Ingeniería de Control
Exam November 2020

Consider the simplified planar model of a rocket represented in Figure 1, in which the position of the center of mass of the rocket, (x, y), and the inclination angle of the rocket, θ, are the state variables of the system. The thrust force u_1 and its direction u_2 are the control inputs of the system.

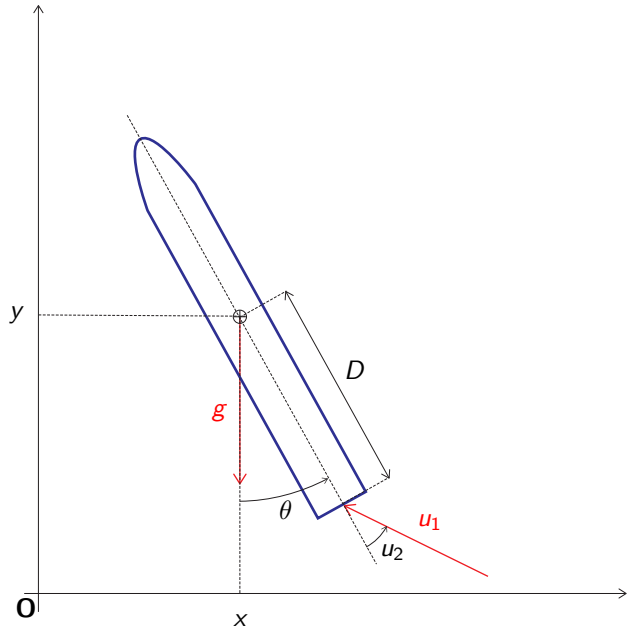

Figure 1: Sketch of the space rocket.

The dynamic model of this system is

$$\begin{aligned} \ddot{x} &= -\frac{1}{m}\sin(\theta + u_2)u_1, \\ \ddot{y} &= -g + \frac{1}{m}\cos(\theta + u_2)u_1, \\ \ddot{\theta} &= -\frac{D}{J}\sin(u_2)u_1, \end{aligned}$$

with the following parameters

- barycentric moment of inertia of the rocket: $J = 1050$ [kg m^2],
- mass of the rocket: $m = 450$ [kg],
- $D = 1.25$ [m],
- gravity acceleration: $g = 9.81$ [m/s^2].

1) Explain the two equations of the dynamic model.

2) Calculate the state space representation of the system, assuming that $\mathbf{x} = (x, y, \theta, \dot{x}, \dot{y}, \dot{\theta})^T = (x_1, x_2, x_3, x_4, x_5, x_6)^T$, where distances are measured in [m], angles in [rad], linear velocities in [m/s] angular velocities in [rad/s].

3) Calculate all the operating points of the system and explain the obtained result.

4) Find the operating point that corresponds to $\overline{u}_1 = mg, \overline{u}_2 = 0$. Linearize the system around this operating point.

5) Using the LQR method, design a state feedback controller to control the landing of the rocket. We want to steer the rocket from the state $\mathbf{x} = (x, y, \theta, \dot{x}, \dot{y}, \dot{\theta})^T = (0.25, 2.25, 0.0174533, -0.1, -0.2, 0.00174533)^T$ to the state $\mathbf{x} = (0, 1.25, 0, 0, 0, 0)^T$. Give the matrices Q and R that have been assigned to the regulator and illustrate the behaviour of the controller by plotting the relevant state and control variables and by a graphical simulation.

Originality and completeness of the answers will be the aspects that will be taken into account in the grading of the project, and therefore, the `Matlab` code alone will not be considered.