Model of a tank robot[1]

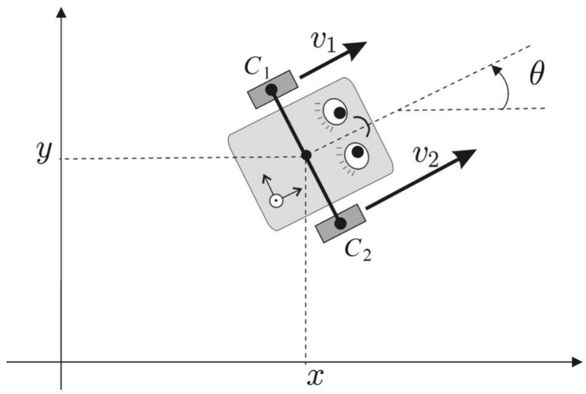

Figure 1: Robot tank viewed from above

Let us choose as state vector the vector $\mathbf{x} = (x, y, \theta, v_1, v_2)^T$.

Let l be the distance between points C_1 and C_2. The tank robot rotates around point C, the center of the axis of the wheels. Thus $l/2$ will be the distance between point C and the points C_1 and C_2.

Consider the Varignon's formula applied to the velocities of the points C_1 and C_2 that coincide with the linear velocities of the centers of the two wheels $\mathbf{v}_1$ and $\mathbf{v}_2$. Note that this relation is a vector relation that depends on the frame. Let us express it in the frame of the tank represented in Figure 2:

$$\mathbf{v}_2 = \mathbf{v}_1 + \overrightarrow{C_2C_1} \wedge \overrightarrow{\omega}.$$

In this equation, $\overrightarrow{C_2C_1}$ is the vector from C_2 to C_1, $\overrightarrow{\omega}$ is the (counterclockwise) angular velocity of the tank robot which, in this case, is a vector in the direction of Z pointing towards the viewer. The module of vector $\overrightarrow{\omega}$ is $\dot{\theta}$. Symbol $\wedge$ represents the cross product between vectors. If we express the vectors explicitly, we obtain:

$$\begin{pmatrix} v_2 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} v_1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ l \\ 0 \end{pmatrix} \wedge \begin{pmatrix} 0 \\ 0 \\ \dot{\theta} \end{pmatrix}$$

. Calculating the cross product using, for instance, the determinant rule, we easily get the scalar relation $v_2 = v_1 + l\dot{\theta}$. From this relation we can compute $\dot{\theta}$ as follows: $\dot{\theta} = \frac{v_2 - v_1}{l}$.

The Varignon's formula can be written with respect to points C and C_2. We get

$$\mathbf{v}_2 = \mathbf{v} + \overrightarrow{C_2C} \wedge \overrightarrow{\omega},$$

[1]Adapted from https://www.ensta-bretagne.fr/jaulin/automooc.pdf

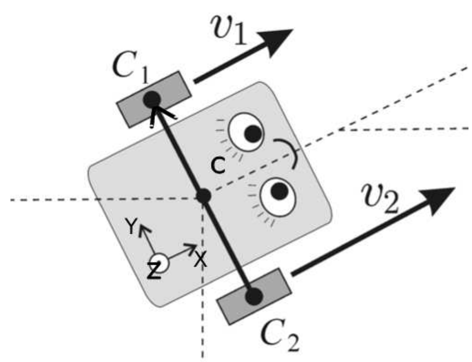

Figure 2: Detail of the robot tank viewed from above. The point C is the center of the axis of the wheels. The reference frame has a Z vertical axis pointing towards the viewer.

being $\mathbf{v}$ the linear velocity of point C. With the same procedure as above, we get the scalar equation: $v_2 = v + \frac{l}{2}.\dot{\theta}$

The Varignon's formula can also be written with respect to points C and C_1. We get

$$\mathbf{v} = \mathbf{v_1} + \overrightarrow{CC_1} \wedge \overrightarrow{\omega},$$

being $\mathbf{v}$ the linear velocity of point C. Proceeding as above, we get the scalar equation: $v = v_1 + \frac{l}{2}\dot{\theta}$, which can be rewritten as $v_1 = v - \frac{l}{2}\dot{\theta}$.

Summing the terms of equations $v_2 = v + \frac{l}{2}\dot{\theta}$ and $v_1 = v - \frac{l}{2}\dot{\theta}$ we get

$$v = \frac{v_1 + v_2}{2}.$$

Let us suppose that the control inputs of the tank robot are the angular acceleration of the axis of the two wheels denoted by u_1 ad u_2. The relation between the angular acceleration u_1 of the axis of Wheel 1 and the linear acceleration $\dot{v}_1$ of the center of the same wheel is $\dot{v}_1 = Ru_1$. Likewise, $\dot{v}_2 = Ru_2$.

Thus, the state equations of the system are

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{v}_1 \\ \dot{v}_2 \end{pmatrix} = \begin{pmatrix} \frac{v_1+v_2}{2}\cos\theta \\ \frac{v_1+v_2}{2}\sin\theta \\ \frac{v_2-v_1}{l} \\ Ru_1 \\ Ru_2 \end{pmatrix},$$

where $\dot{x}$ and $\dot{y}$ are the velocities of the tank robot in the positive direction of x and y, respectively.