Exercise 5.6. Kalman decomposition[1]

A linear system can always be decomposed, after a suitable change of basis, into four subsystems $\mathcal{S}_1$, $\mathcal{S}_2$, $\mathcal{S}_3$, $\mathcal{S}_4$ where $\mathcal{S}_1$ is controllable and observable, $\mathcal{S}_2$ is non-controllable and observable, $\mathcal{S}_3$ is controllable and non-observable and $\mathcal{S}_4$ is neither controllable nor observable. The dependencies between the subsystems can be summarized in Figure 1

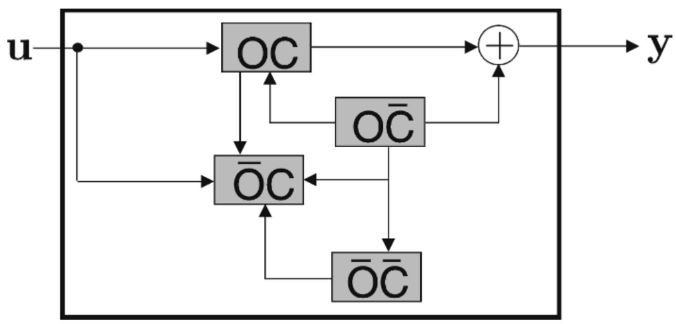

Figure 1: Principle of the Kalman decomposition: C controllable, O observable, $\overline{C}$ non-controllable, $\overline{O}$ non-observable.

Let us note that, in the figure, there is no path (respecting the direction of the arrows) leading from the input u to a non-controllable system. Similarly, there is no path leading from a non-observable system to y. We consider the system described by the state equation:

$$
\begin{aligned}
\dot{\mathbf{x}}(t) &= \begin{pmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} & 0 & 0 \\ 0 & \mathbf{A}_{22} & 0 & 0 \\ \mathbf{A}_{31} & \mathbf{A}_{32} & \mathbf{A}_{33} & \mathbf{A}_{34} \\ 0 & \mathbf{A}_{42} & 0 & \mathbf{A}_{44} \end{pmatrix} \mathbf{x}(t) + \begin{pmatrix} \mathbf{B}_1 \\ 0 \\ \mathbf{B}_3 \\ 0 \end{pmatrix} \mathbf{u}(t) \\
\mathbf{y}(t) &= \begin{pmatrix} \mathbf{C}_1 & \mathbf{C}_2 & 0 & 0 \end{pmatrix} \mathbf{x}(t) + \mathbf{D}\,\mathbf{u}(t)
\end{aligned}
$$

Draw a wiring diagram of the system. Show a decomposition in subsystems $\mathcal{S}_i$ corresponding to the Kalman decomposition.

[1] Adapted from https://www.ensta-bretagne.fr/jaulin/automooc.pdf

Solution of Exercise 5.6

The requested diagram is represented in Figure 2. We can see the four subsystems: $\mathcal{S}_1$ (controllable and observable), $\mathcal{S}_2$ (non-controllable and observable), $\mathcal{S}_3$ (controllable and non-observable) and $\mathcal{S}_4$ (neither controllable nor observable).

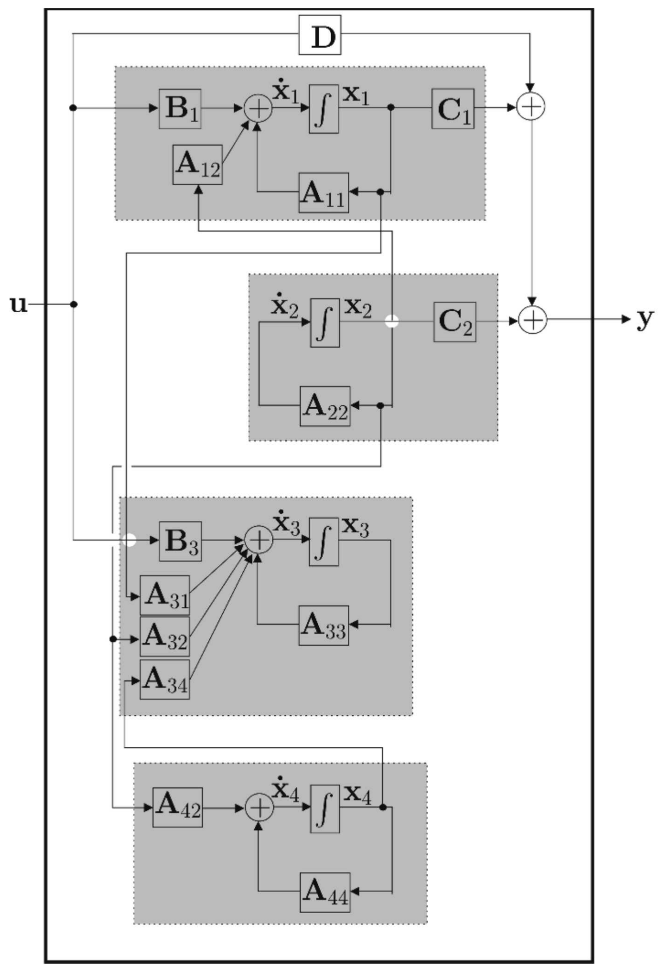

Figure 2: Kalman decomposition for linear systems.