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Q1 Describe the Joux's three-party Diffie-Hellman protocol

Q2 What is the (min) order of the following multiplicative groups defined by the chosen parameters p and g :
[hint: 593, 1187, 3559 primes, with $1187 = 2 \times 593 + 1$ and $3559 = 593 \times 2 \times 3 + 1 \rightarrow$ with the exception of (2), no need to do any modular exponentiation! – **NOTE: EXPLAIN YOUR ANSWER otherwise answer is not valid**]

1) $g^x \bmod p$, with $g=1186$, $p=1187$

2) $g^x \bmod p$, with $g=7$, $p=1187$

3) $g^x \bmod p$, with $g=9$, $p=1187$

4) $g^x \bmod p$, with $g=64$, $p=3559$

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Q3 Describe the Pedersen Verifiable Secret sharing

Q4 Being $e: G \times G \rightarrow G$ a bilinear map, and g a generator of G , simplify the expression:

$$e(g^a g^b, g^c g^d) / e(g^{ac}, g^{bd})$$

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E1. Consider the Elliptic curve $y^2 = x^3 - 2x - 3$ defined over the modular integer field \mathbb{Z}_5 .

A. find all the points $EC(\mathbb{Z}_5)$ and state what is the order of the corresponding group

$$\begin{aligned} P &= (x_1, y_1) \\ Q &= (x_2, y_2) \\ R &= P + Q = (x_3, y_3) \\ x_3 &= \lambda^2 - x_1 - x_2 \\ y_3 &= \lambda(x_1 - x_3) - y_1 \\ \lambda &= \begin{cases} \frac{y_2 - y_1}{x_2 - x_1} & P \neq Q \\ \frac{3x_1^2 + a}{2y_1} & P = Q \end{cases} \end{aligned}$$

B. Compute $[4](1,1)$

[HELP: possibly useful mnemonic hints reported here on the right;

MUST-DO: show step-by-step detailed computations; try to minimize the number of EC additions]

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E2 – Secret Sharing and Threshold RSA. A group of four parties forms a (3,4) secret sharing scheme. Using ordinary arithmetic (no moduli) parties $P_i = \{1,2,3,4\}$ are dealt with shares σ_i of a key SK, which is the private key of an RSA scheme with modulus $n=91$ and public key $PK=5$.

A. Using shares $\sigma_1=34$, $\sigma_2=41$, $\sigma_4=61$, reconstruct the secret key SK and verify that it is the correct one for the considered RSA public key PK;

B. If the attacker only knows shares σ_1 and σ_4 , and does not yet know share σ_2 , what's his/her advantage in terms of chances to guess the secret key versus a pure random guess? *[ignore here the further info that SK is an RSA secret key]*

C. Using the Shoup construction, show step by step how the three parties P1, P2 and P4 can distributely compute a threshold RSA signature for the message $m=15$, and verify that the result is correct by directly computing the signature using the SK value computed before.

[NOTE: if you prefer, to simplify computation instead of using the full $L!$ you can use the minimum value that permits to make all lambda coefficients integer] [HINT: $15^{-1} \bmod 91 = 85$]