Problem 2: Simulation of a car-trailer system[1]

Let us consider the car-trailer system represented in Figure 1 which has the following state equations:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\theta}_r \\ \dot{v} \\ \dot{\delta} \end{pmatrix} = \begin{pmatrix} v\cos\delta\cos\theta \\ v\cos\delta\sin\theta \\ \frac{v\sin\delta}{L} \\ \frac{v\cos\delta\sin(\theta-\theta_r)}{L_r} \\ u_1 \\ u_2 \end{pmatrix}.$$

The state vector is given by

$$\mathbf{x} = \begin{pmatrix} x \\ y \\ \theta \\ \theta_r \\ v \\ \delta \end{pmatrix},$$

where x, y, θ corresponds to the pose of the car (in other words, its position and orientation), θ_r is the orientation of the trailer, v is the speed and δ is the angle of the front wheels of the car. The parameter $L = 3$ [m] is the distance between the two axles of the car. The parameter $L_r = 5$ [m] represents the distance between the attachment point and the middle of the axle of the trailer.

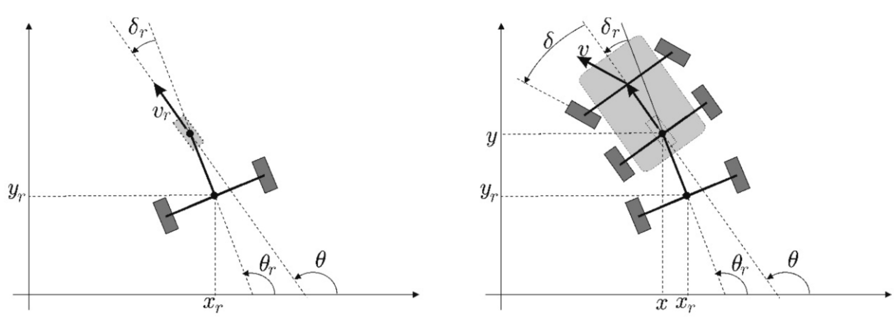

Figure 1: The car-trailer system.

1) Using homogeneous coordinates, design a function which draws a car-trailer system in a state $\mathbf{x} = (x, y, \theta, \theta_r, v, \delta)^T$ using the sketch of the car-trailer system represented in Figure 2 whose vertices in homogeneous coordinates are represented by the columns of the following matrix:

[1] Adapted from https://www.ensta-bretagne.fr/jaulin/automooc.pdf

$$M_{\text{trailer}} = \begin{pmatrix} -1 & 2 & 5 & 5 & 2 & -1 & -1 & -1 & 0 & 0 & -1 & 1 & 0 & 0 & -1 & 1 & 0 & 0 & 2 \\ -2 & -2 & 0 & 0 & 2 & 2 & -2 & -2 & -2 & -3 & -3 & -3 & -3 & 3 & 3 & 3 & 3 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

The corresponding `Matlab` code is:

```
M_trailer=[-1  2  5  5 2 -1 -1 -1 0  0  -1  1  0 0 -1 1 0 0 2;
           -2 -2  0  0 2 2  -2 -2 -2 -3 -3 -3 -3 3  3 3 3 2 2;
           ones(1,19)];
```

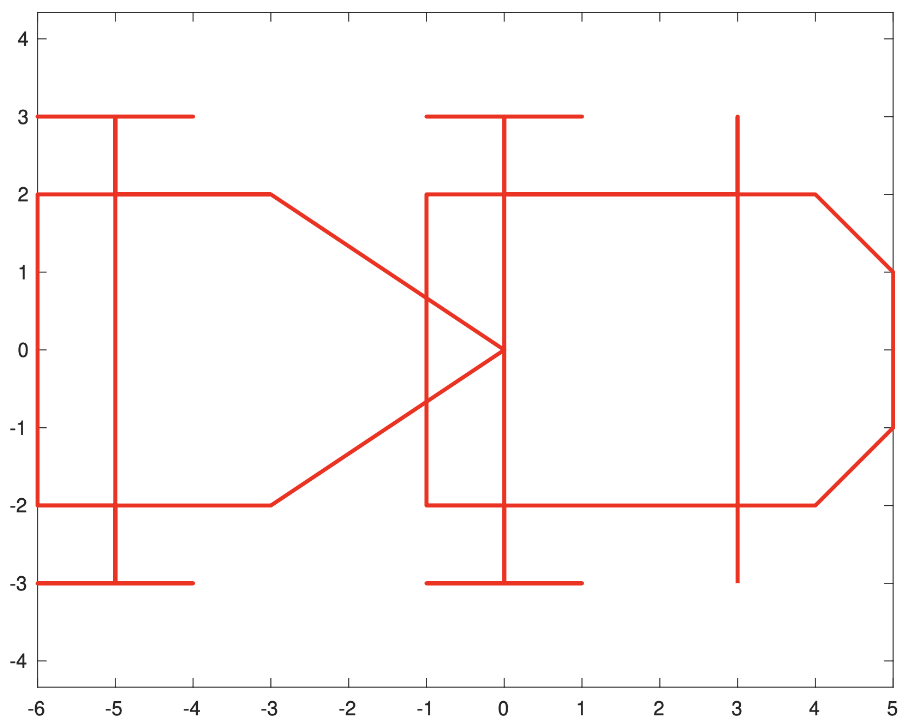

Figure 2: Sketch of the car-trailer system.

2) Propose a program which simulates the dynamic evolution of this car-trailer system during 5 seconds with Euler's method and a sampling step of 0.01 [s]. Take the initial state for the car-trailer system as $\mathbf{x}(0) = (0, 0, 0, 0, 50, 0)^T$, which means that, at time $t = 0$,

- the car is centered around the origin, with a zero orientation angle, a speed of 50 [m s^{-1}] and the front wheels parallel to the axis of the car.
- the trailer has zero orientation angle.

We assume that the vectorial control $u(t)$ remains constant and equal to $(0, 0.05)$. Which means that the car does not accelerate (since $u_1 = 0$) and that the steering wheel is turning at a constant speed of 0.05 [rad s^{-1}].

Solution of Problem 2

1) Using homogeneous coordinates, design a function which draws a car-trailer system in a state $\mathbf{x} = (x, y, \theta, \theta_r, v, \delta)^T$ using the sketch of the car-trailer system represented in Figure 2 whose vertices in homogeneous coordinates are represented by the columns of the following matrix:

$$M_{\text{trailer}} = \begin{pmatrix} -1 & 2 & 5 & 5 & 2 & -1 & -1 & -1 & 0 & 0 & -1 & 1 & 0 & 0 & -1 & 1 & 0 & 0 & 2 \\ -2 & -2 & 0 & 0 & 2 & 2 & -2 & -2 & -2 & -3 & -3 & -3 & -3 & 3 & 3 & 3 & 3 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

The corresponding `Matlab` code is:

```
M_trailer=[-1  2  5  5 2 -1 -1 -1 0  0  -1  1  0 0 -1 1 0 0 2;
           -2 -2  0  0 2 2  -2 -2 -2 -3 -3 -3 -3 3  3 3 3 2 2;
           ones(1,19)];
```

Your solution goes here

File `namefile1.m`

```
Your Matlab code goes here
```

2) Propose a program which simulates the dynamic evolution of this car-trailer system during 5 seconds with Euler's method and a sampling step of 0.01 [s]. Take the initial state for the car-trailer system as $\mathbf{x}(0) = (0, 0, 0, 0, 50, 0)^T$, which means that, at time $t = 0$,

   - the car is centered around the origin, with a zero orientation angle, a speed of 50 [m s^{-1}] and the front wheels parallel to the axis of the car.
   - the trailer has zero orientation angle.

   We assume that the vectorial control $u(t)$ remains constant and equal to $(0, 0.05)$. Which means that the car does not accelerate (since $u_1 = 0$) and that the steering wheel is turning at a constant speed of 0.05 [rad s^{-1}].

File `namefile2.m`

```
Your Matlab code goes here
```

File `namefile3.m`

```
Your Matlab code goes here
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File `namefile4.m`

```
Your Matlab code goes here
```

Write a detailed report answering each question in a different section. Originality and completeness of the answers will be the aspects that will be taken into account in the grading of the report. Include the Matlab code in the report. Additionally, upload the Matlab code in a separate folder.