Exercise 6.6. Controlling an inverted rod pendulum[1]

Let us consider the inverted rod pendulum represented on Figure 1, composed of a pendulum placed in unstable equilibrium on a carriage.

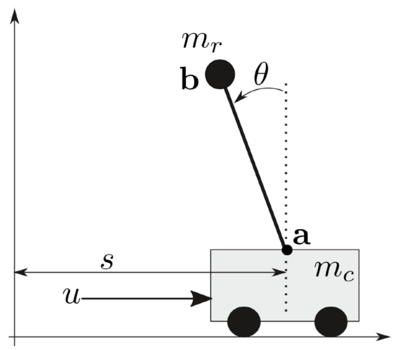

Figure 1: Inverted rod pendulum.

The value u is the force exerted on the carriage of mass $m_c = 5$ [kg], s indicates the position of the carriage, θ is the angle between the pendulum. At the tip b of the pendulum of length $l = 4$ [m] is a fixated mass $m_r = 1$ [kg]. Finally, a is the point of articulation between the rod and the carriage. As seen in Exercise 2.5, the state equations of this system are:

$$
\begin{aligned}
\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{pmatrix} &= \begin{pmatrix} x_3 \\ x_4 \\ \frac{-m_r \sin x_2 (l x_4^2 - g \cos x_2)}{m_c + m_r \sin^2 x_2} \\ \frac{\sin x_2 ((m_c + m_r) g - m_r l x_4^2 \cos x_2)}{l(m_c + m_r \sin^2 x_2)} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \frac{1}{m_c + m_r \sin^2 x_2} \\ \frac{\cos x_2}{l(m_c + m_r \sin^2 x_2)} \end{pmatrix} u, \\
y &= x_1
\end{aligned}
$$

where $\mathbf{x} = (s, \theta, \dot{s}, \dot{\theta})^T = (x_1, x_2, x_3, x_4)^T$. The observation equation indicates that only the position of the carriage x_1 is measured.

1) Calculate all the operating points of the system.

2) Linearize the system around the operating point $\overline{\mathbf{x}} = (0, 0, 0, 0)$ and $\overline{u} = 0$.

3) Obtain an output feedback controller which sets all the poles of the looped system to -2: We propose a control in position, which means that the setpoint w corresponds to x_1. Illustrate by a simulation the behavior of the controller.

4) Intentionally omitted.

Solve using `Matlab`.

[1] Adapted from https://www.ensta-bretagne.fr/jaulin/automooc.pdf