Assignment Due: Thursday March 17, 2025, 5:30pm

The total number of marks possible marks for the assignment is 60. All students should attempt all questions. Make sure you show all your work, and make sure that your work is your own. Your reasoning and work is more important than your answer.

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1. In class, we defined the closeness centrality of a node v as

$$c(v) = \frac{n-1}{\sum_i d(i,v)}.$$

There is a very similar definition with n in the numerator instead of $n-1$. Consider an undirected tree of n nodes. A particular edge in the tree joins nodes 1 and 2, and divides the tree into two subtrees, with n_1 nodes on 1's side and n_2 nodes on 2's side.

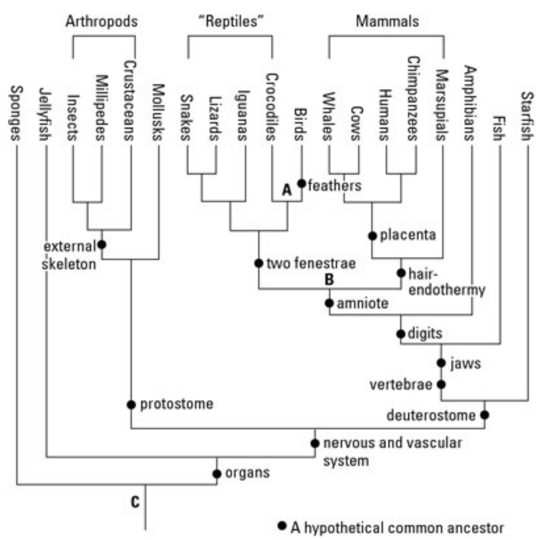
   (a) Make a sketch of the above.

   (b) Show that the closeness centralities C_1 and C_2 of the two nodes, defined using n instead of $n-1$, are related to each other by:

$$\frac{1}{C_1} + \frac{n_1}{n} = \frac{1}{C_2} + \frac{n_2}{n}.$$

[10]

2. Here is a phylogenetic tree:



   (a) Do the reptiles form a clade?

   (b) Do the mammals form a clade?

   (c) What is the closest relative of the fish?

(d) According to the groupings on the tree, what feature separates mammals from other organisms?

(e) What feature separates sponges from other organisms?

(f) Are snakes closer to mollusks or to amphibians in this tree?

[12]

3. A *tree shape* refers to the structure of the tree, without the labels. For example, a tree shape might be a fully symmetric tree, or a caterpillar tree. A tree shape corresponds to more than one phylogenetic tree, because the phylogenetic tree has labels at the tips, and there are many ways to assign the labels to the tips.

(a) How many rooted caterpillar phylogenetic trees are there for 8 taxa?

(b) How many rooted fully symmetric phylogenetic trees are there for 8 taxa?

(c) How many *ranked* rooted caterpillar phylogenetic trees are there for 8 taxa?

(d) How many *ranked* rooted fully symmetric phylogenetic trees are there for 8 taxa?

[12]

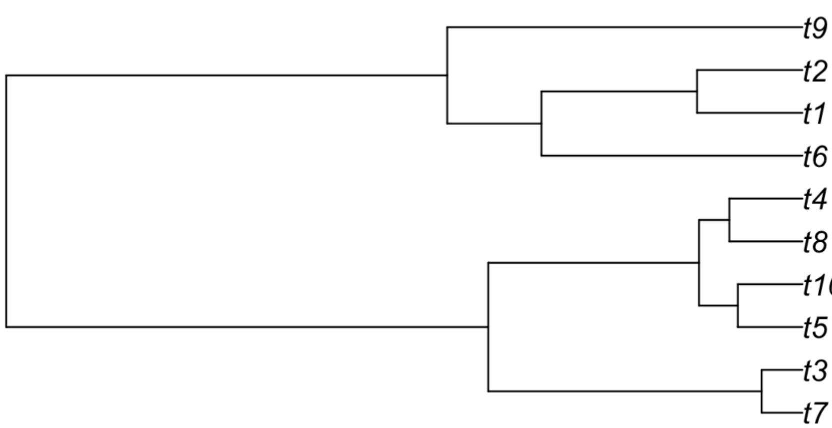
4. In class, we proved that (i) an unrooted binary tree T with $n \geq 2$ leaves has $2n-3$ and $n-2$ interior vertices and (ii) if T is not binary it has fewer than $2n-3$ edges and fewer than $n-2$ interior vertices. In this question you'll do similarly for *rooted* binary trees.

(a) Show (without using the result for unrooted binary trees) that a rooted binary tree has $2n-2$ edges and $n-1$ interior vertices.

(b) Show that if the tree is not binary it has fewer than $2n-2$ edges and fewer than $n-1$ interior vertices.

[10]

5. In the stochastic process known as 'the coalescent', we imagine a process moving back in time from the present to the past. At any time, each pair of lineages (branches) may coalesce (which means join). The process continues until they have all merged:



The result is a ranked phylogeny. In the tree above, tips 3 and 7 coalesce 'first' (thinking backwards in time). Then 5 and 10 do, and then 8 and 4, and so on.

Consider a phylogenetic tree with n taxa.

(a) How many choices (of which lineages will merge) are there for the first coalescent event?

(b) How many choices are there for the second coalescent event?

(c) Based on this, how many ranked phylogenies are there for n tips?

[10]

6. In class we defined a *split* of the taxa set X (a bipartition, or a split into two disjoint parts). How many splits are possible for n taxa? [6]