MATLAB © Exercise II: Active Noise Control with an FIR filter

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Contents

Chapter 1

The problem of Active Noise Cancellation

1.1 Theoretical Reference

Consider the signal configuration displayed in Figure 1.1 (see also Figure 7.8 on page 349 of [1]).

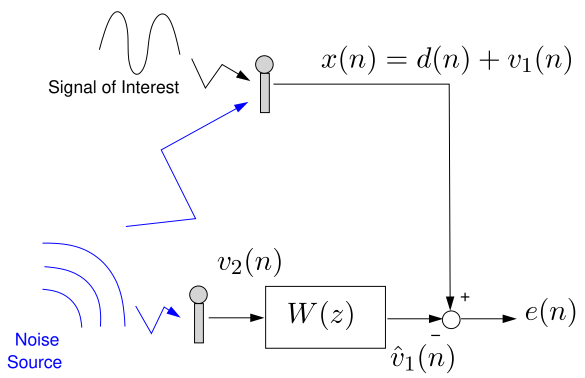

Figure 1.1: The Signal configuration of the Active Noise Control Problem.

In the configuration in Figure 1.1 both discrete time signals $x(n) = d(n) + v_1(n)$ (the corrupted signal of interest) and $v_2(n)$ (the signal correlated with $v_1(n)$ **but** uncorrelated with $d(n)$) are given. All signals are real.

The goal is to design an FIR filter with $W(z)$ given as:

$$W(z) = w(0) + w(1)z^{-1} + \cdots + w(m-1)z^{-m+1} \tag{1.1}$$

such that *using the 'measurable" signals* $x(n)$ *an* $v_2(n)$ **only** the following criterium is minimized,

$$\sum_{n=1}^{N} \Big(v_1(n) - \hat{v}_1(n)\Big)^2 \tag{1.2}$$

with $\hat{v}_1(n) = \sum_{\ell=0}^{m-1} w(\ell)v_2(n-\ell)$.

The theory for solving this exercise is given in section 7.2 of [1].

Chapter 2

Active Noise Control: the MATLAB © exercise

2.1 The data

Referring to Figure 1.1 the signals $x(n) = d(n)+v_1(n)$ and $v_2(n)$ can be generated using the `MATLAB` © template `Matlab2template2018.m` resp. as variables `x` and `v2`. This template can be downloaded from the `Brightspace` website. The template contains for verification **only** the original source signal (sound) $d(n)$[1]. The sound is stored in the variable `d`. The goal of this exercise is to design an FIR filter to filter the noise signal $v_1(n)$ to retrieve the original sound signal.

2.2 Exercise

This exercise will consist of 2 parts. In Part 1, you will create a (corrupted) sound signal for another group to filter. In part 2, you will filter the signal created by another group to reconstruct the original sound file.

2.2.1 Part 1: signal construction

In this part of the exercise, the signals $x(n)$ and $v_2(n)$, as shown in Figure 1.1 are created and sent to another group. In order to achieve this, the following tasks need to be solved.

1. Choose a sound signal that you have available (for example as an mp3 or wav file). If you feel truly uninspired, you may also use the standard file provided on Brightspace. Load this file into MATLAB. Hint: you may use the command `audioread`.

   Name this your signal of interest $d(n)$. You can check the conversion went succesfully by using the command `sound`. Make sure your sound signal has an appropriate length (**maybe put a limit on the time length of the**

[1] You can listen to this sound within `MATLAB` © by the command `sound`

sound?). If necessary, crop the sound to obtain a shorter signal, or repeat the sound to achieve a longer signal.

2. Consider that the sequences $v_1(n)$ and $v_2(n)$ are generated as follows:

$$\begin{aligned} v_1(n) - \alpha v_1(n-1) &= g(n) - \beta g(n-1) \\ v_2(n) - 0.95\alpha v_2(n-1) &= g(n) - 1.2\beta g(n-1) \end{aligned} \tag{2.1}$$

with $g(n)$ a (discrete-time) zero-mean white noise sequence with standard deviation 0.4 that is the common input to generate both sequences $v_1(n)$ and $v_2(n)$. The variables α and β need to be adjusted in order to achieve a desired signal-to-noise ratio (SNR).

Use Eq. (2.1) and the MATLAB function `filter` to generate the signals `v1` and `v2`. Start by choosing $\alpha = 0.9$ and $\beta = 0.5$. Generate signal `x` as defined in Section 2.1.

1 point

3. Determine the SNR of the signal $x(n)$ obtained in the previous task. What is the effect of increasing or decreasing α and β on the SNR? Find values for α and β that result in a SNR between 0.1 and 0.2. Report the values of α and β as well as your SNR using Brightspace **we still have to determine how exactly!**.

[Hint: the SNR of a signal can be determined using the command `SNR`]

4. Send your corrupted signal $x(n)$, as well as the noise measurement $v_2(n)$ to the other group **we still have to determine how exactly this will go**.

5. (*This and the next task can maybe be left out if we feel like it is too much work for the students*) The goal of the exercise is to derive the Wiener-Hopf equations to estimate the filter parameters $w(j)$ for $j = 0, \cdots, m-1$, such that Eq. (1.2) is minimized using the signals $x(n)$ and $v_2(n)$ **only**.
Your answer should consist of

(1) The derivation of the Wiener-Hopf equation. A hint for deriving this equation is given by the following recipe:

(a) Knowing the ARMA(1,1) model in (2.1) for $v_2(n)$ determine the Auto-correlation function $r_{v_2}(k)$ for $k = -200 : 200$ via the Yule-Walker equations (see Chapter 4).

2 points

(b) To determine the cross-correlation function $r_{v_1 v_2}(k)$ for $k = -200 : 200$ determine the coefficients of the transfer function of the a system that has $v_2(n)$ as input and $v_1(n)$ as output. Show, using properties of both systems in (2.1), that it suffices to calculate one side of the auto-correlation function and mirror it. Then you can use the insights from Sec. 4.2 from the reader.

3 points

(2) The MATLAB © script that determines the m coefficients $w(j)$ for $j = 0, \cdots, m-1$ of the optimal FIR Wiener filter using the Wiener-Hopf equation, where m is the filter order.

2 points

[Hint: for part 1(1)-a you can use the MATLAB© command `flip`. and for part 1(1)-b you can use the MATLAB© command `conv`.]

6. Use the calculated coefficients $w(j)$ for $j = 0, \cdots, m-1$ of Part 2 of the Exercise, to generate the estimated sequence $\hat{v}_1(n)$ (see Figure 1.1) and subsequently estimate the standard deviation of the error signal $d(n) - (x(n) - \hat{v}_1(n))$ using the MATLAB© command `std`.
   Your answer should contain the computed standard deviation of the error signal $d(n) = (x(n) - \hat{v}_1(n))$ for the order m of the FIR filter $W(z)$ resp. equal to, **2 points**
   $$m = 1, 2, 4, 6$$

2.2.2 Part 2: signal filtering

In this part, you will try to filter a corrupted radio signal sent to you by another group. To this effect, the following tasks need to be completed.

1. Intercept the corrupted radio signal, as well as the noise measurement, through the dongels (**we still need to work out how this will be done**). Name the signal of interest `y` and the noise measurement `v2` in MATLAB. Listen to the radio signal. Can you already make out what the sound might be?

2. An approximation to the Wiener Hopf equation (derived in Part 1) should be determined when the Auto- and Cross correlation function are approximated as follows. Let N samples of two zero-mean stochastic process $x(n)$ and $v_2(n)$ be given, then the following approximation of the Auto- and Cross correlation is used to solved the Active Noise Control Problem:
   $$\hat{r}_{v_2}(\tau) = \frac{1}{N-\tau} \sum_{n=\tau+1}^{N} v_2(n)v_2(n-\tau)$$
   $$\hat{r}_{xv_2}(\tau) = \frac{1}{N-\tau} \sum_{n=\tau+1}^{N} x(n)v_2(n-\tau) \qquad (2.2)$$
   Use these approximations to compute an estimate of the parameters $w(j)$ for $j = 0, \cdots, m-1$ as done in part 2 of the exercise.
   Your answer should consists of (see next page):
   (1) the MATLAB © script with inputs the given time sequences $x(n)$ and $v_2(n)$ and the order m of the FIR filter $W(z)$ and as output the m estimates of $w(j)$ for $j = 0, \cdots, m-1$. **2 points**
   (2) the estimated standard deviation of $d(n) = (x(n) - \hat{v}_1(n))$ with $\hat{v}_1(n)$ derived from estimated coefficients $w(j)$ for $j = 0, \cdots, m-1$ computed in Part 4(1) of the exercise for the same orders m considered in Part 3 of the Exercise. **1 point**

3. Qualify your perception in one word of the quality of your recovered signal $\hat{d}(n) = x(n) - \hat{v}_1(n)$ using the MATLAB © `sound` command. Can you make out what the original sound might have been? **1 point**

4 points

4. **Do we want to keep this in? We can also take this out to compensate for the added questions.**Solve exercises 4.12 and 6.7 from the reader.

2.3 Reporting

The report of this MATLAB © exercise should consist of the requested answers to the above 5 parts also nicely ordered in 5 corresponding parts, together with your answers to exercises 4.12 and 6.7 from the reader. The MATLAB © scripts should be included as text in your reports as well as your MATLAB © plots.

This report containing the names of the members of your group and their student number should be dated and handed in following the Course Schedule (see Brightspace).

Bibliography

[1] M. Hayes, *Statistical Digital Signal Processing and Modeling*, John Wiley & Sons, New York, 1996.