

### Práctica 3. Tracking and pick-and-place problems for a planar vertical robot manipulator.

Consider the planar robot manipulator represented in Figure 1, which moves in a vertical plane.

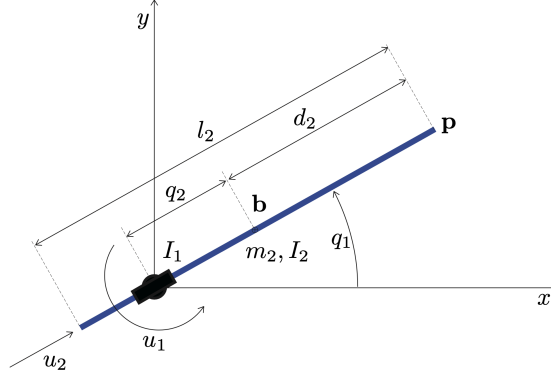


Figure 1: Planar robot manipulator that moves in a vertical plane.

The dynamic model of this robotic system is represented by the second order differential equation

$$\mathbf{B}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{N}(\mathbf{q}) = \mathbf{u},$$

where the two matrices  $\mathbf{B}(\mathbf{q})$  and  $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$  have the following expressions

$$\begin{aligned}\mathbf{B}(\mathbf{q}) &= \begin{bmatrix} I_1 + I_2 + m_2 q_2^2 & 0 \\ 0 & m_2 \end{bmatrix}, \\ \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) &= \begin{bmatrix} 2m_2 q_2 \dot{q}_1 \dot{q}_2 \\ -m_2 q_2 \dot{q}_1^2 \end{bmatrix}, \\ \mathbf{N}(\mathbf{q}) &= m_2 g \begin{bmatrix} q_2 \cos q_1 \\ \sin q_1 \end{bmatrix}.\end{aligned}$$

$\mathbf{q} = (q_1, q_2)^T$  is the vector of configuration variables, where  $q_1$  is the angular position of the link with respect to the  $x$  axis of the reference frame  $\{x, y\}$  and  $q_2$  is the linear position of the center of mass  $\mathbf{b}$  of the link with respect to the origin of the reference frame. The vector  $\dot{\mathbf{q}} = (\dot{q}_1, \dot{q}_2)^T$  is the vector of joint velocities, where  $\dot{q}_1$  is an angular velocity and  $\dot{q}_2$  is a linear velocity. The vector  $\ddot{\mathbf{q}} = (\ddot{q}_1, \ddot{q}_2)^T$  is the vector of accelerations, where  $\ddot{q}_1$  is an angular acceleration and  $\ddot{q}_2$  is a linear acceleration. The control inputs of the system are  $\mathbf{u} = (u_1, u_2)^T$ , where  $u_1$  is the torque applied by the angular actuator to the link and  $u_2$  is the force applied by the linear actuator to the link.  $I_1$  is the barycentric moment of inertia of the angular and linear actuators,  $I_2$  is the barycentric moment of inertia of the link, and  $m_2$  is the mass of the link.

Consider the following two robotic tasks.

- 1) Tracking task: follow with the tip  $\mathbf{p}$  a setpoint describing a circle centred at  $(0.5, 0.5)$  m with radius 0.25 m which moves clockwise with angular velocity 0.25 rad/s.
- 2) Pick and place task: move iteratively the tip  $\mathbf{p}$  between the setpoints  $\mathbf{p}_A = (-0.5, 0.75)$  m and  $\mathbf{p}_B = (0.5, 0.25)$  m. The motions must be rest to rest.

Assume that the initial state of the manipulator is  $(q_1, q_2, \dot{q}_1, \dot{q}_2)^T = (0, 0, 0, 0)^T$ . Assume that  $I_1 = 1 \text{ kg m}^2$ ,  $I_2 = 1 \text{ kg m}^2$ ,  $m_2 = 1 \text{ kg}$ ,  $l_2 = 1 \text{ m}$ ,  $d_2 = 0.5 \text{ m}$ ,  $g = 9.81 \text{ m/s}^2$ .

- a. Demonstrate the equations of the dynamic model using the Lagrange method. (Copia la solución del la Práctica 1)
- b. Compute the state space representation of the dynamics of the manipulator in which  $\mathbf{x} = (x_1, x_2, x_3, x_4)^T = (q_1, q_2, \dot{q}_1, \dot{q}_2)^T$ . Take the coordinates of point  $\mathbf{p}$  as output variables. (Copia la solución del la Práctica 1)
- c. Compute the relation between the configuration variables  $\mathbf{q} = (q_1, q_2)^T$  and the position of the tip  $\mathbf{p} = (p_1, p_2)^T$ . (Copia la solución del la Práctica 1)
- d. Compute the Jacobian matrix of the relation determined in c. (Contesta en el informe y sube el código Matlab a Aula Virtual en el fichero `j_manipulator.m`)
- e. Compute the time derivative of the Jacobian matrix determined in d. (Contesta en el informe y sube el código Matlab a Aula Virtual en el fichero `dot_j_manipulator.m`)
- f. Design a controller based on the transpose Jacobian method to execute the tracking task. Write a Matlab code that implements the controller to execute the task. Show, plotting the relevant variables and an animation, that the controller satisfies the specifications. (Contesta en el informe y sube el código Matlab a Aula Virtual en la carpeta `controller_jt`)
- g. Design a controller based on the feedback linearization method to execute the pick and place task. Write a Matlab code that implements the controller to execute the task. Show, plotting the relevant variables and an animation, that the controller satisfies the specifications. (Contesta en el informe y sube el código Matlab a Aula Virtual en la carpeta `controller_fl`)

Write a detailed report answering each question in a different section. Explain each result obtained and motivate each choice made during the design process. Include the Matlab code in the corresponding section in the written report. Additionally, upload the Matlab code of each section of the project in a separate folder (uncompressed, please). Originality and completeness of the answers will be the aspects that will be taken into account in the grading of the project, and therefore, the Matlab code alone will not be considered.

## Solution of Práctica 3

- a. Demonstrate the equations of the dynamic model using the Lagrange method. (Copia la solución del la Práctica 1)

Your answer goes here

- b. Compute the state space representation of the dynamics of the manipulator in which  $\mathbf{x} = (x_1, x_2, x_3, x_4)^T = (q_1, q_2, \dot{q}_1, \dot{q}_2)^T$ . Take the coordinates of point  $\mathbf{p}$  as output variables. (Copia la solución del la Práctica 1)

Your answer goes here

- c. Compute the relation between the configuration variables  $\mathbf{q} = (q_1, q_2)^T$  and the position of the tip  $\mathbf{p} = (p_1, p_2)^T$ . (Copia la solución del la Práctica 1)

Your answer goes here

- d. Compute the Jacobian matrix of the relation determined in c. (Contesta en el informe y sube el código Matlab a Aula Virtual en el fichero `j_manipulator.m`)

Your answer goes here

File `j_manipulator.m`

Your Matlab code goes here

- e. Compute the time derivative of the Jacobian matrix determined in d. (Contesta en el informe y sube el código Matlab a Aula Virtual en el fichero `dot_j_manipulator.m`)

Your answer goes here

File `dot_j_manipulator.m`

Your Matlab code goes here

- f. Design a controller based on the transpose Jacobian method to execute the tracking task. Write a Matlab code that implements the controller to execute the task. Show, plotting the relevant variables and an animation, that the controller satisfies the specifications. (Contesta en el informe y sube el código Matlab a Aula Virtual en la carpeta controller\_jt)

Your answer goes here

File controller\_jt\_init.m

Your Matlab code goes here

File controller\_jt\_f.m

Your Matlab code goes here

File controller\_jt\_draw.m

Your Matlab code goes here

File controller\_jt\_main.m

Your Matlab code goes here

Your figures go here

- g. Design a controller based on the feedback linearization method to execute the pick and place task. Write a Matlab code that implements the controller to execute the task. Show, plotting the relevant variables and an animation, that the controller satisfies the specifications. (Contesta en el informe y sube el código Matlab a Aula Virtual en la carpeta controller\_fl)

Your answer goes here

File controller\_fl\_init.m

Your Matlab code goes here

File controller\_fl\_f.m

Your Matlab code goes here

File controller\_fl\_draw.m

Your Matlab code goes here

File controller\_fl\_main.m

Your Matlab code goes here

Your figures go here