

Ingeniería de Control

Exam November 24, 2022

Self balancing motorcycle

Consider the simplified planar model of the control system for self balancing of a motorcycle represented in Figure 1, in which the motorcycle is represented by a bar that rotates around point b . Points c_1 and c_2 are the positions of the centers of mass of the motorcycle and the reaction wheel, respectively. The control system is composed of an actuated reaction wheel, which rotates around point c_2 . A motor controls the rotation of the reaction wheel applying a torque τ , which is the only input of the control system. Angles q_1 , q_2 and their time derivatives are the state variables of the system.

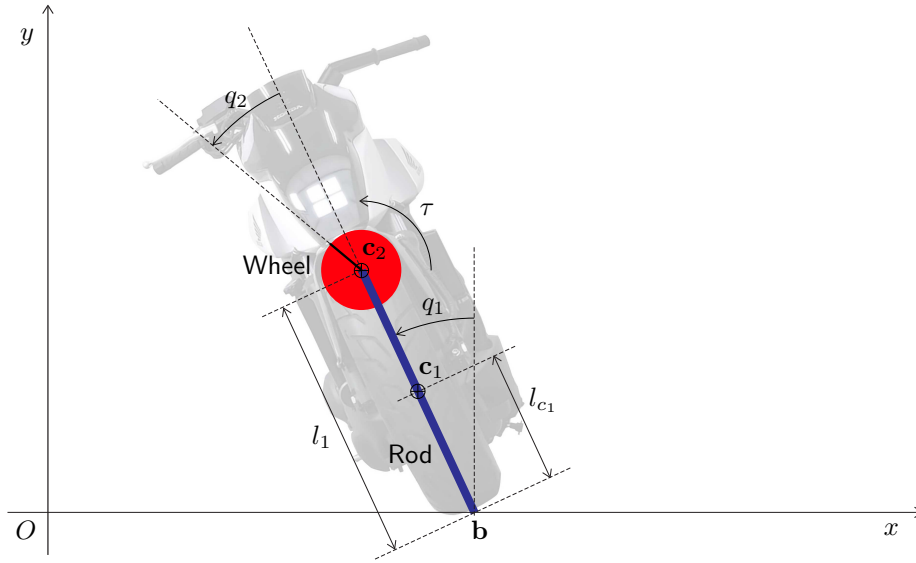


Figure 1: Sketch of the control system for self balancing of a motorcycle

The dynamic model of this system is

$$\begin{aligned} (m_1 l_{c_1}^2 + m_2 l_1^2 + I_1 + I_2) \ddot{q}_1 + I_2 \ddot{q}_2 - (m_1 l_{c_1} + m_2 l_1) g \sin(q_1) &= 0, \\ I_2 \ddot{q}_1 + I_2 \ddot{q}_2 &= \tau, \end{aligned}$$

with the following parameters

- mass of the motorcycle $m_1 = 200$ [kg],

- mass of the reaction wheel $m_2 = 50$ [kg],
- barycentric moment of inertia of the motorcycle $I_1 = 25$ [kg m²],
- barycentric moment of inertia of the reaction wheel $I_2 = 5$ [kg m²],
- length of the rod $l_1 = 1$ [m],
- $l_{c_1} = \frac{l_1}{2} = 0.5$ [m],
- gravity acceleration: $g = 9.81$ [m/s²].

- 1) Demonstrate the equations of the dynamic model using the Lagrange method. (Contesta en el cuadernillo)
- 2) Calculate the state space representation of the system, assuming that $\mathbf{x} = (x_1, x_2, x_3, x_4)^T = (q_1, q_2, \dot{q}_1, \dot{q}_2)^T$, where angles are expressed in [rad] and angular velocities in [rad/s]. (Contesta en el cuadernillo y sube el código Matlab a Aula Virtual en el fichero `statespace.m`)
- 3) Calculate all the operating points of the system and explain the obtained result. (Contesta en el cuadernillo y sube el código Matlab a Aula Virtual en el fichero `equilibrium.m`)
- 4) Find the operating point that corresponds to $\bar{x}_1 = 0, \bar{u} = 0$. Linearize the system around this operating point. (Contesta en el cuadernillo y sube el código Matlab a Aula Virtual en el fichero `linearization.m`)
- 5) Is the linearized system controllable? (Contesta en el cuadernillo y sube el código Matlab a Aula Virtual en el fichero `controllability.m`)
- 6) Is the linearized system observable by means of the output $y = x_1$? Is the linearized system observable by means of the output $y = x_2$? (Contesta en el cuadernillo y sube el código Matlab a Aula Virtual en el fichero `observability.m`)
- 7) Using the LQR method, design a state feedback controller to control the inclination of the motorcycle. We want to steer the motorcycle from the state $\mathbf{x} = (q_1, q_2, \dot{q}_1, \dot{q}_2)^T = (0.261799, 0, 0, 0)^T$ to the state $\mathbf{x} = (0, 0, 0, 0)^T$. Give the eigenvalues that have been assigned to the controlled system and illustrate the behaviour of the controller by plotting the relevant state and control variables and by a graphical animation. (Contesta en el cuadernillo y sube el código Matlab a Aula Virtual en la carpeta `controller`)

Originality and completeness of the written answers will be the aspects that will be taken into account in the grading of the exam, and therefore, the Matlab code alone will not be considered.