THEORETICAL COMPUTER SCIENCE TUTORING (6)

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Problem 3 from the exam held on September 13, 2019

Consider the following decision problem: given an undirected graph $G = (V, E)$ and an integer k, decide whether G has a vertex cover of at most k nodes or contains an independent set of at least k nodes.
After formalizing the problem using the triple $\langle I, S, \pi \rangle$, answer the following questions (in the order deemed appropriate), providing justification for each response.

- Is the problem in P?
- Is the problem in NP?
- Is the problem in coNP?

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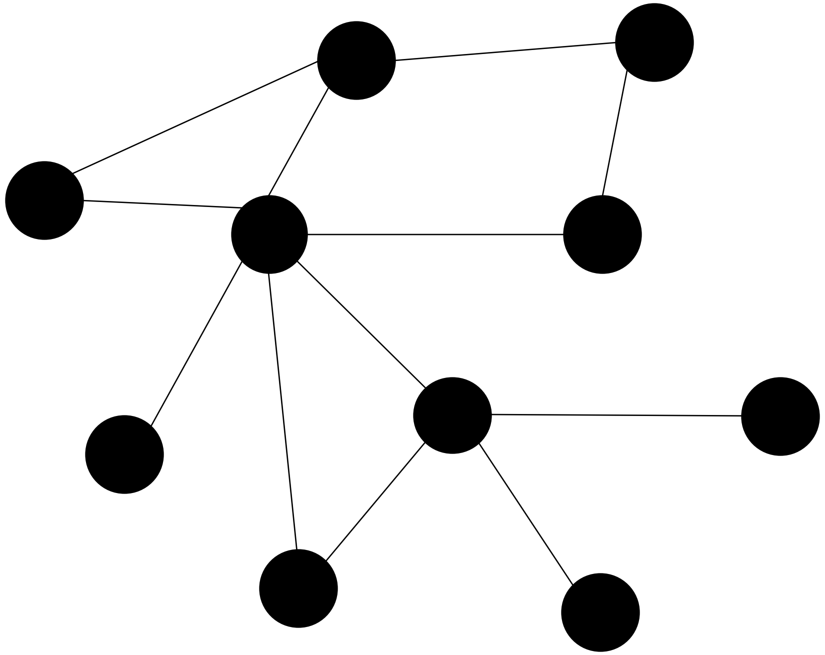
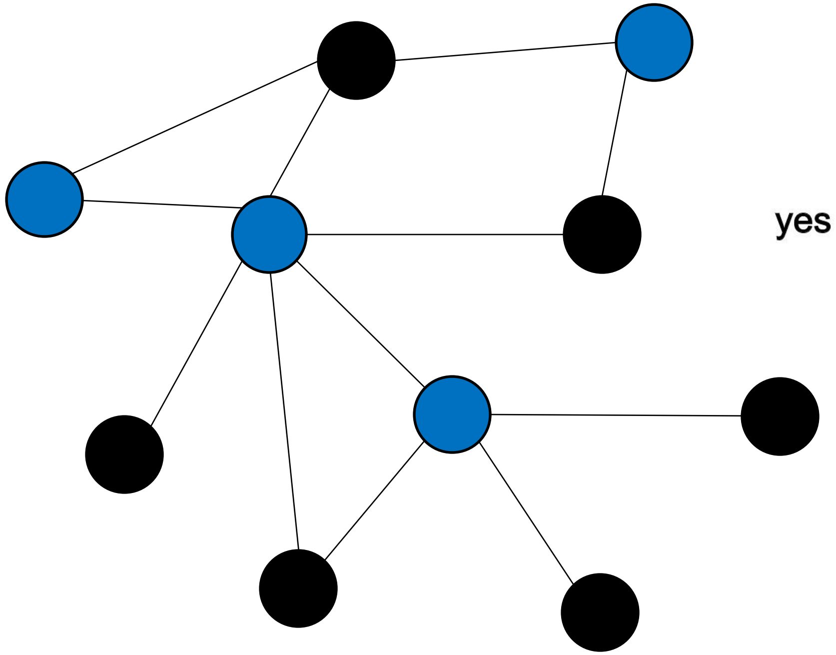
Let's review **Vertex Cover**

- $I_{VC} = \{\langle G = (V, E), k\rangle : \ G$ is an undirected graph $\wedge\, k \in \mathbb{N}\}$
- $S_{VC}(G, k) = \{V' \subseteq V\}$
- $\pi_{VC}\big(G, k, S_{VC}(G, k)\big) = \exists\, V' \in S_{VC}(G, k) : [|V'| \leq k \ \wedge \forall \{u, v\} \in E \ u \ \in V' \wedge v \in V']$





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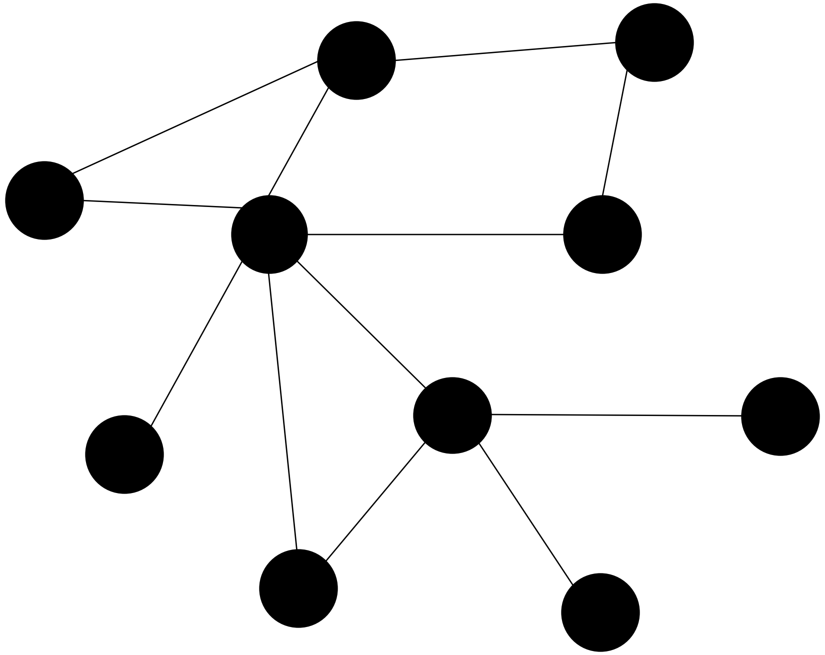

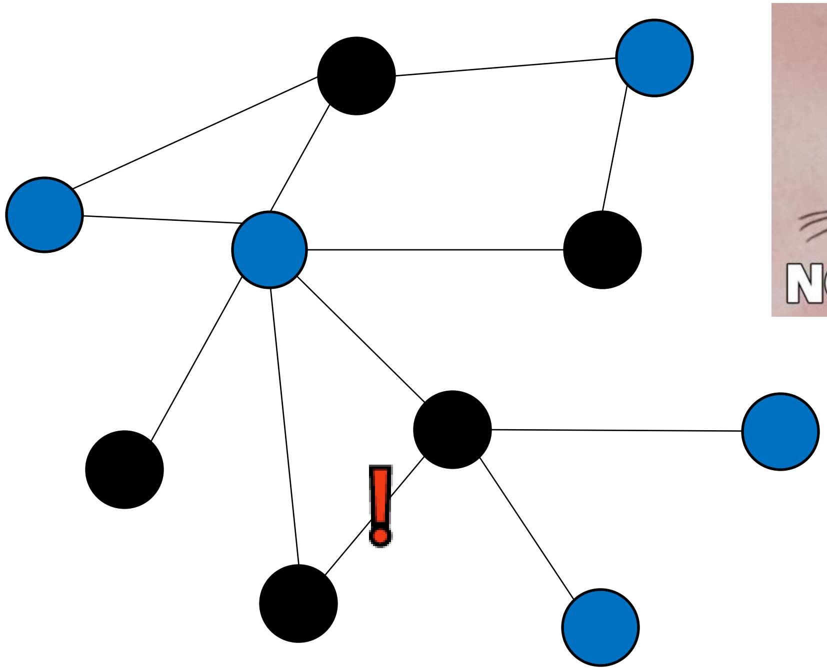

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Let's review **Independent Set**

- $I_{IS} = \{\langle G = (V, E), k\rangle :\ G \text{ is an undirected graph} \wedge k \in \mathbb{N}\}$
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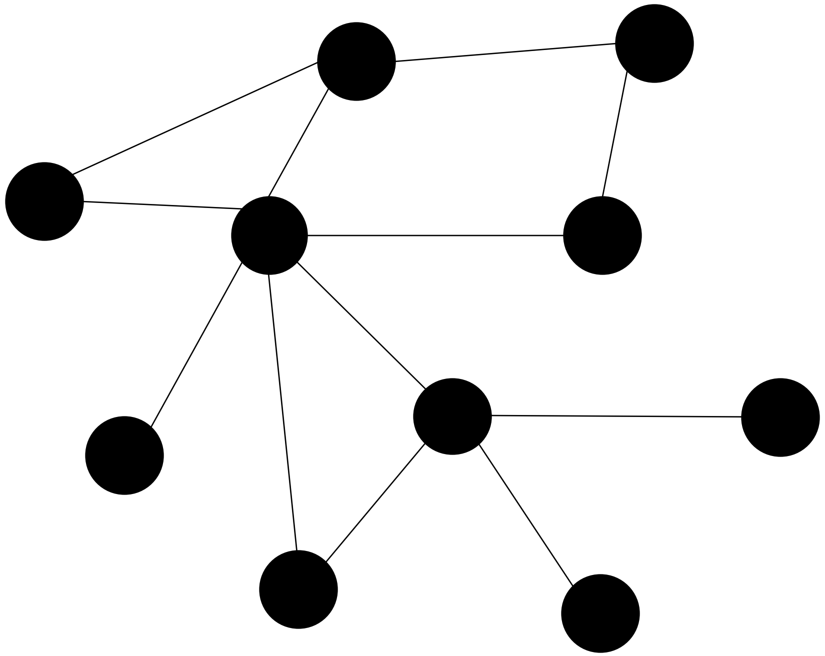

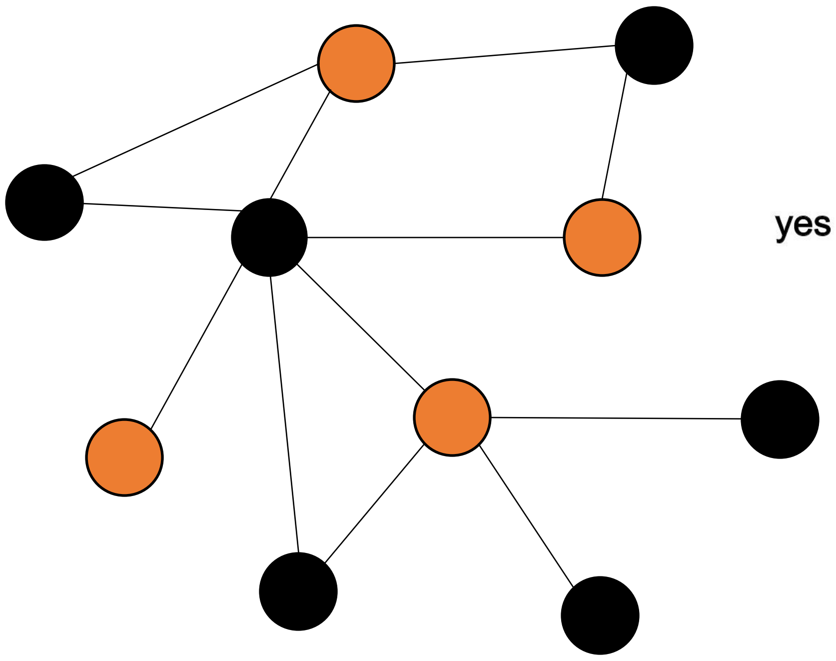

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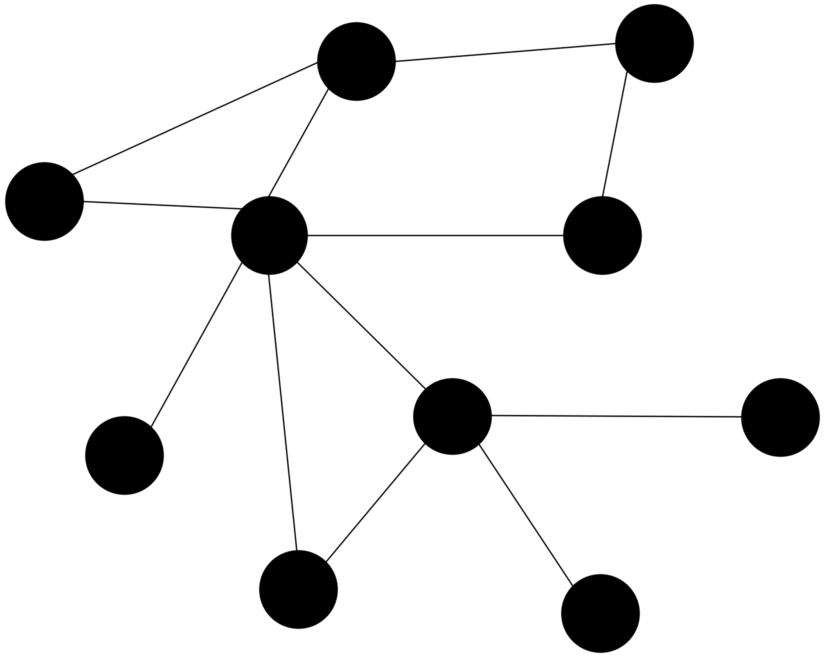

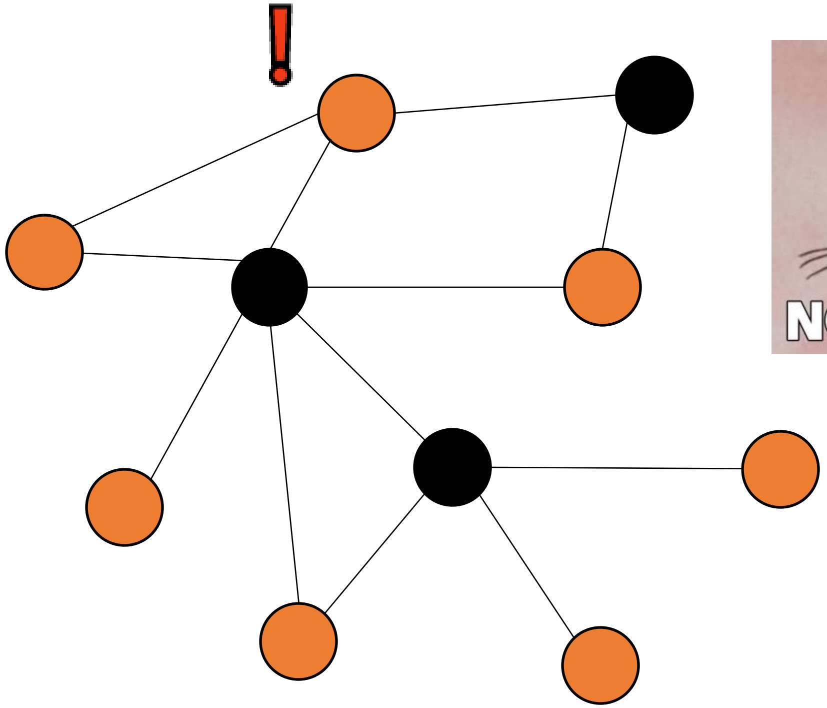

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- $I_{exam} = \{\langle G = (V, E), k\rangle : G$ is an undirected graph $\wedge\, k \in \mathbb{N}\}$
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A certificate is a pair (V', V''), verifying whether it satisfies the predicate requires polynomial time.

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Let's reduce from **Independet Set**

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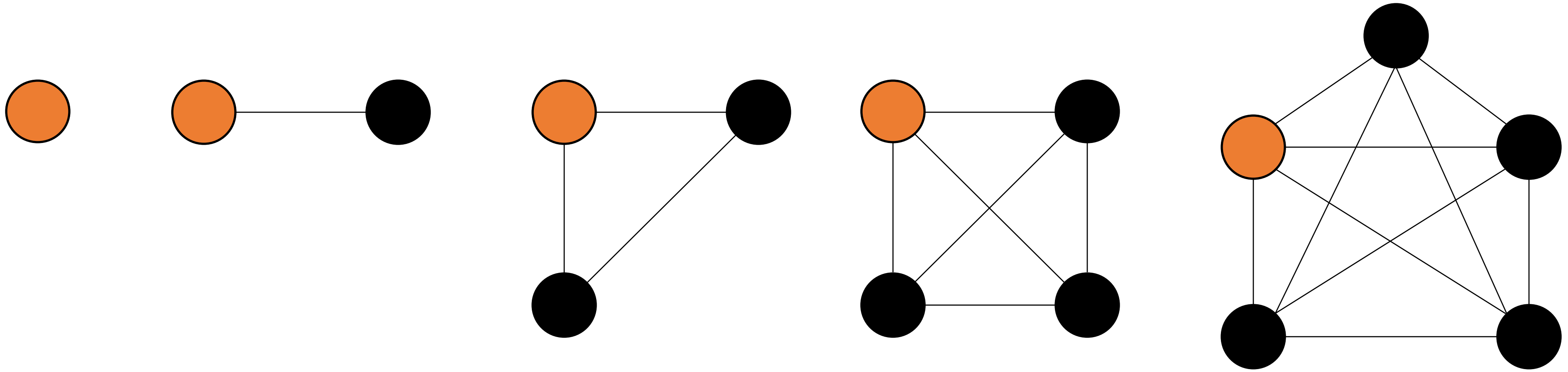
IS on complete graphs

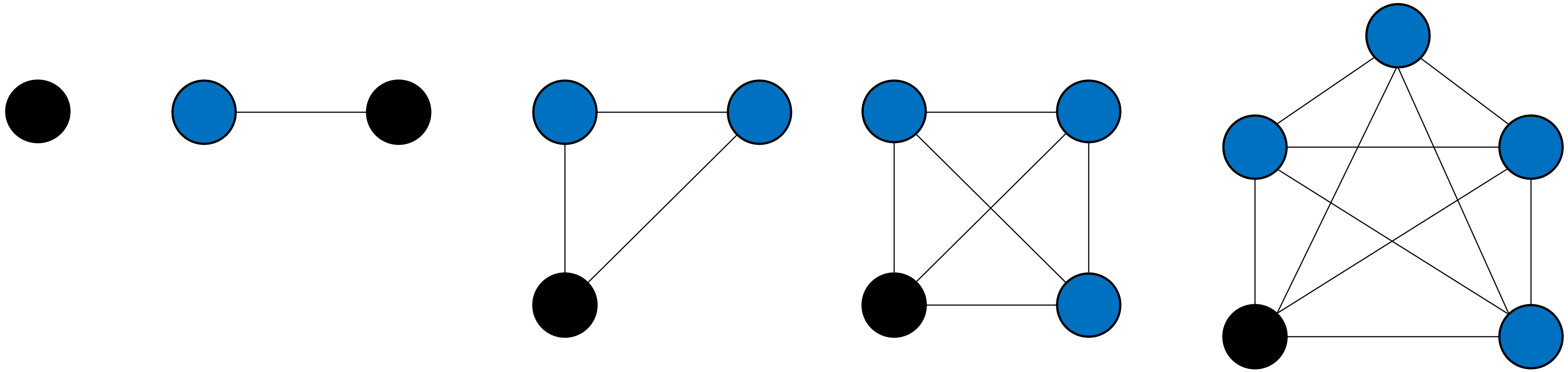
🔑 The Maximum Independent Set on a complete graph consists of a single vertex



Problem 3 from the exam held on September 13, 2019

VC on complete graphs

The minimum vertex cover on a complete graph with k nodes is of k-1 nodes



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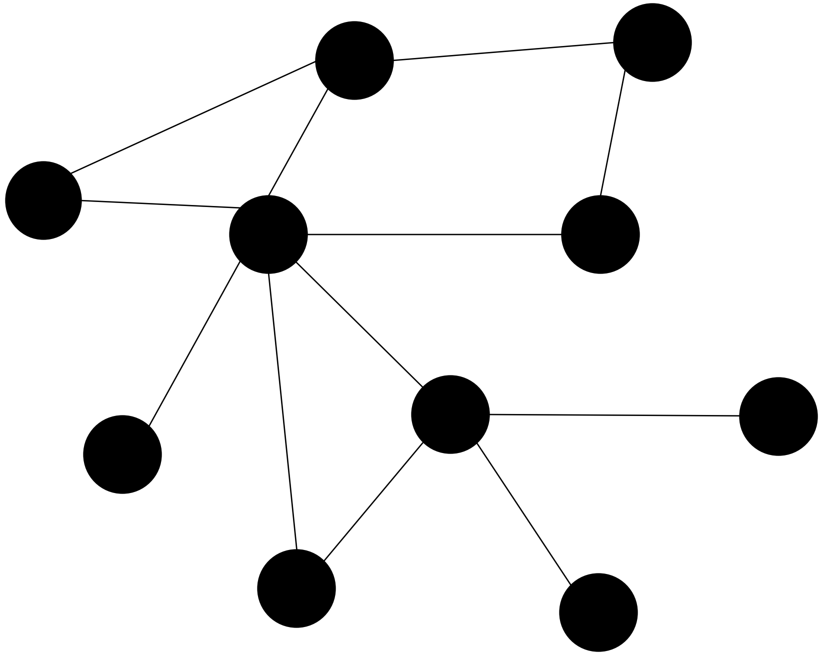
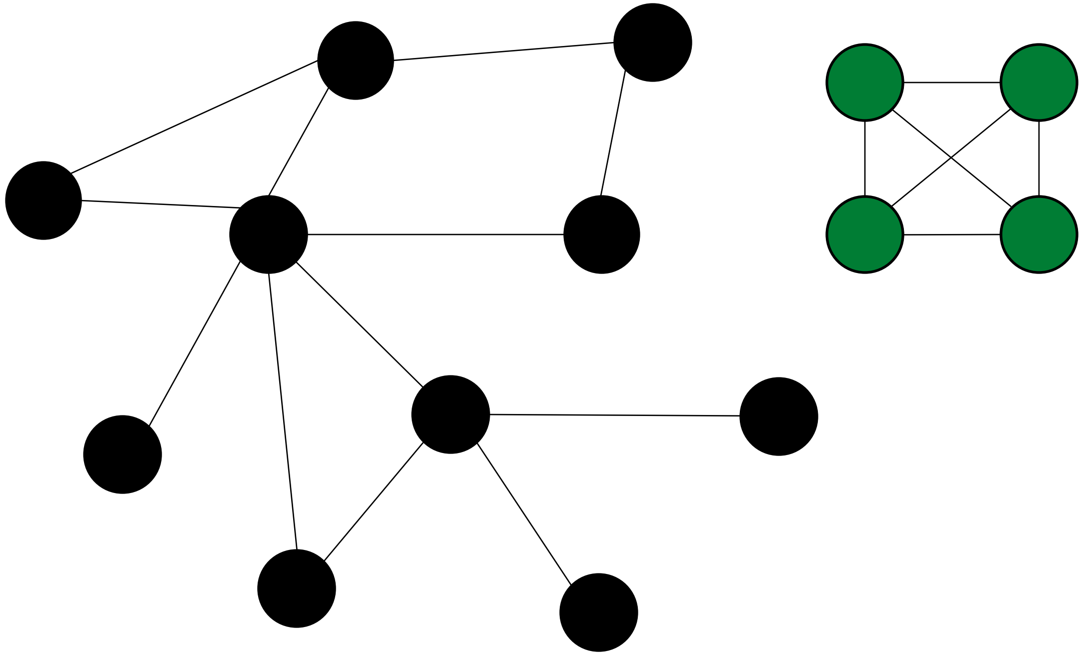


Instance of **IS**

$k + 3$ clique



Instance of our problem

VC ∨ **IS**

Problem 3 from the exam held on September 13, 2019

Instance of **IS**

$I_{IS} = \{\langle G = (V, E), k\rangle : G$ is an undirected graph $\wedge\, k \in \mathbb{N}\}$

Instance of **VC** ∨ **IS**

$I_{ex} = \{\langle G_{ex} = (V_{ex}, E_{ex}), k+1\rangle :$ G_{ex} is an undirected graph $\wedge\, k \in \mathbb{N}\}$

G_{ex} is obtained by adding a clique of $k+3$ nodes to G

*

G_{ex} doesn't have a **VC** of at most $k+1$ vertices $\Longrightarrow$ ~~**VC**~~ ∨ **IS**

If G has an **IS** of at least k vertices, G_{ex} has an **IS** of at least $k+1$ vertices, I_{ex} is a "yes" instance of **VC** ∨ **IS**

If G doesn't have an **IS** of at least k vertices, G_{ex} doesn't have an **IS** of at least $k+1$ vertices (and *), I_{ex} is a "no" instance of **VC** ∨ **IS**

Problem 3 from the exam held on July 4, 2019

Consider the following decision problem: given an undirected graph $G = (V, E)$ and an integer k, decide whether G has a vertex cover of at most k nodes and contains an independent set of at least k nodes.
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Let's reduce from **Vertex Cover**

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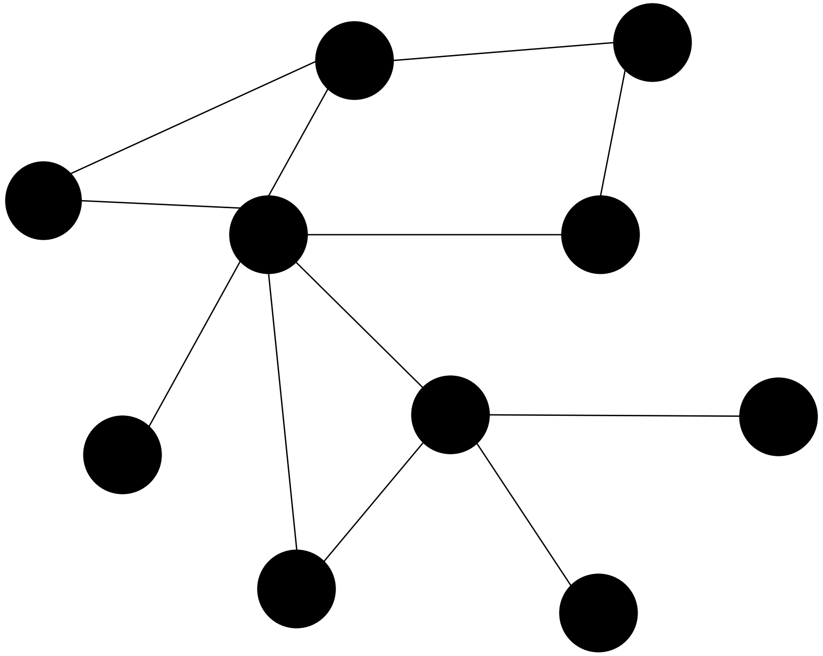
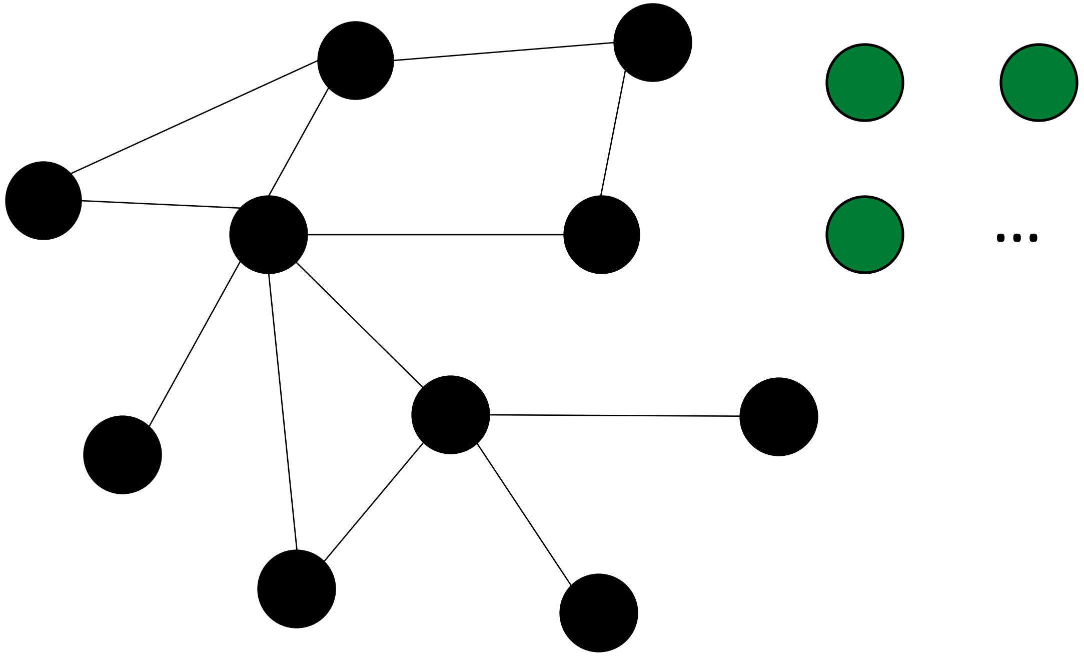


Instance of **VC**

$\#k$ new **vertices**



Instance of our problem

VC $\wedge$ **IS**

Problem 3 from the exam held on July 4, 2019

Instance of **VC**

$I_{IS} = \{\langle G = (V, E), k\rangle : G$ is an undirected graph $\wedge\ k \in \mathbb{N}\}$

Instance of **VC** ∧ **IS**

$I_{exam} = \{\langle G_{exam} = (V_{exam}, E), k\rangle : G_{exam}$ is an undirected graph $\wedge\ k \in \mathbb{N}\}$

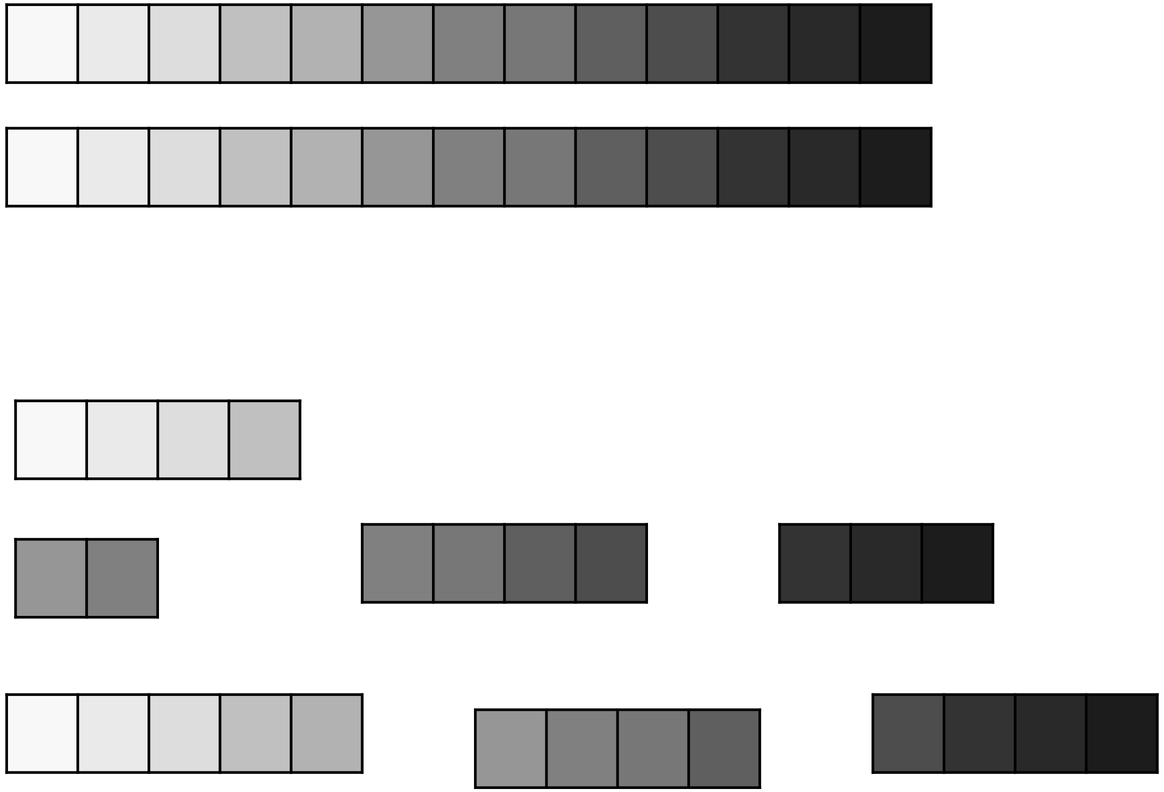
$V_{exam} = V \cup \{v_1, v_2, \dots, v_k\}$

G_{exam} has an **IS** of at least k vertices

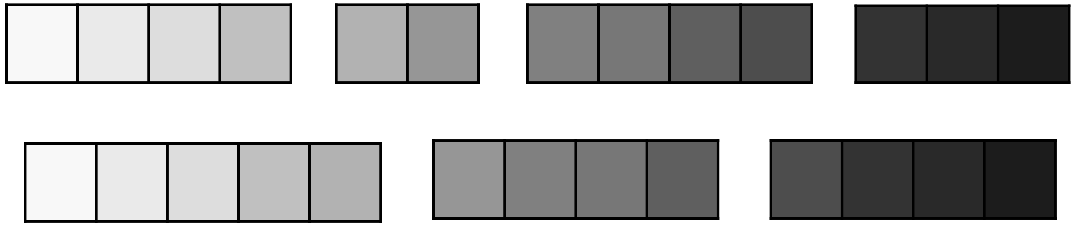
If G has a **VC** of at most k vertices, G_{exam} has a **VC** of at most k vertices, having an **IS** of k nodes, I_{exam} is a "yes" instance of **VC** ∧ **IS**

If G doesn't have a **VC** of at most k vertices, G_{exam} doesn't have a **VC** of at most k vertices, I_{exam} is a "no" instance of **VC** ∧ **IS**

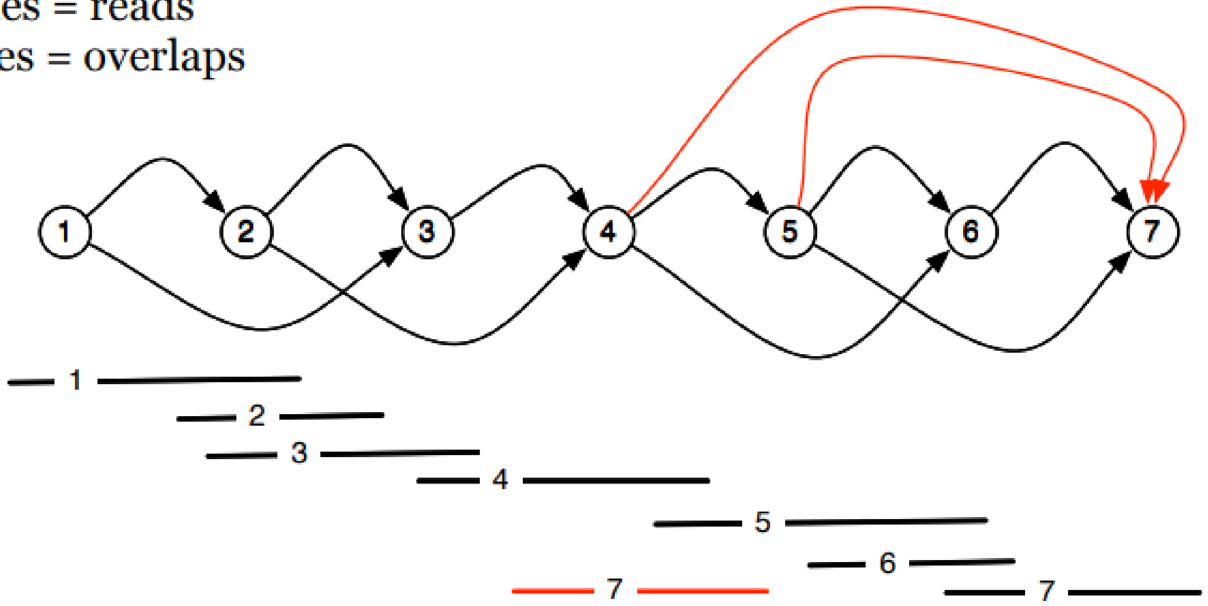
Genome Assembly



Genome Assembly



Overlap graph:
Nodes = reads
Edges = overlaps



assembly.pdf (cmu.edu)

GRAZIE A TUTTI 

https://forms.gle/94iKk8qS587MPsYb6