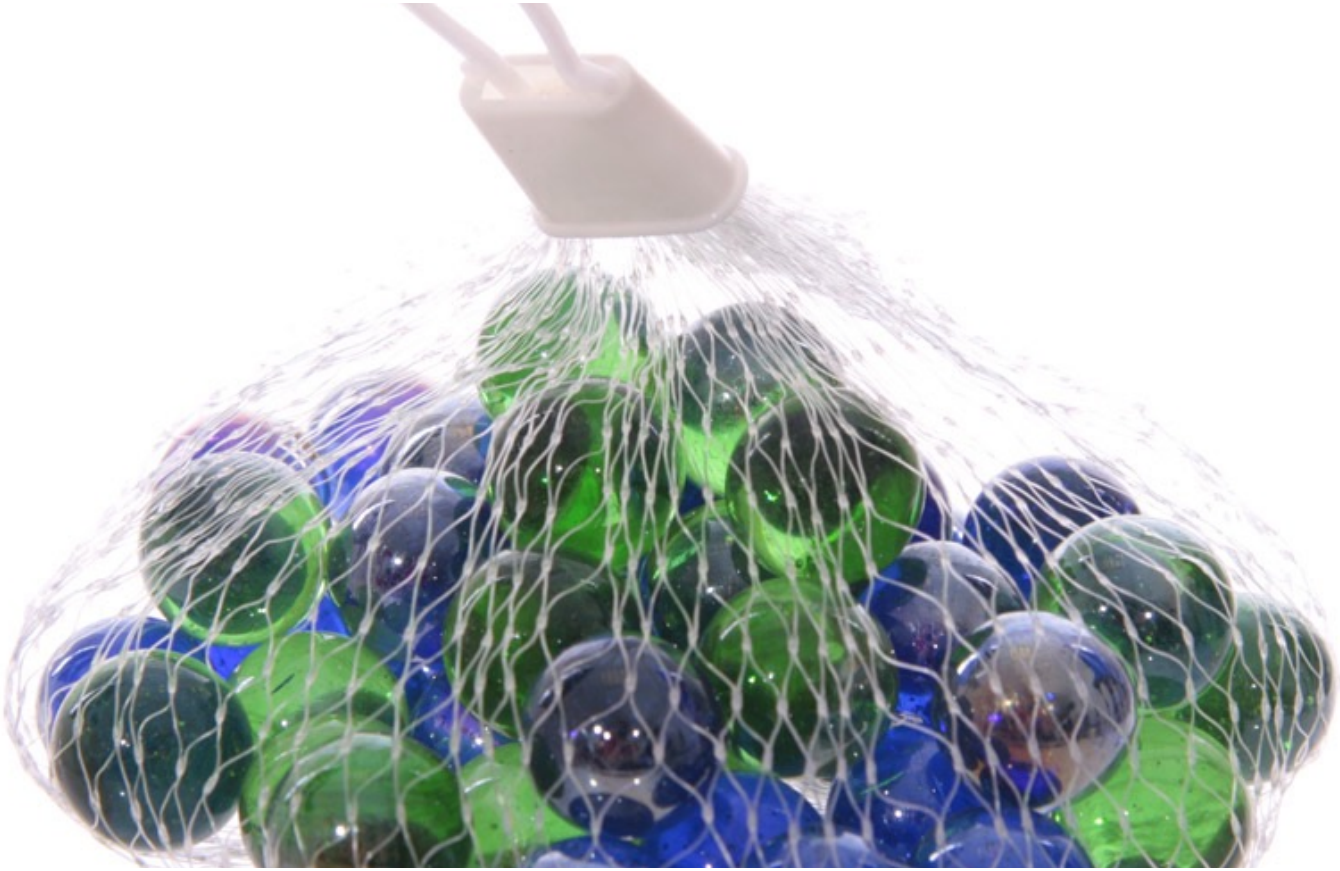
Bag of Words Model



Overview of today's lecture

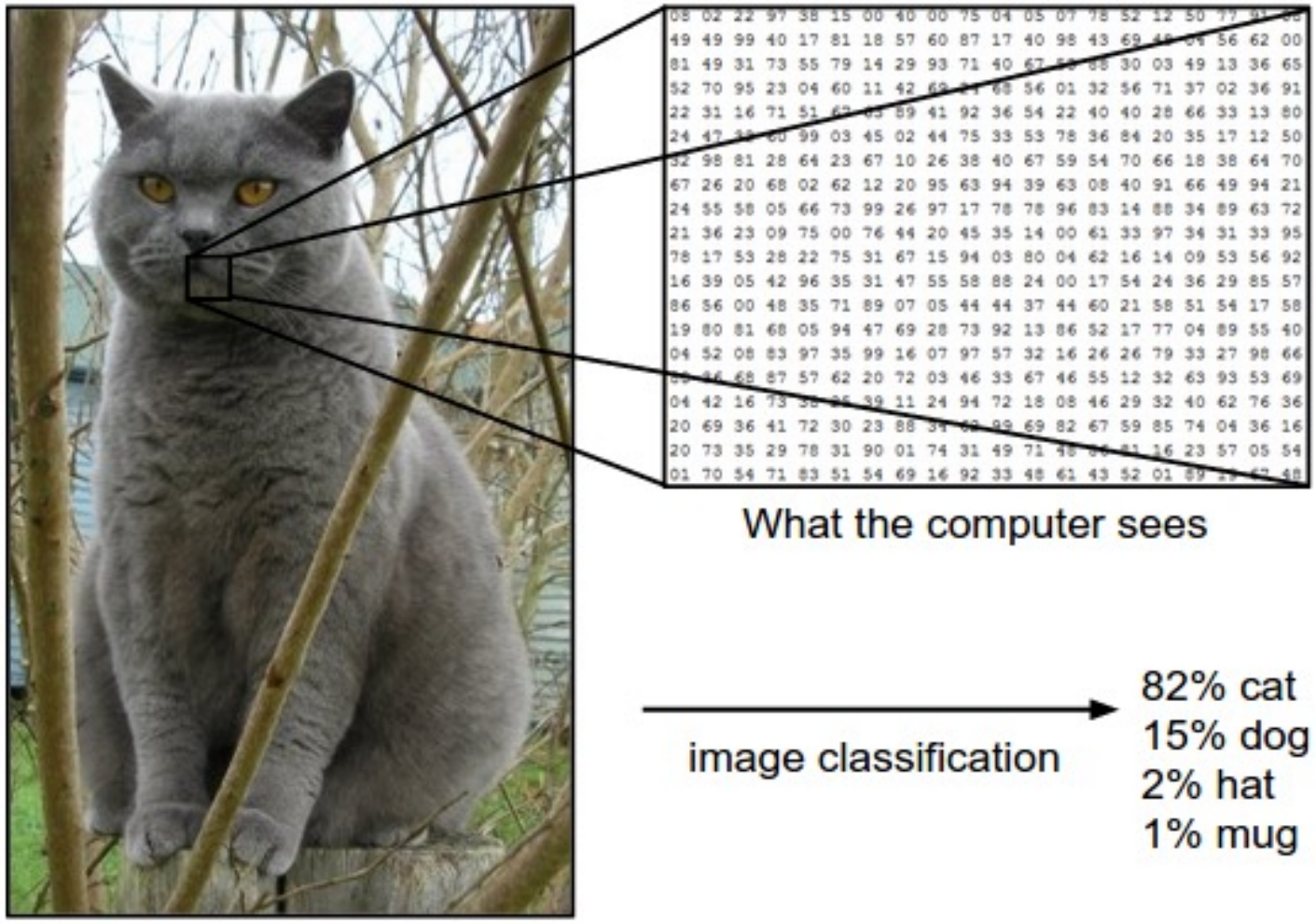
- Bag-of-words.
- K-means clustering.
- Classification.
- K nearest neighbors.
- Support vector machine.

Image Classification

(assume given set of discrete labels)
{dog, cat, truck, plane, ...}

cat

Image Classification: Problem

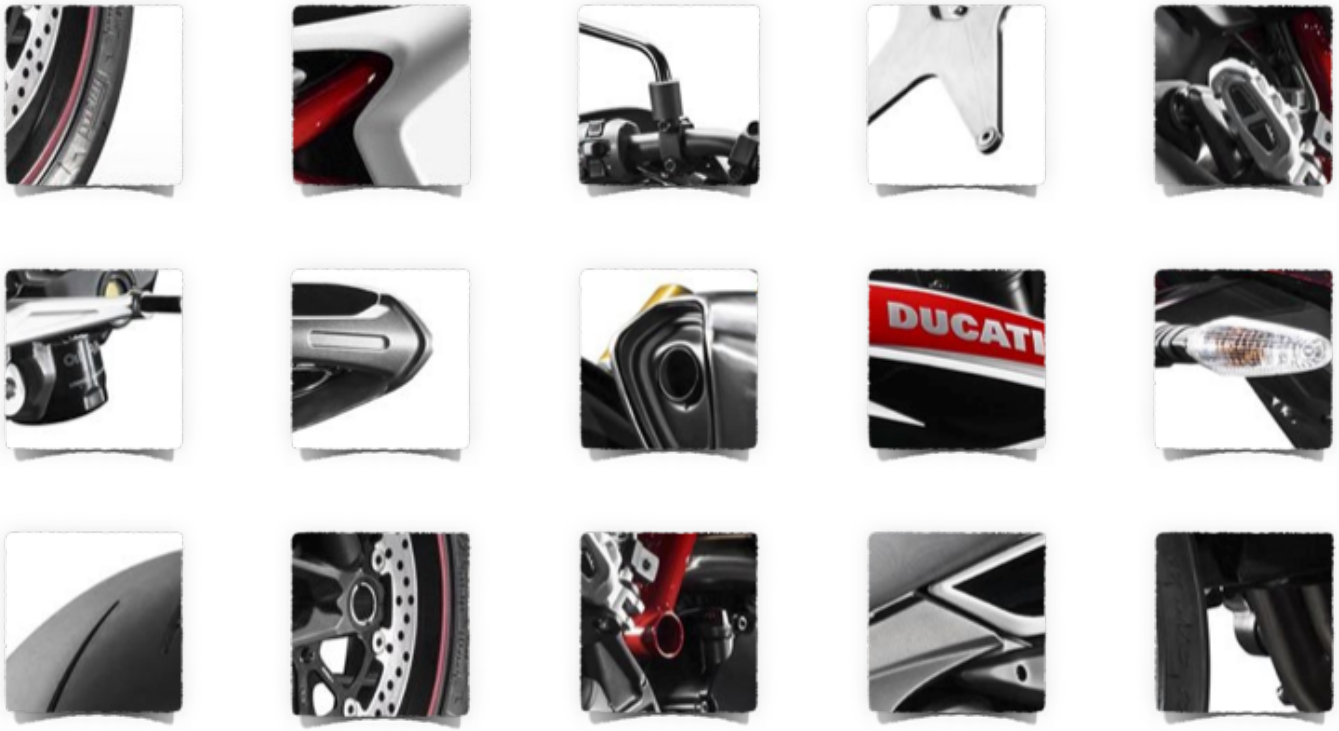


Data-driven approach

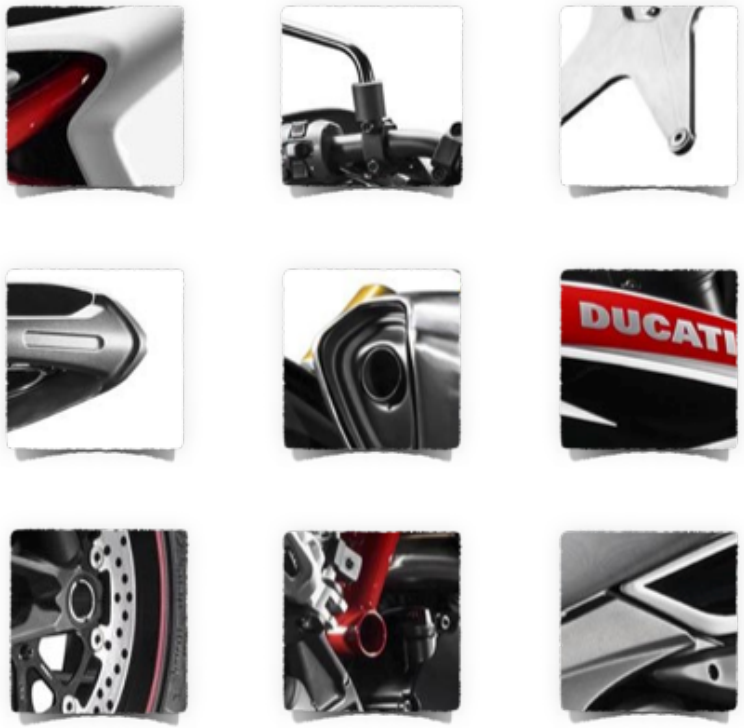
- Collect a database of images with labels
- Use ML to train an image classifier
- Evaluate the classifier on test images

Bag of words

What object do these parts belong to?



Some local feature are very informative



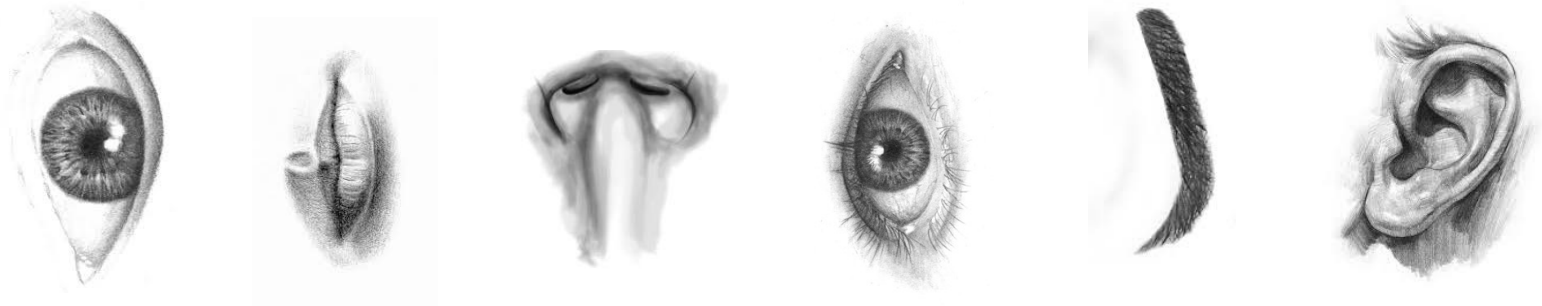
An object as

a collection of local features

(bag-of-features)

- deals well with occlusion
- scale invariant
- rotation invariant

(not so) crazy assumption



spatial information of local features
can be ignored for object recognition (i.e., verification)

CalTech6 dataset






class	bag of features	bag of features	Parts-and-shape model
	Zhang et al. (2005)	Willamowski et al. (2004)	Fergus et al. (2003)
airplanes	**98.8**	97.1	90.2
cars (rear)	98.3	**98.6**	90.3
cars (side)	**95.0**	87.3	88.5
faces	**100**	99.3	96.4
motorbikes	**98.5**	98.0	92.5
spotted cats	**97.0**	—	90.0

Works pretty well for image-level classification

Csurka et al. (2004), Willamowski et al. (2005), Grauman & Darrell (2005), Sivic et al. (2003, 2005)

Standard BOW pipeline

(for image classification)

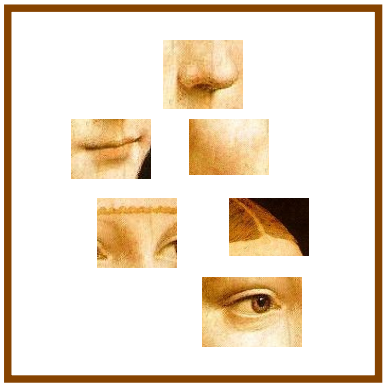
Dictionary Learning:
Learn Visual Words using clustering

Encode:
build Bags-of-Words (BOW) vectors
for each image

Classify:
Train and test data using BOWs

Dictionary Learning:
Learn Visual Words using clustering

1. extract features (e.g., SIFT) from images

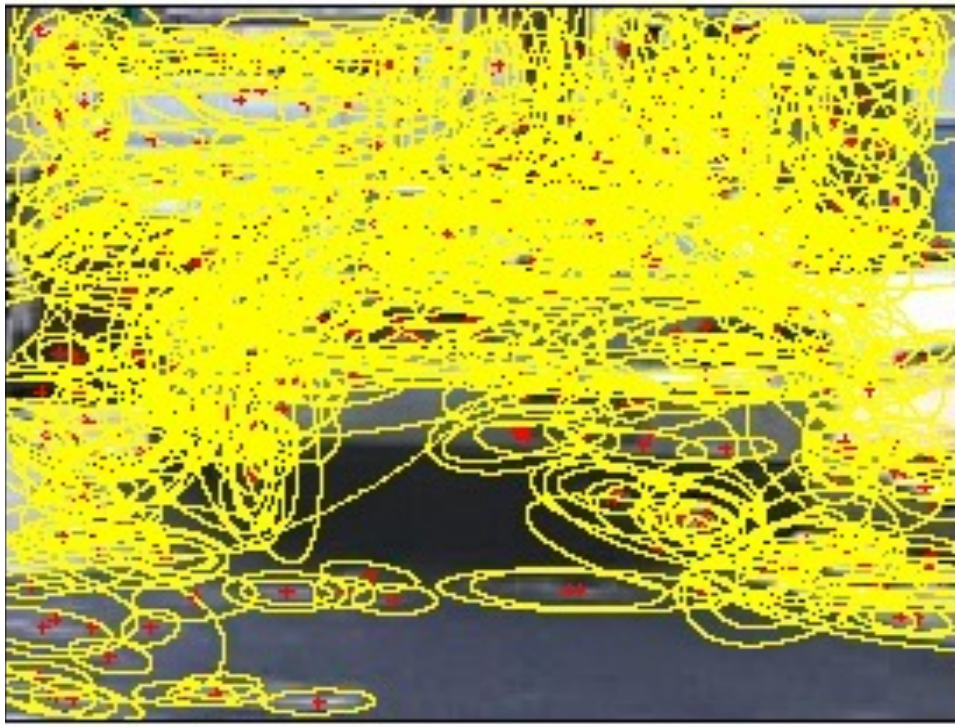


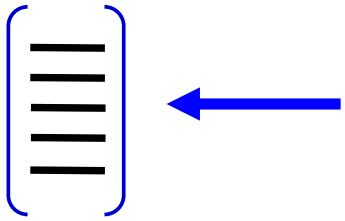
Dictionary Learning:
Learn Visual Words using clustering

2. Learn visual dictionary (e.g., K-means clustering)

What kinds of features can we extract?

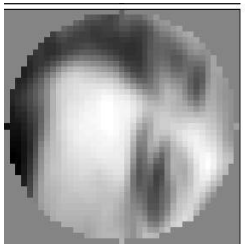
- Regular grid
  - Vogel & Schiele, 2003
  - Fei-Fei & Perona, 2005
- Interest point detector
  - Csurka et al. 2004
  - Fei-Fei & Perona, 2005
  - Sivic et al. 2005
- Other methods
  - Random sampling (Vidal-Naquet & Ullman, 2002)
  - Segmentation-based patches (Barnard et al. 2003)



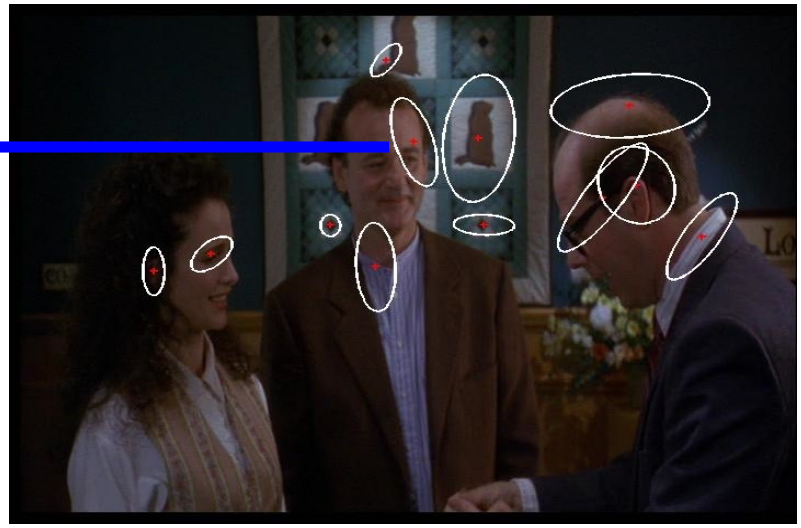


Compute SIFT descriptor

[Lowe'99]



Normalize patch

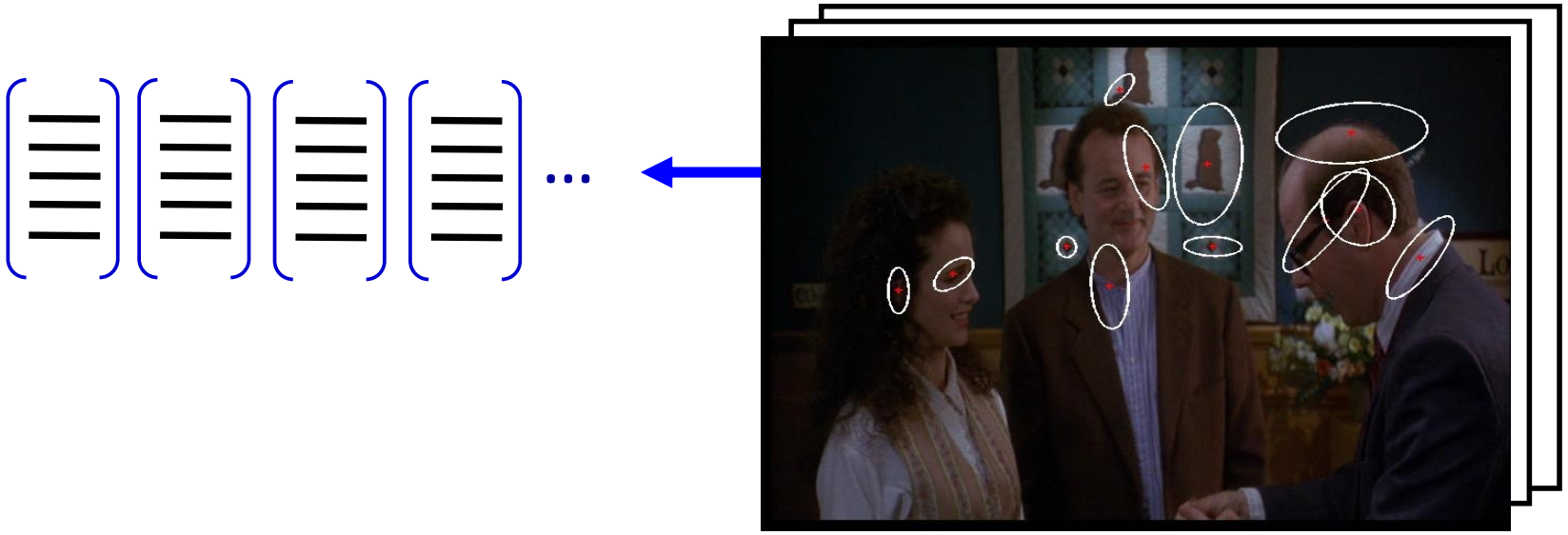


Detect patches

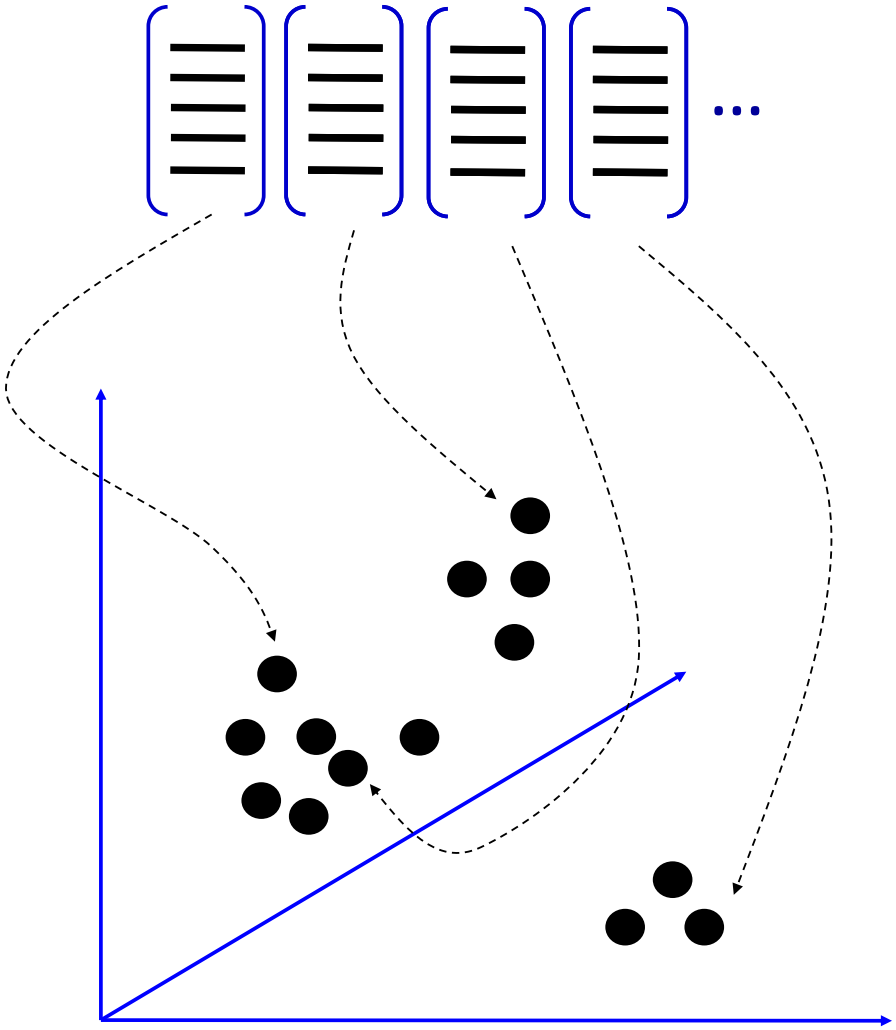
[Mikojaczyk and Schmid '02]

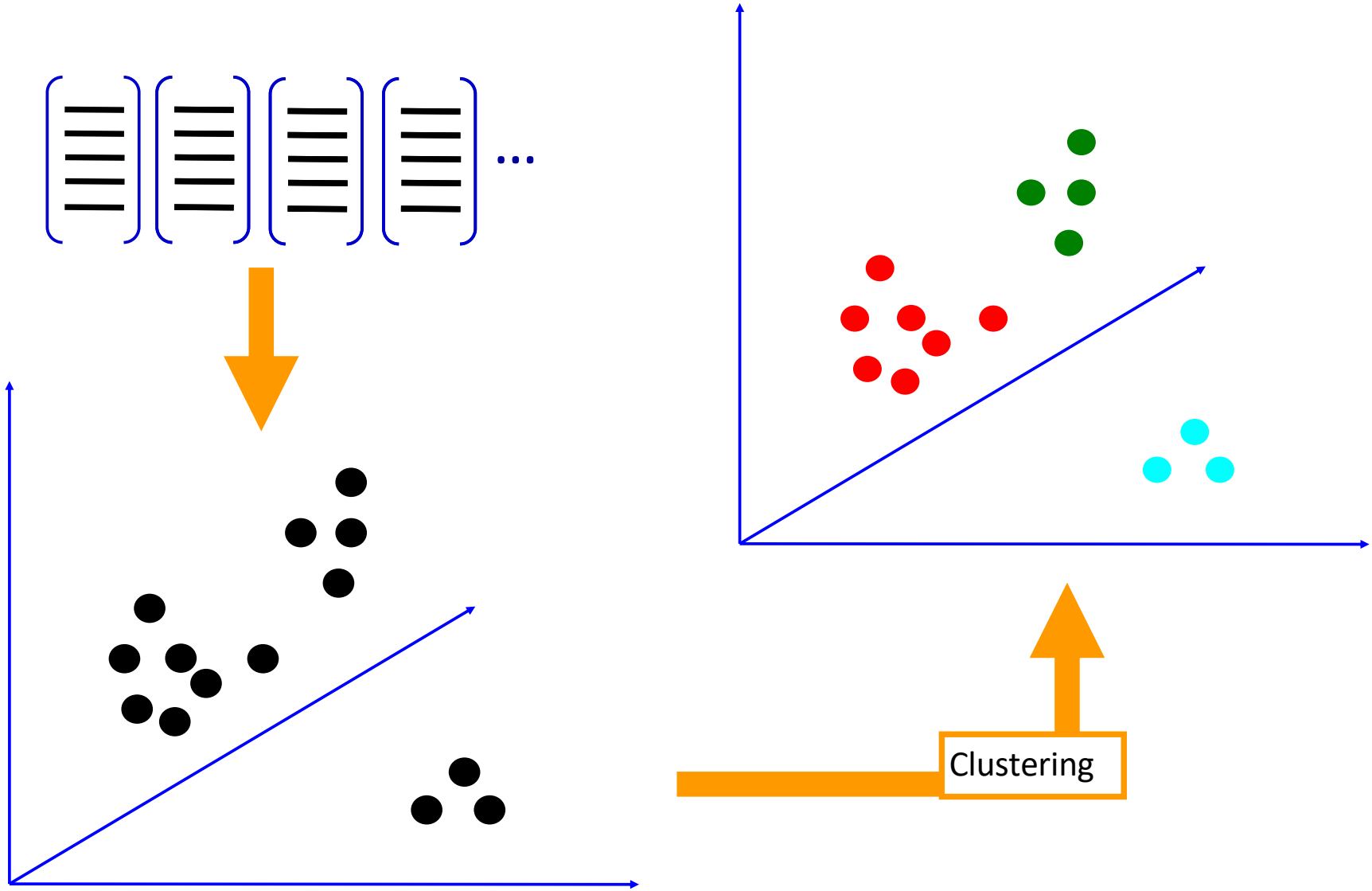
[Mata, Chum, Urban & Pajdla, '02]

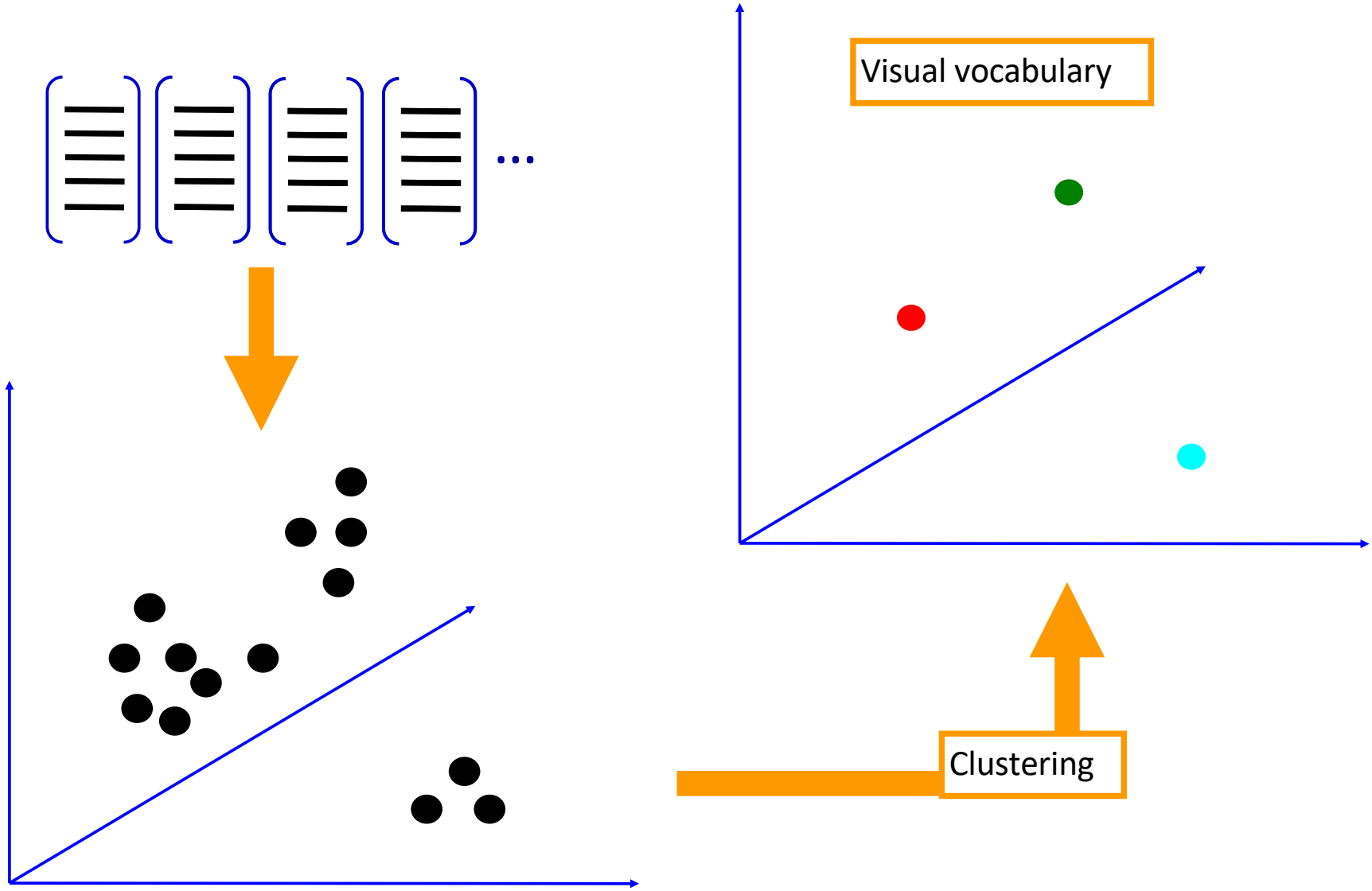
[Sivic & Zisserman, '03]



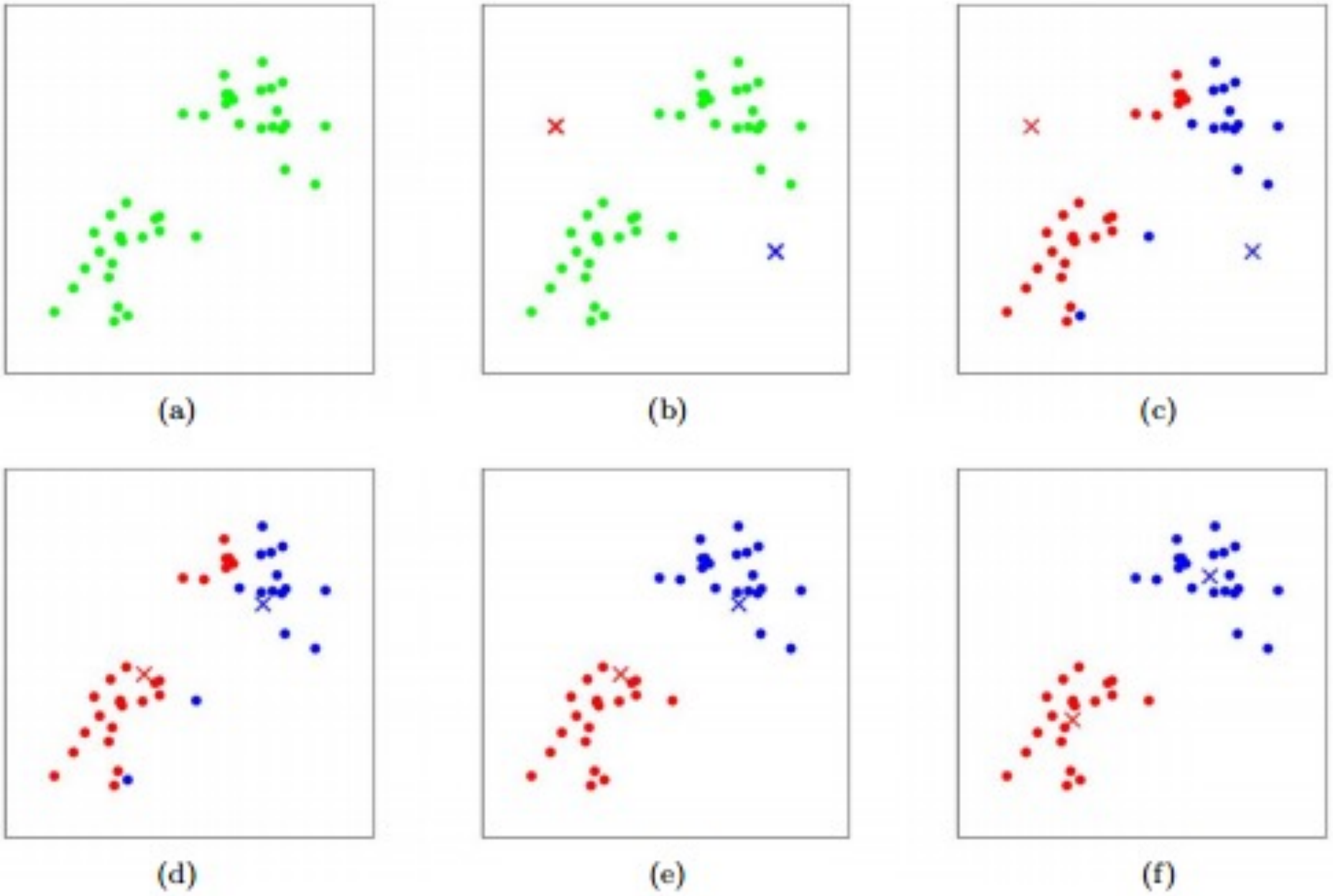
How do we learn the dictionary?




...
Clustering


Visual vocabulary
Clustering

K-means clustering



[Stanford CS221]

K-means Clustering

Given k:

1. Select initial centroids at random.

2. Assign each object to the cluster with the nearest centroid.

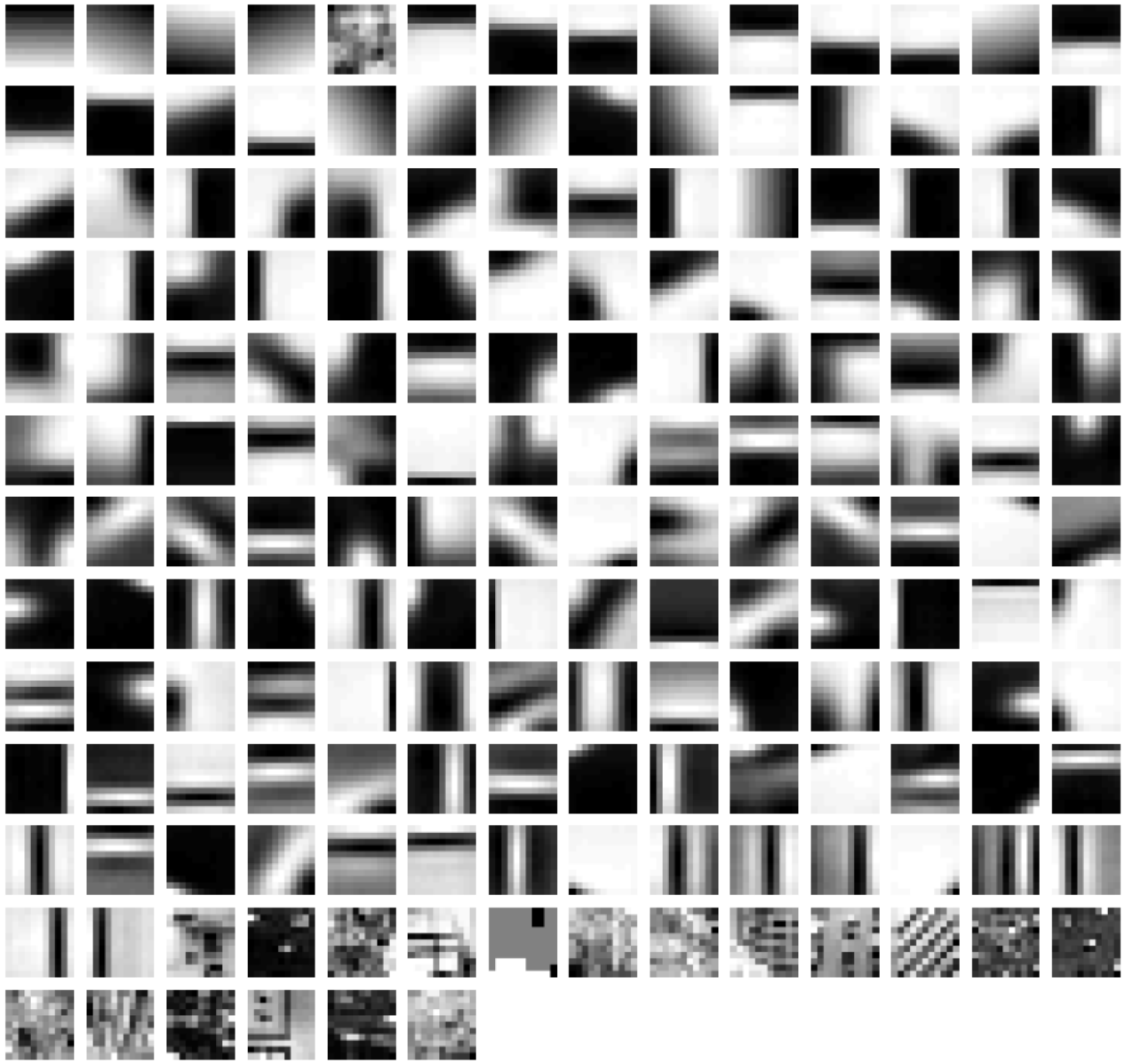
3. Compute each centroid as the mean of the objects assigned to it.

4. Repeat previous 2 steps until no change.

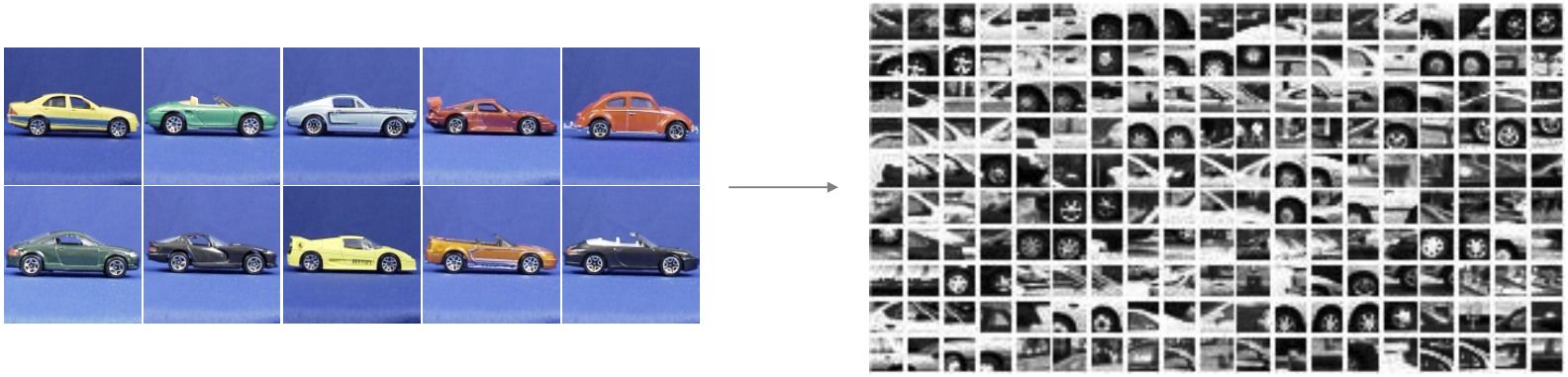
*From what **data** should I learn the dictionary?*

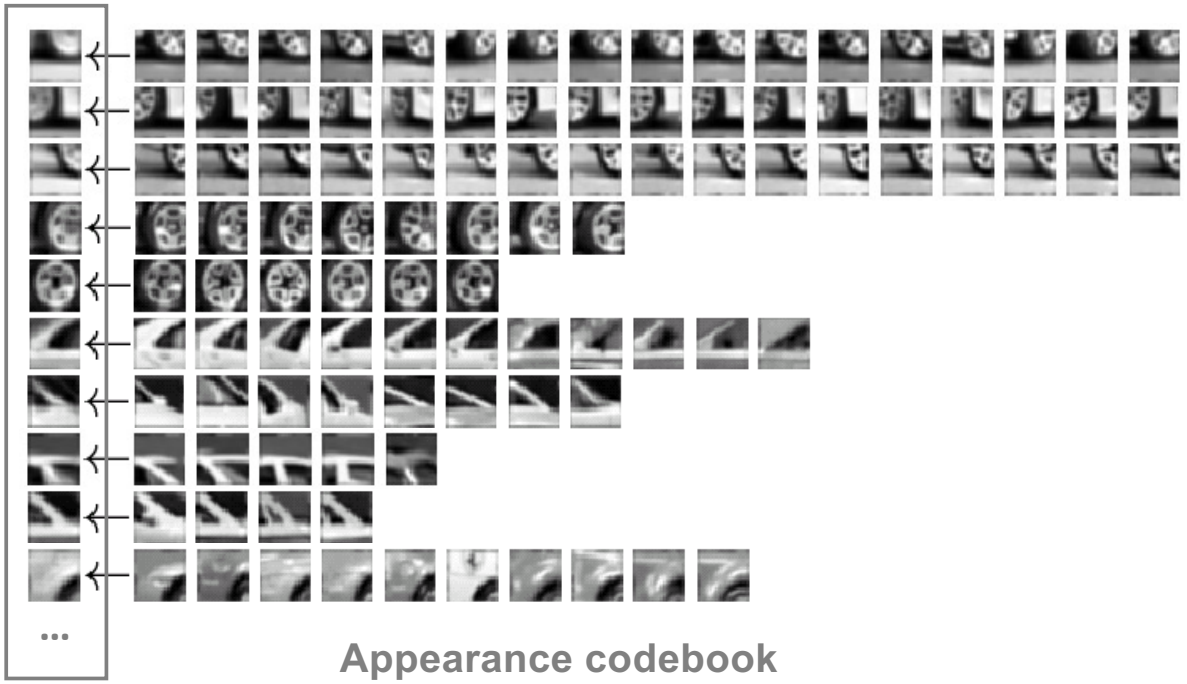
- Dictionary can be learned on separate training set
- Provided the training set is sufficiently representative, the dictionary will be “universal”

Example visual dictionary



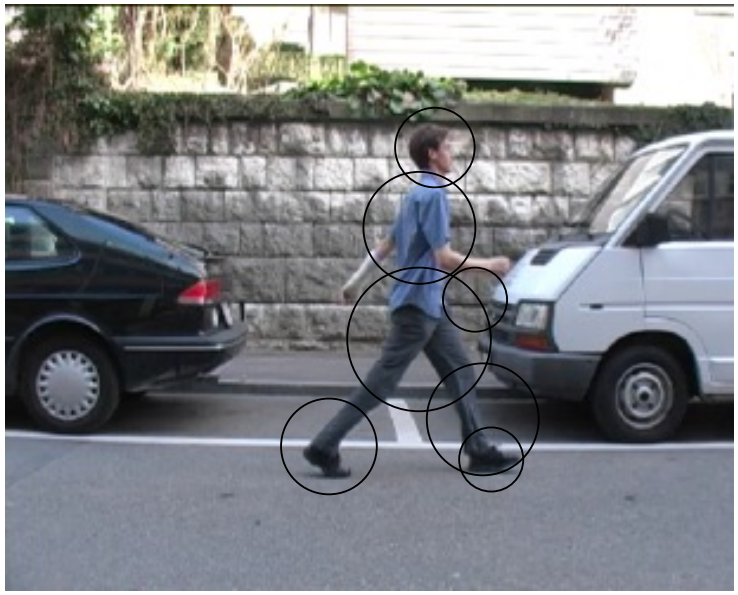
Example dictionary

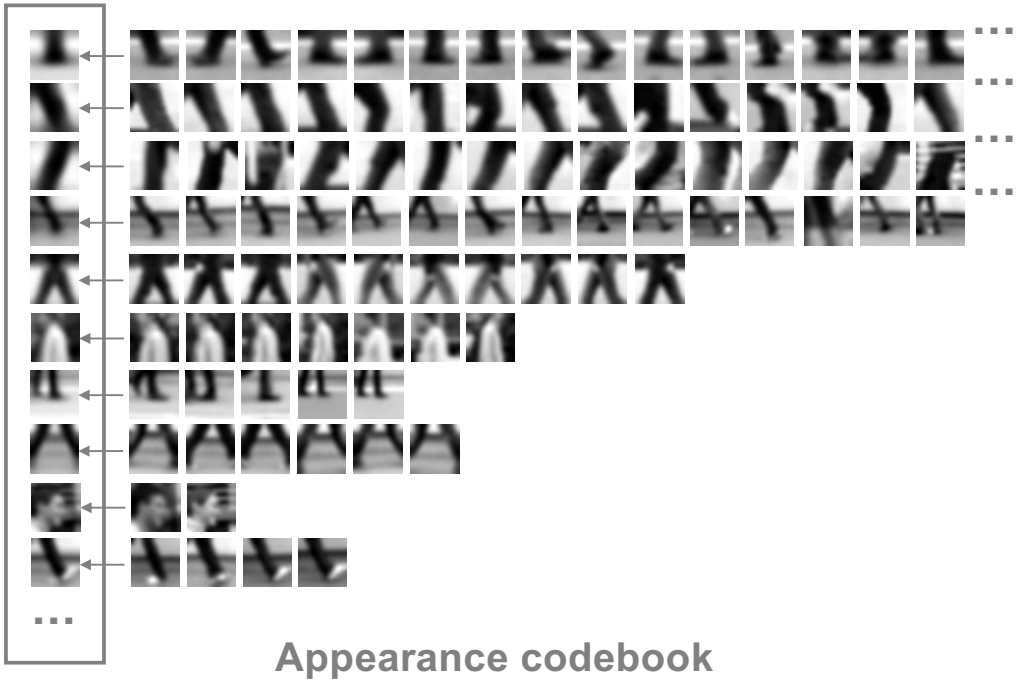




Source: B. Leibe

Another dictionary





Appearance codebook

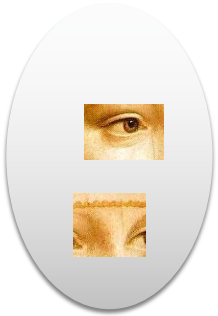
Source: B. Leibe

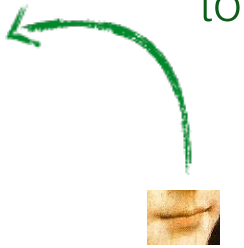
Dictionary Learning:
Learn Visual Words using clustering

Encode:
build Bags-of-Words (BOW) vectors
for each image

Classify:
Train and test data using BOWs

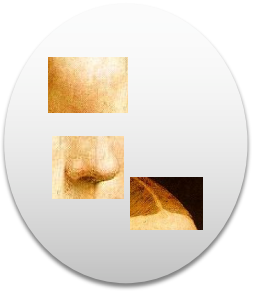
1. Quantization: image features gets associated to a visual word (nearest cluster center)





Encode:

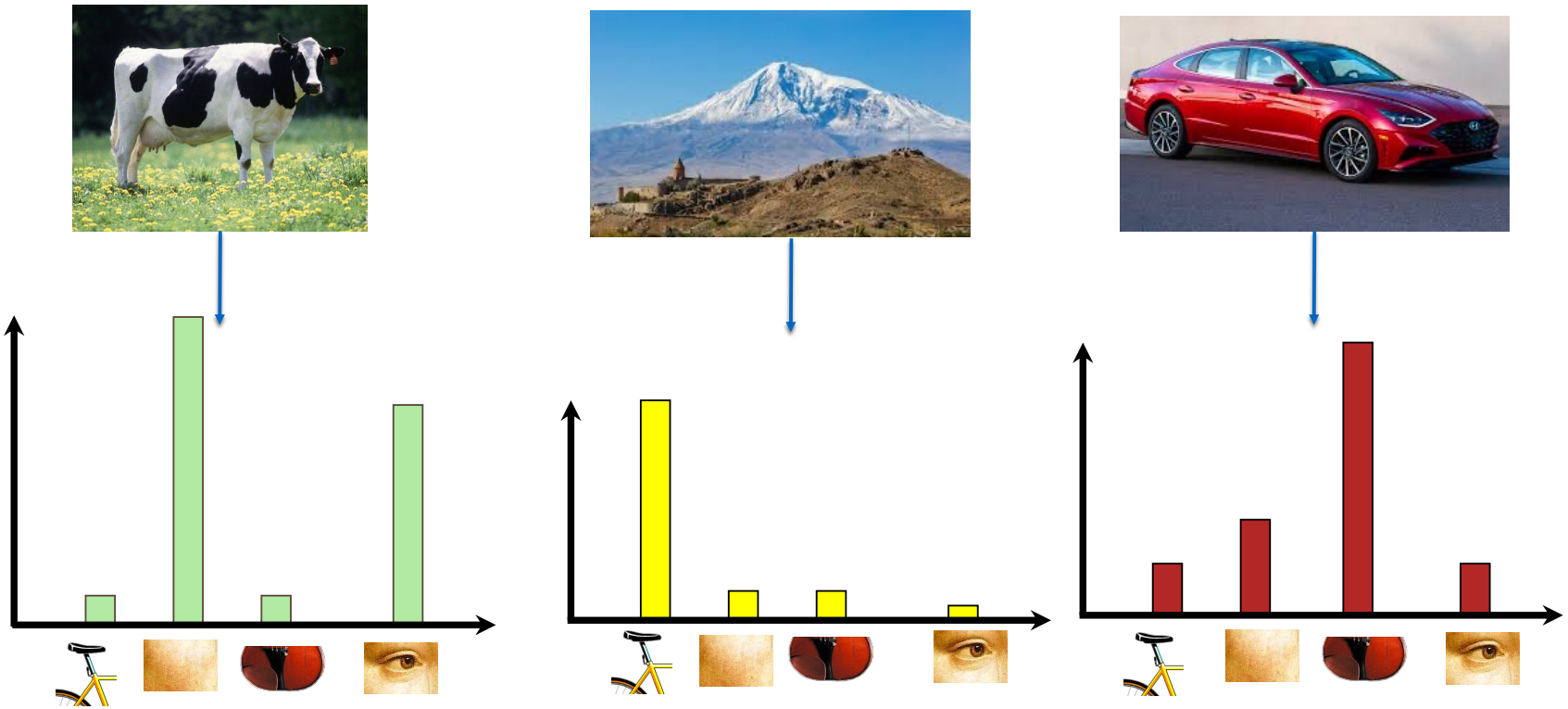
build Bags-of-Words (BOW) vectors for each image

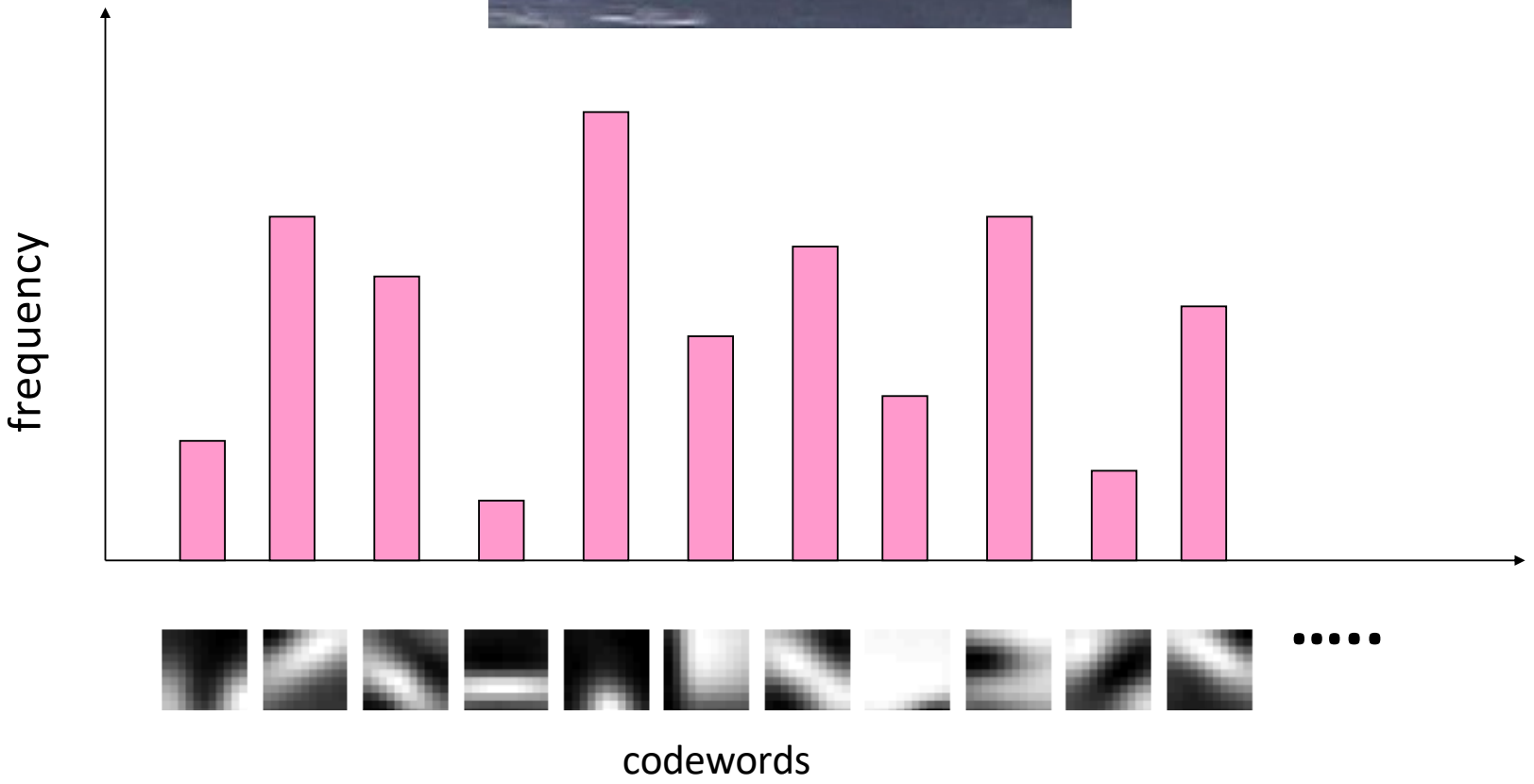


2. Histogram: count the number of visual word occurrences

Encode:

build Bags-of-Words (BOW) vectors for each image




frequency
codewords

Dictionary Learning:
Learn Visual Words using clustering

Encode:
build Bags-of-Words (BOW) vectors
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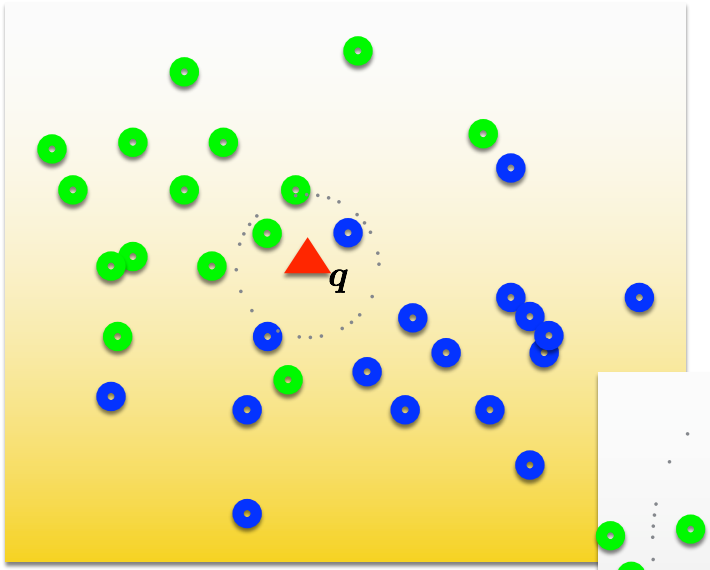
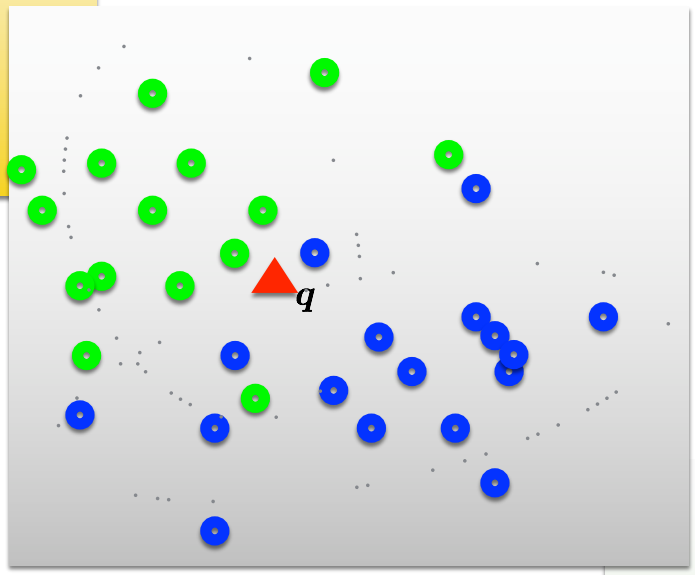
Classify:
Train and test data using BOWs

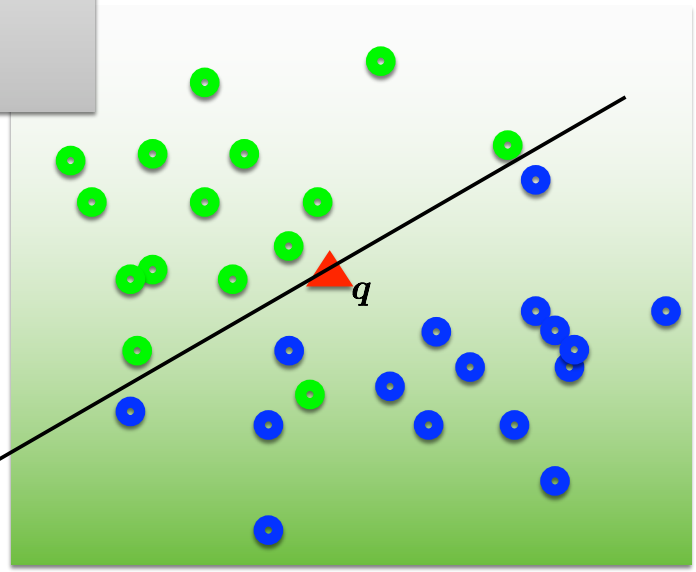


K nearest neighbors



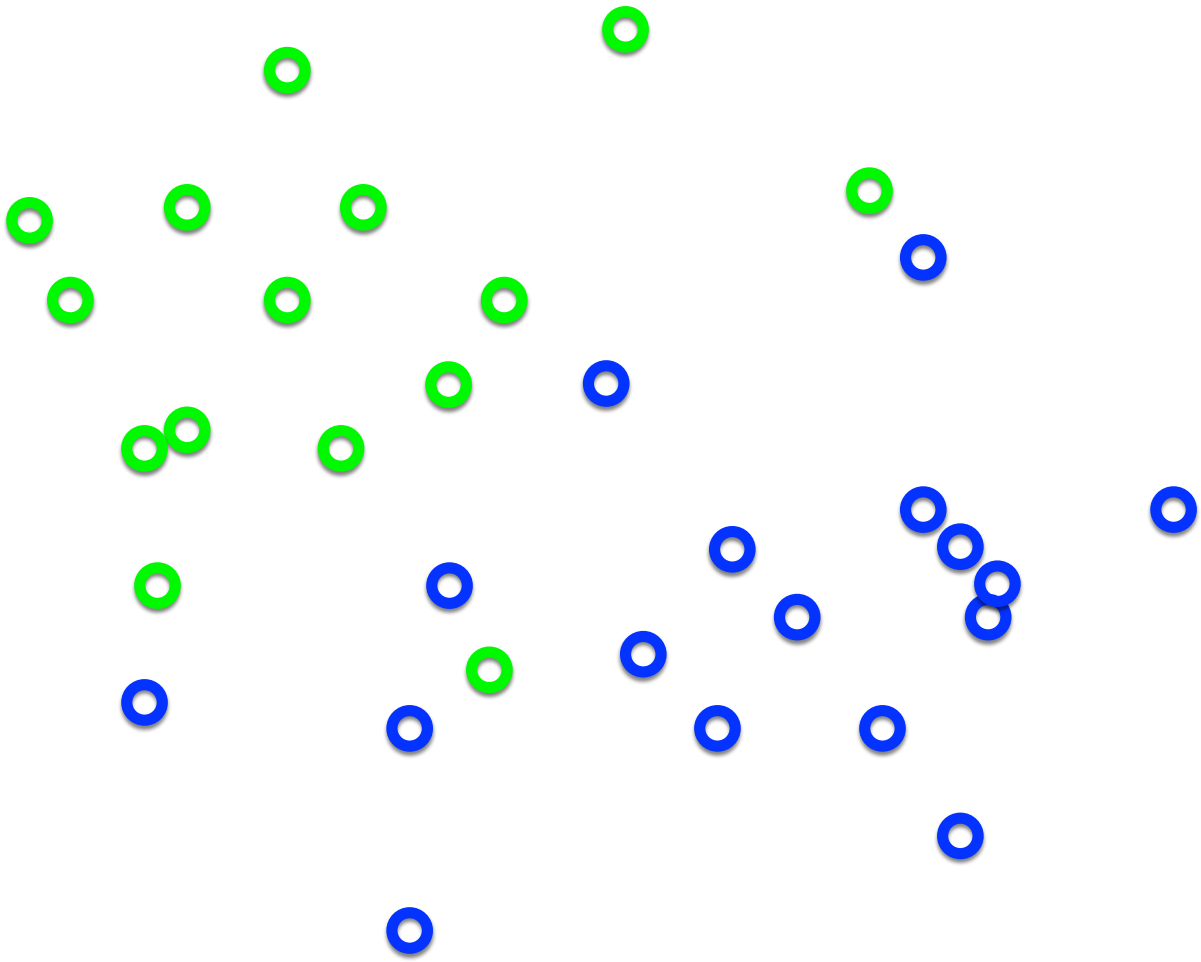
Naïve Bayes

Support Vector Machine

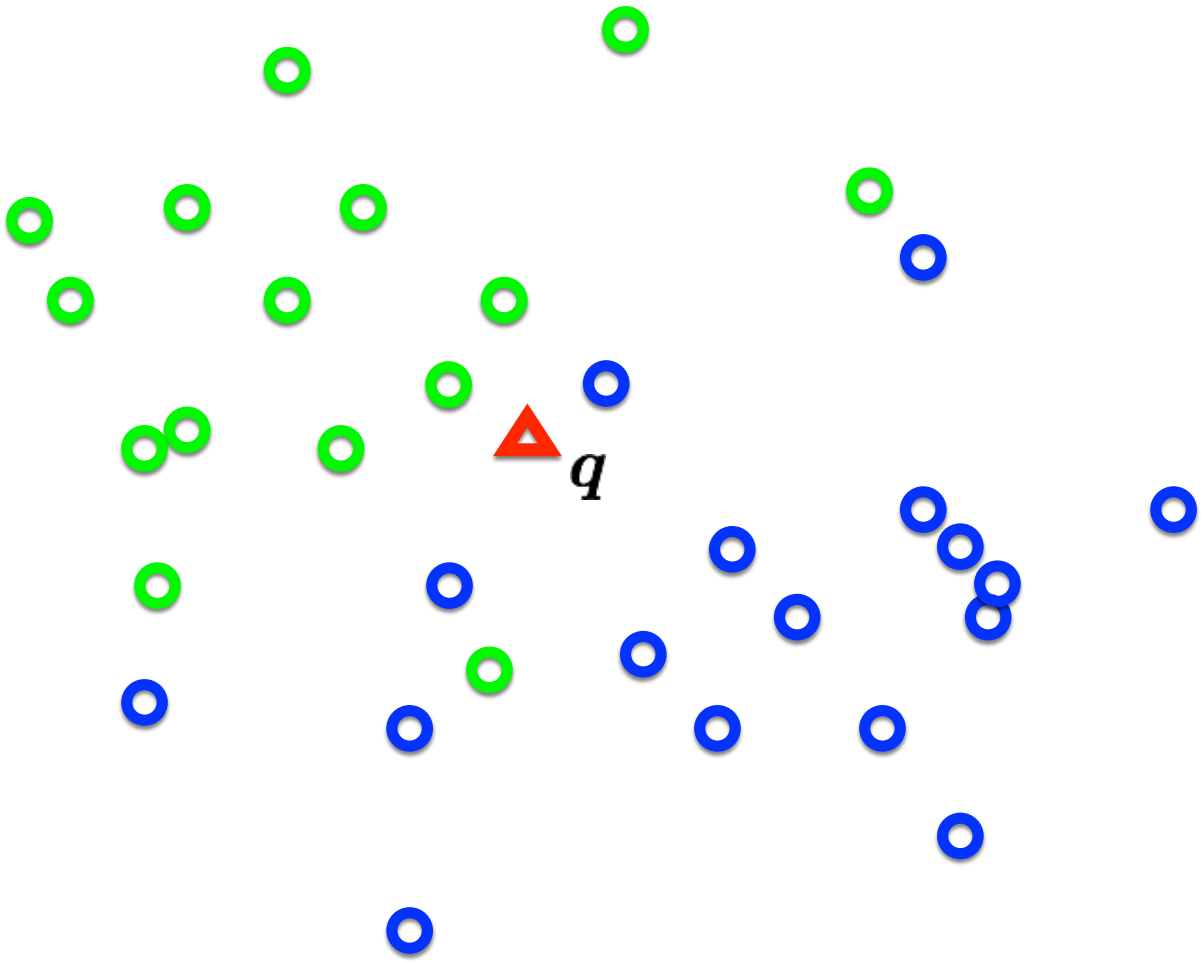


K nearest neighbors

Distribution of data from two classes

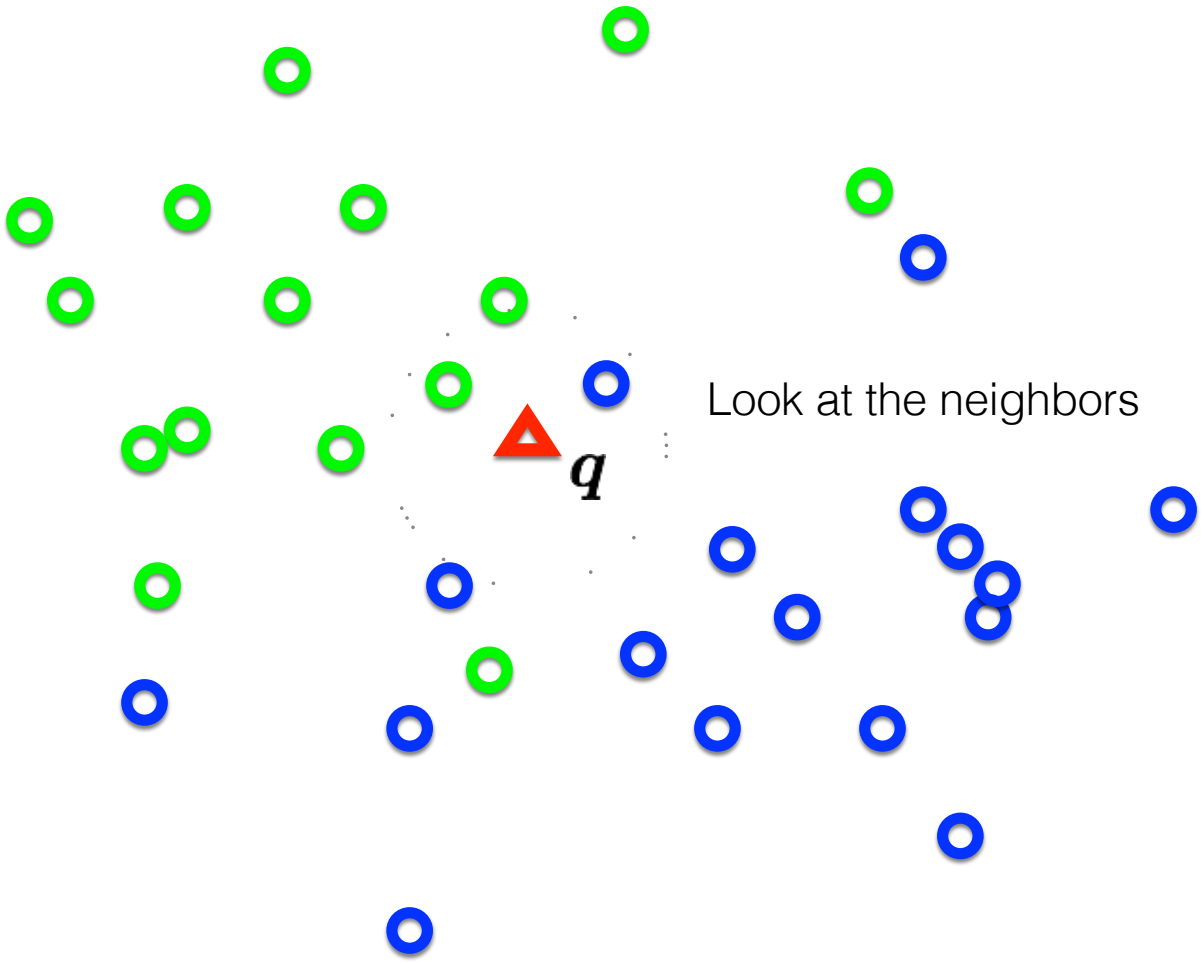


Distribution of data from two classes

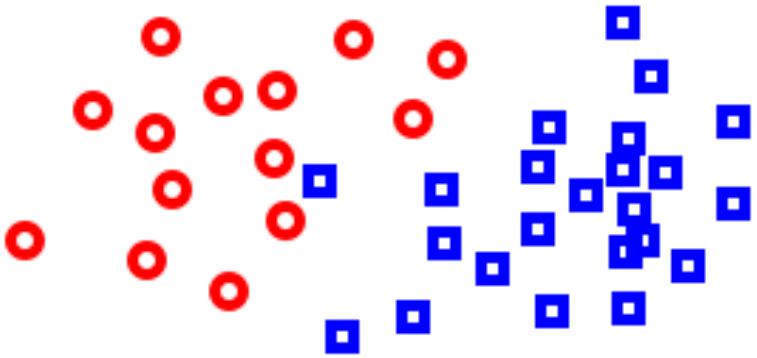


Which class does q belong too?

Distribution of data from two classes



K-Nearest Neighbor (KNN) Classifier

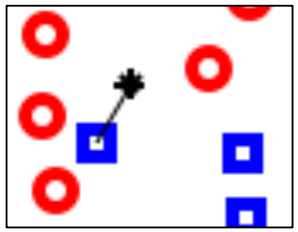


Non-parametric pattern classification approach

Consider a two class problem where each sample consists of two measurements (x,y).

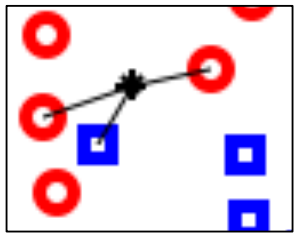
For a given query point q, assign the class of the nearest neighbor

k = 1

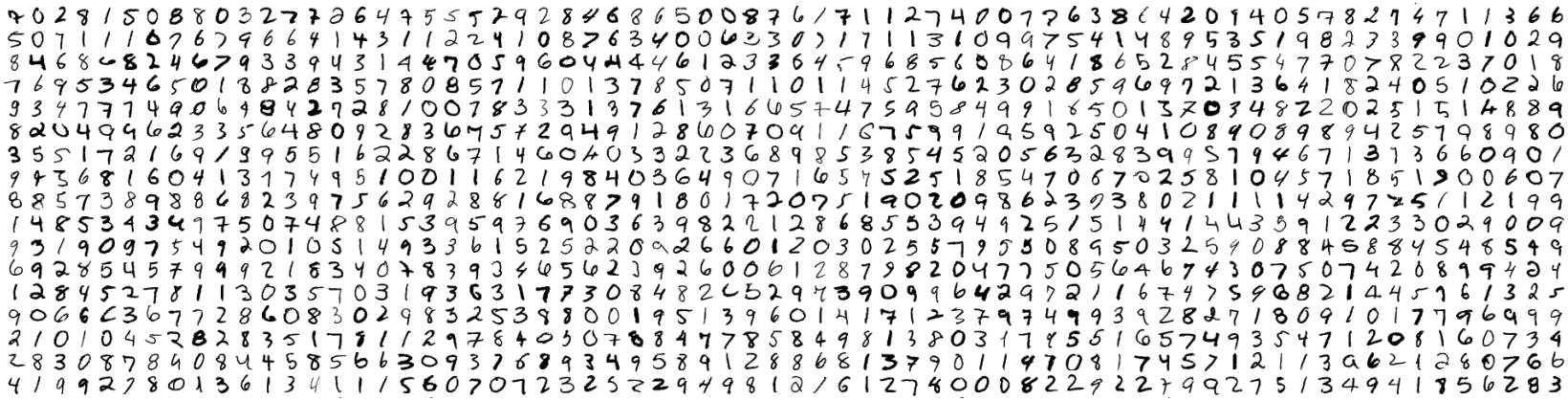


Compute the k nearest neighbors and assign the class by majority vote.

k = 3



Nearest Neighbor is competitive



MNIST Digit Recognition

- Handwritten digits
- 28x28 pixel images: d = 784
- 60,000 training samples
- 10,000 test samples

Yann LeCunn

	Test Error Rate (%)
Linear classifier (1-layer NN)	12.0
K-nearest-neighbors, Euclidean	5.0
K-nearest-neighbors, Euclidean, deskewed	2.4
K-NN, Tangent Distance, 16x16	1.1
K-NN, shape context matching	0.67
1000 RBF + linear classifier	3.6
SVM deg 4 polynomial	1.1
2-layer NN, 300 hidden units	4.7
2-layer NN, 300 HU, [deskewing]	1.6
LeNet-5, [distortions]	0.8
Boosted LeNet-4, [distortions]	0.7

What is the best distance metric between data points?

- Typically Euclidean distance
- Important to normalize.
  Dimensions have different scales

How many K?

- Typically k=1 is good
- Cross-validation (try different k!)

Distance metrics

$$D(\boldsymbol{x}, \boldsymbol{y}) = \sqrt{(x_1 - y_1)^2 + \cdots + (x_N - y_N)^2}$$ Euclidean

$$D(\boldsymbol{x}, \boldsymbol{y}) = \frac{\boldsymbol{x} \cdot \boldsymbol{y}}{\|\boldsymbol{x}\| \|\boldsymbol{y}\|} = \frac{x_1 y_1 + \cdots + x_N y_N}{\sqrt{\sum_n x_n^2} \sqrt{\sum_n y_n^2}}$$ Cosine

$$D(\boldsymbol{x}, \boldsymbol{y}) = \frac{1}{2} \sum_n \frac{(x_n - y_n)^2}{(x_n + y_n)}$$ Chi-squared

Distance metrics

L1 (Manhattan) distance

$$d_1(I_1, I_2) = \sum_p |I_1^p - I_2^p|$$

L2 (Euclidean) distance

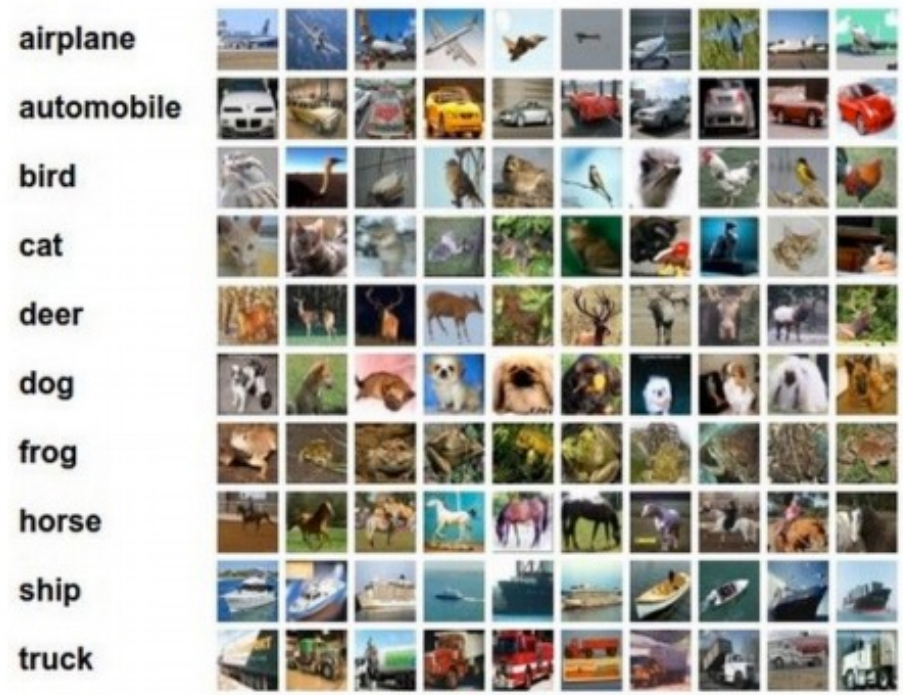
$$d_2(I_1, I_2) = \sqrt{\sum_p \left(I_1^p - I_2^p\right)^2}$$

- Two most commonly used special cases of p-norm

$$||x||_p = \left(|x_1|^p + \cdots + |x_n|^p\right)^{\frac{1}{p}} \qquad p \geq 1, x \in \mathbb{R}^n$$

CIFAR-10 and NN results

Example dataset: **CIFAR-10**
10 labels
50,000 training images
10,000 test images.



For every test image (first column), examples of nearest neighbors in rows

kNN

Pros

- simple yet effective

Cons

- search is expensive (can be sped-up)
- storage requirements
- difficulties with high-dimensional data

kNN -- Complexity and Storage

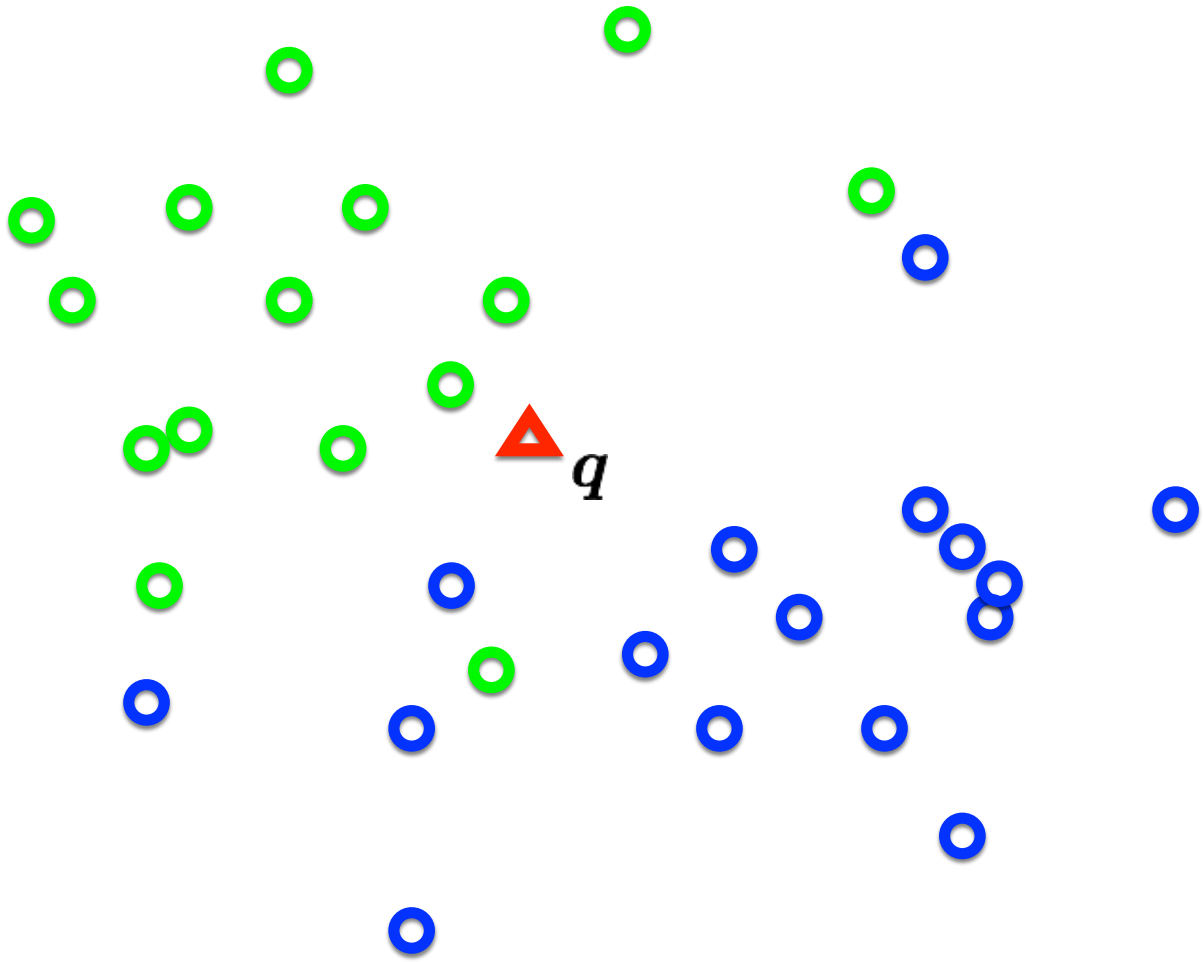
- N training images, M test images

- Training: O(1)
- Testing: O(MN)

- Hmm…
  - Normally need the opposite
  - Slow training (ok), fast testing (necessary)

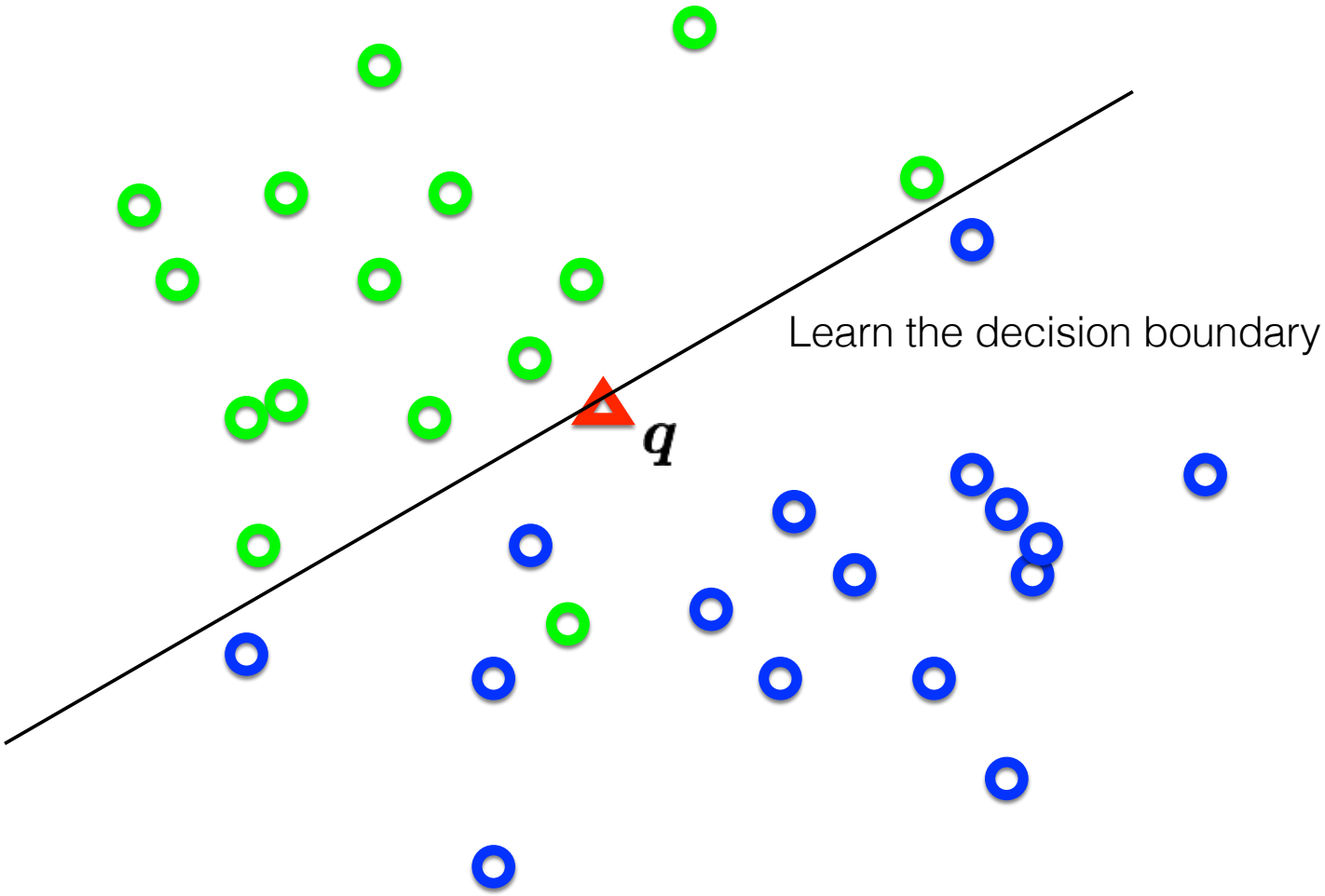
Support Vector Machine

Distribution of data from two classes

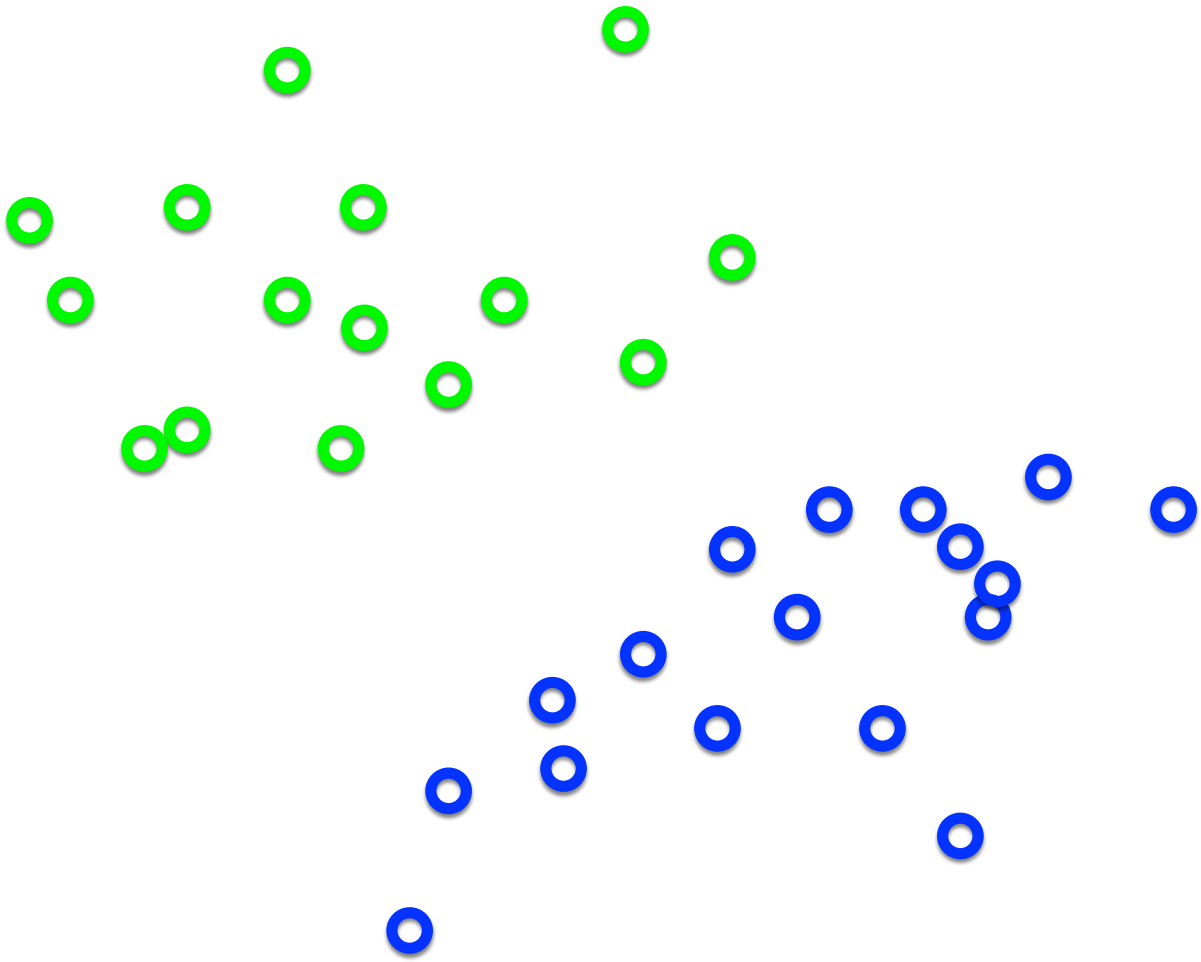


Which class does q belong too?

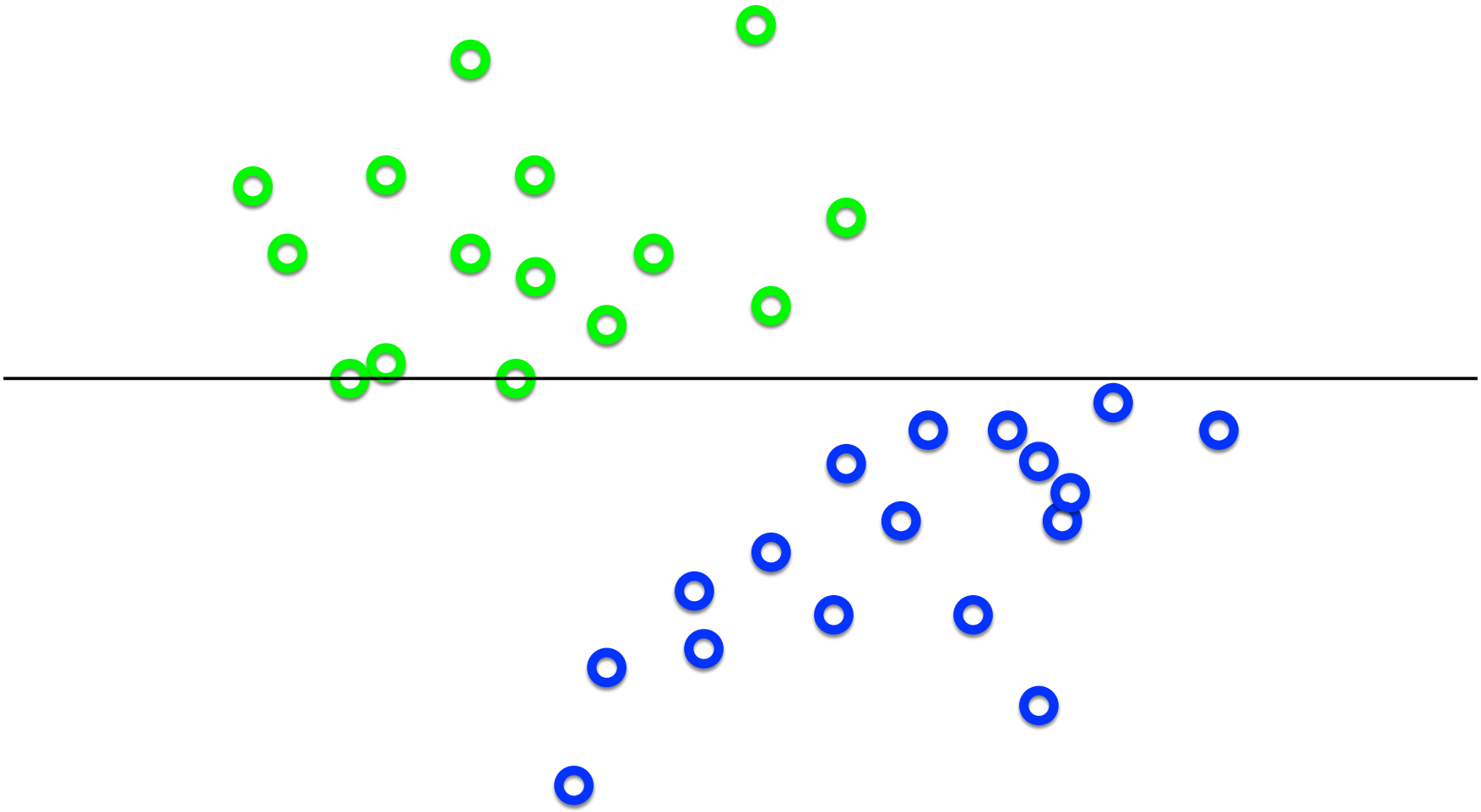
Distribution of data from two classes



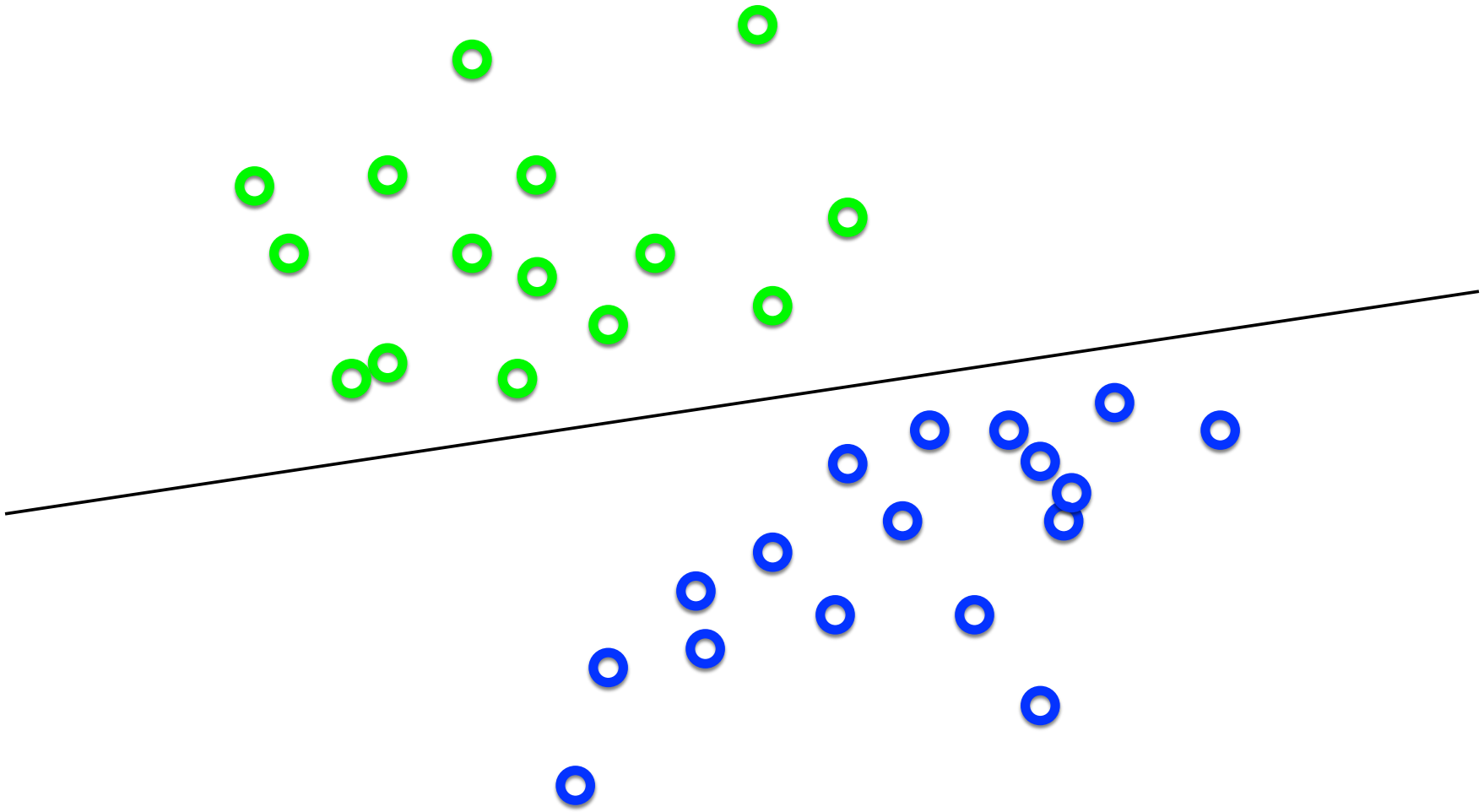
What's the best **w**?



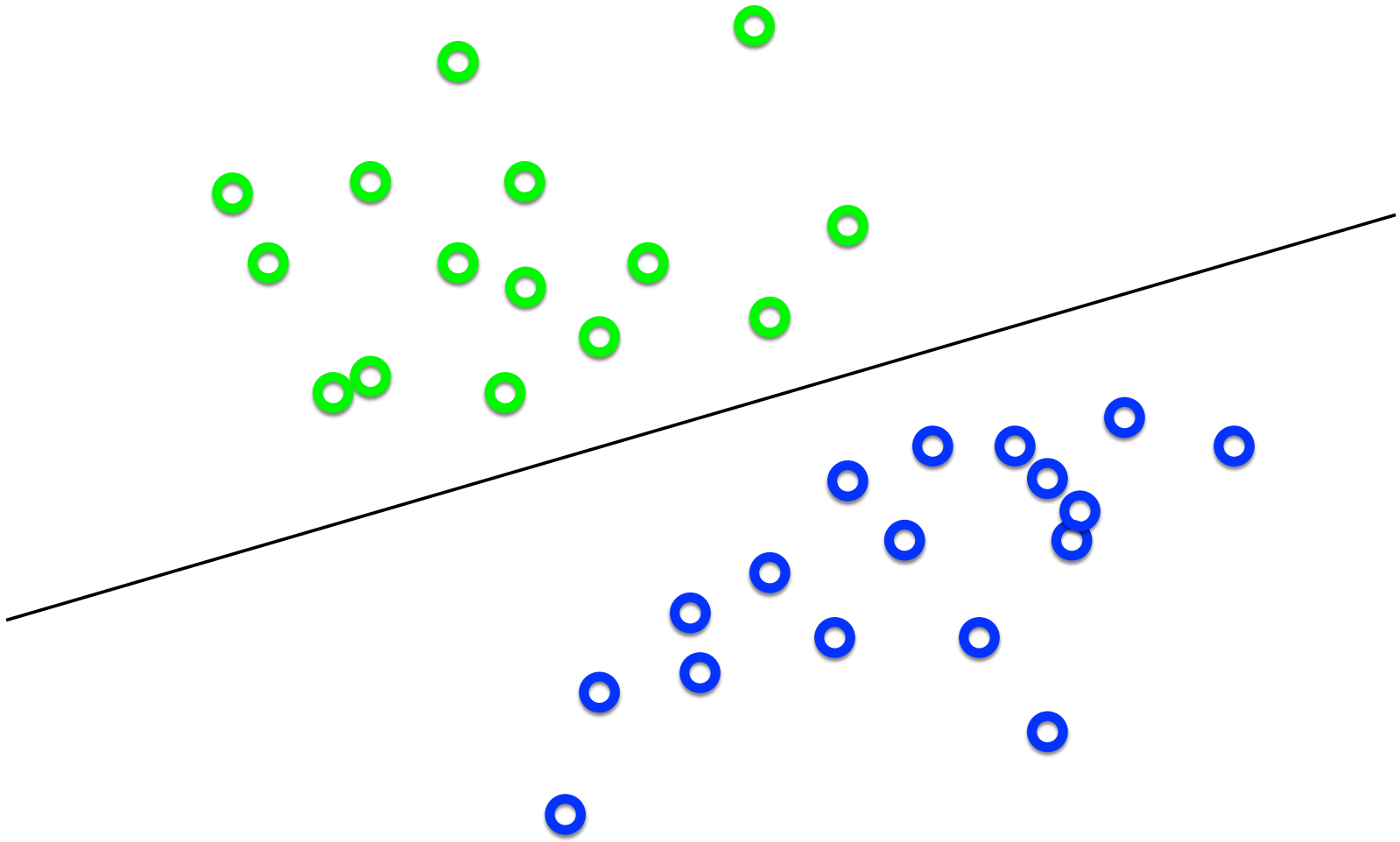
What's the best **w**?



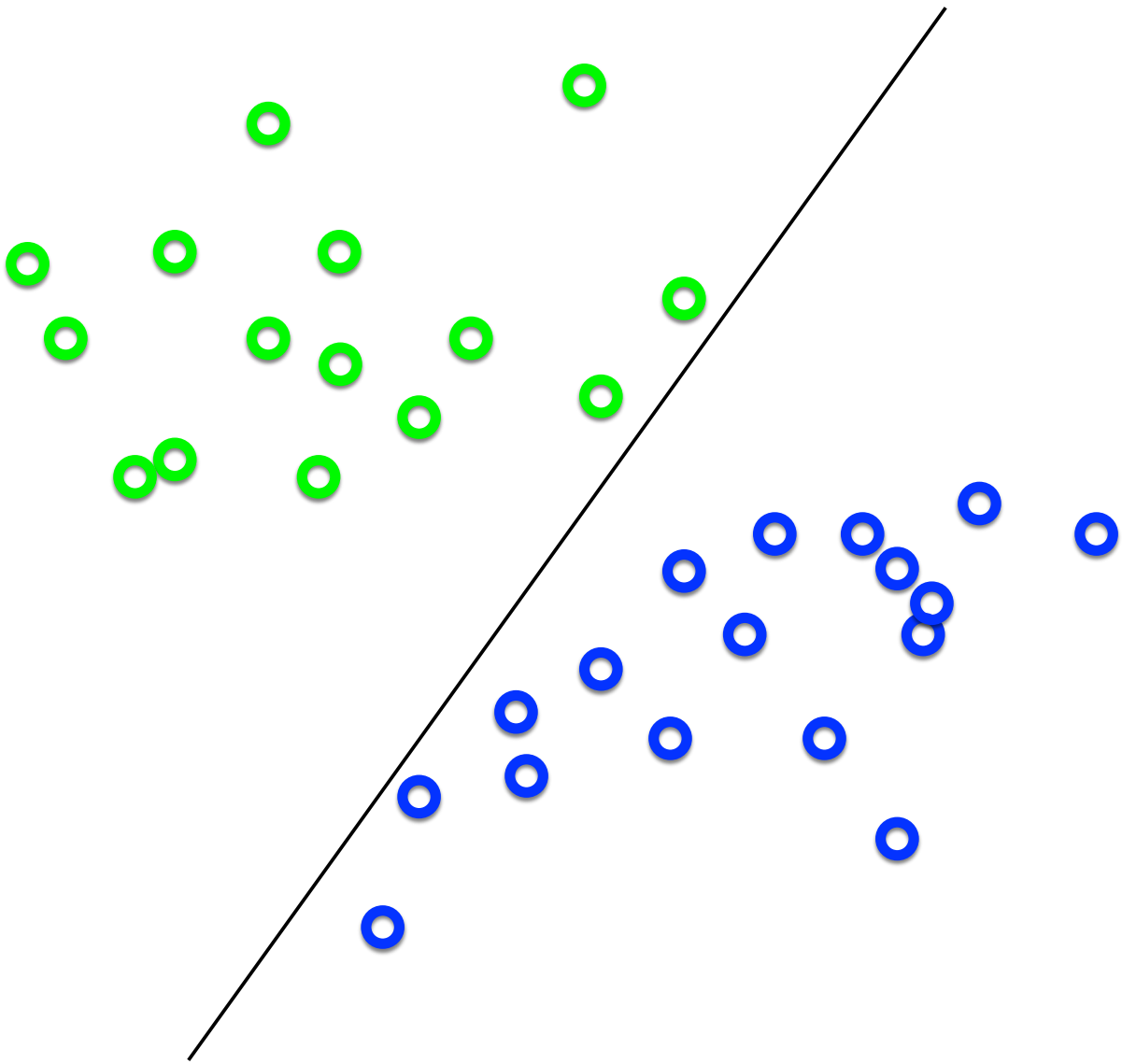
What's the best **w**?



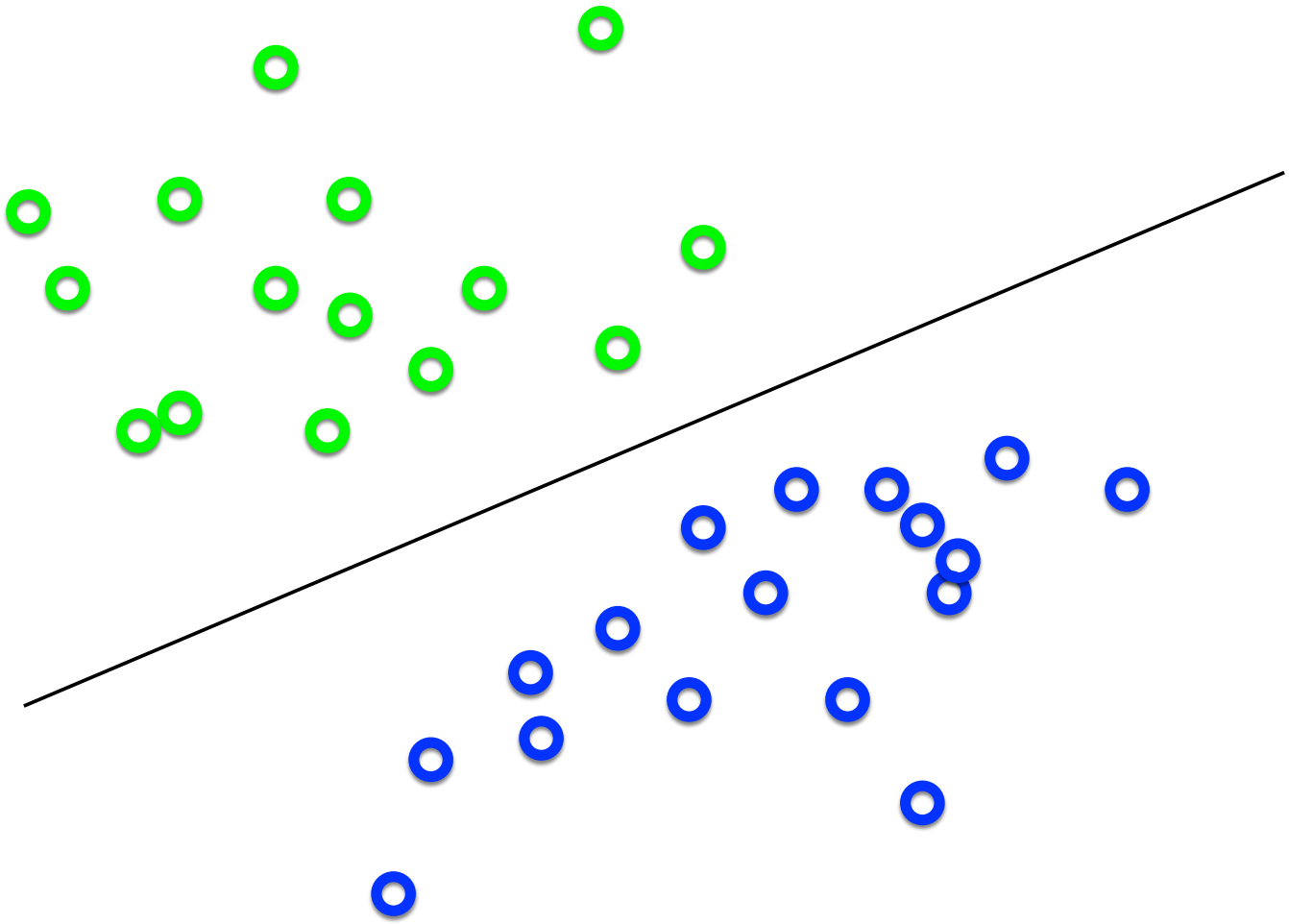
What's the best **w**?



What's the best **w**?

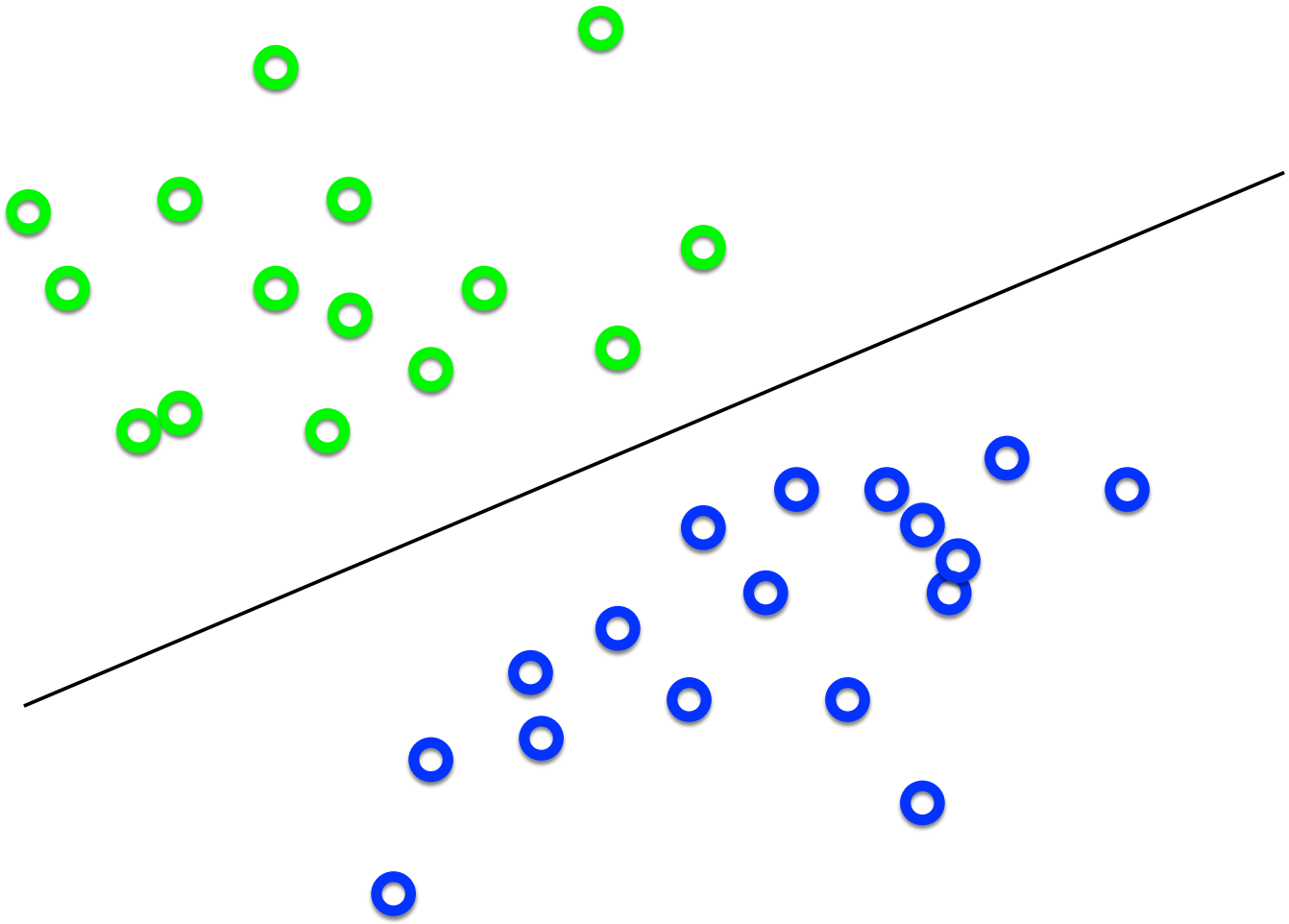


What's the best **w**?



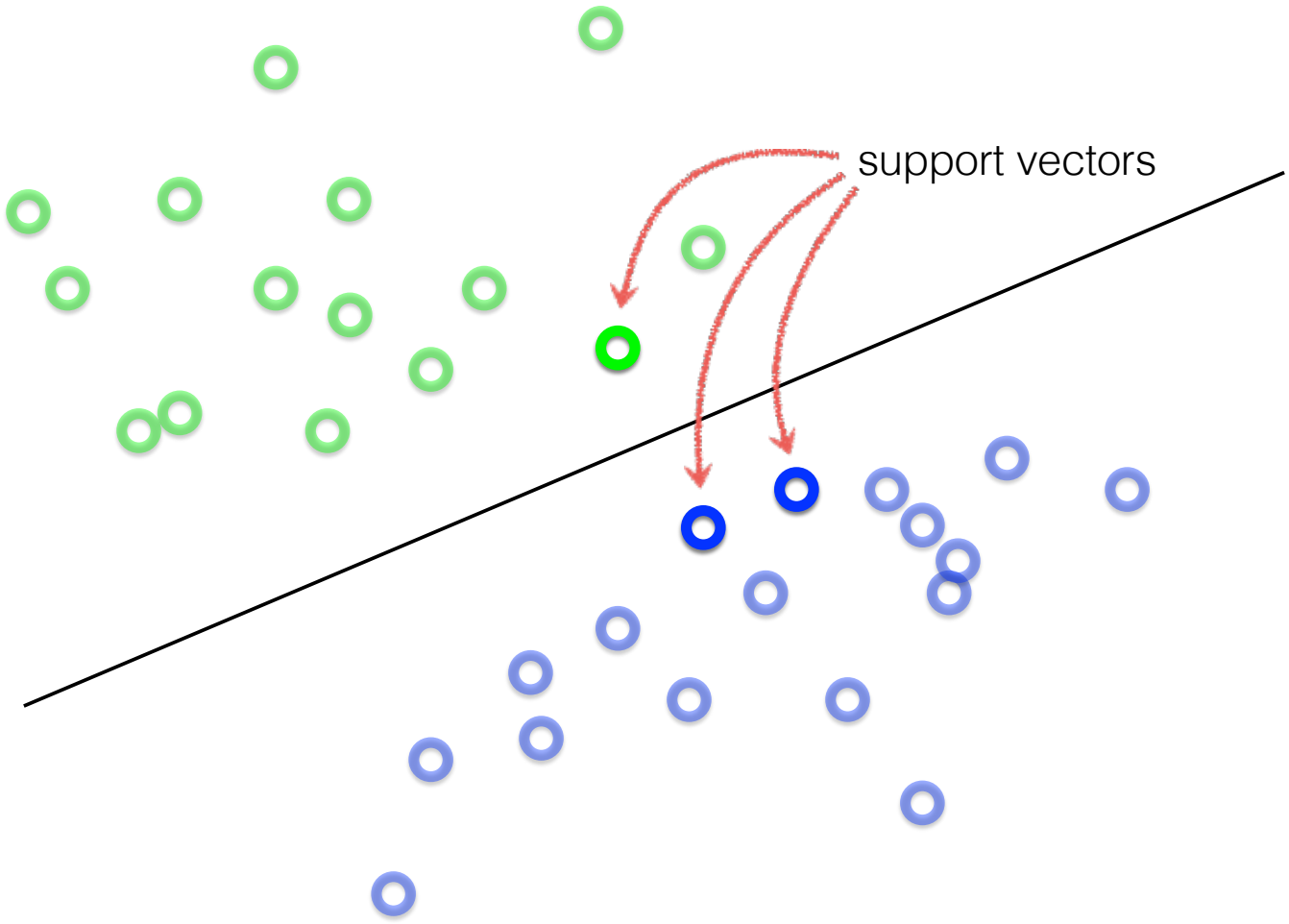
Intuitively, the line that is the farthest from all interior points

What's the best **w**?



Maximum Margin solution:
most stable to perturbations of data

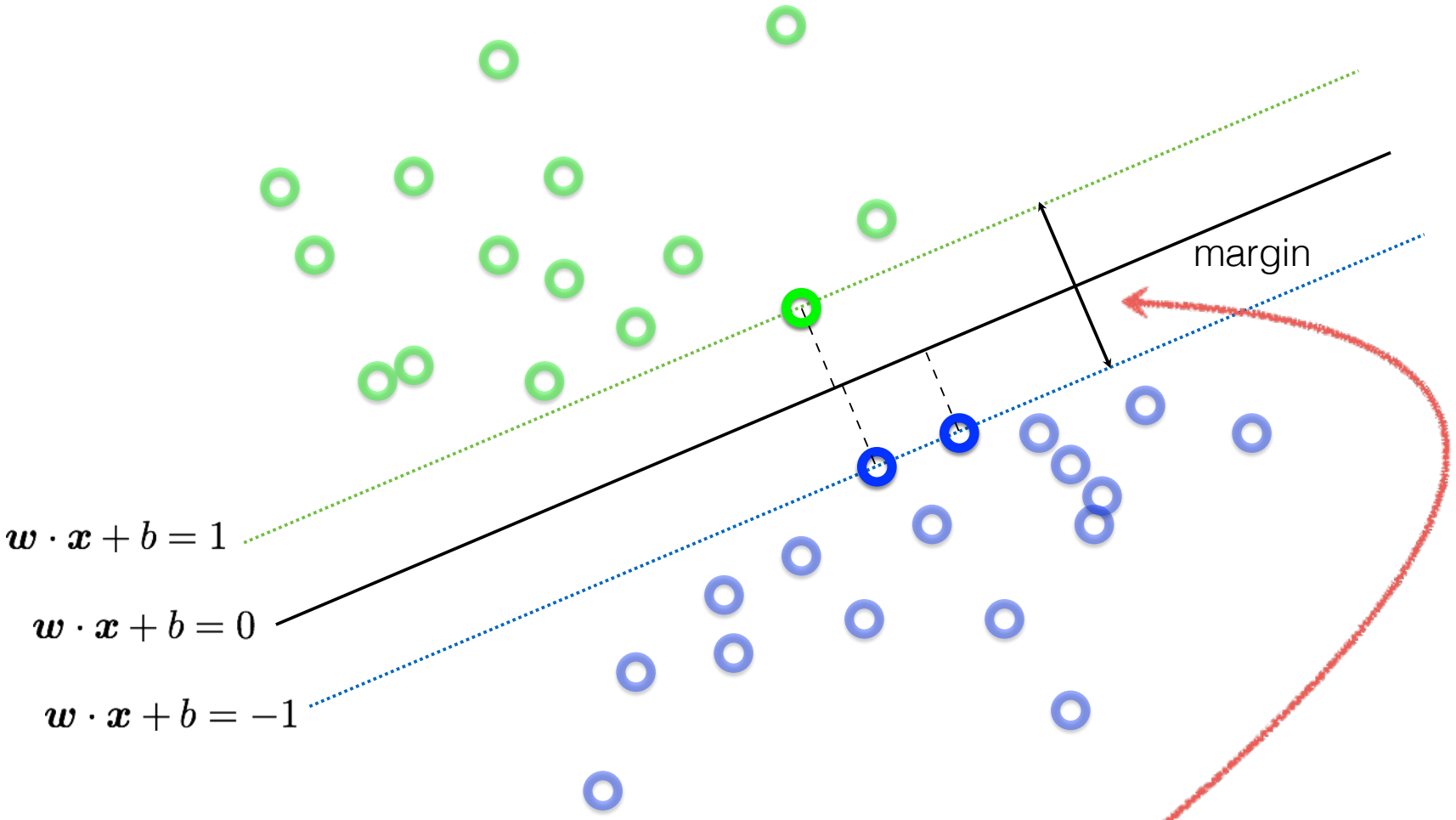
What's the best **w**?



Want a hyperplane that is far away from 'inner points'

Skipping some details.

Find hyperplane **w** such that …



the gap between parallel hyperplanes $\frac{2}{\|\boldsymbol{w}\|}$ is maximized

Can be formulated as a maximization problem

$$\max_{\boldsymbol{w}} \frac{2}{\|\boldsymbol{w}\|}$$

$$\text{subject to } \boldsymbol{w} \cdot \boldsymbol{x}_i + b \begin{cases} \geq +1 & \text{if } y_i = +1 \\ \leq -1 & \text{if } y_i = -1 \end{cases} \text{ for } i = 1, \ldots, N$$

What does this constraint mean?

label of the data point

Why is it +1 and -1?

Can be formulated as a maximization problem

$$\max_{\boldsymbol{w}} \frac{2}{\|\boldsymbol{w}\|}$$

$$\text{subject to } \boldsymbol{w} \cdot \boldsymbol{x}_i + b \begin{matrix} \geq +1 & \text{if } y_i = +1 \\ \leq -1 & \text{if } y_i = -1 \end{matrix} \text{ for } i = 1, \ldots, N$$

Equivalently,

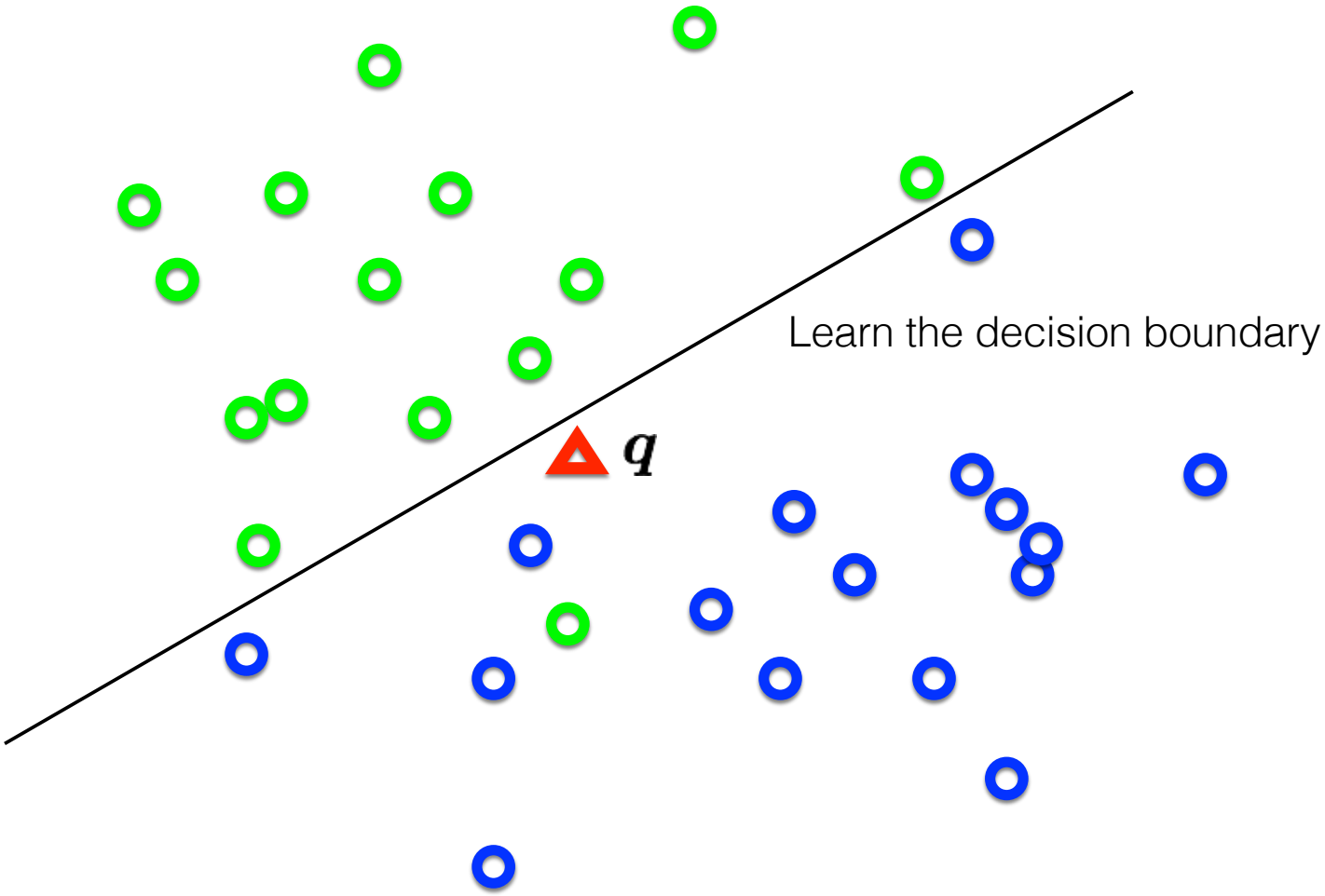
Where did the 2 go?

$$\min_{\boldsymbol{w}} \|\boldsymbol{w}\|$$

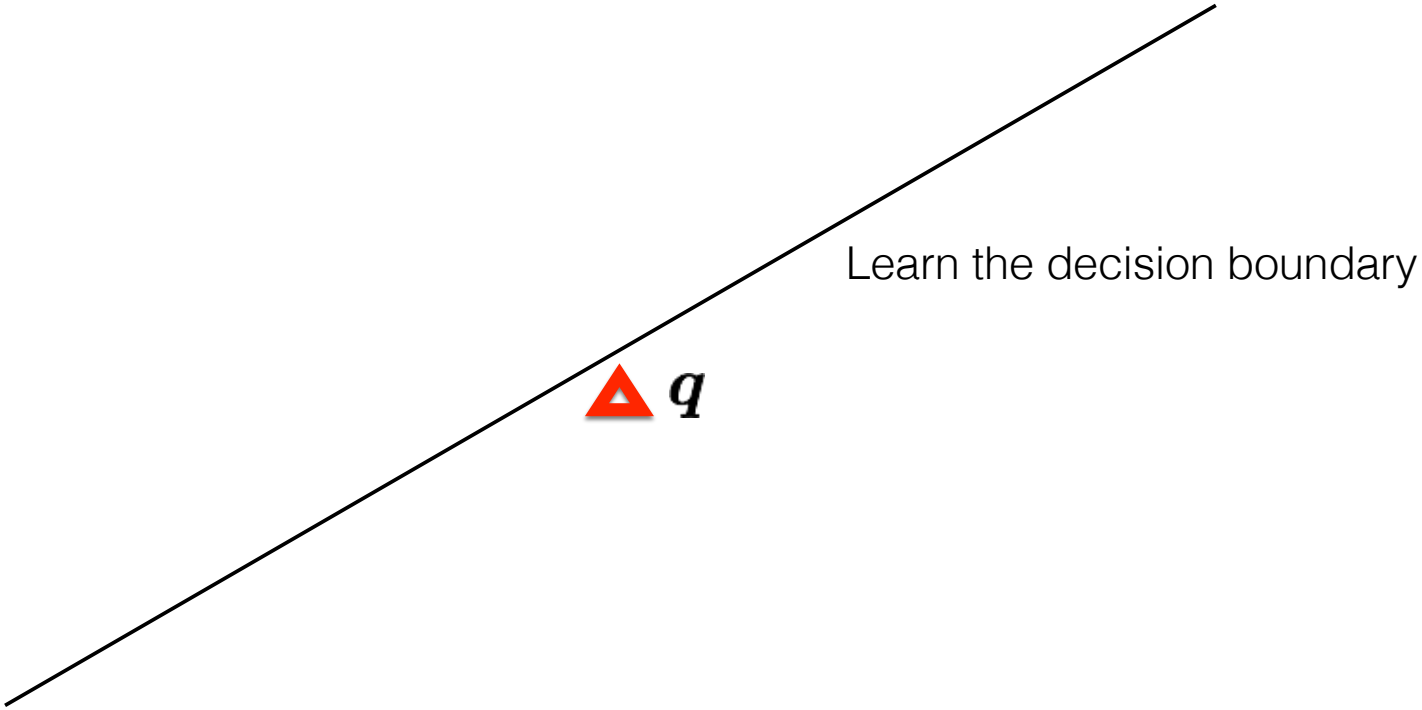
$$\text{subject to } y_i(\boldsymbol{w} \cdot \boldsymbol{x}_i + b) \geq 1 \text{ for } i = 1, \ldots, N$$

What happened to the labels?

Distribution of data from two classes



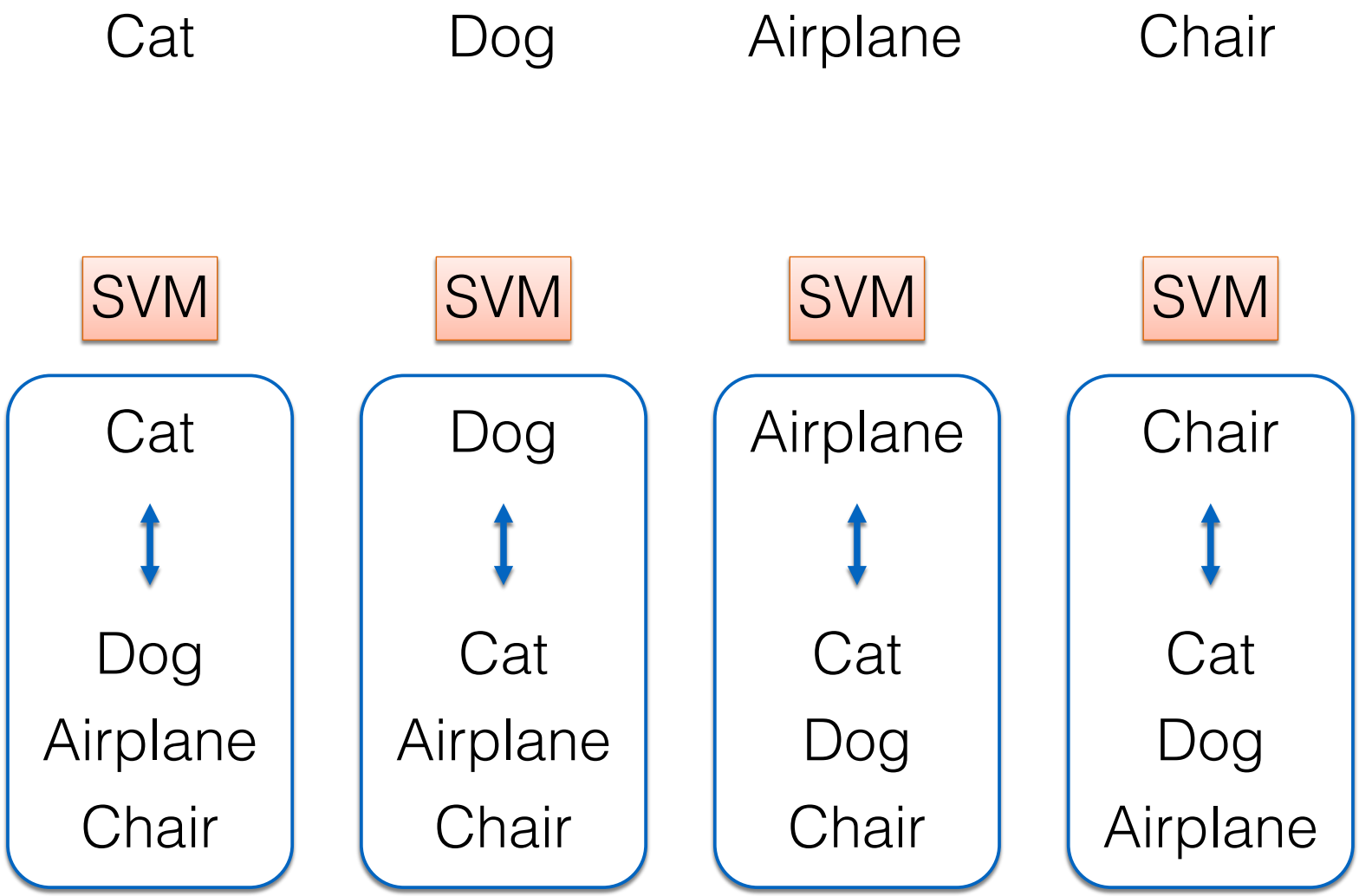
Distribution of data from two classes



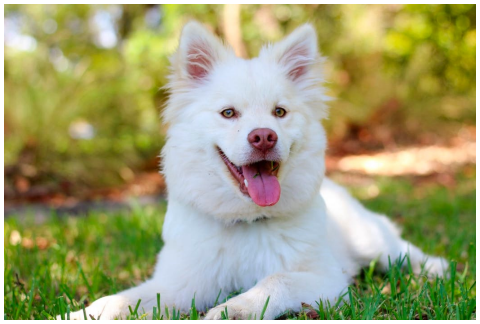
Multi-class case

Cat Dog Airplane Chair

Multi-class case



Multi-class case



0.5	0.9	0.1	0.2
SVM	SVM	SVM	SVM
Cat ↕ Dog	Dog ↕ Cat	Airplane ↕ Cat	Chair ↕ Cat
Airplane	Airplane	Dog	Dog
Chair	Chair	Chair	Airplane

References

Basic reading:

- Szeliski, Chapter 14.