# MATH 469/769 ASSIGNMENT #1

Assignment Due: Monday, January 27 2025, 5:30pm

The total number of marks possible marks for the assignment is 60. All students should attempt all questions. Make sure you show all your work, and make sure that your work is your own. Your reasoning and work is more important than your answer.

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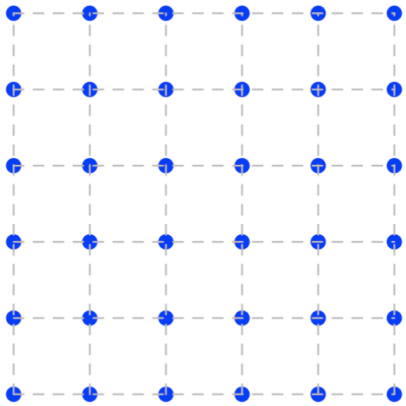
1. Consider this graph:



Without computing or squaring the adjacency matrix $A$, find $\sum_{i=1}^{n}(A)_{ii}^2$. [10]

2. There is a rare and scary cancer, zyphomelanoma[1]. Only 0.1% of the population has this cancer. There is a test for it with the following properties:

   - If a person has zyphomelanoma, it detects this correctly 97% of the time.
   - If not, there is a 2% probability of a false positive (wrongly saying that the person has zyphomelanoma).

   (a) Your friend receives a positive test without prior screening. What's the probability that they have zyphomelanoma?

   (b) Your friend is at risk because of a family history. One in 10 people with this history have zyphomelanoma, and this is why they get tested. The test is positive. What's the probability that they have zyphomelanoma?

   What might you conclude about this "97% accurate" test and how it should or should not be used? [10]

[1] This is completely fictional

3. A random Erdos-Renyi network is created with $N$ nodes, in which each pair of nodes is connected with probability $p$.

   (a) State the expected degree of a node in this network.

   (b) What is the condition for the existence of a giant component in terms of $p$ and $N$? (you may restate the result we had in class).

   (c) For $N = 1500$ and $p = 0.003$, determine whether a giant component is likely to exist in this network. Explain your reasoning.

[10]

4. The *local clustering coefficient* for a vertex $v$ of a graph $G$ is defined as the number of links among the neighbours of $v$ divided by the number of possible links among them:

$$C_i = \frac{|\{e_{iv}\}|}{\binom{k_i}{2}}$$

   (a) What is the mean clustering coefficient for a vertex in an Erdos-Renyi network with connectivity $c$?

   (b) The *global clustering coefficient* for a graph $G$ is the fraction of paths of length two in $G$ that are closed over all paths of length two in $G$. In an Erdos-Renyi graph with $N$ nodes and edge probability $p$, what is the expected global clustering coefficient? Relate this to the local clustering coefficient.

[10]

5. Perhaps we like triangles! So, consider this new random graph model: beginning with $N$ nodes, we go through each distinct set of 3 nodes, and connect them in a triangle with probability $p = c/\binom{N-1}{2}$. $c$ is a constant. (We allow multiple edges between a pair of nodes, so if nodes $i$ and $j$ occur in two triangles and both are added, $i$ and $j$ have two edges linking them).

   (a) Will any node have an odd degree?

   (b) Show that the mean degree of a node is $\bar{k} = 2c$

   (c) What is the probability of a node $i$ having $m$ triangles?

   (d) What is the distribution, in the limit of large $N$, for the number of triangles of a node $i$?

   (e) Hence find the degree distribution (in the limit of large $N$).

   (f) Another way to describe the global clustering coefficient is 3 times the (expected) number of triangles divided by the (expected) number of connected triples. Here, a connected triple $A \sim v \sim B$ is counted as distinct from $v \sim A \sim B$, so that in a complete graph (all possible edges are present) the global clustering coefficient is 1, as is the local clustering coefficient. Find the expected global clustering coefficient for the random graph model in this question.

[20]