

Stock Market Trading and Investing

Notes

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Investment Strategies

Foundations and Principles

Definitions

Bid¹: highest price a buyer is willing to pay (\times number of people looking at that price);

Ask: lowest price a seller is willing to accept (\times number of people selling);

$\Delta = \text{Ask} - \text{Bid}$: spread;

Volume: number of transactions over the day;

Avg. Volume: average transactions per day;

Market Cap: useful to compare a company with another, the bigger is that value the bigger is the company in the stock market;

Beta (5Y monthly): measure of the risk $\in [0, 1]$;

- beta = 1.0 same price movements as the wider market;
- beta > 1.0 swings wider (up or down) than the wider market;
- beta < 1.0 swings less (up or down) than the wider market (less risky, volatile);

EPS (TTM): earnings per share (trailing twelve months), if 1 share is \$50, then then EPS represents the earning per share;

Earning Date: this date represents when company will update the earnings;

Forward Dividend & Yield: dividend per share (the higher the better);

Ex-Dividend Date: dividend per share when company pay money (dividend)).

Type of Traders

- Scalper (Arbitrage): minutes
 - Day Trading: minutes to hours
- } require technical analysis, charting
-
- Swing Trading: hours to days
 - Position Trading: days to weeks
 - Part Time Investing: weeks to months
 - Buy & Hold: entry with fewer exits
- } require fundamental analysis

Type of (Main) Ratios to evaluate a stock

Price to Earning ratio (P/E ratio)

It is the price to pay for \$1.00,

$$\text{P/E ratio} = \frac{\text{PRICE PER SHARE}}{\text{EARNINGS PER SHARE}} = \frac{\text{PRICE PER SHARE}}{\text{EPS}}.$$

¹Reference is <https://finance.yahoo.com/>

A general rule of thumb is that shares traded at a "low" P/E are a value, though the definition for "low" can vary from industry to industry.

Price to Earning with growth factored in ratio (PEG ratio)

PEG ratio uses the basic format of the P/E ratio for a numerator and then divides by the potential growth for EPS. P/E and PEG ratios are very similar but the PEG can take into account future earnings growth. A general rule of thumb is that any PEG below 1.0 is considered to be a good value,

$$\text{PEG ratio} = \frac{\text{P/E RATIO}}{\text{PROJECTED ANNUAL GROWTH IN EARNINGS PER SHARE}}.$$

Price to Sales ratio (P/S ratio)

$$\text{P/S ratio} = \frac{\text{PRICE PER SHARE}}{\text{ANNUAL SALES PER SHARE}}.$$

Comparing two different firms, the investors are willing to pay for the company with a higher P/S ratio. P/S ratio is a great tool because the sales figures are considered to be relatively reliable, while other income statements, like earnings, can be easily manipulated by using different accounting rules.

Price to Book ratio (P/B ratio)

It is the ratio of price to book value per share. Book value is the value of an asset according to its balance sheet account. It's a company's value if it liquidates its assets and paid back all its liabilities.

PROS

P/B is a stable metric, this value does not fluctuate as much like others, ex. P/E ratio.

CONS

Accounting differences can be hard to compare, harder to compare companies with little assets (tech firms, service providers) against those with lots of inventory and equipment (retailers, automobile).

A good P/B ratio is between 1.0 and 3.0, P/B < 1.0 is excellent.

Enterprise Value to Revenue ratio (EV/Revenue ratio)

$$\begin{aligned} \text{EV} &= \text{MARKET CAPITALIZATION (MARKET CAP)} + \text{DEBT} - \text{CASH \& CASH EQUIVALENT}, \\ \text{REVENUE} &= \text{TOTAL ANNUAL REVENUE}, \end{aligned}$$

it is used when company is at early stage and does not have yet positive earnings or for high growth business. Alternative to P/E, PEG, EV/EBITDA ratios.

- Beneficial when there are significant differences between accounting policies of companies. P/E ratio, on the opposite hand, can vary dramatically with changes in the accounting policies.
- Used for companies with negative free cash flows or unprofitable companies, ex. tech startup.
- It's easy to find revenue data for most businesses, making it easy to calculate this ratio.
- Usually is listed in statistics (Yahoo Finance).
- Note,
 - does not take into account the cost structure of the company;
 - ignores profitability and generation of cash flow;
 - hard to compare across different industries and different company phases (early vs mature).

EV/Revenue has generally a range between 1.0 and 3.0. The lower the better.

Enterprise Value to EBITDA ratio (EV/EBITDA ratio)

The EV/EBITDA (earnings before interest, taxes, depreciation and amortization) looks at the company the way a potential acquirer may. Good and bad enterprise depends on the industry,

$$\text{ENTERPRISE VALUE} = \frac{\text{EV}}{\text{EBITDA}},$$

generally that value is less than 10.0, the lower the better (undervalued).

Price to cash flow ratio (P/CF ratio)

It measures how much cash a company is generating relative to its market value.

P/CF is a good alternative to P/E as cash flows are less susceptible to manipulation than earnings.

– cash flow does not incorporate no-cash expense items, like depreciation or amortization (income statement metrics) which can be subject to various accounting rules.

$$\text{P/CF} = \frac{\text{SHARE PRICE}}{\text{OPERATING CASH FLOW PER SHARE}},$$

in order to avoid volatility in the multiple, a 30-, 60-day average price can be used to deliver a more stable stock value that is not skewed by random market movements.

The Operating Cash flow (OCF) used in the denominator is obtained through a calculation of the TTM OCFs generated by the firm divided by the number of shares outstanding.

In addition, the calculation can also be done on a whole-company basis by

$$\text{OCF} = \frac{\text{FIRM'S TOTAL MARKET VALUE (MARKET CAP)}}{\text{TOTAL OPERATING CASH FLOWS}},$$

$$\text{OCF PER SHARE} = \frac{\text{OPERATING CASH FLOW (TTM)}}{\text{SHARES OUTSTANDING}}.$$

Price to free cash flow ratio (P/FCF ratio)

A more rigorous measure than P/CF ratio, though very similar to it, this metric is considered a more exact measure as it uses free cash flow (FCF), which subtracts capital expenditures (CapEx) from a company's total OCF, thereby reflecting actual cash flow available to fund non-asset related growth.

Companies use this metric when they need to expand their asset bases either to grow their business or supply to maintain acceptable levels of FCF.

$$\text{FCF PER SHARE} = \frac{\text{(LEVERED) FREE CASH FLOW (TTM)}}{\text{SHARES OUTSTANDING}},$$

$$\text{P/FCF} = \frac{\text{PRICE PER SHARE}}{\text{FCF PER SHARE}}.$$

PROS

- hard to manipulate than ratios using earnings;
- useful for companies that have a positive cash flow but are not profitable yet.

CONS

- more difficult to calculate.

Good P/CF ratio is generally < 10.0, the lower the better.

Good P/FCF ratio is generally < 5.0, the lower the better.

Useless to say that in order to evaluate a company, it is a best practice to evaluate more ratios and not only relying only on one.

(Main) Financial ratios summary

RATIO	FORMULA	BEST VALUE (RULE OF THUMB)	BEST INTERVAL
Price to Earnings (P/E)	$\frac{\text{PRICE PER SHARE}}{\text{EARNINGS PER SHARE}}$	Lower is better (depends on industry).	Varies widely by industry; < 20 for stable inds., < 30 for growth inds.
Price to Earnings with Growth (PEG)	$\frac{\text{P/E}}{\text{EARNING GROWTH RATE}}$	Lower is better.	< 1 is often considered good (indicating undervaluation relative to growth).
Price to Sales (P/S)	$\frac{\text{PRICE PER SHARE}}{\text{REVENUE VALUE PER SHARE}}$	Lower is better.	< 2 is often considered good.
Price to Book (P/B, Pr/Bk)	$\frac{\text{PRICE PER SHARE}}{\text{BOOK VALUE PER SHARE}}$	Lower is better.	< 1 is generally favorable.
Enterprise Value to Revenue (EV/Revenue)	$\frac{\text{ENTERPRISE VALUE}}{\text{REVENUE}}$	Lower is better.	< 5 – 10 is often considered good.
Enterprise Value to EBITDA (EV/EBITDA)	$\frac{\text{ENTERPRISE VALUE}}{\text{EBITDA}}$	Lower is better.	< 10 is often considered good.
Price to Cash Flow (P/CF)	$\frac{\text{PRICE PER SHARE}}{\text{OPERATING CASH FLOW PER SHARE}}$	Lower is better.	< 10 is often considered good.
Price to Free Cash Flow (P/FCF)	$\frac{\text{PRICE PER SHARE}}{\text{FREE CASH FLOW PER SHARE}}$	Lower is better.	< 10 is often considered good.
OTHERS			
Dividend Yield	$\frac{\text{DIVIDEND PER SHARE}}{\text{PRICE PER SHARE}}$	Higher is better.	3 – 6% is often considered good.
Return on Equity (ROE)	$\frac{\text{NET INCOME}}{\text{SHAREHOLDER'S EQUITY}}$	Higher is better.	15 – 20% is often considered good.
Debt to Equity	$\frac{\text{TOTAL DEBT}}{\text{SHAREHOLDER'S EQUITY}}$	Lower is better.	1% is often considered good. (Varies by ind.)
Current Ratio	$\frac{\text{CURRENT ASSETS}}{\text{CURRENT LIABILITIES}}$	Ideally 1 or higher.	1.5-3 is often considered good.

Table 1: Ratios recap

Pricing Stock using technical analysis

Important factors to consider,

- Supply & demand, a lot of demand indicates price is going up;
- There can be predictable behaviors but sometimes unexpected (random) events can occur at
 - company level;
 - country level;
 - global level;
- Breakouts and retracements (pullbacks), normally they have a trading range (low-high), price tends to stay in that range;
- General trends
 - flat
 - upward
 - downward.

Price Bars

There are different price bars to consider.

- Daily Open High Low Close (OHLC) - candlestick;
- Area chart;
- Line Chart;
- Price histogram;

- The close price is the most important;
- Summary of trading sentiment;
- Decide: hold stock overnight? Yes (Positive); No (Not so much).

If close higher than yesterday, day after day, more demand and traders willing to pay higher prices (buying trend).

Strong buying signal when close is the highest of the day.

When close lower than yesterday then selling trend.

UPTREND

High today is higher than yesterday. Additional confirmations if lows are also higher.

DOWNTREND

Low today is lower than yesterday. Additional confirmations if highs are also lower.

THE TREND IS YOUR FRIEND.

Candlestick Price Bars

It is a very important way to check the trend for a security.

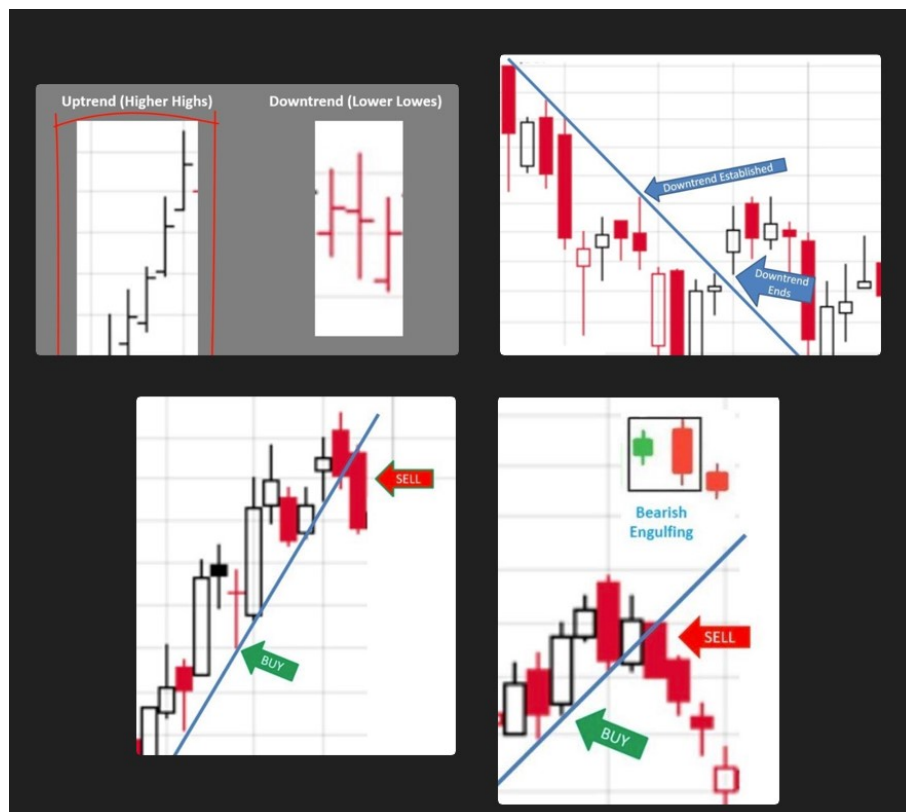


Figure 1: Support Lines and Buy-Sell conditions

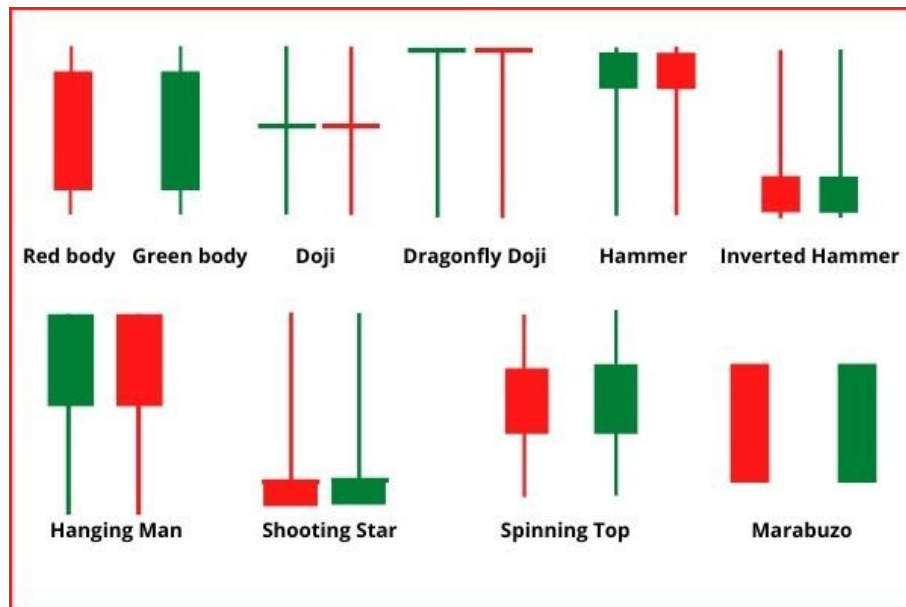


Figure 2: Candlestick Types

Channels

It is another tool to check the trend. It simply consists of drawing a Support and a Resistance line on the same chart.



Figure 3: Channels

Identify trendlines and Exit Strategies

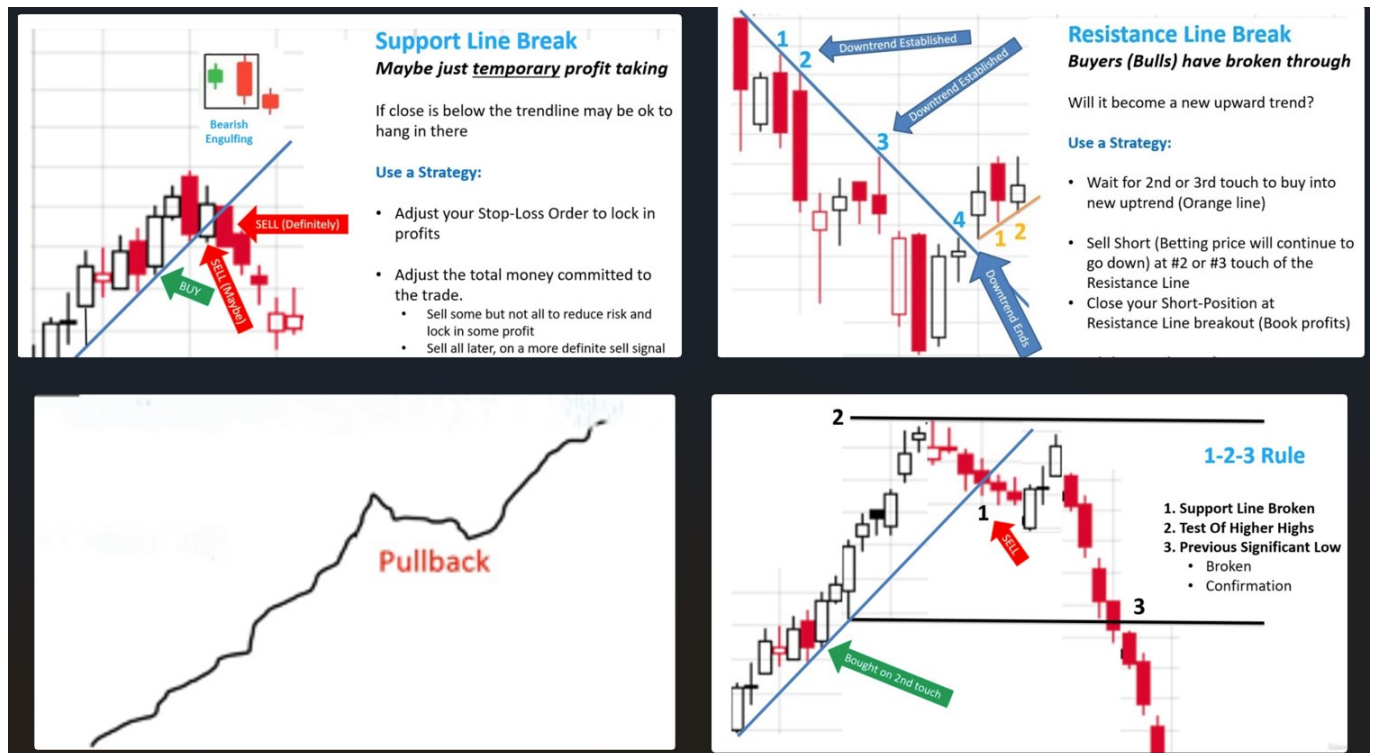


Figure 4: Exit strategies and false trend line breaks

Filter Strategy with Trendlines

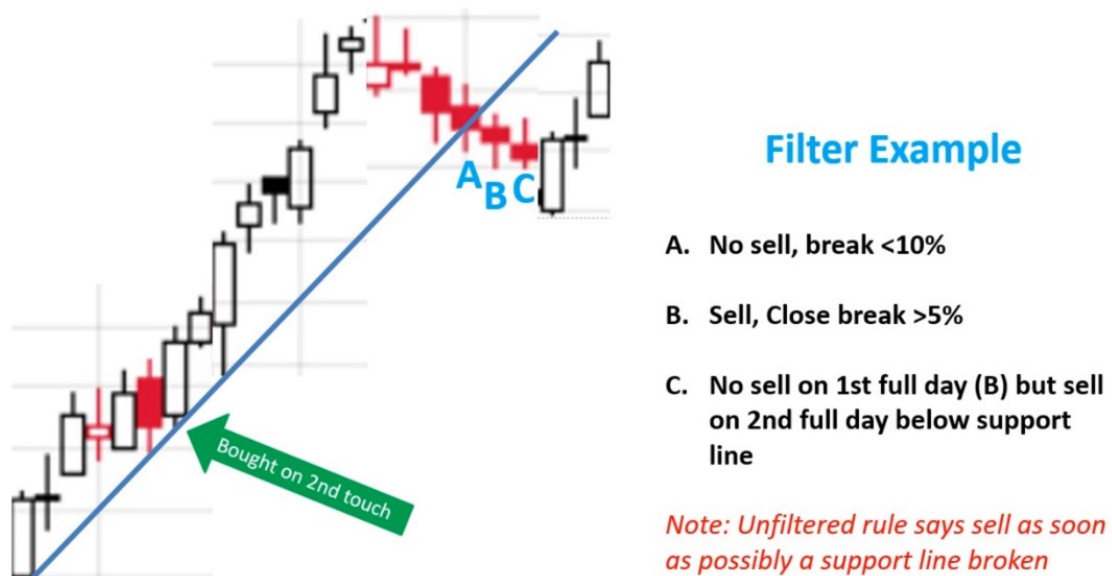


Figure 5: Filter strategies

- Set a predetermined rule on action when certain conditions are met;
- With trendline breakage can modify:
 - amount (%) of break;
 - duration (time) of the break.

As a rule of thumb, for the Support Line (SL),

– Sell ASAP after low falls under the SL.

The Filter can be on:

– Amount,

- Price range has to break the SL by 10%;
- Close has to break the SL by 5%.

– Duration,

- Allow one full break of SL but not two.

Chart Patterns

They can indicate that something in the past will do so again in the future.

– Most employ a straight line. Pattern lines follow either highs or lows;

Pattern Type organized whether they forecast a **continuation** or **reversal** of the current price move.

A pattern can take time to develop, but when you get it it can be a powerful indicator tool.

It is possible to confirm chart patterns with:

- other indicators (ex. moving average9);
- trading volume changes, higher volume when a pattern reaches completion as other traders recognize the pattern.

Continuation Pattern (CP)

In an established trend, there may be a "Pause" in buying and selling \implies trend is losing steam.

When you see a CP that is indicating that the trend will accelerate again after the pause \implies you identified the trend correctly.

Reversal Pattern (RP)

When you see a RP that is indicating that the trend will end and a new one is beginning \implies need to take appropriate action.

Triangle Pattern

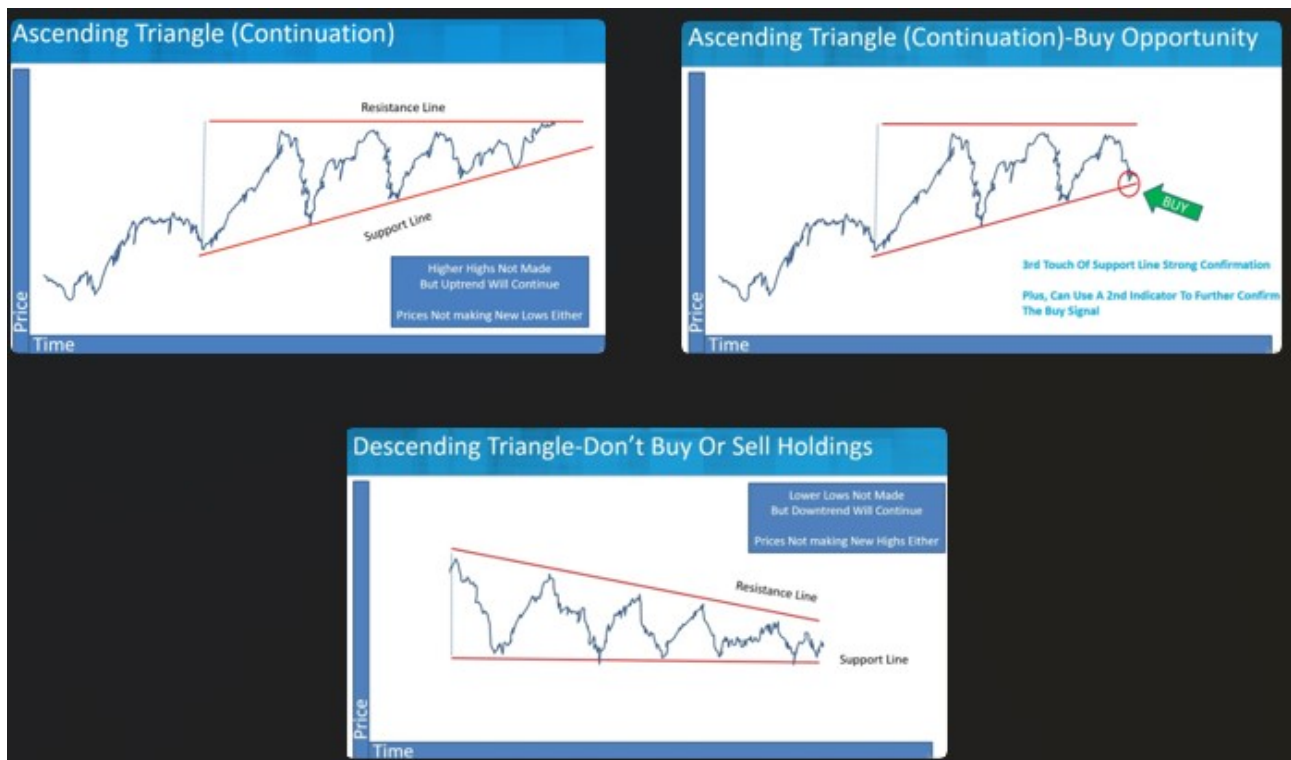


Figure 6: Buy or sell according to pattern

Rectangular Pattern

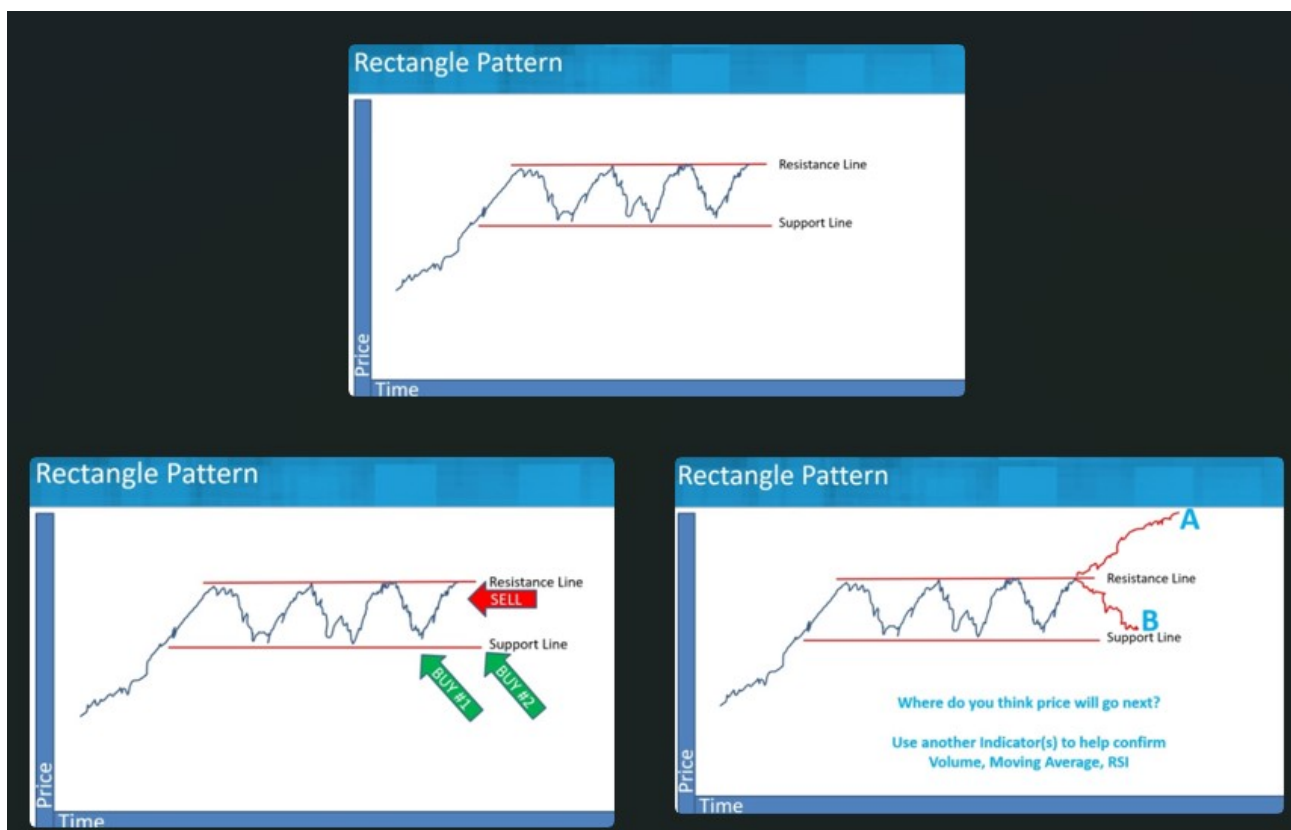


Figure 7: Buy or sell according to pattern

Simple Moving Average (SMA)

- lagging indicator (not forecasting) or trend following;
- no need to try and figure out start and end points (like draw a trendline);
- subjective part is choosing type of MA and how many periods.

The Crossing Rule

Buy

- Price crosses above the MA line.

Sell

- Price crosses below the MA line.



Figure 8: Crossing Rule according to SMA

Moving Average Rule for Decision Making

Call end to the trend:

- **Uptrend:** MA today is less than yesterday;
- **Downtrend:** MA today is higher than yesterday.

It can call the end of a trend faster than the crossover rule, nevertheless, the crossover rule may be more reliable for decision making.

Weighted, Exponential and Adaptive MA

- MA more reflective of the most recent price;
- Weighted MA (WMA) puts more weight on the more recent days;
- Exponential MA (EMA) puts even more weight to recent days;
- Adaptive MA (AMA) uses complex automated calculations by detrending the series to make the current price more important.
- An improvement of AMA is the Kaufman adaptive moving average (KAMA)

Adaptive Moving Averages



Kaufman Adaptive Moving Average (KAMA)

Developed by Perry Kaufman

KAMA will closely follow prices when the price swings are relatively small and the noise is low.

Complex formula with multiple layers.

Kaufman Recommends Settings of:

- 10 is the number of periods for the Efficiency Ratio (ER).
- 2 is the number of periods for the fastest Exponential Moving Average EMA constant.
- 30 is the number of periods for the slowest EMA constant.

To learn more on manual

calculation: https://school.stockcharts.com/doku.php?id=technical_indicators:kaufman_s_adaptive_moving_average

Figure 9: KAMA best settings

0.0.1 Dealing with MAs

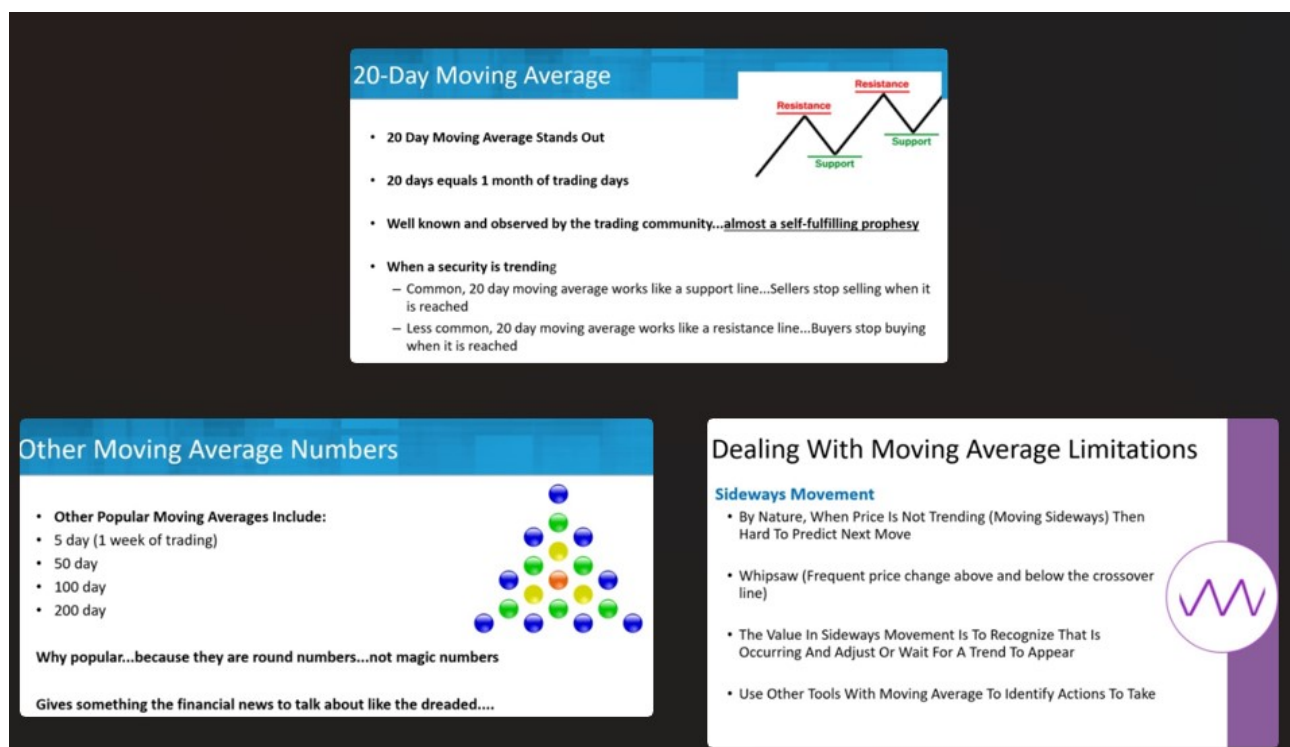


Figure 10: MAs params and limitations

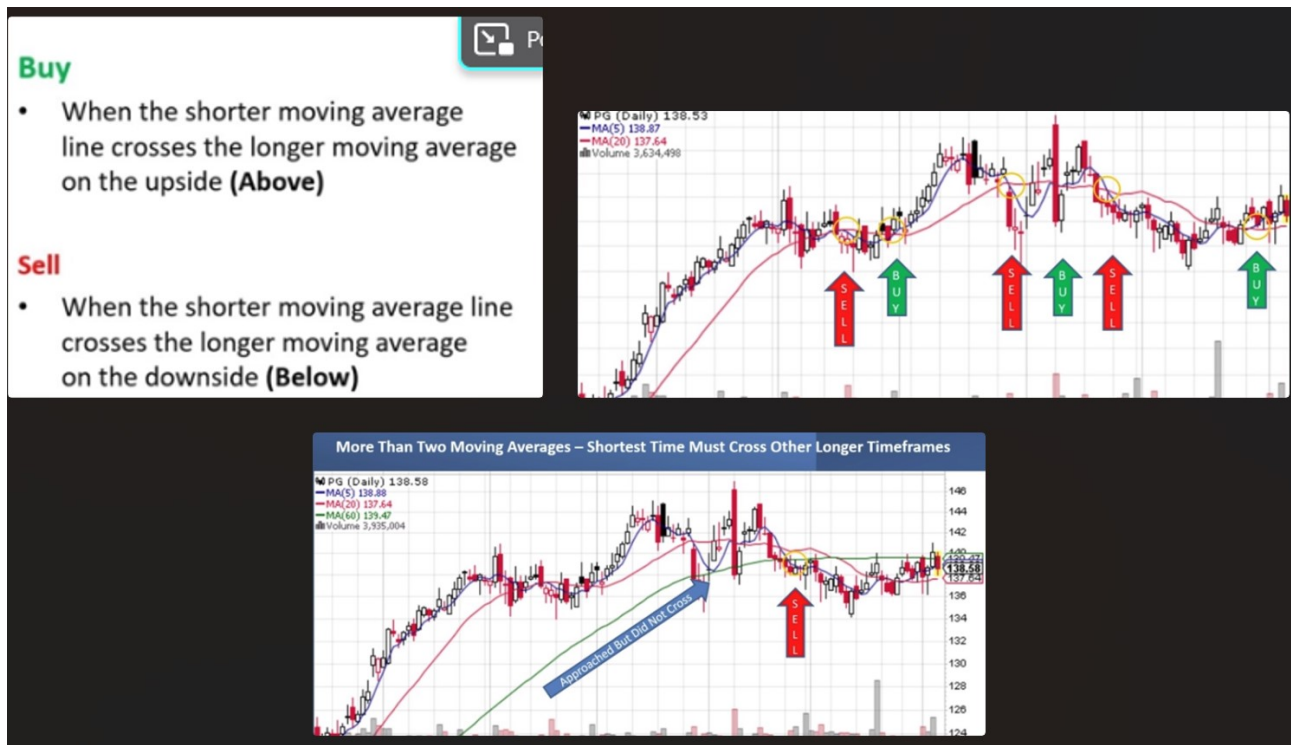


Figure 11: MAs cross rules

0.0.2 Candlestick Signals

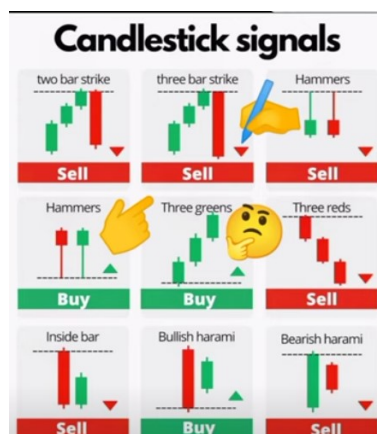


Figure 12: Candlestick Signals

0.0.3 Screenshots

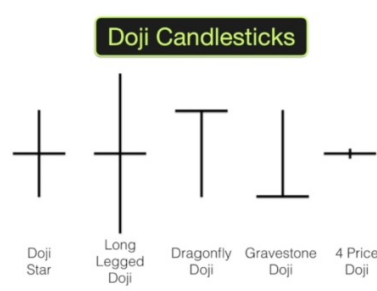


Figure 13: Doji Types

- ▶ When you see a significant increase in bids (buyers) appearing on your level 2, that indicates that there may be more buyers at that price than sellers. This can be a great opportunity to buy.
- ▶ When you see a significant increase on asks (sellers) appearing on your level 2, that indicates that there may be more sellers at that price than buyers. This can be a great opportunity to sell if you previously bought or short sell.

Figure 14: Level 2 Time & Sales

- ▶ Level 2: Provides realtime price quotes displaying where people are buying and selling as well as how many shares they're buying or selling.
- ▶ The bids (buyers) are shown on the left side of level 2, while the asks (sellers) are shown on the right.

View: Quote Montage - Fixed Symbol										
Symbol	Last Tr.	% Chg C	% Chg	High	Low	Vol	Volume	Exch	Last	Size
MMO	0.99	-0.22	-0.01	0.99	0.91	147	150	AMEX	0.99	100
MMO	0.99	0.00	0.00	0.99	0.91	147	150	AMEX	0.99	100
MMO	0.99	0.00	0.00	0.99	0.91	147	150	AMEX	0.99	100
BATS	0.99	0.00	0.00	0.99	0.91	147	150	AMEX	0.99	100
BATS	0.99	0.00	0.00	0.99	0.91	147	150	AMEX	0.99	100
BATS	0.99	0.00	0.00	0.99	0.91	147	150	AMEX	0.99	100
EDGX	0.99	0.00	0.00	0.99	0.91	147	150	AMEX	0.99	100
EDGX	0.99	0.00	0.00	0.99	0.91	147	150	AMEX	0.99	100
EDGX	0.99	0.00	0.00	0.99	0.91	147	150	AMEX	0.99	100
ARCA	0.97	0.00	0.00	0.97	0.92	200	200	AMEX	0.97	100
ARCA	0.97	0.00	0.00	0.97	0.92	200	200	AMEX	0.97	100
ARCA	0.97	0.00	0.00	0.97	0.92	200	200	AMEX	0.97	100
NSDQ	0.96	0.00	0.00	0.96	0.92	200	200	AMEX	0.96	100
NSDQ	0.96	0.00	0.00	0.96	0.92	200	200	AMEX	0.96	100
NSDQ	0.96	0.00	0.00	0.96	0.92	200	200	AMEX	0.96	100

Figure 15: Level 2 Time & Sales 2

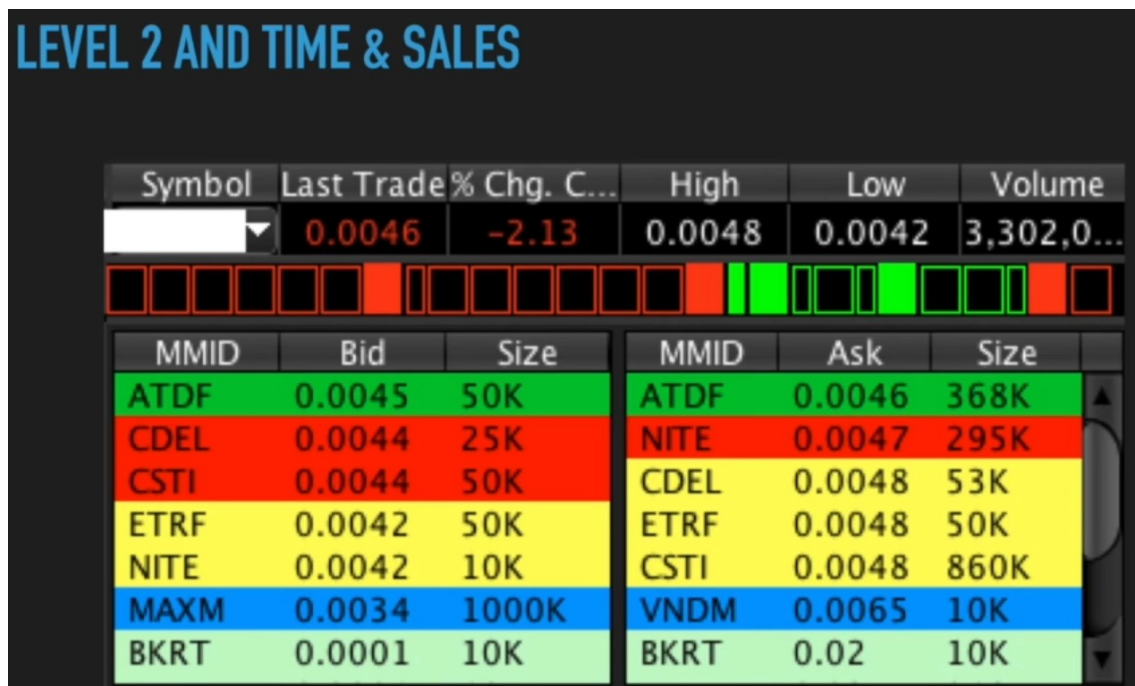


Figure 16: Level 2 Time & Sales 3

Basic Strategy to be supported by more in-depth analysis:

- When Bids (Buyers) > Asks (Sellers) \implies Price \uparrow ;
- When Bids (Buyers) < Asks (Sellers) \implies Price \downarrow ;

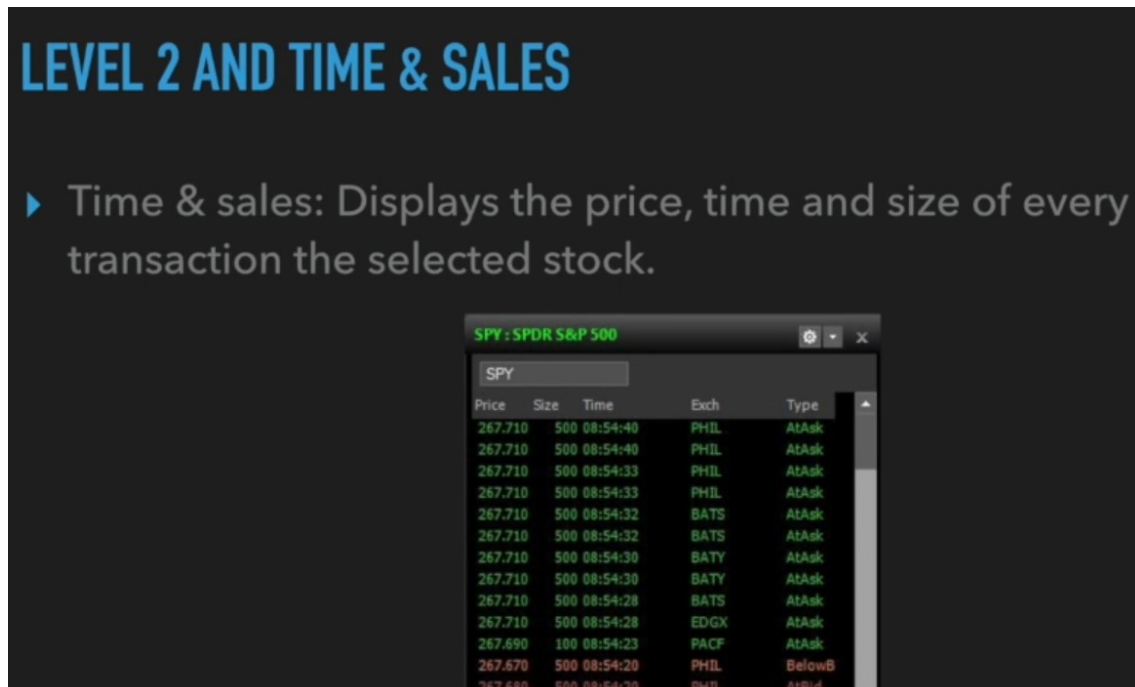


Figure 17: Level 2 Time & Sales 4

LEVEL 2 AND TIME & SALES

- ▶ Unfortunately, advanced trading software allows some traders to hide their orders from level 2.
- ▶ So, you may see that level 2 is showing only 100 shares of stock ABC being bought at \$10.00, but on the time and sales you see 1,000s of shares being traded at that price.
- ▶ This can indicate that there is a “hidden buyer” at \$10.00 and can cause the stock’s price to rise as long as it stays above \$10.00.

Figure 18: Level 2 Time & Sales 5

THE BEST CHART INDICATORS/STUDIES

- ▶ RSI: Relative Strength Index. A momentum indicator that measure the magnitude of recent price changes to evaluate overbought and oversold areas.
- ▶ Overbought: When a stock’s RSI is over 70, it is considered overbought. Overbought stocks have recently had a strong move up and are due for a pullback or a reversal back down.

Figure 19: Best Indicators

BUYING BREAKOUTS

- ▶ **Breakout:** When the price of a stock moves outside a defined level of resistance.
- ▶ This can be a sign of strength and can offer an opportunity to buy into the stock's momentum.

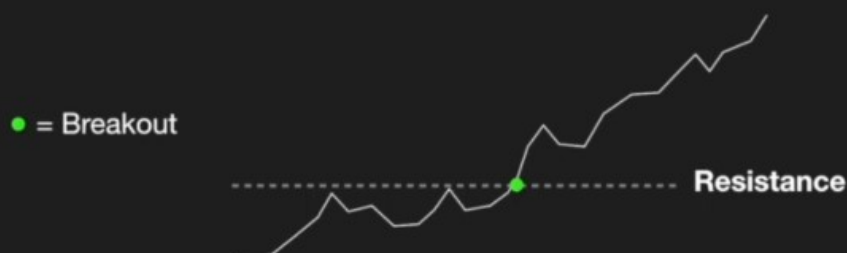


Figure 20: Breakout Strategy (conversely, same logic to sell)

BUYING BREAKOUTS (CONT.)

- ▶ **False Breakout:** A short-term, temporary breakout above resistance or below support often causing inexperienced traders to buy/sell at the worst time.
- ▶ This is why it's crucial to only buy a breakout "the right way."

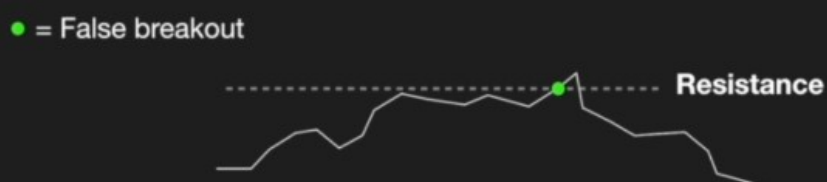


Figure 21: False Breakouts

Dividends

- Amount: paid per share, annual, paid quarterly, the company can decide to raise it or to eliminate it;
- Yield: it is equal to (annual dividend per share) / (current share price), it is easier to compare, higher the number the better;
- Payout ratio: it is equal to (dividend paid) / (net income), lower the number the better.

Intrinsic Value of a Company (IV)

It is a measure of what an asset is worth.

Change in book value = Current Years Earnings - Dividends

IV can be calculated in different ways, as follows,

- Dividend Discount Model (DDM)

1. Focused on Cash

$$\text{Value of Stock} = \frac{EDPS}{(CCE - DGR)}$$

where:

- EDPS = Expected dividend per share;
- CCE = Cost of capital equity;
- DGR = Dividend growth rate.

- Gordon Growth Model (GGM)

$$P = \frac{D_1}{r - g}$$

where: – P = Present value of stock;

- D_1 = Expected dividends one year from the present;
- r = required rate of return for equity investors;
- g = Annual growth rate in dividends in perpetuity.

- Residual Income Model (RIM)

$$V_0 = BV_0 + \sum \frac{RI_t}{(1+r)^t}$$

where:

- BV_0 = Current book value of the company's equity;
- RI_t = Residual income of a company at time period t ;
- r = Cost of equity.

- Discounted Cash Flow Model (DCFM)

$$DCF = \frac{CF_1}{1+r} + \frac{CF_2}{(1+r)^2} + \frac{CF_3}{(1+r)^3} + \dots + \frac{CF_n}{(1+r)^n}$$

where:

- CF_n = Cash flow in period n ;
- $d = 1 + r$ = Discounted rate, Weighted average cost of capital (WACC).

Intrinsic Value Online Calculators

Discounted Cash Flow Calculator Moneychimp.com

<http://moneychimp.com/articles/valuation/dcf.htm>

Warren Buffet Formula (?) Moneychimp.com

http://www.moneychimp.com/articles/valuation/buffett_calc.htm

Dividend Discount Model (DDM)

<https://dqydj.com/dividend-discount-model-calculator/>

Gordon Growth Model

<https://studyfinance.com/gordon-growth-model/#4-gordon-growth-model-calculator>

Residual Income Calculator

<https://studyfinance.com/residual-income/#4-residual-income-calculator>

Figure 22: False Breakouts

0.0.4 Computing DCFM

OnLine Calculator: <http://www.moneychimp.com/articles/valuation/dcf.htm>

Website for data: <https://finviz.com> – search for the company (ex. P&G) Find Parameters:

- EPS(ttm);
- EPS(next 5 Y)
- Set a value for growth assumption (according to your estimation);
- Set a value for the discount rate (according to your estimation).

0.0.5 Computing DDM

OnLine Calculator: <https://dqydj.com/dividend-discount-model-calculator>

Website for data: <https://www.gurufocus.com> – search for the company (ex. P&G)

Or

<https://finance.yahoo.com/>

Find Parameters:

- Current or last Low value;
- Set a value for discount rate (according to your estimation);
- Set a value for dividend growth (according to your estimation);
- Set a value for growth assumption (according to your estimation);
- Set a value for the discount rate (according to your estimation).

Quantitative Finance

Time Value of Money

Discrete model

$$FV = PV(1 + r)^n,$$

where

- r : interest rate;
- n : duration in years.

Continuous model

Let x_0 , (PV) the initial investment, $x(t)$ the amount of money at time $t \implies$

$$\begin{aligned} x(t + dt) - x(t) &= \frac{dx(t)}{dt} dt \implies x(t + dt) - x(t) = r x(t) dt \implies \frac{dx(t)}{dt} dt = r x(t) dt \implies \\ \frac{dx(t)}{dt} &= r x(t) \implies \int \frac{dx(t)}{x(t)} = \int r dt \implies \ln x(t) = rt + x_0 \implies x(t) = x_0 e^{rt} \implies \\ FV &= PV e^{rt}. \end{aligned}$$

Stocks and Shares

- Stocks: ownership of a small portion of a company; the owner of a stock is entitled to a proportion of the company's asset and profit equal to how much stock they own;
- Shares: units of a stock.

Measuring the risk of a stock - Volatility

- Statistical measure of the dispersion of a given security, which is the amount of uncertainty (or risk) about the size of changes in the value of a given security (stock, bond, etc.);
- Volatility can be measured with standard deviation or variance (higher the volatility, higher the risk). To measure the volatility different models can be used, eg. Capital Asset Pricing Model (CAPM), Markowitz model. etc.

An investment can be

- Long: buying a stock and selling in future, hoping stock price will increase (this must include a commission to be paid to the stock broker);
- Short: buying a stock and selling in future hoping the price will decrease (this includes an interest rate for the loaned money plus a commission to the stock broker).

This result is that a short investment is generally more expensive and riskier than a long investment.

Modern Portfolio Theory - Long Position

$$\text{Daily return} = \frac{P(t+1) - P(t)}{P(t)}, \quad t \text{ in days, } P(t) \text{ is the price of the stock.}$$

$$\text{Usually for normalization logarithm is used: } \ln \frac{P(t+1)}{P(t)}.$$

The expected return for a portfolio can be given using the Markowitz model. In this model the variance measures the risk of a portfolio, while the mean μ can be expressed in the following way ($E[\dots]$ is the expected value)

$$\mu_{\text{PORTFOLIO}} = E\left[\sum_i w_i \cdot r_i\right] = \sum_i w_i \cdot E[r_i] = \sum_i w_i \mu_i = \mathbf{w}^T \cdot \boldsymbol{\mu} = \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{pmatrix} \cdot (\mu_1 \quad \mu_2 \quad \dots \mu_n)$$

The risk of a portfolio can be measured by the volatility, which can be approximated by the standard deviation or the variance.

$$\sigma_{ij} = E[(r_i - \mu_i) \cdot (r_j - \mu_j)]$$

the covariance defines how two variables move together.

$$\begin{cases} \sigma_{ij} < 0 \implies \text{negative covariance, returns move inversely,} \\ \sigma_{ij} > 0 \implies \text{positive covariance, returns move together.} \end{cases}$$

The aim of diversification is to eliminate fluctuations in the long term.

It is important to keep uncorrelated stocks or shares in the portfolio.

The variance of a portfolio is given by the following computation

$$\begin{aligned} \sigma_i^2 &= E[(r_i - \mu_i)^2] \implies \\ \sigma_{\text{PORTFOLIO}}^2 &= E[(r_i - \mu_i)^2] = \sum_i \sum_j w_i w_j \sigma_{ij} = \mathbf{w}^T \cdot \boldsymbol{\Sigma} \cdot \mathbf{w}, \text{ where} \end{aligned}$$

$$\boldsymbol{\Sigma} = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \dots & \sigma_{1n} \\ \sigma_{21} & \sigma_2^2 & \dots & \sigma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \dots & \sigma_n^2 \end{bmatrix}$$

Sharpe Ratio - Efficient Frontier

It is one of the most important risk and return used in quantitative finance - used first time by William Sharpe.

$$S(x) = \frac{r_x - R_f}{\sigma_x},$$

where

- r_x : average rate of ROI (return of investment) of the stock x ;
- R_f : rate of return of a risk-free security (ex. bond, treasury bill, bank interest)

It describes how much excess return an investor can receive for extra volatility while holding a riskier asset (stocks or shares). The optimal Sharpe ratio is to maximize r_x and minimize σ_x (standard deviation). When $S(x) > 1$ then the stock can be considered a good investment.

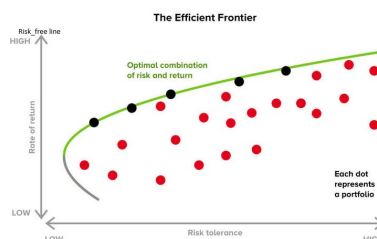


Figure 23: Efficient Frontier

Capital Asset Pricing Model (CAPM)

– Risk in finance

- **Unsystematic (Specific) Risk:** This is specific to individual stock. It can be diversified away by holding multiple stocks in a portfolio; this risk is a component of a stock's return that is not correlated with the market moves.
- **Systematic (Market) Risk:** It cannot be diversified away. This can be due to interest rate changes, recessions and wars. The CAPM measures this risk with β parameter.

The CAPM can be calculated in the following way

$$E[r_a] = r_f + \beta_a(E[r_m] - r_f),$$

where

- $E[r_a]$ is the expected return of investment (sub. a stands for asset) - it may be a single stock or a portfolio;
- r_f is the risk-free rate (can be set to zero or a low value);
- The last part is the so called market premium or market excess return multiplied by the factor β_a , while $E[r_m]$ is the expected market return. As index for r_m we can use the market fluctuations of the stock S&P 500.

There is a linear relationship between any stock expected return and the market premium, we have

$$\beta_a = \frac{\text{cov}(r_a, r_m)}{\text{var}(r_m)} = \frac{\text{cov}(r_a, r_m)}{\sigma_{r_m}} = \frac{\sum_i^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_i^n (x_i - \bar{x})^2},$$

where

- n : number of stocks;
- σ : standard deviation;
- x y : the stock price for the stocks in the portfolio and the price for the referenced market (e.g. S&P 500) respectively, during a chosen period.

β_a represents the ratio between the covariance of the investment return and the market return divided by the variance of the market return. The parameter β_a defines how risky a portfolio is relative to the market. According to CAPM the β parameter is the only relevant measure of a stock's risk.

β measures the stock's relative volatility, that is how much the price of a stock goes up or down compared to market's price (e.g. share price of S&P 500).

For multiple stocks in a portfolio, we have

$$\beta_a = \sum_i^n w_i \beta_i.$$

As reference value in the CAPM, we can use the Adjusted Closing Price, which has something to do with the Closing Price but it takes into account factors, such as dividends, stock splits, etc. Adjusted closing price is more accurate reflection of the stock's value. In this case, rather than using daily returns, which is good for short term tactical forecasting, it is advice to use monthly returns for long-term models such as CAPM, as the monthly returns are at least approximately normally distributed. Most of the models assume normal distribution.