|\psi\rangle$

$$(|0\rangle\langle 0|\otimes I)|1\rangle|\psi\rangle + (|1\rangle\langle 1|\otimes U)|1\rangle|\psi\rangle$$

$$|0\rangle\overset{\text{zero}}{\cancel{\langle 0|1\rangle}}\otimes I|\psi\rangle + |1\rangle\overset{\text{one}}{\cancel{\langle 1|1\rangle}}\otimes U|\psi\rangle$$

$$|1\rangle\otimes U|\psi\rangle = |1\rangle U|\psi\rangle \checkmark$$

ii| $|0\rangle\langle 0|\otimes I + |1\rangle\langle 1|\otimes H$

Controlled gates followed by measurement (by def^n)

3) $c\text{-}U|0\rangle|\psi\rangle = |0\rangle|\psi\rangle = |\psi\rangle$

$c\text{-}U|1\rangle|\psi\rangle = |1\rangle U|\psi\rangle = U|\psi\rangle$

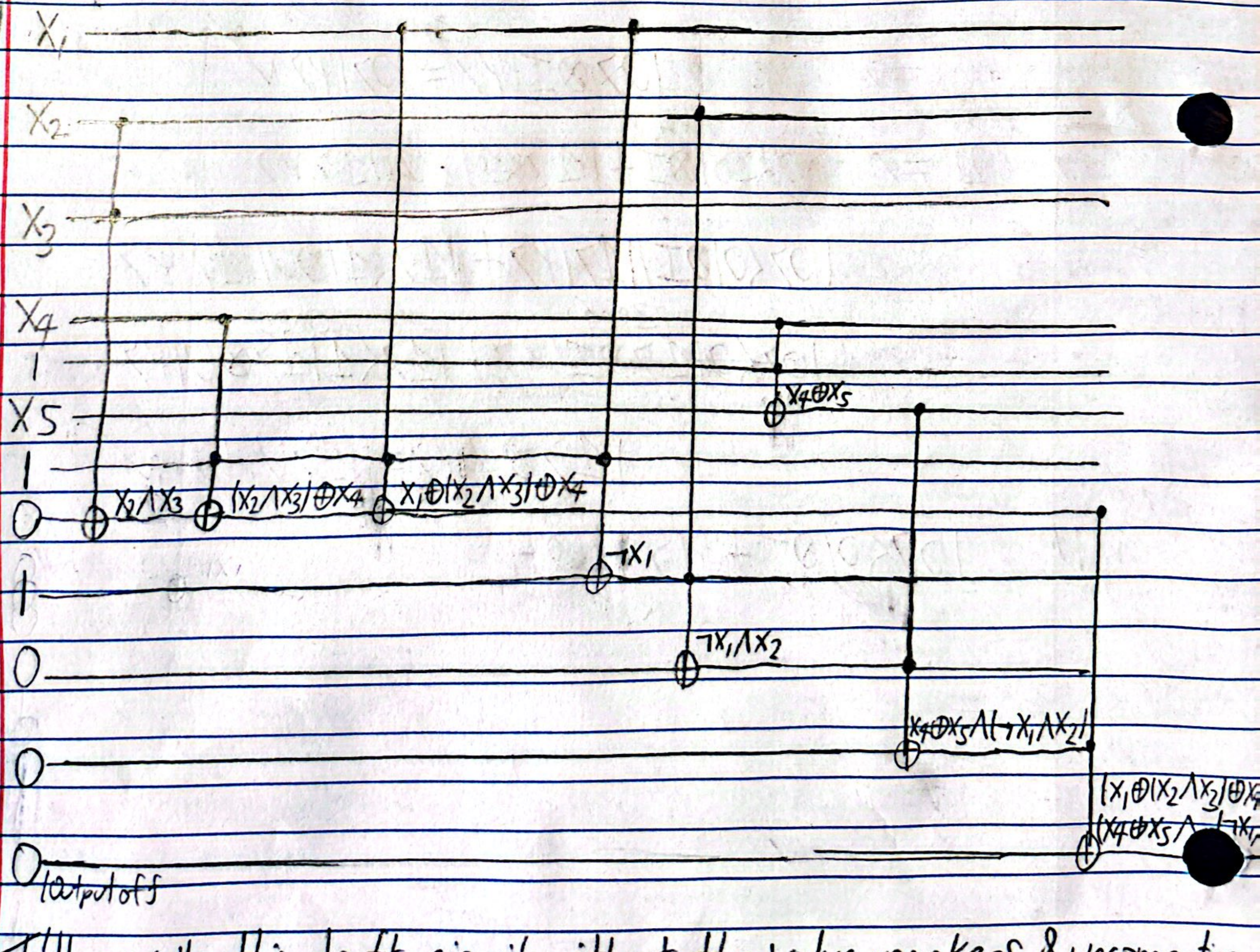
Controlled by measurement (from question)

$|0\rangle|\psi\rangle \Rightarrow$ 0 measured, don't apply $U \Rightarrow |\psi\rangle$

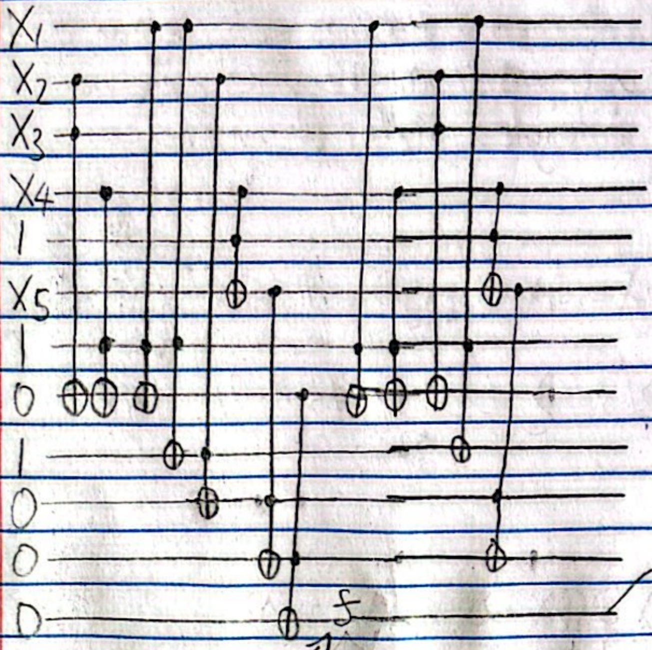
$|1\rangle|\psi\rangle \Rightarrow$ 1 measured, apply $U \Rightarrow U|\psi\rangle$

For arbitrary states $|0\rangle|\psi\rangle$ & $|1\rangle|\psi\rangle$, the two cases have equivalen[t] results $\Rightarrow$ they're equivalent

4)



I'll rewrite this draft circuit without the value markers & uncomputing garbage values

4/ 

Don't uncompute f! We need that

Up to here, we have computed f in the last temporary value
Let's uncompute temp values

5/ $U_f : |x\rangle|0\rangle \to |x\rangle|f(x)\rangle$ for some $f : \{0,1\} \to \{0,1\}$

i/ $|01\rangle + |10\rangle$ is an entangled state

$$f: \begin{matrix} 0 \to 1 \\ 1 \to 0 \end{matrix} \Rightarrow U_f\left(\frac{1}{\sqrt{2}}(|0\rangle|0\rangle + |1\rangle|0\rangle)\right) = \frac{1}{\sqrt{2}}(|0\rangle|1\rangle + |1\rangle|0\rangle)$$

$$= \frac{1}{\sqrt{2}}|01\rangle + \frac{1}{\sqrt{2}}|10\rangle \checkmark$$

ii/ We have $\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$

Measurement of 0 in second qubit ⇒ must be $|10\rangle$ ⇒ collapses to $|1\rangle$ after measurement

Measurement of 1 in second qubit ⇒ must be $|01\rangle$ ⇒ collapses to $|0\rangle$ after measurement

They both collapse to a set state in first qubit instead of being a superposition like before measurement

iii) As we saw in the second part of this question, measuring a qubit could collapse other qubits in superposition. We don't measure to reset as that could affect other entangled states within the computation.

6) $f: \mathbb{Z}_2^n \to \mathbb{Z}_2$ where,

$$f(x) = s \cdot x = s_1 x_1 \oplus s_2 x_2 \oplus \ldots \oplus s_n x_n$$

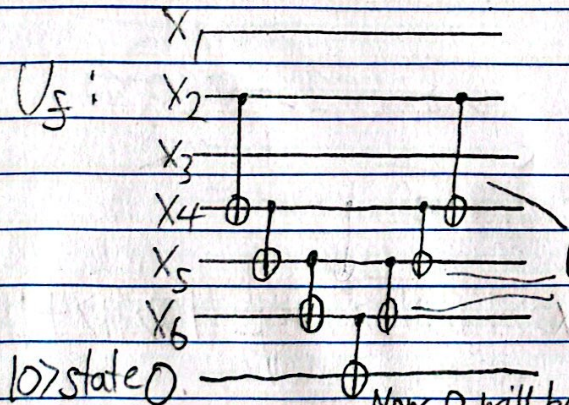
Using $U_f: |x\rangle|0\rangle \mapsto |x\rangle|f(x)\rangle$, $U_f: |x\rangle \mapsto (-1)^{f(x)}|x\rangle$

Let $n=6$, $s=010111$

i) $f(x) = s_1x_1 \oplus s_2x_2 \oplus s_3x_3 \oplus s_4x_4 \oplus s_5x_5 \oplus s_6x_6$

$$f(x) = 0x_1 \oplus 1x_2 \oplus 0x_3 \oplus 1x_4 \oplus 1x_5 \oplus 1x_6$$
$$= x_2 \oplus x_4 \oplus x_5 \oplus x_6$$

U_f:



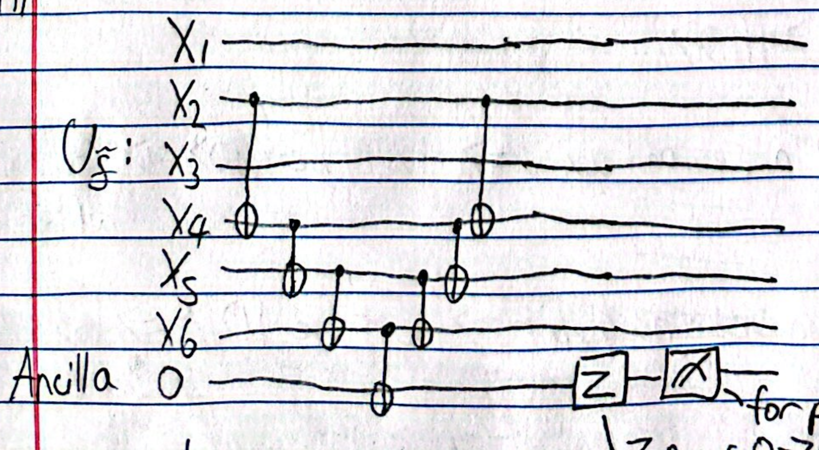
Uncompute to get $|x\rangle$ back

$|x\rangle|0\rangle \mapsto |x\rangle|f(x)\rangle$ ✓

Now 0 will be $0 \oplus x_2 \oplus x_4 \oplus x_5 \oplus x_6 = f(x)$

ii)

U_f:



for phase change

get f in 0 state (U_f)

Z maps $0 \to 0$, $1 \to -1$ exactly what we want for $|x\rangle \mapsto (-1)^{f(x)}|x\rangle$

iii) The value of s can be computed in $O(n)$ for a classical f. My implementa[tion] was serveral bitwise operations & mappings so yes, it could be.

Query complexity is not a good characterization as it's the same as DJ, $2^{n-1}+1$. The query complexity doesn't tell us much revelant info about the speed/computational time of the algo.

U_f being any poly-sized oracle, it would still be in P & it's query complexi[ty] would still be irrelevant

7) $f: \{0,1\}^3 \to \{0,1\}^3$, $f(a,b,c) = (b(\neg a) \oplus b(\neg c), b(\neg a \oplus c), a \oplus c)$

i) $$\frac{1}{\sqrt{2}^3} \sum_{x \in \{0,1\}^3} |x\rangle = (H \otimes H \otimes H)|0\rangle|0\rangle|0\rangle$$

here, $n=3$. We have to add ancillas & U_f to get $|x\rangle|f(x)\rangle$ within the sum

$$\Rightarrow \frac{1}{\sqrt{2}^n} \sum_{x \in \{0,1\}^3} |x\rangle|f(x)\rangle = U_f(H \otimes H \otimes H \otimes I \otimes I \otimes I)|0\rangle|0\rangle|0\rangle|0\rangle|0\rangle|0\rangle$$

ii) Non-zero amplitude: let $c = (0,1,0) \Rightarrow (a,b,c) = (0,1,0)$

iii) $$H^{\otimes 3}|x\rangle \Rightarrow (H \otimes H \otimes H)|x_1 x_2 x_3\rangle = \frac{1}{\sqrt{2}^3} \sum_{z \in \{0,1\}^3} (-1)^{x \cdot z} |z\rangle \text{ by def}^n$$

So we can apply $H^{\otimes 3}$ to our x registers after simulating measurement

$\Rightarrow H^{\otimes 3} \otimes I (|x\rangle + |x \oplus s\rangle)|c\rangle$, paths cancel iff $x \cdot z \neq (x \oplus s) \cdot z \Rightarrow 0 \neq s \cdot z$
only z's that don't destructively interfere are z's orthogonal to $s \Rightarrow z \in s^{\perp}$

$$\Rightarrow \frac{1}{\sqrt{|s^{\perp}|}} \sum_{z \in s^{\perp}} (-1)^{x \cdot z} |z\rangle|f(x)\rangle$$

iv) take $|z_1\rangle = \frac{1}{\sqrt{2^2}}|0,1,1|$, $|z_2\rangle = \frac{1}{\sqrt{2^2}}|1,1,0|$

V) $\begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix} \begin{pmatrix} s_1 \\ s_2 \\ s_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow$