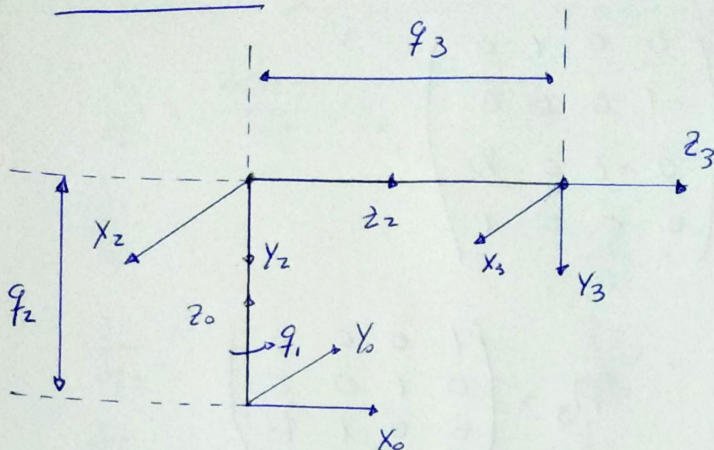


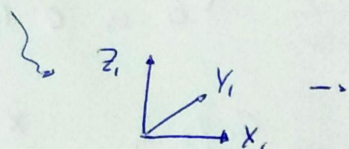
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Problema



1 articulación rotacional
2 articulaciones prismáticas



El sistema 1 está en la misma posición que el sistema 0

• Cinemática directa:

$${}^0A_1 = \begin{pmatrix} \cos q_1 & -\sin q_1 & 0 & 0 \\ \sin q_1 & \cos q_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^1A_2 = \begin{pmatrix} \cos(-90^\circ) & -\sin(-90^\circ) & 0 & 0 \\ \sin(-90^\circ) & \cos(-90^\circ) & 0 & 0 \\ 0 & 0 & 1 & q_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(-90^\circ) & -\sin(-90^\circ) & 0 \\ 0 & \sin(-90^\circ) & \cos(-90^\circ) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & q_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & q_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^2A_3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$T = {}^0A_1 \cdot {}^1A_2 \cdot {}^2A_3$$

$${}^0A_1 \cdot {}^1A_2 = \begin{pmatrix} C_1 & -S_1 & 0 & 0 \\ +S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & q_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} S_1 & 0 & C_1 & 0 \\ -C_1 & 0 & S_1 & 0 \\ 0 & -1 & 0 & q_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^2A_3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$T = {}^0A_1 \cdot {}^1A_2 \cdot {}^2A_3 = \begin{pmatrix} S_1 & 0 & C_1 & C_1 q_3 \\ -C_1 & 0 & S_1 & S_1 q_3 \\ 0 & -1 & 0 & q_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$x = C_1 q_3$$

$$y = S_1 q_3$$

$$z = q_2$$

• Cinemática inversa:

$$\boxed{z = q_2}$$

$$x^2 + y^2 = C_1^2 q_3^2 + S_1^2 q_3^2 = q_3^2 (C_1^2 + S_1^2) \stackrel{L}{=} q_3^2$$

$$\boxed{q_3 = \sqrt{x^2 + y^2}}$$

$$\begin{aligned} x &= C_1 q_3 \rightarrow \cos q_1 = \frac{x}{q_3} \\ y &= S_1 q_3 \rightarrow \sin q_1 = \frac{y}{q_3} \end{aligned} \left\{ \begin{aligned} \cos q_1 &= \frac{x}{q_3} \\ \sin q_1 &= \frac{y}{q_3} \end{aligned} \right. \quad \left\{ \begin{aligned} \tan q_1 &= \frac{y/q_3}{x/q_3} = \frac{y}{x} \end{aligned} \right.$$

$$\boxed{q_1 = \arctan\left(\frac{y}{x}\right)}$$