Exercise 5.8. Output feedback of a scalar system[1]

Let us consider the following state equation:

$$\begin{aligned} \dot{x} &= 3x + 2u, \\ y &= 4x. \end{aligned}$$

1) Propose an output feedback controller that puts all the poles in -1 and such that the setpoint variable corresponding to x (in other words if we fix the setpoint at $\overline{w}$, we want the state x to converge toward $\overline{w}$).

2) Give the state equations of the looped system. What are the poles of the looped system?

Solution of Exercise 5.8

1) Propose an output feedback controller that puts all the poles in -1 and such that the setpoint variable corresponding to x (in other words if we fix the setpoint at $\overline{w}$, we want the state x to converge toward $\overline{w}$).

   We apply the KLH method. In order to find K and L, we need to solve:

$$\begin{aligned} \det(s\mathbf{I} - \mathbf{A} + \mathbf{BK}) &= P_{\text{con}}, \\ \det(s\mathbf{I} - \mathbf{A} + \mathbf{LC}) &= P_{\text{obs}}, \end{aligned}$$

   which in this case become

$$\begin{aligned} s - 3 + 2K &= s + 1, \\ s - 3 + 4L &= s + 1. \end{aligned}$$

   We obtain K $= 2$ and L $= 1$. For the calculation of the precompensator, we will take E $= 1$ (since the setpoint variable is $x_c = x$). Thus:

$$H = -(E(A - BK)^{-1}B)^{-1} = \frac{-1}{1 \cdot (3 - 2 \cdot 2)^{-1} \cdot 2} = \frac{1}{2}.$$

   The controller we are looking for is, therefore, given by:

$$\begin{aligned} \frac{d}{dt}\hat{\mathbf{x}} &= (\mathbf{A} - \mathbf{BK} - \mathbf{LC})\hat{\mathbf{x}} + \mathbf{BHw} + \mathbf{Ly} \\ \mathbf{u} &= -\mathbf{K}\hat{\mathbf{x}} + \mathbf{Hw} \end{aligned}$$

   which in this case become

[1] Adapted from https://www.ensta-bretagne.fr/jaulin/automooc.pdf

$$\begin{aligned} \frac{d}{dt}\hat{x} &= -5\hat{x} + w + y, \\ u &= -2\hat{x} + \frac{1}{2}w. \end{aligned}$$

2) Give the state equations of the looped system. What are the poles of the looped system?

We have two systems

$$\begin{aligned} \dot{x} &= 3x + 2u, \\ y &= 4x. \end{aligned}$$

and

$$\begin{aligned} \frac{d}{dt}\hat{x} &= -5\hat{x} + w + y, \\ u &= -2\hat{x} + \frac{1}{2}w. \end{aligned}$$

By replacing the expressions of u and y, we obtain that the looped system is described by the following evolution equations:

$$\begin{aligned} \dot{x} &= 3x - 4\hat{x} + w, \\ \frac{d}{dt}\hat{x} &= 4x - 5\hat{x} + w. \end{aligned}$$

The evolution matrix of this system is

$$\mathbf{A} = \begin{pmatrix} 3 & -4 \\ 4 & -5 \end{pmatrix}$$

whose eigenvalues are -1 and -1.

```
clear all
close all
clc

A=[3 -4;
   4 -5];

eigA=eig(A)
```

```
eigA =

  -1.0000 + 0.0000i
  -1.0000 - 0.0000i
```

We verified that the poles of this system are the ones we have placed, which is a consequence of the separation principle (see Exercise 5.9).

Simulate the step response of the system using Euler and Runge-Kutta methods