Ingeniería de Control
Exam January 10, 2023

Tracking and pick and place problems for a planar vertical manipulator.

Consider the robot manipulator represented in Figure 1, which moves in a vertical plane.

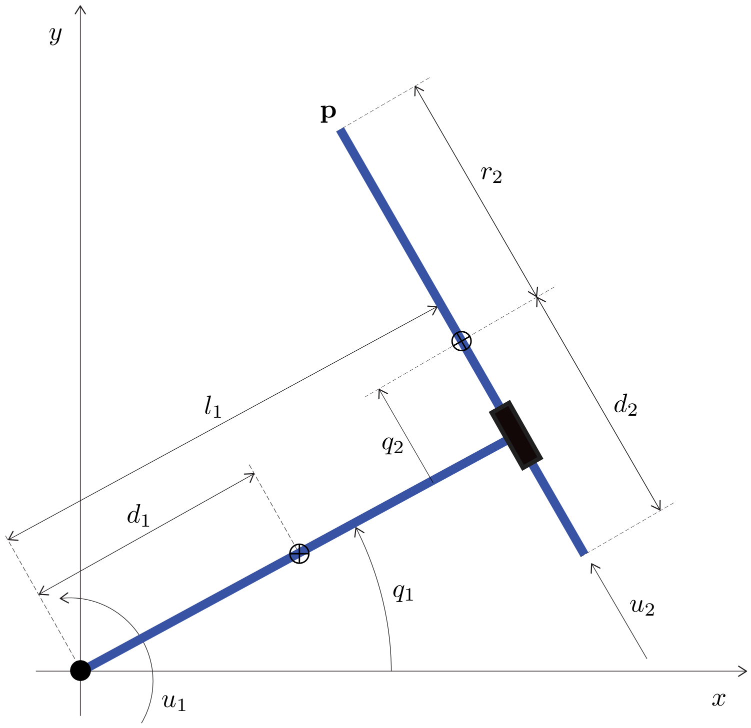

Figure 1: Planar vertical robot manipulator.

The dynamic model of this robotic system is represented by the second order differential equation

$$\mathbf{B}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{N}(\mathbf{q}) = \mathbf{u},$$

where the matrices $\mathbf{B}(\mathbf{q})$, $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$, and $\mathbf{N}(\mathbf{q})$ have the following expressions

$$\begin{aligned}
\mathbf{B}(\mathbf{q}) &= \begin{bmatrix} I_1 + m_1 d_1^2 + I_2 + m_2 l_1^2 + m_2 q_2^2 & m_2 l_1 \\ m_2 l_1 & m_2 \end{bmatrix}, \\
\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) &= \begin{bmatrix} 2 m_2 q_2 \dot{q}_1 \dot{q}_2 \\ -m_2 q_2 \dot{q}_1^2 \end{bmatrix}, \\
\mathbf{N}(\mathbf{q}) &= \begin{bmatrix} (m_1 d_1 + m_2 l_1) g \cos q_1 - m_2 g q_2 \sin q_1 \\ m_2 g \cos q_1 \end{bmatrix}.
\end{aligned}$$

The vector $\mathbf{q} = (q_1, q_2)^T$ is the vector of configuration variables, where q_1 is the angular position of link 1 with respect to the x axis of the reference frame $\{x, y\}$ and q_2 is the linear position of the center of mass of link 2 with respect to link 1 as illustrated in Figure 1. The vector $\dot{\mathbf{q}} = (\dot{q}_1, \dot{q}_2)^T$ is the vector of joint velocities, where $\dot{q}_1$ is the angular velocity of link 1 and $\dot{q}_2$ is the linear velocity of link 2. The vector $\ddot{\mathbf{q}} = (\ddot{q}_1, \ddot{q}_2)^T$ is the vector of joint accelerations, where $\ddot{q}_1$ is the angular acceleration of link 1 and $\ddot{q}_2$ is the linear acceleration of link 2. The control inputs of the system are $\mathbf{u} = (u_1, u_2)^T$, where u_1 is the torque applied by the actuator at joint

1, and u_2 is the force applied by the actuator at joint 2. l_1 is the length of link 1, l_2 is the length of link 2, m_1 is the mass of link 1, m_2 is the mass of link 2, I_1 is the barycentric moment of inertia of link 1, and I_2 is the barycentric moment of inertia of link 2. Distances d_1, d_2, and r_2 are defined in Figure 1. The parameters of the dynamic model of the robot manipulator are $l_1 = 1$ [m], $l_2 = 1$ [m], $m_1 = 1$ [kg], $m_2 = 1$ [kg], $I_1 = 1$ [kg m^2], $I_2 = 1$ [kg m^2], $d_1 = \frac{l_1}{2}$ [m], $d_2 = \frac{l_2}{2}$ [m]. $r_2 = \frac{l_2}{2}$ [m], Consider the following two robotic tasks.

a) Pick and place task: move iteratively the end effector $\mathbf{p}$ between the setpoints $\mathbf{p}_A = (0.4, 1.2)^T$ [m] and $\mathbf{p}_B = (1.3, 0.3)^T$ [m]. The motions must be rest to rest.

b) Tracking task: follow with the end effector a setpoint $\mathbf{w}$ describing the target line segment from $\mathbf{p}_A = (0.4, 1.2)^T$ [m] to $\mathbf{p}_B = (1.3, 0.3)^T$ [m], which moves with constant linear velocity of 0.1 [m/s].

Suppose that, in both cases, at $t = 0$ [s] the position of the end effector is $\mathbf{p} = \mathbf{p}_A$.

1) Compute the kinetic and potential energy of the two links of the robot manipulator. (Contesta en el cuadernillo)

2) Compute the state space representation of the dynamics of the robot manipulator in which $\mathbf{x} = (x_1, x_2, x_3, x_4)^T = (q_1, q_2, \dot{q}_1, \dot{q}_2)^T$, where distances are measured in [m], angles in [rad], linear velocities in [m/s], and angular velocities in [rad/s]. (Contesta en el cuadernillo y sube el código Matlab a Aula Virtual en el fichero answer_2.m)

3) Compute the relation between the configuration variables $\mathbf{q}$ and the position of the end effector $\mathbf{p} = (p_1, p_2)^T$. (Contesta en el cuadernillo)

4) Compute the relation between the position of the end effector $\mathbf{p}$ and the configuration variables $\mathbf{q}$. Compute the values of the configuration variables that correspond to $\mathbf{p} = \mathbf{p}_A$ and $\mathbf{p} = \mathbf{p}_B$. (Contesta en el cuadernillo y sube el código Matlab a Aula Virtual en el fichero answer_4.m)

5) Compute the relation between the joint velocities $\dot{\mathbf{q}}$ and the velocity of the end effector $\dot{\mathbf{p}}$. (Contesta en el cuadernillo y sube el código Matlab a Aula Virtual en el fichero answer_5.m)

6) Compute the relation between the velocity of the end effector $\dot{\mathbf{p}}$ and the joint velocities $\dot{\mathbf{q}}$. Suppose that the robot manipulator is ready to execute the tracking task, that is, the position of the end effector is $\mathbf{p} = \mathbf{p}_A$. Compute the initial joint velocities to execute this task. (Contesta en el cuadernillo y sube el código Matlab a Aula Virtual en el fichero answer_6.m)

7) Compute the time derivative of the Jacobian matrix. (Contesta en el cuadernillo y sube el código Matlab a Aula Virtual en el fichero answer_7.m)

8) Design a controller based on the feedback linearization method to execute the pick and place task. Write a `Matlab` code that implements the controller to execute the task. Show, plotting the relevant variables and an animation, that the controller satisfies the specifications. (Contesta en el cuadernillo y sube el código `Matlab` a Aula Virtual en la carpeta answer_8)

9) Design a controller based on the feedback linearization method to execute the tracking task. Compute the duration of the manoeuvre. Write a `Matlab` code that implements the controller to execute the task. Show, plotting the relevant variables and an animation, that the controller satisfies the specifications. (Contesta en el cuadernillo y sube el código `Matlab` a Aula Virtual en la carpeta answer_9)

Originality and completeness of the written answers will be the aspects that will be taken into account in the grading of the exam, and therefore, the `Matlab` code alone will not be considered.