Peer-graded Assignment: Regression Models Course Project

Alejandro Osorio

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Motor Trend (a magazine about the automobile industry) is interested in exploring the relationship between a set of variables and miles per gallon (MPG as outcome), by looking at a data set of a collection of cars. They are particularly interested in the following two questions:

1. "Is an automatic or manual transmission better for MPG"
2. "Quantify the MPG difference between automatic and manual transmissions"

Take the 'mtcars' data set and write up an analysis to answer their question using regression models and exploratory data analyses.

General Considerations

Fuel efficiency may depend on many variables, out of which the type of transmission may not be statistically relevant among them. In spite of that, and given the need to have a reasonable understanding of the impact that specific variable has in gas consumption, the problem was approached as an inference one, based on a multivariable regression analysis. The structure of the analysis consisted on the following steps:

1. Preliminary data analysis
2. Model selection
3. Preliminary regressors
4. Regression modeling
5. Regression diagnostics
6. Confidence intervals
7. Conclusions

Preliminary Data Analysis

Basic properties of the 'mtcars' dataset:

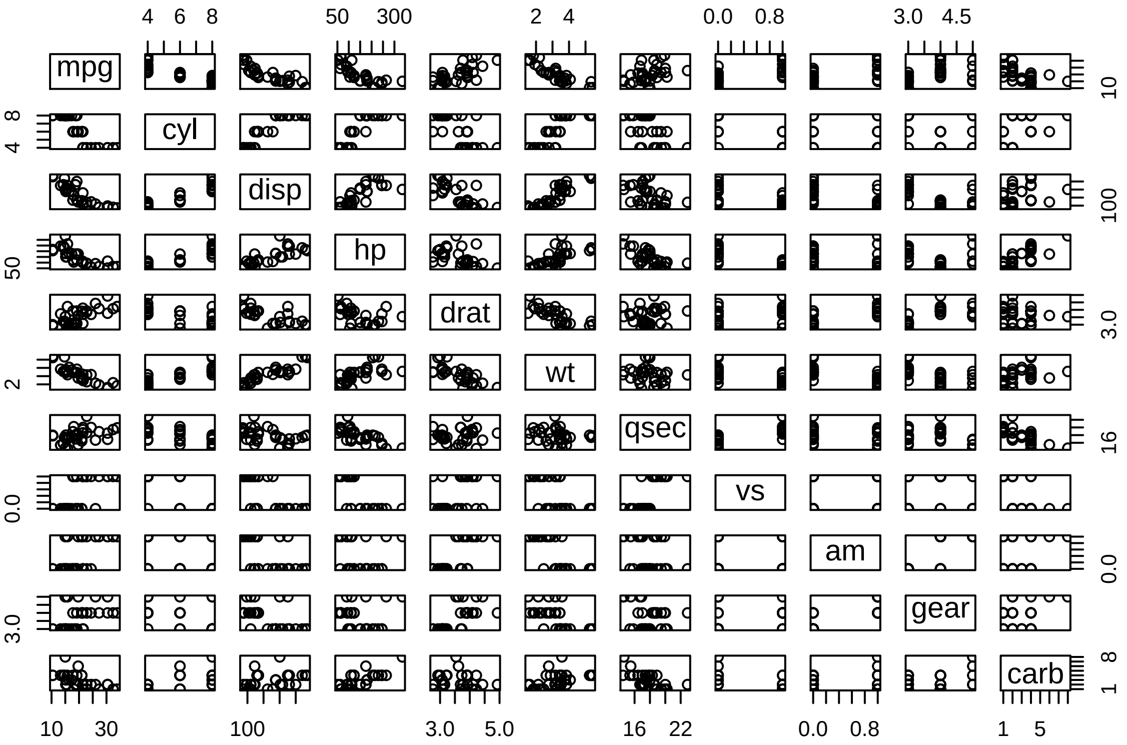
```
## 'data.frame':    32 obs. of  11 variables:
##  $ mpg : num  21 21 22.8 21.4 18.7 18.1 14.3 24.4 22.8 19.2 ...
##  $ cyl : num  6 6 4 6 8 6 8 4 4 6 ...
##  $ disp: num  160 160 108 258 360 ...
##  $ hp  : num  110 110 93 110 175 105 245 62 95 123 ...
##  $ drat: num  3.9 3.9 3.85 3.08 3.15 2.76 3.21 3.69 3.92 3.92 ...
##  $ wt  : num  2.62 2.88 2.32 3.21 3.44 ...
##  $ qsec: num  16.5 17 18.6 19.4 17 ...
##  $ vs  : num  0 0 1 1 0 1 0 1 1 1 ...
##  $ am  : num  1 1 1 0 0 0 0 0 0 0 ...
##  $ gear: num  4 4 4 3 3 3 3 4 4 4 ...
##  $ carb: num  4 4 1 1 2 1 4 2 2 4 ...
```

So, we're talking of a small sample of only 32 observations, with 11 variables, which are described as follows:

[, 1] mpg Miles/(US) gallon [, 2] cyl Number of cylinders [, 3] disp Displacement (cu.in.) [, 4] hp Gross horsepower [, 5] drat Rear axle ratio [, 6] wt Weight (1000 lbs) [, 7] qsec 1/4 mile time [, 8] vs V/S [, 9]

am Transmission (0 = automatic, 1 = manual) [,10] gear Number of forward gears [,11] carb Number of carburetors

Additionally, visual correlations among pairs of variables from the dataset, can be seen as follows:



Model Selection

Given the nature of the analysis (continuous outcome, obtained from discrete and continuous regressors), the size of the dataset (only 32 observations), plus the visual correlations observed between 'mpg' (the outcome) and its potential regressors, the model chosen was of a linear (lm) type, with 'am' (type of transmission) as the main binary-factor variable, among any additional statistically significant variables for the outcome.

Preliminary Regressors

A preliminary analysis considered all mtcars variables as potential regressors. With variables 'cyl', 'vs', 'am', 'gear' and 'carb' as factors, the following lm function was analysed:

```r
fitAll <- lm(mpg ~ ., mtcars2)
summary(fitAll)$coef
```

```
##                Estimate  Std. Error     t value   Pr(>|t|)
## (Intercept) 23.87913244 20.06582026  1.19004018 0.25252548
## cyl6        -2.64869528  3.04089041 -0.87102622 0.39746642
## cyl8        -0.33616298  7.15953951 -0.04695316 0.96317000
## disp         0.03554632  0.03189920  1.11433290 0.28267339
## hp          -0.07050683  0.03942556 -1.78835344 0.09393155
## drat         1.18283018  2.48348458  0.47627845 0.64073922
## wt          -4.52977584  2.53874584 -1.78425732 0.09461859
## qsec         0.36784482  0.93539569  0.39325050 0.69966720
## vs1          1.93085054  2.87125777  0.67247551 0.51150791
## am1          1.21211570  3.21354514  0.37718957 0.71131573
## gear4        1.11435494  3.79951726  0.29328856 0.77332027
## gear5        2.52839599  3.73635801  0.67670068 0.50889747
## carb2       -0.97935432  2.31797446 -0.42250436 0.67865093
## carb3        2.99963875  4.29354611  0.69863900 0.49546781
## carb4        1.09142288  4.44961992  0.24528452 0.80956031
## carb6        4.47756921  6.38406242  0.70136677 0.49381268
## carb8        7.25041126  8.36056638  0.86721532 0.39948495
```

With a P-value based criteria, no variable would make the cut (with 'hp' and 'wt' the closest, though). Therefore, the first conclusion was that some industry research was required in order to determine the best regressor candidates for 'mpg' outcome.

After some web research (such as http://www.driverside.com/auto-library/top_10_factors_contributing_to_fuel_economy-317 and https://www.quora.com/On-what-factors-does-mileage-of-a-vehicle-depend), the main variables suggested (and therefore candidates for main regresors), were: displacement ('disp'), power ('hp'), aerodynamics, weight ('wt') and number of forward gears ('gear').

Besides aerodynamics (not available in the dataset), the visual correlations between them and 'mpg' are pretty clear, as seen in Preliminary Data Analysis (such as the inverse relation between 'mpg' and both 'hp' or 'wt'). Therefore, including 'am' (required in order to answer the main question), they were selected as candidates for regression modeling in the next appendix.

Finally, given the lowest P-value results obtained by 'wt' and 'hp' regressors (which interestingly enough build up the well known Weight/Power KPI) from the automotive industry, in further analysis both 'wt' and 'hp' were considered primary potential regressors while 'disp' and 'gear' were considered secondary potential regressors (just to prioritize further analysis. Cuantitative analysis would finally determine which regressors to keep).

Regression Modeling

For the regression analysis, two types of models were used.

The first one with different intersection points but the same slope for all regression lines (no interaction between 'am' -the binary factor regressor- and the rest of the regressors), such as:

```
## lm(formula = mpg ~ hp + wt + factor(am), data = mtcars)
```

The second one, with different intersects AND slopes for regression lines (factor(am):regressor type interactions), such as:

```
## lm(formula = mpg ~ hp + wt + factor(am) + factor(am):hp + factor(am):wt,
##     data = mtcars)
```

Additionally, ANOVA comparisons (incrementally adding regressors) were used in order to determine the best regressor mix for each model. The order in which regressor candidates were added, followed the primary and secondary criteria specified before. Therefore, the regressor sequences that were finally considered, were the following (all with 'am' included):

1. 'wt' - 'hp' - 'disp'
2. 'wt' - 'hp' - 'gear'
3. 'hp' - 'disp' - 'gear'
4. 'wt' - 'gear' - 'disp'

Model 1: Different intersection points, with same slopes

```
## Analysis of Variance Table
## 
## Model 1: mpg ~ factor(am)
## Model 2: mpg ~ wt + factor(am)
## Model 3: mpg ~ hp + wt + factor(am)
## Model 4: mpg ~ wt + hp + disp + factor(am)
##   Res.Df    RSS Df Sum of Sq       F    Pr(>F)    
## 1     30 720.90                                   
## 2     29 278.32  1    442.58 66.4206 9.394e-09 ***
## 3     28 180.29  1     98.03 14.7118 0.0006826 ***
## 4     27 179.91  1      0.38  0.0576 0.8122229    
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

ANOVA test with 2nd sequence (replacing 'disp' by 'gear') resulted in almost same outcome.

Just in case, ANOVA analysis of the first sequence was carried on, reordering the sequence as follows: 'wt' - 'disp' - 'hp'

```
## Analysis of Variance Table
## 
## Model 1: mpg ~ factor(am)
## Model 2: mpg ~ wt + factor(am)
## Model 3: mpg ~ wt + disp + factor(am)
## Model 4: mpg ~ wt + hp + disp + factor(am)
##   Res.Df    RSS Df Sum of Sq      F    Pr(>F)    
## 1     30 720.90                                  
## 2     29 278.32  1    442.58 66.421 9.394e-09 ***
## 3     28 246.56  1     31.76  4.767  0.037878 *  
## 4     27 179.91  1     66.65 10.002  0.003842 ** 
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Interestingly enough, the result sugests that all 4 regressors are relevant for the outcome. Same test was carried on with the sequence number 2, with less promising results.

Sequences 3 and 4 were not relevant as soon as regressors 'disp' or 'gear' were included (including sequence reordering).

Therefore, model 4 passed the test for final P-value analysis of its coefficients, as follows:

```
##                 Estimate Std. Error    t value     Pr(>|t|)
## (Intercept) 34.209443370 2.82282610 12.1188632 1.979953e-12
## wt          -3.046747000 1.15711931 -2.6330448 1.382936e-02
## hp          -0.039323213 0.01243358 -3.1626624 3.842032e-03
## disp         0.002489354 0.01037681  0.2398959 8.122229e-01
## factor(am)1  2.159270737 1.43517565  1.5045341 1.440531e-01
```

Given the P-value > 0.05 for the 'hp:factor(am)'disp' regressor ('am' regressor is included anyway, being required as binary factor), the final type 1 model is as follows:

```
## lm(formula = mpg ~ hp + wt + factor(am), data = mtcars)
```

Model 2: Different intersection points and slopes

ANOVA analysis to sequences 1 and 2 suggests ‘wt’, ‘hp’ and ‘am’ are the only relevant regressors (including ‘disp’ or ‘gear’ in any order, didn’t make a difference) when including variable slopes.

```
## Analysis of Variance Table
##
## Model 1: mpg ~ factor(am)
## Model 2: mpg ~ wt + factor(am) + factor(am):wt
## Model 3: mpg ~ hp + wt + factor(am) + factor(am):hp + factor(am):wt
## Model 4: mpg ~ wt + hp + factor(gear) + factor(am) + factor(am):wt + factor(am):hp +
##     factor(am):factor(gear)
##   Res.Df    RSS Df Sum of Sq       F    Pr(>F)
## 1     30 720.90
## 2     28 188.01  2    532.89 49.5549 3.013e-09 ***
## 3     26 135.90  2     52.11  4.8456   0.01707 *
## 4     24 129.04  2      6.86  0.6378   0.53720
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Therefore, model 3 passed the test for final P-value analysis of its coefficients, as follows:

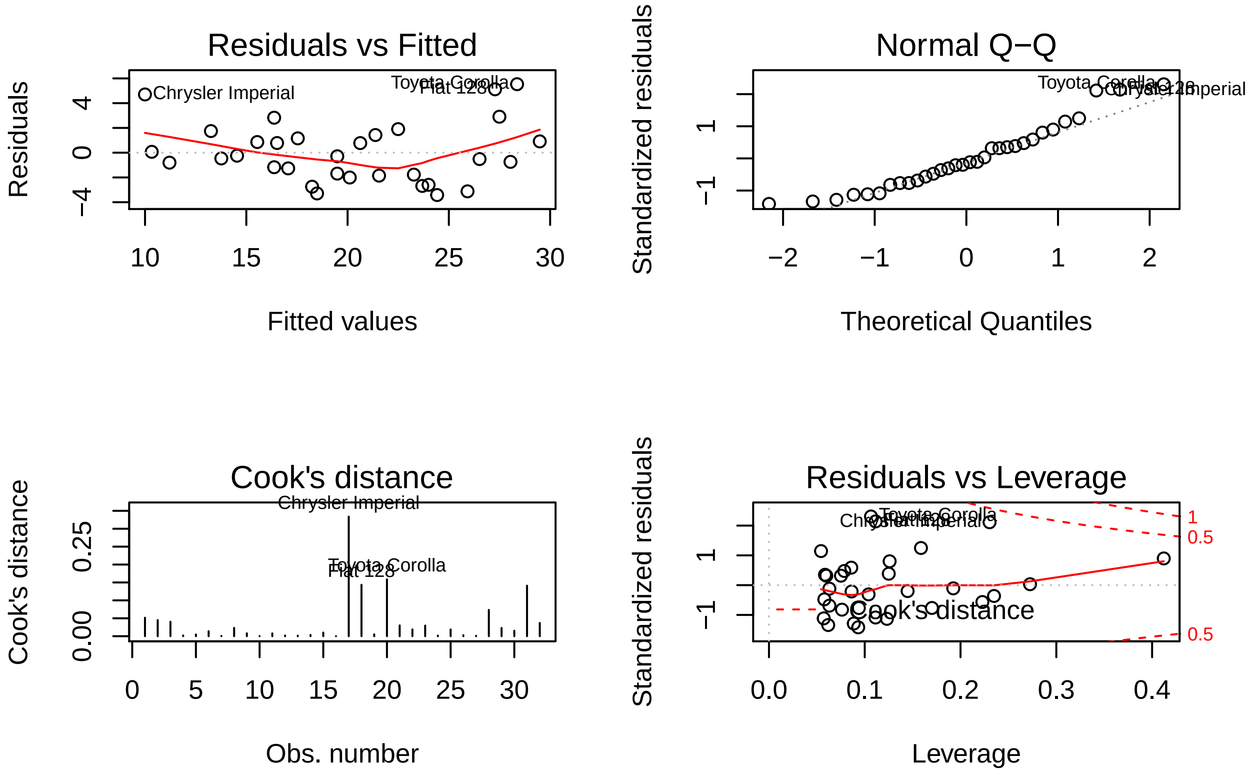
```
##                   Estimate Std. Error   t value     Pr(>|t|)
## (Intercept)    30.70392721 2.67515435 11.477441 1.117089e-11
## hp             -0.04094406 0.01362921 -3.004142 5.826559e-03
## wt             -1.85591121 0.94510642 -1.963706 6.034159e-02
## factor(am)1    13.74000384 4.22337051  3.253327 3.155621e-03
## hp:factor(am)1  0.02779357 0.01920705  1.447050 1.598330e-01
## wt:factor(am)1 -5.76894729 2.07200930 -2.784228 9.870579e-03
```

Given the P-value > 0.05 for the hp:factor(am) coefficient, the final type 2 model is as follows:

```
## lm(formula = mpg ~ hp + wt + factor(am) + factor(am):wt, data = mtcars)
```

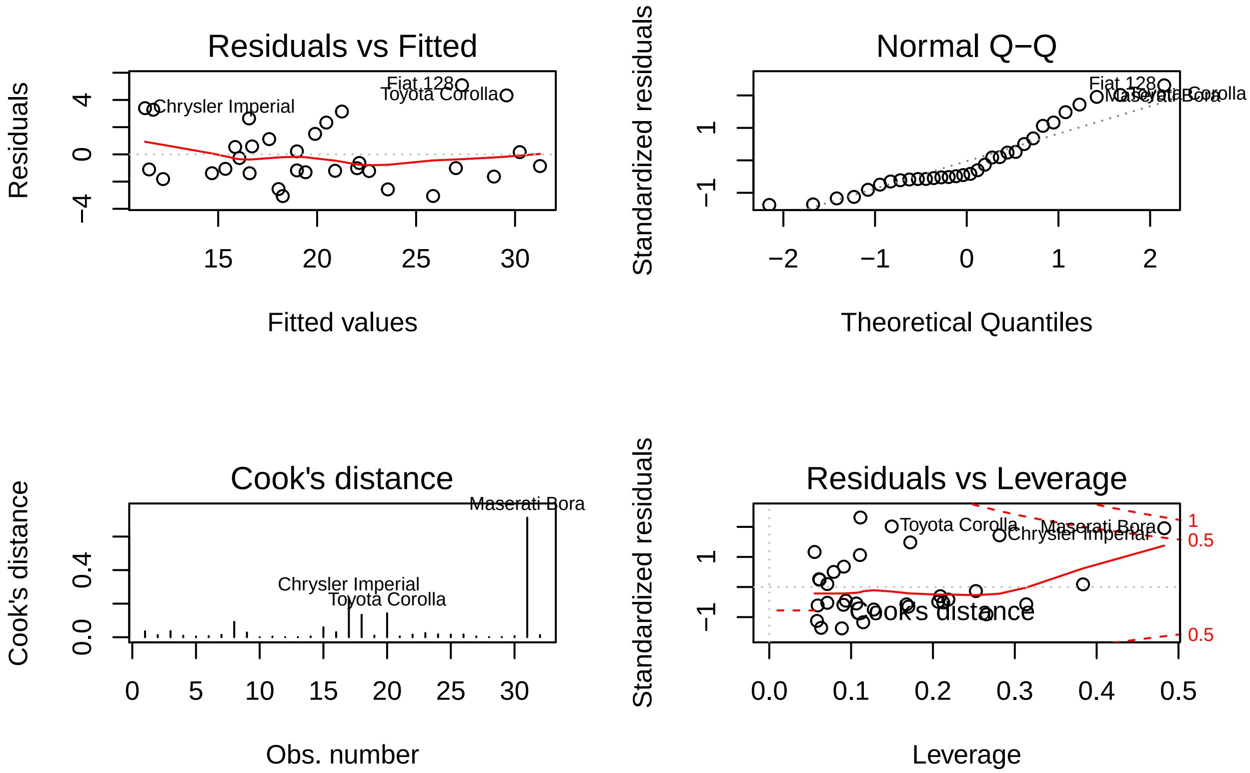
Regression Diagnostics

Model 1: Different intersection points, with same slopes



Redisual analysis looks good enough (no visible unbalanced patterns). Normal distribution of residuals, as well as their leverage, look a bit off though. It probably requires further analysis of cases such as Chrysler Imperial.

Model 2: Different intersection points and slopes



Redisual analysis looks good enough (no visible patterns). Normal distribution of residuals, as well as their leverage, look a bit off though. It probably requires further analysis of cases such as Masseratti Bora.

Confidence Intervals

Model 1: Different intersection points, with same slopes

```
##                  Estimate  Std. Error   t value     Pr(>|t|)
## (Intercept) 34.00287512 2.642659337 12.866916 2.824030e-13
## hp          -0.03747873 0.009605422 -3.901830 5.464023e-04
## wt          -2.87857541 0.904970538 -3.180850 3.574031e-03
## factor(am)1  2.08371013 1.376420152  1.513862 1.412682e-01
```

Considering that the 4th row represents the impact on the intercept once we include the factor that the transmission is manual (without changing the slopes), the confidence interval for that impact is:

```
sumCoef1[4,1] + c(-1,1) * qt(.975, df = model1$df) * sumCoef1[4,2]
```

```
## [1] -0.7357587  4.9031790
```

Which means that, with a 95% confidence, we estimate that cars with a manual transmission result in a -0.74 to 4.9 general impact in MPG, compared to those with automatic transmission, at the average value of any of this model's regressors ('wt' or 'hp').

The fact that the confidence interval includes the value 0, implies that, based on this model, the type of transmission could impact the MPG outcome in any possible way, which doesn't help answer the central question.

Model 2: Different intersection points and slopes

```
##                     Estimate  Std. Error   t value     Pr(>|t|)
## (Intercept)    30.94733319 2.723410935 11.363446 8.546944e-12
## hp             -0.02694935 0.009795903 -2.751084 1.047673e-02
## wt             -2.51558550 0.844496532 -2.978799 6.051842e-03
## factor(am)1    11.55481296 4.023276579  2.871991 7.844579e-03
## wt:factor(am)1 -3.57790980 1.442795585 -2.479845 1.967639e-02
```

```
sumCoef2[4,1] + c(-1,1) * qt(.975, df = model2$df) * sumCoef2[4,2]
```

```
## [1]  3.299731 19.809895
```

In this case, with a 95% confidence, we estimate that cars with a manual transmission result in a 3.3 to 19.8 general impact in MPG, compared to those with automatic transmission, at the average value of any of this model's regressors ('wt' or 'hp'). Additionally, considering the wt:factor(am)1 interaction:

```
sumCoef2[5,1] + c(-1,1) * qt(.975, df = model2$df) * sumCoef2[5,2]
```

```
## [1] -6.5382818 -0.6175378
```

With a 95% confidence, we also estimate that cars with a manual transmission result in an extra -6.53 to -0.618 impact in MPG for each 1000 lbs increase in weight.

Based on this model, then, it can be said that the type of transmission does have an observable impact on MPG, improving it when it comes down to a manual type.

Conclusions

1. Based on multivariable linear regression, it went down to 2 main regressors when modeling the outcome. Those were 'wt' and 'hp' (both with significant p-values). Interestingly enough, those 2 regressors are widely used by the automotive industry in the well known 'Weight/Power' KPI.

2. The third regressor, 'am', was imposed as a binary factor, due to the main objective of the analysis (otherwise, given its high p-value, it would have been discarded as a relevant regressor for the outcome).

3. Out of the two type of models used for the analysis (with and without interaction between the binary factor 'am' and the other regressors):

- The one without interactions suggested a confidence interval for the intersection, that included 0. Therefore no relevant information could be obtained from it regarding the central question of the analysis.
- Model 2 (with interactions) suggested a positive impact of a manual transmission on the outcome (an expected increase on intercept values when the transmission is manual). The confidence interval for the positive impact, with a 95% confidence, consists of a 3.3 to 19.8 extra MPG impact (for the intercepts). Additionally, it suggests an additional negative impact on MPG, for every 1000 lbs of weight.

4. Further leverage analysis is recomended, due to the probable impact of a couple of outliers in the model.

5. Given the fact that the regression was based on linear models, further analysis with non linear models is also recomended.

6. Finally, the small size of the dataset must also be taken into account before jumping into more serious conclusions.