Soluciones ejercicios tema 2

2.1

a) $(s^2+23s+131)X(s) = 7F(s) \Rightarrow d^2x/dt^2 + 23\,dx/dt + 131x(t) = 7\epsilon(t)$

b) $(s^3+10s^2+11s+18)X(s) = (s+2)F(s) \Rightarrow d^3x/dt^3 + 10\,d^2x/dt^2 + 11\,dx/dt + 18x(t) = d\epsilon/dt + 2\epsilon(t)$

2.2

a) $(s^2+12s+32)Y(s) = 32F(s) \Rightarrow G(s) = Y(s)/F(s) = 32/s^2+12s+32$

b) $32/(s+4)(s+8) = A/(s+4) + B/(s+8) = A(s+8)+B(s+4)/(s+4)(s+8) \Rightarrow \begin{matrix} A+B=0 \\ 8A+4B=32 \end{matrix} \Rightarrow A=8,\ B=-8 \Rightarrow$
$\Rightarrow 8/(s+4) - 8/(s+8) = 8\cdot 1/(s+4) - 8\cdot 1/(s+8) = 8e^{-4t} - 8e^{-8t}$

c) $32/s(s+4)(s+8) = A/s + B/(s+4) + C/(s+8) = A(s^2+12s+32) + B(s^2+8s) + C(s^2+4s)/s(s+4)(s+8) \Rightarrow$
$\Rightarrow \begin{matrix} A+B+C=0 \\ 12A+8B+4C=0 \\ 32A=32 \end{matrix} \Rightarrow A=1,\ B=-2,\ C=1 \Rightarrow 1/s - 2/(s+4) + 1/(s+8) = 1 - 2e^{-4t} + e^{-8t}$

2.3

a) $5s+500/s(s+10)(s+50) = A/s + B/(s+10) + C/(s+50) = A(s^2+60s+500) + B(s^2+50s) + C(s^2+10s)/s(s+10)(s+50) \Rightarrow$
$\Rightarrow \begin{matrix} A+B+C=0 \\ 60A+50B+10C=5 \\ 500A=500 \end{matrix} \Rightarrow A=1,\ B=-9/8,\ C=1/8 \Rightarrow 1/s - 9/8/(s+10) + 1/8/(s+50) = 1 - 9/8\,e^{-10t} + 1/8\,e^{-50t}$

b) Teorema del valor final: $\lim_{t\to\infty} \epsilon(t) = \lim_{t\to\infty} 1 - 9/8\,e^{-10t} + 1/8\,e^{-50t} = 1$

2.4

a) $5s+10/s(s+3)(s+5) = A/s + B/(s+3) + C/(s+5) = A(s^2+8s+15) + B(s^2+5s) + C(s^2+3s)/s(s+3)(s+5) \Rightarrow$
$\Rightarrow \begin{matrix} A+B+C=0 \\ 8A+5B+3C=5 \\ 15A=10 \end{matrix} \Rightarrow A=2/3,\ B=5/16,\ C=-3/2 \Rightarrow 2/3/s + 5/16/(s+3) - 3/2/(s+5) = 2/3 + 5/16\,e^{-3t} - 3/2\,e^{-5t}$

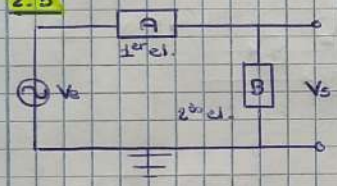
b) $5s+10/s(s+3)^2 = A/s + B/(s+3) + C/(s+3)^2 = As(s+3)^2 + Bs(s+3)^2 + Cs(s+3)^2/s(s+3)^2 \Rightarrow A=10/9,\ B=-10/9,\ C=5/3$
$\Rightarrow 10/9/s - 10/9/(s+3) + 5/3/(s+3)^2 = 10/9 - 10/9\,e^{-3t} + 5/3\,te^{-3t}$

c) $2s+12/s^2+2s+5 = 2s/s^2+2s+5 + 12/s^2+2s+5 = 2\left[s/s^2+2s+5\right] + 12/s^2+2s+5 =$
$= 2\left(s/(s+1)^2+2^2\right) + 12/(s+1)^2+2^2 = 2\left((s+1)/(s+1)^2+2^2\right) + 11\cdot 1/4\left(4/(s+1)^2+2^2\right) = 2e^{-t}\cos(2t) + 5e^{-t}\operatorname{sen}(2t)$

d) $s-11/(s+3)^2+5^2 = s/(s+3)^2+5^2 - 11/(s+3)^2+5^2 = (s+3)-3/(s+3)^2+5^2 - 11/(s+3)^2+5^2 =$
$= (s+3)/(s+3)^2+5^2 - 14\cdot 1/5\cdot 5/(s+3)^2+5^2 = e^{-3t}\cos(5t) - 14/5\operatorname{sen}(5t)$

e) $5s+10/s(s+3)^2+5^2 = A/s + Bs+C/(s+3)^2+5^2 = A(s^2+6s+34) + (Bs+C)s/s(s^2+6s+34) \Rightarrow$
$A=5/17,\ B=-5/17,\ C=55/17 \Rightarrow 5/17/s + -5/17s+55/17/(s+3)^2+5^2 = 5/17/s - 5/17\,(s+3)/(s+3)^2+5^2 + (55/17+3\cdot 5/17)\cdot$
$\cdot 1/s\ 5/(s+3)^2+5^2 = 5/17 - 5/17\,e^{-3t}\cos(5t) + 14/17\,e^{-3t}\operatorname{sen}(5t)$

2.5



$RC \triangleq 1/sC/R + 1/sC = 1/RCs + 1$

$CR \triangleq R/1/sC + R = RCs/1 + RCs$

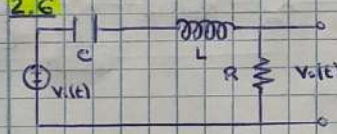
$RL \triangleq sL/R + sL$

$LR \triangleq R/sL + R$

$CL \triangleq sL/1/sC + sL = s^2LC/s^2LC + 1 = s^2/s^2 + 1/LC$

$LC \triangleq 1/sC/sL + 1/sC = 1/s^2LC + 1 = 1/LC/s^2 + 1/LC$

2.6

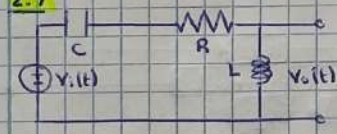


$Vo(s)/Vi(s) = R/1/sC + sL + R = RCs/1 + s^2LC + RCs = R/Ls/s^2 + R/Ls + 1/LC$

Con $R=5\,\Omega$, $L=1H$, $C=0'25F \Rightarrow Vo(s)/Vi(s) = 5s/s^2+5s+4$

$5s/(s+1)(s+4) = A/s+1 + B/s+4 = A(s+4)+B(s+1)/(s+1)(s+4) \Rightarrow \begin{matrix} 4A+B=5 \\ A+B=0 \end{matrix} \Rightarrow A=-5/3,\ B=20/3$
$\Rightarrow -5/3/s+1 + 20/3/s+4 = -5/3\,1/s+1 + 20/3\,1/s+4 = -5/3\,e^{-t} + 20/3\,e^{-4t}$

2.7

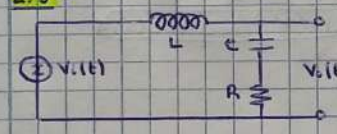


$Vo(s)/Vi(s) = sL/1/sC + R + sL = s^2LC/1 + RCs + s^2LC = s^2/s^2 + R/Ls + 1/LC$

Con $R=2\,\Omega$, $L=1H$, $C=0'2H \Rightarrow Vo(s)/Vi(s) = s^2/s^2+2s+5$

$s/(s+1)^2+2^2 = s+1/(s+1)^2+2^2 - 1/2\cdot 2/(s+1)^2+2^2 = e^{-t}\left[\cos(2t) - 1/2\operatorname{sen}(2t)\right]$

2.8

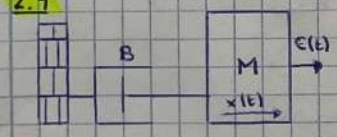


$Vo(s)/Vi(s) = R + 1/sC/sL + R + 1/sC = RCs + 1/s^2LC + RCs + 1 = R/Ls + 1/LC/s^2 + R/Ls + 1/LC$

Con $R=20\,\Omega$, $L=10H$ y $C=0'1F \Rightarrow Vo(s)/Vi(s) = 2s+1/s^2+2s+1$

$2s+1/(s+1)^2 = 2s/(s+1)^2 + 1/(s+1)^2 = 2(s/(s+1)^2) - 1/(s+1)^2 = 2e^{-t} - te^{-t}$

2.9

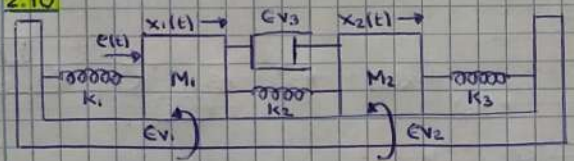


a) $\epsilon(t) - B\,dx(t)/dt = M\,d^2x(t)/dt^2 \Rightarrow F(s) - BsX(s) = Ms^2X(s) \Rightarrow$
$\Rightarrow F(s) = X(s)\left[Ms^2+Bs\right] \Rightarrow X(s)/F(s) = 1/Ms^2+Bs = 1/s(Ms+B)$

b) $1/M/s^2+B/Ms = A/s + B/s+B/M = 1/B/s - 1/B/s+B/M = 1/B(1-e^{-B/M\cdot t})$

c) $x(t) = \int_0^t 1/B\,dt - \int_0^t 1/B\,e^{-B/Mt}\,dt = 1/B\left[t + M/Be^{-B/Mt} - M/B\right]$

2.10



$G(s) = X_2(s)/F(s) = (\epsilon V_3 s + K_2)/\Delta$, donde $\Delta \triangleq$

$$\begin{vmatrix} M_1s^2 + (\epsilon V_1 + \epsilon V_3)s + (K_1+K_2) & -(\epsilon V_3 s + K_2) \\ -(\epsilon V_3 s + K_2) & M_2s^2 + (\epsilon V_2 + \epsilon V_3)s + (K_2+K_3) \end{vmatrix}$$

2.11

Figura A

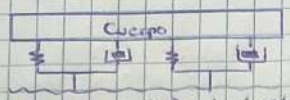
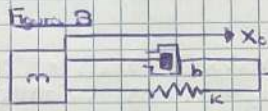


Figura B

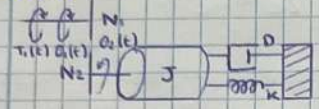


a) $ms^2X_o(s) = k(X_i(s) - X_o(s)) + bs(X_i(s) - X_o(s)) \Rightarrow ms^2X_o(s) + kX_o(s) + bsX_o(s) = kX_i(s) + bsX_i(s) \Rightarrow$
$\Rightarrow X_i(s)[ms^2 + bs + k] = X_i(s)[bs + k] \Rightarrow X_o(s)/X_i(s) = bs + k / ms^2 + bs + k$

b) $4000s + 4000 / 1000s^2 + 4000s + 4000 = 4(s+1)/s^2 + 4s + 1 \cdot 1/s = 4(s+1)/s(s+2)^2 \Rightarrow x_o(t) = 1 - e^{-2t} + 2te^{-2t}$

c) $1/s^2 + 1 \cdot 1/s = As + B/s^2 + 1 + C/s \Rightarrow A = 1, B = 0, C = 1 \Rightarrow -s/s^2 + 1 + 1/s = 1 - \cos(t)$

2.12



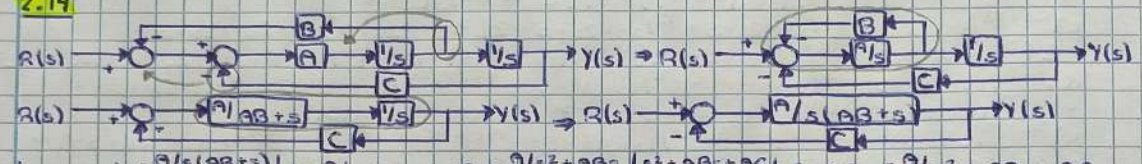
$G(s) = \Theta_o(s)/T_i(s) = (N_2/N_1)^2 / Js^2 + Ds + K$

2.13

a) $G(s) = \Theta(s)/V_e(s) = Km / LeJs^3 + (ReJ + LeB)s^2 + ReBs$

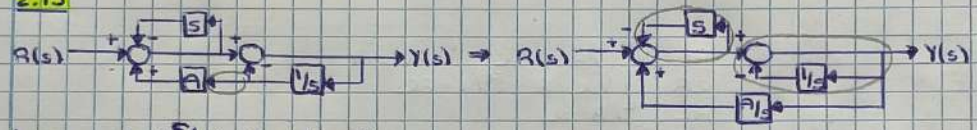
b) $G(s) = Km/s(Re + Les)(Js + B) = 2/s(s+1)(s+3) = 2/3 - e^{-t} + 1/3e^{-3t}$

2.14



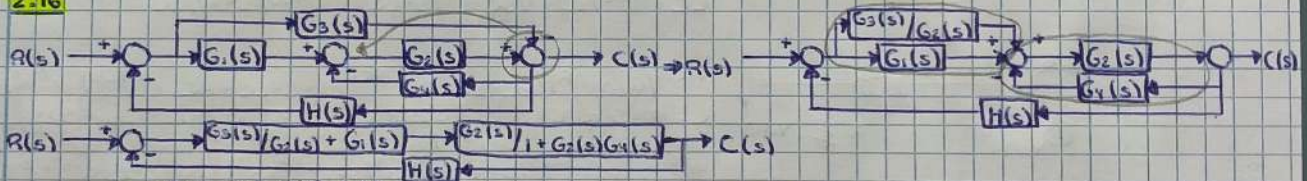
Lazo cerrado: $A/s(AB+s) / 1 + A/s(AB+s) \cdot C = A/s^2 + ABs / s^2 + ABs + AC / s^2 + ABs = A/s^2 + ABs + AC$

2.15



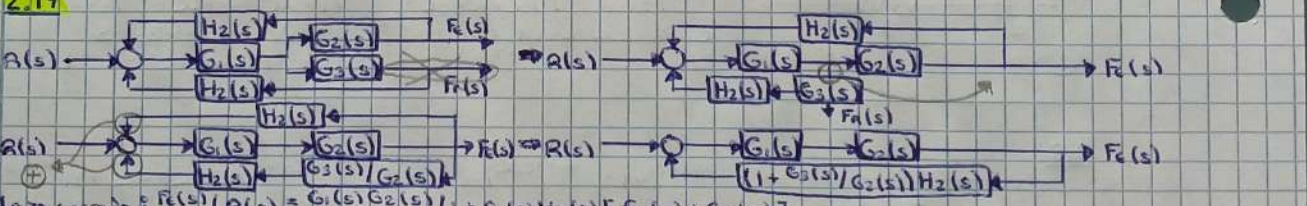
Lazo cerrado: $S/s^2 + 2s + (1 - A)$

2.16



Lazo cerrado: $G_3(s) + G_1(s)G_2(s) / 1 + G_2(s)G_4(s) + H(s)[G_3(s) + G_1(s)G_2(s)]$

2.17

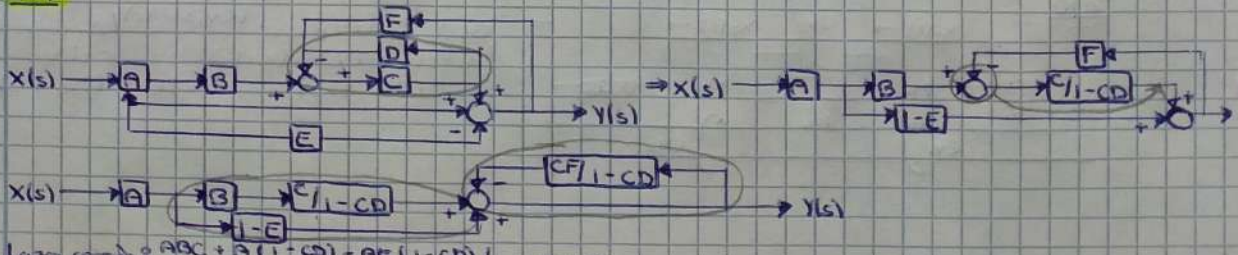


Lazo cerrado: $F_e(s) / R(s) = G_1(s)G_2(s) / 1 + G_1(s)H_2(s)[G_2(s) + G_3(s)]$

2.18

Lazo cerrado: $KG_1(s)G_2(s)/s / 1 + G_1(s)H_3(s) + G_1(s)G_2(s)H_2(s) + G_1(s)G_2(s)H_1(s)$

2.19



Lazo cerrado: $ABC + A(1 + CD) + AE(1 - CD) / 1 - CD + FC$