Práctica 1. Dynamic model of a robot manipulator with the Lagrange method.

Consider the robot manipulator represented in Figure 1, which moves in a vertical plane.

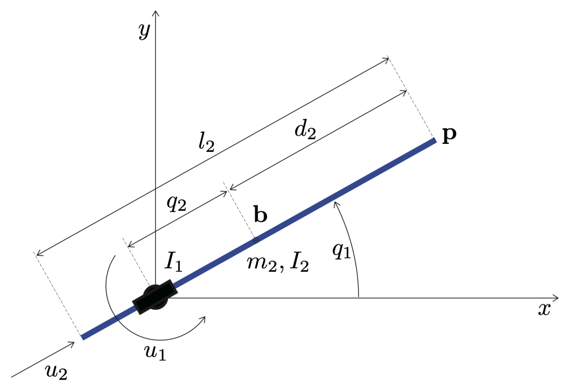

Figure 1: Planar robot manipulator that moves in a vertical plane.

The dynamic model of this robotic system is represented by the second order differential equation

$$\mathbf{B}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{N}(\mathbf{q}) = \mathbf{u},$$

where the two matrices $\mathbf{B}(\mathbf{q})$ and $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ have the following expressions

$$\begin{aligned}
\mathbf{B}(\mathbf{q}) &= \begin{bmatrix} I_1 + I_2 + m_2 q_2^2 & 0 \\ 0 & m_2 \end{bmatrix}, \\
\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) &= \begin{bmatrix} 2 m_2 q_2 \dot{q}_1 \dot{q}_2 \\ -m_2 q_2 \dot{q}_1^2 \end{bmatrix}, \\
\mathbf{N}(\mathbf{q}) &= m_2 g \begin{bmatrix} q_2 \cos q_1 \\ \sin q_1 \end{bmatrix}.
\end{aligned}$$

$\mathbf{q} = (q_1, q_2)^T$ is the vector of configuration variables, where q_1 is the angular position of the link with respect to the x axis of the reference frame $\{x, y\}$ and q_2 is the linear position of the center of mass $\mathbf{b}$ of the link with respect to the origin of the reference frame. The vector $\dot{\mathbf{q}} = (\dot{q}_1, \dot{q}_2)^T$ is the vector of joint velocities, where $\dot{q}_1$ is an angular velocity and $\dot{q}_2$ is a linear velovity. The vector $\ddot{\mathbf{q}} = (\ddot{q}_1, \ddot{q}_2)^T$ is the vector of accelerations, where $\ddot{q}_1$ is an angular acceleration and $\ddot{q}_2$ is a linear acceleration. The control inputs of the system are $\mathbf{u} = (u_1, u_2)^T$, where u_1 is the torque applied by the angular actuator to the link and u_2 is the force applied by the linear actuator to the link. I_1 is the barycentric moment of inertia of the angular and linear actuators, I_2 is the barycentric moment of inertia of the link, and m_2 is the mass of the link.

a. Demonstrate the equations of the dynamic model using the Lagrange method.

b. Compute the state space representation of the dynamics of the manipulator in which $\mathbf{x} = (x_1, x_2, x_3, x_4)^T = (q_1, q_2, \dot{q}_1, \dot{q}_2)^T$. Take the coordinates of point $\mathbf{p}$ as the output variables.

Write a detailed report answering each question in a different section. Originality and completeness of the answers will be the aspects that will be taken into account in the grading of the report.

Solution of Práctica 1

a. Demonstrate the equations of the dynamic model using the Lagrange method.

   The solution goes here

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