Four Ways to Represent a Function

Chapter 1, Section 1

Here are some important ideas from lecture:

- **Circle one**: A function is a rule that assigns to each element x from its domain [more than one / exactly one / less than one] element in its range.
- We can represent functions verbally (words), numerically (table of values), visually (graph), or algebraically (explicit formula).
- The Vertical Line Test is a way to tell whether or not a graph in the xy-plane is a function.

Vertical Line Test

An xy-curve is the graph of a function *if and only if* no vertical line intersects the curve more than once.

- **Fill in the blanks**: A function f is *even* if ________________. A function f is *odd* if __________________. These rules must hold for all x.

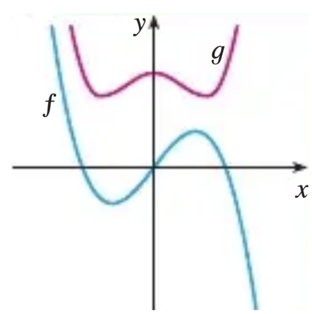
Mnemonic for even and odd functions

Symmetry has to do with the behavior of the function for input values of $-x$ as opposed x. One way to remember what even and odd functions do is **E**ven **E**ats the negative while **O**dd spits it **O**ut.

- A function f is called *increasing* on an interval I if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ in I.
- A function f is called *decreasing* on an interval I if ____.

. . . .

Problem 1: (Stewart Section 1.1) Consider the following graph, which depicts the functions f and g. If f even, odd, or neither? Why? Is g even, odd, or neither? Why?



Problem 2: (Borcherds '05 Midterm 1) Find the domain of the function $g(u) = \sqrt{u} + \sqrt{2-u}$.

Problem 3: (Stewart Section 1.1) Recall that a *piecewise function* splits its domain into pieces and is defined by different formulas for each piece. Sketch the graph of the following piecewise function:

$$f(x) = \begin{cases} x+1 & \text{if } x \leq -1 \\ x^2 & \text{if } x > -1 \end{cases}.$$

Problem 4: (Stewart Section 1.1) Determine whether $f(x) = x|x|$ is even, odd, or neither.

Problem 5: (Stewart Section 1.1) Does $x^2 + (y-3)^2 = 5$ define a function? Explain why or why not.

Problem 6: (Stewart Section 1.1) An open rectangular box with volume 2m^3 has a square base. Express the surface area of the box as a function of the length of a side of the base.

Challenge problem: Consider the function $f(x) = 4 + 3x - x^2$. Evaluate the difference quotient given by

$$\frac{f(3+h) - f(3)}{h}.$$