Ingeniería de Control
Exam November 25, 2021

Aircraft vertical takeoff and landing

Consider the simplified planar model of the system for vertical takeoff and landing of an aircraft represented in Figure 1, in which the aircraft is represented by a bar. The position of the center of mass of the aircraft, $\mathbf{c} = (x, y)^T$, the roll angle of the aircraft, θ, and their time derivatives are the state variables of the system. The thrust force S, applied to the center of mass of the aircraft, and the forces F, applied to the wing tips, are the control inputs u_1 and u_2 of the system, respectively. The thrust force S keeps the aircraft flying. The forces F, which always act in opposite directions, control the roll of the aircraft.

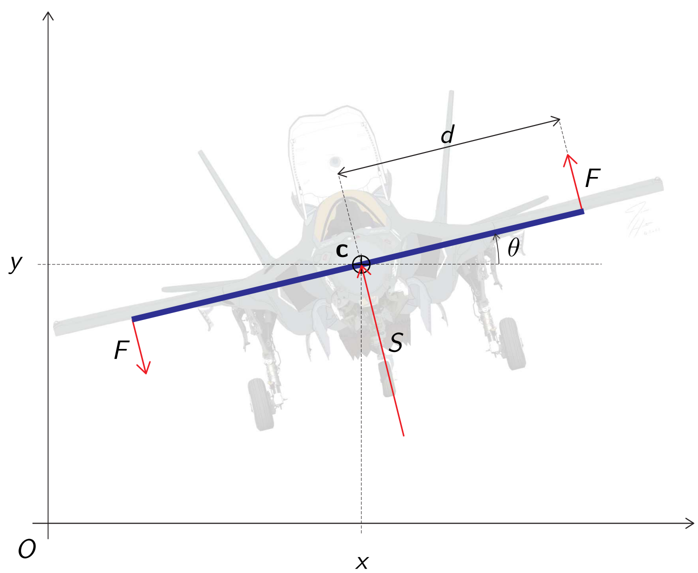

Figure 1: Sketch of the system for aircraft vertical takeoff and landing.

The dynamic model of this system is

$$\begin{aligned}
\ddot{x} &= -\frac{1}{m}\sin(\theta)S, \\
\ddot{y} &= -g + \frac{1}{m}\cos(\theta)S, \\
\ddot{\theta} &= \frac{2d}{J}F,
\end{aligned}$$

with the following parameters

- barycentric moment of inertia of the aircraft: $J = 10000$ [kg m^2],
- mass of the aircraft: $m = 30000$ [kg],
- $d = 5.5$ [m],
- gravity acceleration: $g = 9.81$ [m/s^2].

1) Demonstrate the equations of the dynamic model using the Lagrange method.

2) Calculate the state space representation of the system, assuming that $\mathbf{x} = (x, y, \theta, \dot{x}, \dot{y}, \dot{\theta})^T = (x_1, x_2, x_3, x_4, x_5, x_6)^T$, where distances are measured in [m], angles in [rad], linear velocities in [m/s], and angular velocities in [rad/s].

3) Calculate all the operating points of the system and explain the obtained result. (Contesta en el cuadernillo y sube el código `Matlab` a Aula Virtual en el fichero `equilibrium.m`)

4) Find the operating point that corresponds to $\overline{u}_1 = mg, \overline{u}_2 = 0$. Linearize the system around this operating point. (Contesta en el cuadernillo y sube el código `Matlab` a Aula Virtual en el fichero `linearization.m`)

5) Is the linearized system controllable using both control inputs u_1 and u_2? Is the linearized system controllable using only the control input u_1? (Contesta en el cuadernillo y sube el código `Matlab` a Aula Virtual en el fichero `controllability.m`)

6) Using the LQR method, design a state feedback controller to control the landing of the aircraft. We want to steer the aircraft from the state $\mathbf{x} = (x, y, \theta, \dot{x}, \dot{y}, \dot{\theta})^T = (1, 5, 0.0174533, -0.1, -0.2, 0.00174533)^T$ to the state $\mathbf{x} = (0, 2.4, 0, 0, 0, 0)^T$. Give the matrices Q and R that have been assigned to the controlled system, the corresponding eigenvalues, and illustrate the behaviour of the controller by plotting the relevant state and control variables and by a graphical animation. (Contesta en el cuadernillo y sube el código `Matlab` a Aula Virtual en la carpeta `controller`)

Originality and completeness of the written answers will be the aspects that will be taken into account in the grading of the exam, and therefore, the `Matlab` code alone will not be considered.