Números Complejos

Vienen de no poder resolver problemas del tipo $x^2+1=0$

Se define $i \,/\, i^2=-1$

Número complejo $z = a+ib, \quad a,b \in \mathbb{R}$

a: Parte Real $(\mathrm{Re}(z))$
b: Parte Imaginaria $(\mathrm{Im}(z))$

Operaciones

$z_1 = a+ib \qquad z_2 = c+id$

$z_1+z_2 = (a+c)+i(b+d)$

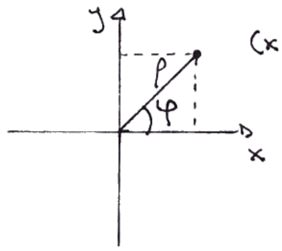
$z_1 \cdot z_2 = (ac-bd)+i(ad+bc)$

$$z_1/z_2 = \frac{a+ib}{c+id} = \frac{a+ib}{c+id}\cdot\frac{c-id}{c-id} = \frac{(ac-bd)+i(ad+bc)}{c^2+d^2} = \frac{ac-bd}{c^2+d^2} + i\,\frac{ad+bc}{c^2+d^2}$$

con $c^2+d^2 \neq 0$

$\overline{z} = \overline{a+ib} = a-ib$

Plano Complejo



$(x,y) \rightsquigarrow x+iy$

Forma Trigonométrica

$\rho = \sqrt{x^2+y^2} = |z| = \sqrt{z\cdot\overline{z}}$

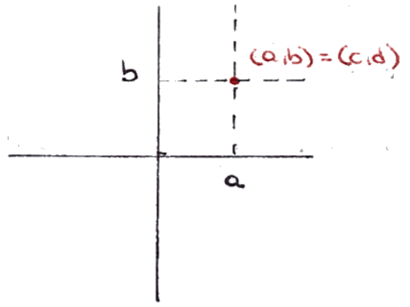
$z = x+iy = |z|\cos(\varphi) + i\,|z|\,\mathrm{sen}(\varphi) = |z|\left(\cos(\varphi)+i\,\mathrm{sen}(\varphi)\right) = \rho\left(\cos\varphi + i\,\mathrm{sen}\,\varphi\right)$

φ debe definirse dentro de un intervalo de 2π. Usamos $\varphi \in [-\pi,\pi)$

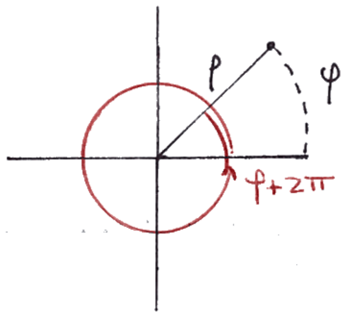
Forma Exponencial

$e^{i\theta} = \cos(\theta) + i\,\mathrm{sen}(\theta) \implies z = \rho\, e^{i\varphi}$

$$\left.\begin{array}{l} z_1 = a+bi \\ z_2 = c+di \end{array}\right\} \quad z_1 = z_2 \iff a+bi = c+di \iff \left\{\begin{array}{l} a=c \\ b=d \end{array}\right.$$



$$\left.\begin{array}{l} z_1 = |z_1| e^{i\varphi_1} \\ z_2 = |z_2| e^{i\varphi_2} \end{array}\right\} \quad z_1 = z_2 \iff \left\{\begin{array}{l} |z_1| = |z_2| \\ \varphi_1 = \varphi_2 + 2k\pi \quad k \in \mathbb{Z} \end{array}\right.$$



<u>Resolvemos $z^2 = 1$</u>

$$z^2 = \rho^2 e^{i2\varphi} \qquad 1 = 1 e^{i0}$$

$$z^2 = 1 \Rightarrow \rho^2 e^{i2\varphi} = 1 e^{i0} \Rightarrow \left\{\begin{array}{l} \rho^2 = 1 \longrightarrow \rho = \sqrt{1} \\ 2\varphi = 0 + 2k\pi \longrightarrow \varphi = k\pi \quad k \in \end{array}\right.$$

<u>$k=0$</u>

$$z_0 = 1 e^{i0} = 1$$

<u>$k=1$</u>

$$z_1 = 1 e^{i\pi} = -1$$

<u>$k=2$</u>

$$z_2 = 1 e^{i2\pi} = 1$$

<u>$k=3$</u>

$$z_3 = 1 e^{i3\pi} = -1$$

Se empiezan a repetir los resultados, $z^2 = 1$ posee 2 soluciones. Limito $k = 0, 1$