Assignment 5

Alen Mehmedbegovic 301476201

1)

$$\begin{array}{c|cc|c|l} & x_1 & x_2 & -1 & \\ \hline y_1 & a & -a & b & = -t_1 \\ y_2 & -a & a & b & = -t_2 \\ \hline -1 & c & c & 0 & = f \\ & s_1 & s_2 & & = g \end{array}$$

a) $b > 0$ & $c > 0$. Since $b > 0$ we have MBFT

$c > 0 \Rightarrow$ we can pivot on either c

Look at $ax_1 - ax_2 - b = -t_1$

$$a(x_1 - x_2) - b = -t_1 \; , \; t_1 \geq 0$$

$$a(x_1 - x_2) - b \geq 0 \Rightarrow a(x_1 - x_2) \geq b$$

$$a(x_1 - x_2) \geq b > 0$$

$$a(x_1 - x_2) > 0 \quad , \; x_1, x_2 \geq 0$$

either $x_2 > x_1 \rightarrow a < 0 \Rightarrow$ this case is impossible as $a(x_1 - x_2) \geq$
 or $x_1 > x_2 \rightarrow a > 0$

$a > 0 \Rightarrow a_{1j}, a_{2j} \leq 0 \Rightarrow$ Primal unbounded

Primal unbounded $\Rightarrow$ dual infeasible

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b/ $b > 0 \Rightarrow$ MBF1

$c < 0 \Rightarrow$ STOP, current solution is optimal

Primal optimal $\Rightarrow$ dual optimal solution possible

c/ $b < 0 \Rightarrow$ M1

$c > 0 \Rightarrow$ look at $ay_1 - ay_2 - c = S_1$

$$a(y_1 - y_2) = S_1 + c \quad , \quad S_1 + c > 0$$

$$a(y_1 - y_2) > 0$$

either $y_1 > y_2 \Rightarrow a > 0$

or $y_2 > y_1 \Rightarrow a < 0$

2) production cost = a · production + b

Minimize maximum error (abs) incurred in estimating daily production costs

if $a=2, b=-1000 \rightarrow pc = 2p - 1000$

$$pc + 1000 = 2p$$

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3) let $A \in \mathbb{R}^{2\times 2}$ be a game matrix

A has a saddle point $\Rightarrow$ A row minimum & column maximum

$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$ a_{ij} is saddle $\Rightarrow$ a_{ij} is smaller than it's row neighbo
a_{ij} is larger than it's column neighbo

Assume WLOG a_{11} is saddle $\Rightarrow a_{11} < a_{12}$, $a_{11} > a_{21}$

$a_{11} > a_{22}$ also as if $a_{11} \leq a_{22}$ then $a_{12} \leq a_{22}$, $a_{11} \geq a_{21}$ and the
$\rightarrow a_{ij}$ wouldn't be a saddle $\Rightarrow \Leftarrow$

$\Rightarrow a_{11} > a_{21}$ & $a_{11} > a_{22}$

$a_{12} > a_{11} > a_{21}$ & $a_{12} > a_{11} > a_{21} \rightarrow a_{12} > a_{22}$

$\Rightarrow a_{11} > a_{21}$ & $a_{12} > a_{22} \rightarrow$ row 2 is dominated by row 1

Now we have $[a_{11} \; a_{12}]$, $a_{11} < a_{12} \rightarrow$ column 2 is dominated by column 1

$[a_{11}] \Rightarrow$ 1x1 matrix

WLOG $[a_{ij}]$ is a saddle point that can be reduced to a 1x1 matrix by dominance

4)

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

a) Column domination: $\nexists$ a pair of columns k, l for $1 \le i \le m$ s.t. $a_{ik} \ge a_{il}$ as we have alternating values for each comparision

Row domination: $\nexists$ a pair of rows k, l for $1 \le i \le m$ s.t. $a_{ik} \le a_{il}$ as we have alternating values for each comparison

b)

$$\begin{array}{c|cccc|c|l}
 & V & q_1 & q_2 & q_3 & -1 & \\
\hline
U & 0 & -1 & -1 & -1 & -1 & = -0 \\
p_1 & -1^* & 1 & 0 & -1 & 0 & = -t_1 \\
p_2 & -1 & 0 & 0 & 0 & 0 & = -t_2 \\
p_3 & -1 & -1 & 0 & 1 & 0 & = -t_3 \\
\hline
-1 & -1 & 0 & 0 & 0 & 0 & = f \\
\hline
 & = 0 & = s_1 & = s_2 & = s_3 & = g &
\end{array}$$

In matrix game A:

max of columns:

$$\begin{matrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{matrix}$$

Choose topleft 1

$\Rightarrow$ pivot on $(2,1) = -1$ & $(1,2) = -1$

$$\begin{array}{c|cccc|c|l}
 & V & q_1 & q_2 & q_3 & -1 & \\
\hline
U & 0 & -1^* & -1 & -1 & -1 & = 0 \\
p_1 & -1 & -1 & 0 & 1 & 0 & = -t_1 \\
p_2 & -1 & -1 & 0 & 1 & 0 & = -t_2 \\
p_3 & -1 & 0 & 0 & 2 & 0 & = -t_3 \\
\hline
-1 & -1 & -1 & 0 & 1 & 0 & = f \\
\hline
 & = 0 & = s_1 & = s_2 & = s_3 & = g &
\end{array}$$

pivot on $(1,2) = -1$

b)

$$\begin{array}{c|cccc|c|l}
 & v & q_1 & q_2 & q_3 & -1 & \\
\hline
v & 0 & -1 & 1 & 1 & 1 & = 0 \\
p_1 & -1 & -1 & 1 & 2 & 1 & = -t_1 \\
p_2 & -1 & -1 & 1 & 2 & 1 & = -t_2 \\
p_3 & -1 & 0 & 0 & 2 & 0 & = -t_3 \\
\hline
-1 & -1 & -1 & 1 & 2 & 1 & = f \\
\hline
 & = 0 & = s_1 & = s_2 & = s_3 & = g &
\end{array}$$

Delete 0-col & row

Now we have MBFT

$$\begin{array}{c|ccc|c|l}
 & q_1 & q_2 & q_3 & -1 & \\
\hline
p_1 & -1 & 1^* & 2 & 1 & = -t_1 \\
p_2 & -1 & 1 & 2 & 1 & = -t_2 \\
p_3 & 0 & 0 & 2 & 0 & = -t_3 \\
\hline
-1 & -1 & 1 & 2 & 1 & = f \\
\hline
 & = s_1 & = s_2 & = s_3 & = g &
\end{array}$$

Pivot on $(1,2) = 1$

$$\begin{array}{c|ccc|c|l}
 & q_1 & t_1 & q_3 & -1 & \\
\hline
s_2 & -1 & 1 & 2 & 1 & = -q_2 \\
p_2 & 0 & -1 & 0 & 0 & = -t_2 \\
p_3 & 0 & 0 & 2 & 0 & = -t_3 \\
\hline
-1 & 0 & -1 & 0 & 0 & = f \\
\hline
 & = s_1 & = p_1 & = s_3 & = g &
\end{array}$$

$c_1, \dots, c_n \leq 0 \Rightarrow$ Optimal

$E_1(p) = p_1 a_{11} + p_2 a_{21} + p_3 a_{31} = -p_1$. $\min E_1(p) = -1$, $(1,0,0)$

$E_2(p) = p_1 a_{12} + p_2 a_{22} + p_3 a_{32} = p_1 - p_2$. $\min E_2(p) = -1$, $(0,1,0)$

$E_3(p) = 2p_1 + 2p_3$. $\operatorname{Min} E_3(p) = 0$, $(0,1,0)$

EV $\max \min\limits_{1 \leq j \leq n} E_j(p) = 0$

b) $f_1(q) = q_1 a_{11} + q_2 a_{12} + q_3 a_{13} = -q_1 + q_2 + 2q_3$ $\max f_1(q) = 2, (0,0,1)$

$f_2(q) = -q_2$ $\max f_2(q) = 0, (\frac{1}{2}, 0, \frac{1}{2})$

$f_3(q) = 2q_3$ $\max f_3(q) = 2, (0,0,1)$

EV $\min_{1 \leq i \leq n} \max f_i(q) = 0$:

Both are 0 → fair game

c) $\begin{bmatrix} \boxed{1} & \boxed{0} & -1 \\ 0 & \boxed{0} & 0 \\ -1 & \boxed{0} & \boxed{1} \end{bmatrix}$ row O row min
column $\square$ col max

Saddle point is middle 0, $(2,2) = 0$

Yes the reduced tableau has $g = f = 0$ as von Neumann

and opt. strats: $\vec{p} = \begin{vmatrix} 0 \\ 1 \\ 0 \end{vmatrix}$ for row & $\vec{q} = \begin{vmatrix} \frac{1}{2} \\ 0 \\ \frac{1}{2} \end{vmatrix}$ for col

as we can overlook the matrix multiplication & see which one maximizes/minimizes payoff

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5) Player 1 write int: 1..15

Player 1 tells player 2 an integer

Player 2 guesses if player 1 was telling the truth:

If 1 gets caught lying: 10\$ to 2
If falsely accused lying: 5\$ to 1
If 1 truth & 2 accused truth: 1\$ to 2
If 1 lie & 2 accused truth: 5\$ to 1

$$\begin{array}{cc|cc} & & 2 & \\ & & T & L \\ \hline 1 & T & -1 & \boxed{5} \\ & L & \boxed{5} & -10 \end{array}$$

No domination rules
row O
column $\square$

$\Rightarrow \nexists$ a saddle point

$E_1(p) = p_1 a_{11} + p_2 a_{21} = -p_1 + 5p_2 \quad \min E_1(p) = -1, (1,0)$

$E_2(p) = p_1 a_{12} + p_2 a_{22} = 5p_1 - 10p_2 \quad \min E_2(p) = -10, (0,1)$

row: $\max \min_{j=1,2} E_j(p) = -1$ EV

$F_1(p) = q_1 a_{11} + q_2 a_{12} = -q_1 + 5q_2 \quad \max F_1(q) = 5, (0,1)$

$F_2(p) = q_1 a_{21} + q_2 a_{22} = 5q_1 - 10q_2 \quad \max F_2(q) = 5, (1,0)$

column: $\min \max_{j=1,2} F_j(q) = 5$ EV

6) $x, y, z \in \mathbb{R}$

$$\begin{vmatrix} x & x \\ y & z \end{vmatrix}$$

if $|x \le y \le z|$ then $\begin{vmatrix} x & x \\ y & z \end{vmatrix}$ q.t. sol $= y$, von. Neumann: y

for $x \square y$, $y \square z$, $z \square x$
there are $3 \cdot 3 \cdot 3 = 27$ combinations of