Ingeniería de Control
Problem 1

Aircraft vertical takeoff and landing

Consider the simplified planar model of the system for vertical takeoff and landing of an aircraft represented in Figure 1, in which the aircraft is represented by a bar. The position of the center of mass of the aircraft, $\mathbf{c} = (x, y)^T$, the roll angle of the aircraft, θ, and their time derivatives are the state variables of the system. The thrust force S, applied to the center of mass of the aircraft, and the forces F, applied to the wing tips, are the control inputs u_1 and u_2 of the system, respectively. The thrust force S keeps the aircraft flying. The forces F, which always act in opposite directions, control the roll of the aircraft.

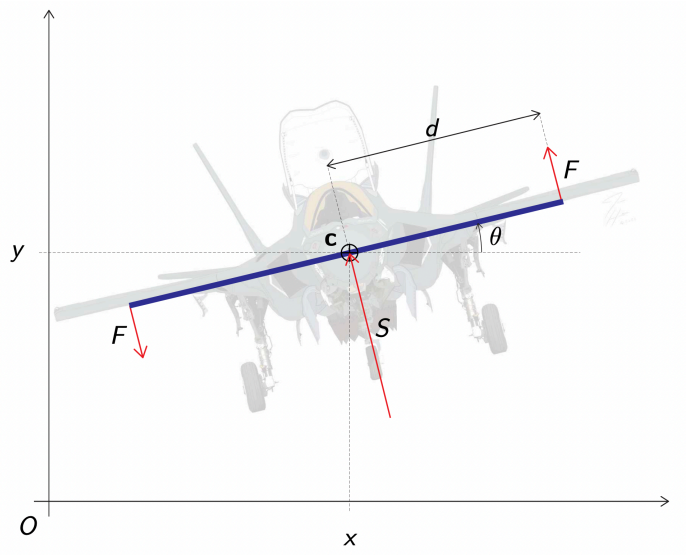

Figure 1: Sketch of the system for aircraft vertical takeoff and landing.

The dynamic model of this system is

$$\begin{aligned} \ddot{x} &= -\frac{1}{m}\sin(\theta)S, \\ \ddot{y} &= -g + \frac{1}{m}\cos(\theta)S, \\ \ddot{\theta} &= \frac{2d}{J}F, \end{aligned}$$

with the following parameters

- barycentric moment of inertia of the aircraft: $J = 10000$ [kg m^2],
- mass of the aircraft: $m = 30000$ [kg],
- $d = 5.5$ [m],
- gravity acceleration: $g = 9.81$ [m/s^2].

1) Demonstrate the equations of the dynamic model using the Lagrange method.
2) Calculate the state space representation of the system, assuming that $\mathbf{x} = (x, y, \theta, \dot{x}, \dot{y}, \dot{\theta})^T = (x_1, x_2, x_3, x_4, x_5, x_6)^T$, where distances are measured in [m], angles in [rad], linear velocities in [m/s], and angular velocities in [rad/s].

Write a detailed report answering each question in a different section. Originality and completeness of the answers will be the aspects that will be taken into account in the grading of the report.