Four Ways to Represent a Function

Chapter 1, Section 1

Here are some important ideas from lecture:

- **Circle one**: A function is a rule that assigns to each element x from its domain [more than one / exactly one / less than one] element in its range.
- We can represent functions verbally (words), numerically (table of values), visually (graph), or algebraically (explicit formula).
- The Vertical Line Test is a way to tell whether or not a graph in the xy-plane is a function.

Vertical Line Test

An xy-curve is the graph of a function *if and only if* no vertical line intersects the curve more than once.

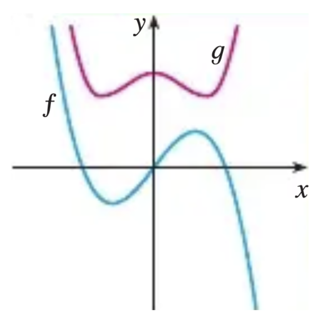
- **Fill in the blanks**: A function f is *even* if $f(-x) = f(x)$. A function f is *odd* if $f(-x) = -f(x)$. These rules must hold for all x.

Mnemonic for even and odd functions

Symmetry has to do with the behavior of the function for input values of $-x$ as opposed x. One way to remember what even and odd functions do is **E**ven **E**ats the negative while **O**dd spits it **O**ut.

- A function f is called *increasing* on an interval I if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ in I.
- A function f is called *decreasing* on an interval I if $f(x_1) > f(x_2)$ whenever $x_1 < x_2$ in I.

Problem 1: (Stewart Section 1.1) Consider the following graph, which depicts the functions f and g. If f even, odd, or neither? Why? Is g even, odd, or neither? Why?



Recall that the symmetry of a function asks what happens when we calculate the function at $-x$ rather than just x.

Testing some values of x, we can see that $f(-x) = -f(x)$ for all x. Thus f is odd. However, $f(-x) \neq f(x)$ for all x, and we can see this by picking $x > 0$. Thus f is not even.

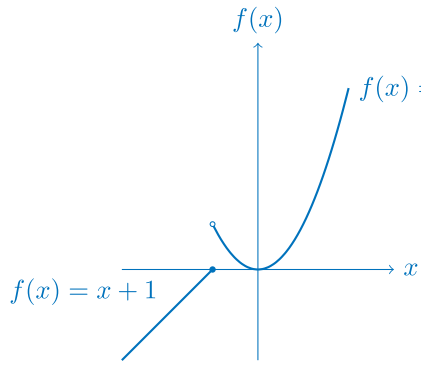
On the other hand, we can see that $g(-x) = g(x)$ for all x, so g is even. However, $g(-x) \neq -g(x)$ for all x, and we can see this by picking $x > 0$. Thus g is not odd.

Problem 2: (Borcherds '05 Midterm 1) Find the domain of the function $g(u) = \sqrt{u} + \sqrt{2-u}$.

Notice that $\sqrt{u}$ is defined for $u \geq 0$, since we cannot take the square root of a negative number. The other term is a little trickier: $\sqrt{2-u}$ is defined for $2 - u \geq 0$, which means $u \leq 2$. In order for the function g to be defined, both of its terms must be defined. Thus we need $u \geq 0$ and $u \leq 2$, which means the domain is in fact $0 \leq u \leq 2$.

Problem 3: (Stewart Section 1.1) Recall that a *piecewise function* splits its domain into pieces and is defined by different formulas for each piece. Sketch the graph of the following piecewise function:

$$f(x) = \begin{cases} x+1 & \text{if } x \leq -1 \\ x^2 & \text{if } x > -1 \end{cases}.$$



Problem 4: (Stewart Section 1.1) Determine whether $f(x) = x|x|$ is even, odd, or neither.

Notice that
$$f(-x) = (-x)|-x| = (-x)|x| = -x|x| = -f(x)$$
Since $f(-x) = -f(x)$, the function is odd.

Problem 5: (Stewart Section 1.1) Does $x^2 + (y-3)^2 = 5$ define a function? Explain why or why not.

The given equation represents a circle centered at $(0,3)$ with radius $\sqrt{5}$. The equation of a circle centered at (h,k) with radius r is given by $(x-h)^2 + (y-k)^2 = r^2$. In this case, the center of the circle is $(0,3)$ and the radius is $\sqrt{5}$. Since for each value of x there are two possible values of y (except at the top and bottom points of the circle), the equation does not define a function by the Vertical Line Test.

Problem 6: (Stewart Section 1.1)
An open rectangular box with volume 2m^3 has a square base. Express the surface area of the box as a function of the length of a side of the base.

Let x be the length of a side of the square base of the open rectangular box, and let the height of the box be h. Thus, we have the equation $x^2 \cdot h = 2$, and solving for h, we get:

$$h = \frac{2}{x^2}$$

The surface area A is the sum of the areas of the five faces of the box (since it is open):

$$A = x^2 + 4(x \times h) = x^2 + 4(x \times \frac{2}{x^2}) = x^2 + \frac{8}{x}$$

So, the surface area of the box is expressed as a function of the length of a side of the base x as $A(x) = x^2 + \frac{8}{x}$.
Challenge problem: Consider the function $f(x) = 4 + 3x - x^2$. Evaluate the difference quotient given by

$$\frac{f(3+h) - f(3)}{h}.$$

