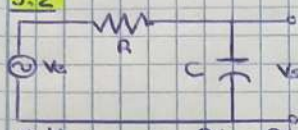
Soluciones ejercicios tema 3 (parte 1) 🔗

3.1

a) Verdadero, $\mathcal{L}\left[\int_0^{\infty} e(\tau)\,d\tau\right] = F(s)/s$

b) Verdadero, $\mathcal{L}\left[\partial^n e(t)/dt^n\right] = s^n F(s)$

3.2



a) $V_s/V_e = 1/sC \,/\, R + 1/sC = 1/1 + RCs$, con $R = 1\,k\Omega$ y $C = 20\,\mu F$ 🔗 $1/1 + 0'02s$ 1er orden

b) Ganancia estática ($K = 1$), polos ($s = -50$), sistema estable.

c) Constante de tiempo ($T = 0'02s$), $t_s = 4T$ ($t_s = 0'08s$), $t_r = 0'0439\,s$

d) $1/s(s+50) = A/s + B/s+50 = A(s+50) + Bs / s(s+50) \Rightarrow \begin{matrix} A+B=0 \\ 50A=1 \end{matrix}$ $A = 0'02$, $B = 0'98$ $\Rightarrow 1 - e^{-50t}$

e) No cambian ni T, ni t_r ni t_s.

f) $V_s/V_e = 1/1 + 50s$, entonces 🔗 $t_r = 109'86s$ y $t_s = 4T = 200s$ y $T = 50$.

3.3

$Y(t)/r(t) = t^2 u_1(t) / u_0(t) = 2/s^3 \,/\, 1/s = 2/s^2$ (Polo doble en 0 $\Rightarrow$ Inestable)

3.4

a) $G_1(s) = 100/s^2 + 12s + 400$

b) $G_2(s) = 3600/s^2 + 90s + 400$

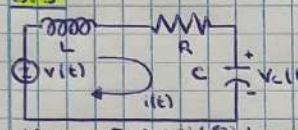
c) $G_3(s) = 75/s^2 + 30s + 225$

d) $G_4(s) = 625/s^2 + 625$

e) $G_5(s) = 128/s^2 - 16s + 64$

f) $G_6(s) = 25/s^2 - 25$

3.5



a) $V_c(s)/V(s) = 1/sC \,/\, sL + R + 1/sC = 1/s^2LC + RCs + 1 = 1/LC \,/\, s^2 + R/L\,s + 1/LC$

Con $R = 1500\,\Omega$, $L = 100\,H$ y $C = 100 \cdot 10^{-6}\,F$ 🔗 $V_c(s)/V(s) = 100/s^2 + 15s + 100$ 2ª orden

b) Polos ($s = -7'5 \pm 6'6\bar{1}$), ganancia estática ($K = 1$).

c) $\omega_n = 10$, $\xi = 0'75$, $K = 1$, sistema subamortiguado.

d) $t_r = \pi - \cos^{-1}|\xi| \,/\, \omega_d = 0'366s$, $t_s = 4/\xi\omega_n = 0'5\bar{3}\,s$, $t_p = \pi/\omega_d = 0'475s$, $M_p = e^{-\pi/tg(\cos^{-1}(\xi))} = 0'0284$, $Y_p = 1 + M_p = 1'0284$

3.6

Con $K = 2$, $\omega_n = 5$ y $\xi = 0'1$ 🔗 $G(s) = 50/s^2 + s + 25$

3.7

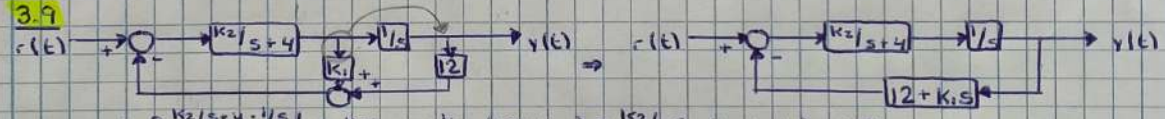
Sabiendo que $M_p = 0'25$ y $t_s = 3'89$, se tiene que 🔗 $\xi = 0'404$ y $\omega_n = 2\ rad/s$.

Con esto se tiene que 🔗 $R = 10\,\Omega$; $L = 6'19\,H$; $C = 0'04\,F$ $\Rightarrow G(s) = 4/s^2 + 1'616s + 4$

3.8

Aplicando el teorema del valor final 🔗 $\lim_{s\to 0} sG(s) = 0$ (no se puede diseñar)

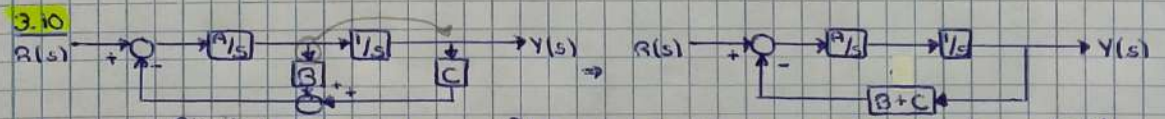
3.9



Lazo cerrado 🔗 $K_2/s+4 \cdot 1/s \,/\, 1 + K_2/s+4 \cdot 1/s \cdot (12 + K_1 s) = K_2/s^2 + (4 + K_2K_1)s + 12K_2$

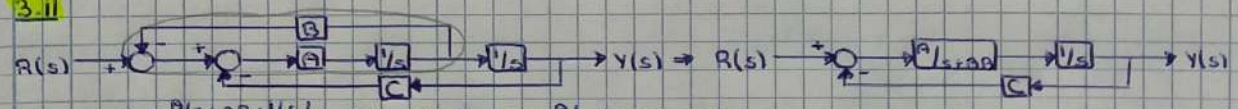
Sabiendo que $\xi = 0'5$ y $t_s = 1$, tenemos que 🔗 $\omega_n = 8\ rad/s$, $K_2 = 5'\bar{3}$, $K_1 = 0'75$

3.10



Lazo cerrado 🔗 $A/s \cdot 1/s \,/\, 1 + A/s \cdot 1/s \cdot (B+C) = A/s^2 + ABs + AC$, $t_s = 4s$, $K = 0'5$, $\xi = 0'1$, $\omega_n = 10\ rad/s$

3.11



Lazo cerrado 🔗 $A/s + AB \cdot 1/s \,/\, 1 + A/s + AB \cdot 1/s \cdot C = A/s^2 + ABs + AC$

$M_p = 0'162 \Rightarrow \xi = 0'5$

$t_p = 0'04534s \Rightarrow \omega_n = 80\ rad/s$

Valor final $= 0'25 \Rightarrow K = 0'25$

$\Big\}\ G(s) = 1600/s^2 + 80s + 6400$, donde $A = 1600$, $B = 0'05$ y $C = 4$.

Soluciones ejercicios tema 3 (parte 2):

3.12

a)
s^4	1	3	1
s^3	3	2	0
s^2	b_1	b_2	0
s^1	c_1	0	0
s^0	d_1	0	0

$b_1 = 3/3 = 2'\hat{3}$; $b_2 = 1$; $c_1 = 1'6/2'3 + 0'7$; $d_1 = 1$; SISTEMA ESTABLE

b)
s^3	1	1
s^2	2	2
s^1	b_1	0
s^0	c_1	0

$b_1 = 0$; $c_1 = 0$; SISTEMA CRÍTICAMENTE ESTABLE

c)
s^5	1	24	25
s^4	2	48	50
s^3	b_1	b_2	0
s^2	c_1	0	0
s^1	d_1	0	0
s^0	e_1	0	0

$b_1 = 0$; $b_2 = 0$; $c_1 = 0$; $d_1 = 0$; $e_1 = 0$; SISTEMA CRÍTICAMENTE ESTABLE

d)
s^3	2	-3	
s^2	1	10	
s^1	b_1	0	
s^0	c_1	0	

$b_1 = -23$; $c_1 = 10$; SISTEMA INESTABLE

e)
s^5	1	2	11
s^4	2	4	10
s^3	b_1	b_2	0
s^2	c_1	0	0
s^1	d_1	0	0
s^0	e_1	0	0

$b_1 = 0$; $b_2 = 6$; $c_1 = 0$; $d_1 = 0$; $e_1 = 0$; SISTEMA INESTABLE

f)
s^5	1	4	3
s^4	1	24	63
s^3	b_1	b_2	0
s^2	c_1	0	0
s^1	d_1	0	0
s^0	e_1	0	0

$b_1 = -20$; $b_2 = -60$; $c_1 = 21$; $d_1 = -60$; $e_1 = 0$; SISTEMA INESTABLE

3.13

s^3	a_0	a_2
s^2	a_1	a_3
s^1	b_1	0
s^0	c_1	0

$b_1 = (a_1a_2 - a_0a_3)/a_1$; $c_1 = a_3$; ESTABLE SI $a_1a_2 > a_0a_3$

3.14

s^3	1	$200+K$
s^2	30	$40K$
s^1	b_1	0
s^0	c_1	0

$b_1 = 200 - 1/3K = 0 \Rightarrow K = 600$

ESTABLE: $0 < K < 600$

CRÍTICAMENTE ESTABLE: $K = 600$

INESTABLE: $K > 600$

3.15

a) Lazo cerrado: $\frac{K/s(s+1)(s+2)}{1 + K/s(s+1)(s+2)} = \frac{K}{s^3+3s^2+2s+K}$

s^3	1	2
s^2	3	K
s^1	b_1	0
s^0	c_1	0

$b_1 = 2 - 1/3K = 0 \Rightarrow K = 6$

ESTABLE: $0 < K < 6$

CRÍTICAMENTE ESTABLE: $K = 6$

INESTABLE: $K > 6$

b) Lazo cerrado: $\frac{K/s(s^2+s+1)(s+2)}{1 + K/s(s^2+s+1)(s+2)} = \frac{K}{s^4+3s^3+3s^2+2s+K}$

s^4	1	3	K
s^3	3	2	0
s^2	b_1	b_2	0
s^1	c_1	0	0
s^0	d_1	0	0

$b_1 = 7/3 = 2'\hat{3}$; $b_2 = K$; $c_1 = 2 - 1'30K = 0 \Rightarrow K = 1'54$

ESTABLE: $0 < K < 1'54$

CRÍTICAMENTE ESTABLE: $K = 1'54$

INESTABLE: $K > 1'54$

c) Lazo cerrado: $\frac{K/s(s+1)(s+2)(s+3)}{1 + K/s(s+1)(s+2)(s+3)} = \frac{K}{s^4+6s^3+11s^2+6s+K}$

s^4	1	11	K
s^3	6	6	0
s^2	b_1	b_2	0
s^1	c_1	0	0
s^0	d_1	0	0

$b_1 = 10$; $b_2 = K$; $c_1 = 6 - 0'6K = 0 \Rightarrow K = 10$

ESTABLE: $0 < K < 10$

CRÍTICAMENTE ESTABLE: $K = 10$

INESTABLE: $K > 10$

3.16

Lazo cerrado: $\frac{s/(s+1)^2}{1 - s/(s+1)^2 - A/s} = \frac{s}{s^2+2s+(1-A)}$

s^2	1	$1-A$
s^1	2	0
s^0	b_1	0

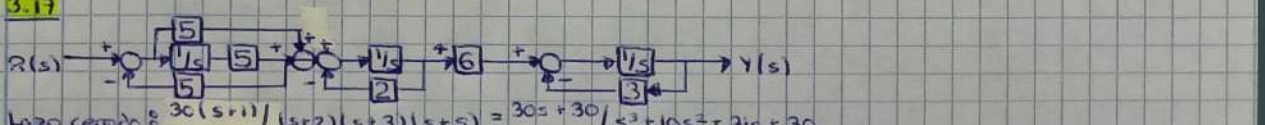
$b_1 = 2 - 2A = 0 \Rightarrow A = 1$

ESTABLE: $0 < A < 1$

CRÍTICAMENTE ESTABLE: $A = 1$

INESTABLE: $A > 1$

3.17



Lazo cerrado: $\frac{30(s+1)}{(s+2)(s+3)(s+5)} = \frac{30s+30}{s^3+10s^2+31s+30}$

a)
s^3	1	31
s^2	10	30
s^1	b_1	0
s^0	c_1	0

$b_1 = 28$; $c_1 = 30$; SISTEMA ESTABLE

b) $\frac{30s+30}{(s+2)(s+3)(s+5)} = \frac{A}{s+2} + \frac{B}{s+3} + \frac{C}{s+5} = \frac{A(s^2+8s+15) + B(s^2+7s+10) + C(s^2+5s+6)}{(s+2)(s+3)(s+5)}$

$A + B + C = 0$

$8A + 7B + 5C = 30$

$15A + 10B + 6C = 30$

$A = -10$; $B = 30$; $C = -20$

$\Rightarrow -10\,1/s+2 + 30\,1/s+3 - 20\,1/s+5 = -10e^{-2t} + 30e^{-3t} - 20e^{-5t}$, $y(\infty) = 0$

3.18

a) Lazo cerrado: $\frac{K/Ts^2+s}{1+K/Ts^2+s} = \frac{K/T}{s^2+1/Ts+K/T} \Leftrightarrow \frac{K\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$

Como $M_p = 3$ y $M_p = 0'25 \Rightarrow$ se tiene que: $\omega_d = 1'05$ rad/s, $\zeta\omega_n = 0'46$, $\zeta = 0'4$, $\omega_n = 1'14$ rad/s

Comparando con la forma de la transferencia de un sistema de segundo orden, obtenemos que: $K = 1'43$, $T = 1'09$

b) $e_{ss} = \lim_{s\to 0} s\,1/s\,\frac{s^2+0'92s}{s^2+0'92s+1'3i} = 0$ (TIPO 1)

c) $e_{ss} = \lim_{s\to 0} s\,1/s^2\,\frac{s^2+0'92s}{s^2+0'92s+1'3i} = 0'70$

3.19

a) Lazo cerrado: $\frac{50/(1+0'1s)(1+2s)}{1+50/(1+0'1s)(1+2s)} = \frac{(1+0'1s)(1+2s)}{(1+0'1s)(1+2s)+50}$

Errores: e_{ss} (escalón) $= 1/51$ (TIPO 0)

b) Lazo cerrado: $\frac{2/s(s^2+4s+200)}{1+2/s(s^2+4s+200)} = \frac{s(s^2+4s+200)}{s(s^2+4s+200)+2}$

Errores: e_{ss} (escalón) $= 0$, e_{ss} (rampa) $= 100$ (TIPO 1)

c) Lazo cerrado: $\frac{K/s(1+0'1s)(1+0'5s)}{1+K/s(1+0'1s)(1+0'5s)} = \frac{s(1+0'1s)(1+0'5s)}{s(1+0'1s)(1+0'5s)+K}$

Errores: e_{ss} (escalón) $= 0$, e_{ss} (rampa) $= 1/K$ (TIPO 2)

d) Lazo cerrado: $\frac{K(1+2s)(1+4s)/s^2(s^2+2s+10)}{1+K(1+2s)(1+4s)/s^2(s^2+2s+10)} = \frac{s^2(s^2+2s+10)}{s^2(s^2+2s+10)+K(1+2s)(1+4s)}$

Errores: e_{ss} (escalón) $= 0$, e_{ss} (rampa) $= 0$, e_{ss} (parábola) $= 10/K$ (TIPO 2)

3.20

Lazo cerrado: $\frac{100/s(s+10)}{1+100/s(s+10)\,1/(s+5)} = \frac{100(s+5)}{s(s+10)(s+5)+100}$

e_{ss} (escalón): $\lim_{s\to 0} s\,1/s\left[1-\frac{100(s+5)}{s(s+10)(s+5)+100}\right] = \frac{-400}{100} = -4$ (TIPO 0)

3.21

Lazo cerrado: $\frac{K/(s+2)}{1+K/(s+2)^2(s+4)} = \frac{K(s+4)}{(s+2)(s+4)+2K}$

e_{ss} (escalón): $\lim_{s\to 0} s\,1/s\left[1-\frac{K(s+4)}{(s+2)(s+4)+2K}\right] = \frac{8-2K}{8+2K} \Rightarrow$ Error nulo: $K = 4$

3.22

a) Lazo cerrado: $\frac{100/s(s+10)}{1+100/s(s+10)\,K/(s+5)} = \frac{100(s+5)}{s^3+15s^2+50s+100K}$

s^3	1	50	$b_1 = 50 - 20/3\,K = 0 \Rightarrow K = 7'5$
s^2	15	100K	ESTABLE: $0 < K < 7'5$
s^1	b_1	0	CRÍTICAMENTE ESTABLE: $K = 7'5$
s^0	C_1	0	INESTABLE: $K > 7'5$

b) e_{ss} (escalón): $\lim_{s\to 0} s\,1/s\left[1-\frac{100(s+5)}{s^3+15s^2+50s+100K}\right] = \frac{100K-500}{100K} \Rightarrow$ Error nulo: $K = 5$

3.23

Lazo cerrado: $\frac{K/s(s+2)(s+3)}{1+K/s(s+2)(s+3)} = \frac{K}{s^3+5s^2+6s+K}$

s^3	1	6	$b_1 = 6 - 0'2K = 0 \Rightarrow K = 30$
s^2	5	K	ESTABLE: $0 < K < 30$
s^1	b_1	0	CRÍTICAMENTE ESTABLE: $K = 30$
s^0	C_1	0	INESTABLE: $K > 30$

e_{ss} (velocidad): $\lim_{s\to 0} 1/Kv \Rightarrow 1/\lim_{s\to 0} s\,K/s(s+2)(s+3) = 1/K/6 = 6/K$

Si: $e_{ss,v} = 0'3 \Rightarrow K = 20$ (ESTABLE), y si $e_{ss,v} = 0'1 \Rightarrow K = 60$ (INESTABLE)

3.24

a) $E_0(s)/D(s) = -G_2H/1+G_1G_2H$

$e_0(\infty) = \lim_{s\to 0} -1/s(s+25)/1+1000\,1/s(s+25) = -1/1000$

b) $e_0(\infty) = \lim_{s\to 0} -\frac{s+2/s+4}{1+1000\,s+2/s+4} = -1/1002$