Exercise Car-trailer system[1]

Let us consider the car studied in Exercise 2.7. Let us add a trailer to this car whose attachment point is found in the middle of the rear axle of the car, as illustrated by Figure 1. Find the state equations of the car-trailer system.

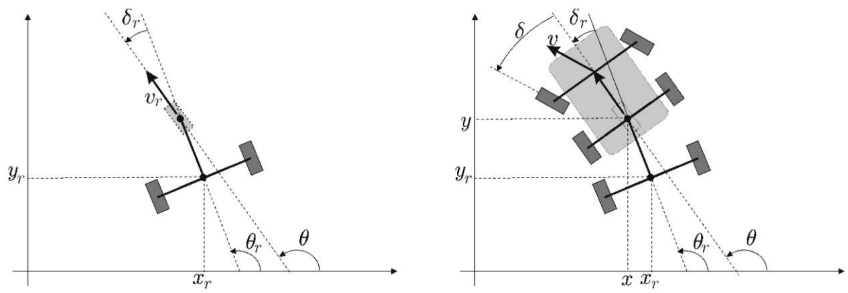

Figure 1: Car with a trailer.

Solution of the exercise

In this system, a trailer is attached to the car. The attachment point is in the middle of the rear axis of the car.

The state equation of the car with a trailer can be easily obtained for the state equations of the car which are

$$\begin{aligned}\dot{x} &= v\cos\delta\cos\theta,\\ \dot{y} &= v\cos\delta\sin\theta,\\ \dot{\theta} &= \frac{v\sin\delta}{L},\\ \dot{v} &= u_1,\\ \dot{\delta} &= u_2,\end{aligned}$$

whose state vector is

$$\mathbf{x} = \begin{pmatrix} x\\ y\\ \theta\\ v\\ \delta \end{pmatrix}.$$

[1]Adapted from L. Jaulin, Automation for Robotics, ISTE-Wiley, 2015.

The state vector is of the car with trailer is

$$\mathbf{x} = \begin{pmatrix} x \\ y \\ \theta \\ \theta_r \\ v \\ \delta \end{pmatrix},$$

in which only the state variable θ_r has been added to those of the car. It is easy to see that the other parameters of the model can be calculated knowing the state of the car and the angle θ_r.

The parameter L_r represents the distance between the attachment point and the middle of the axle of the trailer.

From the state equation of the car

$$\dot{\theta} = \frac{v \sin \delta}{L},$$

we can deduce that

$$\dot{\theta}_r = \frac{v_r \sin \delta_r}{L_r}.$$

From the state equations of the car

$$\begin{aligned} \dot{x} &= v \cos \delta \cos \theta, \\ \dot{y} &= v \cos \delta \sin \theta, \end{aligned}$$

we can deduce that

$$v_r = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{(v \cos \delta \cos \theta)^2 + (v \cos \delta \sin \theta)^2} = \sqrt{v^2 \cos^2 \delta (\cos^2 \theta + \sin^2 \theta)} = v \cos \delta.$$

From the figure we know that

$$\delta_r = \theta - \theta_r.$$

Thus, the state equations of the car-trailer system are

$$\begin{aligned} \dot{x} &= v \cos \delta \cos \theta, \\ \dot{y} &= v \cos \delta \sin \theta, \\ \dot{\theta} &= \frac{v \sin \delta}{L}, \\ \dot{\theta}_r &= \frac{v \cos \delta \sin(\theta - \theta_r)}{L_r}, \\ \dot{v} &= u_1, \\ \dot{\delta} &= u_2. \end{aligned}$$