1

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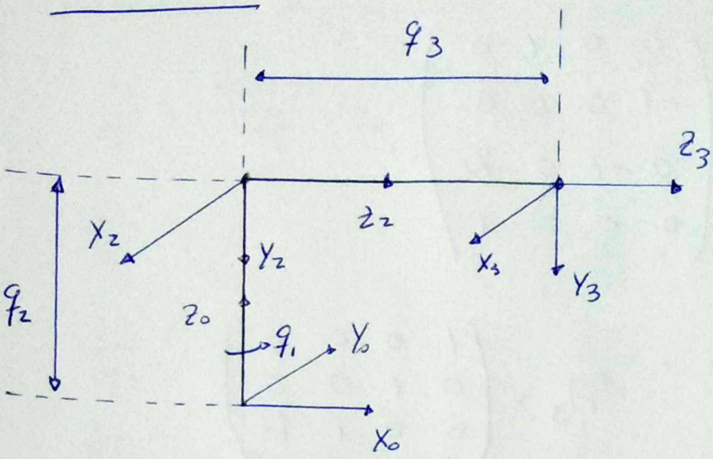
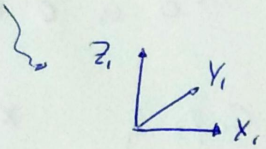
Problema



1 articulación rotación

2 articulaciones prismáticas



→ El sistema 1 está en la misma posición que el sistema 0

- Cinemática directa:

$$ {}^{0}A_1 = \begin{pmatrix} \cos q_1 & -\operatorname{sen} q_1 & 0 & 0 \\ \operatorname{sen} q_1 & \cos q_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^{1}A_2 = \begin{pmatrix} \overset{=0}{\cos(-90)} & \overset{=1}{-\operatorname{sen}(-90)} & 0 & 0 \\ \underset{=-1}{\operatorname{sen}(-90)} & \underset{=0}{\cos(-90)} & 0 & 0 \\ 0 & 0 & 1 & q_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \overset{=0}{\cos(-90)} & \overset{=1}{-\operatorname{sen}(-90)} & 0 \\ 0 & \underset{=-1}{\operatorname{sen}(-90)} & \underset{=0}{\cos(-90)} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = $$

$$ = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & q_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & q_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^{2}A_3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_3 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$T = {}^0A_1 \cdot {}^1A_2 \cdot {}^2A_3$$

$${}^0A_1 \cdot {}^1A_2 = \begin{pmatrix} C_1 & -S_1 & 0 & 0 \\ +S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & q_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} S_1 & 0 & C_1 & 0 \\ -C_1 & 0 & S_1 & 0 \\ 0 & -1 & 0 & q_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad {}^2A_3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$T = {}^0A_1 \cdot {}^1A_2 \cdot {}^2A_3 = \begin{pmatrix} S_1 & 0 & C_1 & C_1 q_3 \\ -C_1 & 0 & S_1 & S_1 q_3 \\ 0 & -1 & 0 & q_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$x = C_1 q_3$$
$$y = S_1 q_3$$
$$z = q_2$$

$\boxed{2}$

- Cinemática diferencial

Calculamos el jacobiano:

$$\frac{\partial x}{\partial q_1} = \frac{\partial \cos q_1}{\partial q_1} \cdot q_3 = -S_1 q_3$$

$$\frac{\partial x}{\partial q_2} = 0 \qquad \frac{\partial y}{\partial q_3} = S_1$$

$$\frac{\partial x}{\partial q_3} = C_1 \qquad \frac{\partial z}{\partial q_1} = 0$$

$$\frac{\partial y}{\partial q_1} = C_1 q_3 \qquad \frac{\partial z}{\partial q_2} = 1$$

$$\frac{\partial y}{\partial q_2} = 0 \qquad \frac{\partial z}{\partial q_3} = 0$$

$$J = \begin{pmatrix} -S_1 q_3 & 0 & C_1 \\ C_1 q_3 & 0 & S_1 \\ 0 & 1 & 0 \end{pmatrix}$$

Calculamos el determinante:

$$|J| = [(C_1 q_3) C_1] - [(-S_1 q_3) S_1] = C_1^2 q_3 + S_1^2 q_3 = q_3 \overbrace{\cancel{(S_1^2 + C_1^2)}}^{1}$$

$$\boxed{|J| = q_3}$$

Las singularidades se producen cuando $|J| = 0$

$\boxed{q_3 = 0}$ La articulación prismática q_3 estaría totalmente contraída y el robot estaría situado verticalmente sobre el eje z. Modificar q_1 no cambiaría la posición del robot por lo que se pierde un grado de libertad.