PROBLEMA 1

a1) $m\dot{v}_x(t) = -b\,v_x(t) \qquad v_x(0) = v_0$

$$\dot{v}_x(t) = -\frac{b}{m} v_x(t) \Rightarrow \boxed{v_x(t) = v_0\, e^{-\frac{b}{m}t}}$$

a2) $$v_0\, e^{-\frac{b}{m}t_1} = \frac{v_0}{2} \Rightarrow e^{-\frac{b}{m}t_1} = \frac{1}{2} \Rightarrow e^{\frac{b}{m}t_1} = 2$$

$$\Rightarrow \frac{b}{m}t_1 = \ln 2 \Rightarrow \boxed{t_1 = \frac{m}{b}\ln 2 \simeq \frac{1\,kg}{0.2\,kg\,s^{-1}}\cdot \ln 2 \simeq 3.47\,s}$$

b1) $$\boxed{a_x(t) = \dot{v}_x(t) = -\frac{b v_0}{m} e^{-\frac{b}{m}t}}$$

b2) $$\boxed{F_x(0) = m\,a_x(0) = -b v_0 = -(0.2\,kg\,s^{-1})\cdot(5\,m\,s^{-1}) = -1\,N}$$

c1) $$\boxed{\begin{aligned} x(t) &= x_0 + \int_0^t v_x(t')\,dt' = \int_0^t v_0\, e^{-\frac{b}{m}t'}\,dt' = \\ &= -\frac{m v_0}{b} e^{-\frac{b}{m}t'}\Big|_0^t = \frac{m v_0}{b}\left(1 - e^{-\frac{b}{m}t}\right) \end{aligned}}$$

c2) $$\boxed{d_M = \lim_{t\to+\infty} x(t) = \frac{m v_0}{b} = \frac{(1\,kg)\cdot(5\,m\,s^{-1})}{0.2\,kg\,s^{-1}} = 25\,m}$$

PROBLEMA 2

a) Equazione dei momenti rispetto al polo P:

$$I\alpha(0) = \frac{L}{2} Mg$$

$$\frac{1}{3} ML^{\cancel{2}} \alpha(0) = \frac{1}{2} Mg\cancel{L} \Rightarrow \boxed{\alpha(0) = \frac{3g}{2L} = \frac{3\cdot 9.81\ m s^{-2}}{2\cdot 1 m} \simeq 14.7 \frac{rad}{s^2}}$$

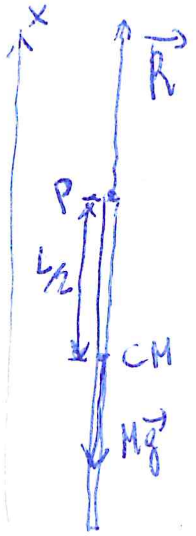
b) Conservazione energia meccanica (la reazione del vincolo non compie lavoro; l'unica forza che compie lavoro e' la forza peso che e' conservativa):

$$\frac{1}{2} I \omega_1^2 - Mg\frac{L}{2} = 0$$

$$I\omega_1^2 = MgL \Rightarrow \frac{1}{3} ML^{\cancel{2}}\omega_1^2 = Mg\cancel{L}$$

$$\omega_1^2 = \frac{3g}{L} \Rightarrow \boxed{\omega_1 = \sqrt{\frac{3g}{L}} = \sqrt{\frac{3\cdot 9{,}81\ m s^{-2}}{1\ m}} \simeq 5{,}42\ rad\ s^{-1}}$$

c)



Prima equazione cardinale (componenti dei vettori lungo l'asse x come in figura):

$$R - Mg = M a_{CM,x} \quad , \quad \text{da cui}$$

$$\boxed{R = M(g + a_{CM,x}) = M\left(g + \omega_1^2 \frac{L}{2}\right) = M\left(g + \frac{\cancel{L}}{2}\cdot\frac{3g}{\cancel{L}}\right) = \frac{5}{2}Mg = \frac{5}{2}\cdot(1\ kg)\cdot(9.81 m s^{-2}) \simeq 24.5\ N}$$

PROBLEMA 3

a) $$\boxed{\Phi(\vec{B}) = |\vec{B}| \cdot h^2 = \beta h^2 t}$$

$$\boxed{\Phi(\vec{B}; t = 10\,s) = \left(2\,\frac{Wb}{m^2 s}\right) \cdot \left(10^{-2}\,m^2\right) \cdot (10\,s) = 0.2\ Wb}$$

$$\boxed{f.e.m.(t) = -\frac{d\,\Phi(\vec{B})}{dt} = -\beta h^2}$$

$$\boxed{f.e.m.(t = 10\,s) = -\beta h^2 = -\left(2\,\frac{Wb}{m^2 s}\right) \cdot \left(10^{-2}\,m^2\right) = -0.02\ V = -20\ mV}$$

b) $$\boxed{i(t) = \frac{f.e.m.(t)}{R} = -\frac{\beta h^2}{R}}$$

$$\boxed{i(t = 10\,s) = -\frac{\beta h^2}{R} = -\frac{\left(2\,\frac{Wb}{m^2 s}\right) \cdot \left(10^{-2}\,m^2\right)}{10\,\Omega} = -2 \cdot 10^{-3}\,A = -2\ mA}$$

[i circola in senso orario, per compensare l'aumento del flusso del campo magnetico esterno al passare del tempo (legge di Lenz)]

c) $$\boxed{\begin{aligned} P(t) &= (i(t))^2 \cdot R = \frac{\beta^2 h^4}{R^{\cancel{2}}} \cdot \cancel{R} = \frac{\beta^2 h^4}{R} = \\ &= \frac{\left(2\,\frac{Wb}{m^2 s}\right)^2 \cdot \left(10^{-4}\,m^4\right)}{10\,\Omega} = 4 \cdot 10^{-5}\ W = 40\ \mu W \end{aligned}}$$