MATH 308 - Assignment 2

1. A post office requires different numbers of full-time employees on different days of the week. The number of full-time employees required on each day is as follows: Day1 (Monday) 17, Day2 (Tuesday) 13, Day3 (Wednesday) 15, Day4 (Thursday) 19, Day5 (Friday) 14, Day6 (Saturday) 16, and Day7 (Sunday) 11.

Union rules state that each full-time employee must work five consecutive days and then have two days off. For example, an emploee working from Monday to Friday must be off on Saturday and Sunday. The post office wants to meet its daily quotas using as few full-time emloyees as possible and still satisfy the requirements of the Union.

Formulate the problem as an LP problem.

2. You are considering to produce two kind of candies: Candy1 and Candy2, both of which consist solely of sugar, nuts, and chocolate. You have in stock 10000 oz of sugar, 80000 oz of nuts, and 50000 oz of chocolate. The mixture used to make Candy1 must contain at least 20% of nuts but not more that 30% of sugar. The mixture used to make Candy2 must contain at least 10% of nuts and 15% of chocolate but not more that 25% of sugar. Each ounce of Candy1 can be sold for 25 cents and each ounce of Candy2 can be sold for 20 cents. Formulate an LP to maximize your profit.

3. A notoriously known secretive company is deciding on producing three new devices code-named A, B, and C. The company assumes it can sell these products in unlimited quantities (yes, we will be buying them :)) at the following unit prices: A \$100, B \$560, and C \$1000. Producing a unit of A requires 1 hour of labor, a unit of B requires 2 hours of labor and 2 units of A, and a unit of C requires 3 hours of labor and 1 unit of B.

Any A that produces B cannot be sold, and any B that produces C cannot be sold. A total of 40000 hours of labor are available. Formulate an LP to maximize company's profit.

4. A cooperative of three farms in a dry region wants to optimize its agricultural production for a year, knowing that there are constraints related to the available irrigable land and the available water. Here are these constraints for each farm

- Farm 1 owns 400 acres of land and have access to 600 tons of water.
- Farm 2 owns 600 acres of land and have access to 800 tons of water.

- Farm 3 owns 200 acres of land and have access to 400 tons of water.

The cooperative grows (and then sells) three crops: cotton, beans, wheat. Each of these crops requires a certain amount of water per acre to grow, and has a specific net return by acre:

- Cotton requires 2 tons of water per acre and returns \$750 per cultivated acre.
- Beans require 3 tons of water per acre and returns \$1000 per cultivated acre.
- Wheat requires 1 ton of water per acre and returns \$250 per cultivated acre.

Moreover, the cooperative has restrictions on the amount of land that can be devoted to each crop (due, for example, to knowledge of the market for each crop):

- at most 500 acres can be devoted to cotton;
- at most 600 acres can be devoted to beans;
- at most 300 acres can be devoted to wheat.

Finally, to ensure equity between farms, each should use the same proportion of its land: if Farm 1 uses only 200 acres of its 400 acres, then Farm 2 will use 300 of its 600 acres and Farm 3 will use 100 of its 200 acres. However, there is no restriction on the kind of crops that can be grown on each farm. The goal of the planning is to maximize the total net return of the three farms together.

5. Apply the Simplex Algorithm using Tucker tableau to the following LP problem. Use x_1 as the first entering variable.

$$\begin{array}{ll} \text{Maximize:} & f(x_1, x_2, x_3) = 3x_1 + 2x_2 + 4x_3 \\ \text{subject to} & x_1 + x_2 + 2x_3 \leq 4 \\ & 2x_1 + 3x_3 \leq 5 \\ & 2x_1 + x_2 + 3x_3 \leq 7 \\ & x_1, x_2, x_3 \geq 0 \end{array}$$

6. Apply the Simplex Algorithm using Tucker tableau to the following LP problem. Use x_1 as the first entering variable.

$$\begin{array}{ll}
\text{Maximize:} & f(x_1, x_2, x_3, x_4) = 5x_1 + 6x_2 + 9x_3 + 8x_4 \\
\text{subject to} & x_1 + 2x_2 + 3x_3 + x_4 \le 5 \\
& x_1 + x_2 + 2x_3 + 3x_4 \le 3 \\
& x_1, x_2, x_3, x_4 \ge 0
\end{array}$$

7. Construct the Tucker tableau for the following LP problem. Then perform the pivot transformation that switches the independent variable x_1 and the dependent variable t_2. Follow by the pivot transformation that switches the independent variable x_3 and the dependent variable t_3. What is an optimal solution for the LP problem? Show that there are infinitely many optimal solutions.

$$\begin{array}{ll}
\text{Maximize:} & f(x_1, x_2, x_3, x_4) = 2x_1 + 3x_2 - x_3 - 12x_4 \\
\text{subject to} & -2x_1 - 9x_2 + x_3 + 9x_4 \le 0 \\
& \frac{1}{3}x_1 + x_2 - \frac{1}{3}x_3 - 2x_4 \le 0 \\
& 2x_1 + 3x_2 - x_3 - 12x_4 \le 2 \\
& x_1, x_2, x_3, x_4 \ge 0
\end{array}$$

Strayer, Chapter 2., p.64–69 Ex. 5a, 5d, 9, 10