Alen Mehmedbegovic
301476201

Assignment 2

1) Monday - 17
Tuesday - 13
Wednesday - 15
Thrusday - 19
Friday - 14
Saturday - 16
Sunday - 11

Let Mon = # of employees that start work on Monday
Define the same for Tuesday, Wednesday, Thrusday, friday, Saturday & Sunday. Each named as first 3 letters of their name.
=> Obj. func f. Minimize f = Mon + Tue + Wed + Thr + Fri + Sat + Sun
Monday will have workers starting from Monday, Thrusday, friday Saturday & Sunday.
Constriants:
=> Mon + Thr + Fri + Sat + Sun ≥ 17
=> Tue + Fri + Sat + Sun + Mon ≥ 13
=> Wed + Sat + Sun + Mon + Tue ≥ 15
=> Thr + Sun + Mon + Tue + Wed ≥ 19
=> Fri + Mon + Tue + Wed + Thr ≥ 14
=> Sat + Tues + Wed + Thr + Fri ≥ 16
=> Sun + Wed + Thr + Fri + Sat ≥ 11

& implied like days cannot be negative
(all days ≥ 0)
- days cant be more than 7

2) 10000 oz of sugar
80000 oz of nuts
50000 oz of chocolate

Candy 1: ≥20% nuts, ≤30% sugar. 25 cents/oz
Candy 2: ≥10% nuts, 15% chocolate, ≤25% sugar. 20 cents/oz

Let C_1 & C_2 be oz's of candy made, $I_\#$ be candy # of ingredient I.
Meaning N_1 be oz's of nuts in candy 1, C_2 be chocolate in Candy 2, etc.

2) Maximize $f = 0.25C_1 + 0.2C_2$
Subject to:

- $0.3C_1 + 0.25C_2 \leq 50000$ Sugar
- $0.2C_1 + 0.1C_2 \leq 80000$ Nuts
- $0.15C_2 \leq 50000$ Chocolate
  $\Rightarrow C_2 \leq 333333.\bar{3}$
- Implied, $C_1, C_2 \geq 0$

3) Let D_x be devices produced: D_A, D_B, D_C

Maximize $f = 100D_A + 560D_B + 1000D_C$
Subject to:

- $1 \cdot D_A + 2 \cdot D_B + 3 \cdot D_C \leq 40000$ Labour
- $2D_B \leq D_A$ Dependancies
- $D_C \leq D_B$
- Implied: $D_A, D_B, D_C \geq 0$

4) Let C_x be crop acre grown on farm x: C_1, C_2, C_3
B_1, B_2, B_3
W_1, W_2, W_3

Maximize $f = 750(C_1+C_2+C_3) + 1000 \cdot (B_1+B_2+B_3) + 250(W_1+W_2+W_3)$

subject to:
- $C_1 + B_1 + W_1 \leq 400$
  $C_2 + B_2 + W_2 \leq 600$ Land space
  $C_3 + B_3 + W_3 \leq 200$
  $2C_1 + 3B_1 + 1 \cdot W_1 \leq 600$
  $2C_2 + 3B_2 + 1W_2 \leq 800$ Water
  $2C_3 + 3B_3 + 1W_3 \leq 400$
  $C_1 + C_2 + C_3 \leq 500$
  $B_1 + B_2 + B_3 \leq 600$ Market
  $W_1 + W_2 + W_3 \leq 300$

$\frac{C_1+B_1+W_1}{400} = \frac{C_2+B_2+W_2}{600} = \frac{C_3+B_3+W_3}{200}$ Proportion

5/ $f(x_1, x_2, x_3) = 3x_1 + 2x_2 + 4x_3$

s.t. $x_1 + x_2 + 2x_3 \leq 4$

$2x_1 + 3x_3 \leq 5$

$2x_1 + x_2 + 3x_3 \leq 7$

$x_1, x_2, x_3 \geq 0$

Introduce slacks $x_1 + x_2 + 2x_3 + t_1 = 4$

$2x_1 + 0 \cdot x_2 + 3x_3 + t_2 = 5$, $t_i \geq 0$

$2x_1 + x_2 + 3x_3 + t_3 = 7$

x_1	x_2	x_3	-1	
1	1	1	4	$= -t_1$
2^*	0	3	5	$= -t_2$
2	1	3	7	$= -t_3$
3	2	4	0	$= f$

Replace x_1 with t_2

t_2	x_2	x_3	-1	
$\frac{1}{2}$	1	$-\frac{1}{2}$	$\frac{3}{2}$	$= -t_1$
$\frac{1}{2}$	0	$\frac{3}{2}$	$\frac{5}{2}$	$= -x_1$
-1	1	0	2	$= -t_3$
$-\frac{3}{2}$	2	$-\frac{1}{2}$	$-\frac{15}{2}$	$= f$

5) Replace x_2 with t_3 => pivot on $[3,2] = 1$

$$\begin{array}{ccc|c|l} t_2 & t_3 & x_3 & -1 & \\ \hline -1 & -1 & -\frac{1}{2} & -\frac{1}{2} & = -t_1 \\ 1 & 0 & \frac{3}{2} & \frac{5}{2} & = -x_1 \\ -1 & 1 & 0 & 2 & = -x_2 \\ \hline -\frac{1}{2} & -2 & -\frac{1}{2} & -\frac{23}{2} & = f \end{array}$$

Not MBFT now, need to pivot on $[3,2] = \frac{3}{2}$

$$\begin{array}{ccc|c|l} t_2 & t_3 & x_1 & -1 & \\ \hline -\frac{2}{3} & -1 & \frac{1}{3} & \frac{1}{3} & = -t_1 \\ \frac{2}{3} & 0 & \frac{2}{3} & \frac{3}{5} & = -x_3 \\ -1 & 1 & 0 & 2 & = -x_2 \\ \hline 0 & -2 & -\frac{1}{3} & -\frac{32}{3} & = f \end{array}$$

, Now it's MBFT

Replace x_1 & t_1, pivot on $\frac{1}{3}$

$$\begin{array}{ccc|c|l} t_2 & t_3 & t_1 & -1 & \\ \hline -2 & -3 & 3 & 1 & = -x_1 \\ 1 & 2 & -2 & \frac{1}{15} & = -x_3 \\ -1 & 1 & 0 & 2 & = -x_2 \\ \hline -\frac{1}{[illegible]} & 2 & 1 & -\frac{329}{[illegible]} & = f \end{array}$$

All of bottom row is ≤ 0
Max Value is $329/30$

6) Maximize $f(x_1, x_2, x_3, x_4) = 5x_1 + 6x_2 + 9x_3 + 8x_4$

s.t. $x_1 + 2x_2 + 3x_3 + x_4 \leq 5$

$x_1 + x_2 + 2x_3 + 3x_4 \leq 3$

$x_1, x_2, x_3, x_4 \geq 0$

Introduce slacks $x_1 + 2x_2 + 3x_3 + x_4 + t_1 = 5$

$x_1 + x_2 + 2x_3 + 3x_4 + t_2 = 3$, $t_i \geq 0$

x_1	x_2	x_3	x_4	-1	
1	2	3	1	5	$= -t_1$
1	1	2	3	3	$= -t_2$
5	6	9	8	0	$= f$

Change x_1 & t_2. pivot on $(2,1) = 1$

t_2	x_2	x_3	x_4	-1	
-1	1	1	-2	2	$= -t_1$
1	1	2	3	3	$= -x_1$
-1	1	-1	-7	-15	$= f$

Now change x_2 & t_1. pivot on $(1,2) = 1$

t_2	t_1	x_3	x_4	-1	
-1	1	1	-2	2	$= -x_2$
2	-1	1	5	1	$= -x_1$
-2	-1	1	-4	-17	$= f$

Almost there, $(3,3) = 1$ is only element that ≥ 0. Pivot on $(2,3) = 1$

Scanned with CamScanner

6)

$$\begin{array}{cccc|c|l} t_2 & t_1 & x_1 & x_4 & -1 & \\ \hline -3 & 0 & -1 & -7 & 1 & = -x_2 \\ 2 & -1 & 1 & 5 & 1 & = -x_3 \\ \hline -4 & 0 & -1 & -9 & -18 & = f \end{array}$$

Optimal form reached, max $f = 18$

7) Max $f(x_1, x_2, x_3, x_4) = 2x_1 + 3x_2 - x_3 - 12x_4$

s.t. $-2x_1 - 9x_2 + x_3 + 9x_4 \leq 0$

$\frac{1}{3}x_1 + x_2 - \frac{1}{3}x_3 - 2x_4 \leq 0$

$2x_1 + 3x_2 - x_3 - 12x_4 \leq 2$

$x_1, x_2, x_3, x_4 \geq 0$

Introduce slacks $-2x_1 - 9x_2 + x_3 + 9x_4 + t_1 = 0$

$\frac{1}{3}x_1 + x_2 - \frac{1}{3}x_3 - 2x_4 + t_2 = 0$, $t_i \geq 0$

$2x_1 + 3x_2 - x_3 - 12x_4 + t_3 = 2$

$$\begin{array}{cccc|c|l} x_1 & x_2 & x_3 & x_4 & -1 & \\ \hline -2 & -9 & 1 & 9 & 0 & = -t_1 \\ \frac{1}{3} & 1 & -\frac{1}{3} & -2 & 0 & = -t_2 \\ 2 & 3 & -1 & -12 & 2 & = -t_3 \\ \hline 2 & 3 & -1 & -12 & 0 & = f \end{array}$$

According to assignment pivot on $(2,1) = \frac{1}{3}$

$$
\begin{array}{cccc|c}
t_2 & x_2 & x_3 & x_4 & -1 \\
\hline
6 & -3 & -1 & -3 & 12 \\
3 & 3 & -1 & -6 & 0 \\
-6 & -3 & 1 & 0 & 2 \\
\hline
-6 & -3 & 1 & 0 & 0
\end{array}
\begin{array}{l}
\\
= -t_1 \\
= -x_1 \\
= -t_3 \\
= f
\end{array}
$$

According to assignment swap x_3 & t_3, Pivot on $(3,3) = 1$

$$
\begin{array}{cccc|c}
t_2 & x_2 & t_3 & x_4 & -1 \\
\hline
0 & -6 & 1 & -3 & 14 \\
-3 & 0 & 1 & -6 & 2 \\
-6 & -3 & 1 & 0 & 2 \\
\hline
0 & 0 & -1 & 0 & -2
\end{array}
\begin{array}{l}
\\
= -t_1 \\
= -x_1 \\
= -x_3 \\
= f
\end{array}
$$

$\max f = 2$. Opt sol.

Optimal solution also has $x_2 = x_4 = t_2 = 0$

$\Rightarrow$ Area spanned by $-2x_1 + x_3 \leq 0, \frac{1}{3}x_1 - \frac{1}{3}x_3 \leq 0, 2x_1 - x_3 \leq$

isn't a single point so $\exists$ a continuous range of reals within the area

$\Rightarrow$ Infinitely many optimal solutions