$$\max = 4x_1 + 3x_2 - x_3$$

$$x_1 + 3x_2 \geq 10$$

$$x_1 - x_2 + 4x_3 \geq 8$$

$$x_1 \geq 0,\ x_2 \leq 0 \quad x_3 \geq 0$$

$$x^1 = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix} \qquad x^2 \begin{bmatrix} 12 \\ 0 \\ 2 \end{bmatrix} \qquad x^3 = \begin{bmatrix} 13 \\ -1 \\ 0 \end{bmatrix} \qquad x^4 = \begin{bmatrix} 14 \\ -1 \\ 2 \end{bmatrix}$$

SBA : 1. AMMISSIBILITÀ

2. BASE

$$x^1 = \begin{array}{cc} \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix} & \begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} \end{array} \qquad \begin{array}{l} 10 \geq 10 \ \checkmark \\ 10 \geq 8 \ \checkmark \end{array}$$

$$x^2 = \begin{bmatrix} 12 \\ 0 \\ 2 \end{bmatrix} \qquad \begin{array}{l} 12 \geq 10 \ \checkmark \\ 20 \geq 8 \ \checkmark \end{array}$$

$$x^3 \begin{bmatrix} 13 \\ -1 \\ 0 \end{bmatrix} \qquad \begin{array}{l} 10 \geq 10 \ \checkmark \\ 14 \geq 8 \ \checkmark \end{array}$$

$$x^4 \begin{bmatrix} 14 \\ -1 \\ 2 \end{bmatrix} \qquad \begin{array}{l} 11 \geq 10 \ \checkmark \\ 23 \geq 8 \ \checkmark \end{array}$$

$$\max -4x_1 - 3\bar{x}_2 - x_3 \qquad \bar{x}_2 = -x_2 \geq 0$$

$$x_1 - 3\bar{x}_2 \qquad - S_1 \qquad = 10$$

$$x_1 + \bar{x}_2 + 4x_3 \quad - S_2 = 8$$

$$x_1, \bar{x}_2, x_3, S_1, S_2 \geq 0$$

$$x^1 = \begin{pmatrix} 10 \\ 0 \\ 0 \end{pmatrix} \begin{cases} S_1 = 0 \\ S_2 = 2 \end{cases} \longrightarrow x^1 \begin{pmatrix} 10 \\ 0 \\ 0 \\ 0 \\ 2 \end{pmatrix}$$

SBA

var. in base $\leq$ # vinc.

$$x^2 = \begin{pmatrix} 12 \\ 0 \\ 2 \end{pmatrix} \begin{cases} S_1 = 2 \\ S_2 = \end{cases}$$

x^2 NON SBA

$$x^3 = \begin{pmatrix} 13 \\ -1 \\ 0 \end{pmatrix} \begin{cases} S_1 = 6 \\ S_2 = 4 \end{cases}$$

x^3 NON SBA $\begin{pmatrix} 13 \\ -1 \\ 0 \\ 6 \\ -1 \end{pmatrix}$

$$x^4 = \begin{pmatrix} 14 \\ -1 \\ 2 \end{pmatrix} \begin{cases} S_1 = 7 \\ S_2 = \end{cases}$$

x^4 NON SBA

b.) $\exists$ SBA con x_2, x_3 in base ?

SBA con x_2, x_3 in base $\longrightarrow \begin{pmatrix} 0 \\ >0 \\ >0 \\ 0 \\ 0 \end{pmatrix}$

$$x^S = \begin{array}{c} \begin{bmatrix} 0 \\ ? \\ ? \\ 0 \\ 0 \end{bmatrix} \end{array} \begin{array}{l} x_1 \\ x_2 \\ x_3 \\ s_1 \\ s_2 \end{array} \longrightarrow \begin{array}{l} -3\bar{x}_2 = 10 \rightarrow x_2 = +\frac{10}{3} \\ x_2 + 4x_3 = 8. \end{array}$$

ass. $\bar{x}_2 = -x_2$

$$4x_3 = 8 - \frac{10}{3} \longrightarrow x_3 = \frac{14}{3} \cdot \frac{1}{4} = \frac{\cancel{14}\,7}{\cancel{12}\,6}$$

SBA con x_2, x_3 in base è $\begin{pmatrix} 0 \\ 10/3 \\ 7/6 \\ 0 \\ 0 \end{pmatrix}$.

---

c.) $\exists$ sol. ott. con x_1 in base ?

SBA con x_1 in base è del tipo

$$\underset{x_0^1}{\begin{pmatrix} >0 \\ >0 \\ 0 \\ 0 \\ 0 \end{pmatrix}} \text{ o } \underset{x_0^2}{\begin{pmatrix} >0 \\ 0 \\ >0 \\ 0 \\ 0 \end{pmatrix}} \text{ o } \underset{x_0^3}{\begin{pmatrix} >0 \\ 0 \\ 0 \\ >0 \\ 0 \end{pmatrix}} \text{ o } \underset{x_0^4}{\begin{pmatrix} >0 \\ 0 \\ 0 \\ 0 \\ >0 \end{pmatrix}}$$

$$x_0^1 = \begin{cases} x_1 - 3\bar{x}_2 = 10 \\ x_1 + \bar{x}_2 = 8 \end{cases} \rightarrow \begin{cases} x_1 = 10 + 3\bar{x}_2 \\ 10 + 3\bar{x}_2 + \bar{x}_2 = 8 \end{cases}$$

$$\rightarrow \begin{cases} x_1 = \frac{17}{2} \\ \bar{x}_2 = -\frac{1}{2} \end{cases} \rightarrow x_2 = \frac{1}{2} \qquad \rightarrow x_0^1 = \begin{pmatrix} 17/2 \\ 1/2 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\max \; -4x_1 - 3\bar{x}_2 - x_3$$

$$\begin{aligned} x_1 - 3\bar{x}_2 - s_1 &= 10 \\ x_1 + \bar{x}_2 + 4x_3 - s_2 &= 8 \end{aligned}$$

$$x_1, \bar{x}_2, x_3, s_1, s_2 \geq 0$$

$\Downarrow$ DUALE

$$\min \; 10y_1 + 8y_2$$

$$\begin{array}{ll} 1 & y_1 + y_2 \leq -4 \\ 2 & -3y_1 + y_2 \leq -3 \\ 3 & 4y_2 \leq -1 \\ 4 & -y_1 \leq 0 \\ 5 & -y_2 \leq 0 \end{array}$$

$$y_1, y_2 \in \mathbb{R}$$

$$\langle s_d , x \rangle = 0 \qquad x = \begin{array}{c} \begin{pmatrix} >0 \\ >0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \end{array} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}$$

$$\begin{cases} -4 - y_1 - y_2 = 0 \\ -3 + 3y_1 - y_2 = 0 \end{cases} \rightarrow \begin{cases} y_1 = -4 - y_2 \\ -3 - 12 - 3y_2 - y_2 = 0 \end{cases}$$

$$\begin{cases} y_1 = -\frac{1}{4} \\ y_2 = -\frac{15}{4} \end{cases}$$

porre in forma canonica

$$\begin{array}{ll} \min & c^T x \\ \text{s.t.} & Ax \geq b \\ & x \geq 0 \end{array} \qquad \begin{array}{ll} \max & c^T x \\ \text{s.t.} & Ax \leq b \\ & x \geq 0 \end{array}$$

$$\begin{array}{lllll} \min & 3x_1 + 4x_2 - 2x_3 \\ \text{s.t.} & x_1 + 2x_2 - x_3 \geq 5 \\ & 2x_1 \qquad + 4x_3 = 12 \\ & x_1 + x_2 + x_3 \leq 15 \\ & x_1, x_2 \geq 0 \quad x_3 \in \mathbb{R} \end{array}$$

$x_3 : x_3^+ - x_3^- \geq 0$

$x_3^+ \geq 0$

$x_3^- \geq 0$

$$\begin{array}{ll} \min & 3x_1 + 4x_2 - 2x_3 \\ & x_1 + 2x_2 - x_3^+ + x_3^- \geq 5 \\ & 2x_1 \qquad + 4x_3^+ - 4x_3^- \geq 12 \\ & -2x_1 \qquad - 4x_3^+ + 4x_3^- \geq -12 \\ & -x_1 - x_2 - x_3^+ + x_3^- \geq -15 \\ & x_1, x_2, x_3^+, x_3^- \geq 0 \end{array}$$

$$\max \; 4x_1 - x_2$$
$$\begin{aligned} x_1 + x_2 - x_3 &= 8 \\ 3x_1 \phantom{{}+x_2} + x_3 &\leq 7 \end{aligned}$$
$$x_1 \geq 0, \quad x_2 \in \mathbb{R}, \quad x_3 \leq 0$$

$$x_2 = x_2^+ - x_2^- \geq 0 \qquad x_2^+ \geq 0, \quad x_2^- \geq 0$$

$$x_3 \qquad \bar{x}_3 = -x_3 \geq 0$$

$$-\min \; -4x_1 + x_2^+ - x_2^-$$
$$\begin{aligned} x_1 + x_2^+ - x_2^- + \bar{x}_3 &\geq 8 \\ -x_1 - x_2^+ + x_2^- - \bar{x}_3 &\geq -8 \\ -3x_1 \phantom{{}+x_2^+ - x_2^-} + 3\bar{x}_3 &\geq -7 \end{aligned}$$

$$\min \; 8x_1 - x_2 + x_3$$
$$\begin{aligned} x_1 \phantom{{}-x_2} + x_3 &\geq 4 \\ x_2 - x_3 &\leq 7 \\ x_1 - x_2 \phantom{{}+x_3} &\leq 2 \end{aligned}$$
$$x_1, x_2 \geq 0 \quad x_3 \leq 0$$

$\overline{x_3} = -x_3 \geq 0$

$$
\begin{aligned}
\min \quad & 8x_1 - x_2 - \overline{x_3} \\
& x_1 - \overline{x_3} \geq 4 \\
& -x_2 - \overline{x_3} \geq -7 \\
& -x_1 + x_2 \geq -2 \\
& x_1, x_2, \overline{x_3} \geq 0
\end{aligned}
$$

$$
\boxed{
\begin{aligned}
& \min \\
& \quad Ax = b \\
& \quad x \geq 0
\end{aligned}
}
$$

$$
\begin{aligned}
\max \quad & 4x_1 - x_2 \\
& x_1 + 2x_2 \leq 2 \\
& 2x_1 + 7x_2 = 8
\end{aligned}
$$

$x_1 \geq 0 \quad x_2 \leq 0 \qquad\qquad x_1 \geq 0 \quad \overline{x_2} = -x_2 \geq 0$

$$
\begin{aligned}
-\min \quad & -4x_1 + \overline{x_2} \\
& x_1 - 2\overline{x_2} + s_1 = 2 \\
& 2x_1 - 7\overline{x_2} = 8 \\
& x_1, \overline{x_2} \geq 0
\end{aligned}
$$

$$
\begin{aligned}
\min \quad & 8x_1 - x_2 + x_3 \\
& x_1 + x_3 \geq 4 \\
& x_2 - x_3 \leq 7 \\
& x_1 - x_2 \leq 2
\end{aligned}
$$

$x_1, x_2 \geq 0 \quad x_3 \leq 0$

$\overline{x_3} := -x_3 \geq 0$

$$\min \quad 8x_1 - x_2 - \overline{x_3}$$

$$x_1 - \overline{x_3} - s_1 = 4$$

$$x_2 + x_3 + s_2 = 7$$

$$x_1 - x_2 + s_3 = 2$$

$$\max \quad 3x_1 + 2x_2 - 5x_3$$

$$4x_1 - 2x_2 + 2x_3 \leq 4$$

$$2x_1 + x_2 + x_3 \leq 1$$

$$x_1, x_2, x_3 \geq 0$$

$$-\min \quad -3x_1 - 2x_2 + 5x_3$$

$$4x_1 - 2x_2 + 2x_3 + s_1 = 4$$

$$2x_1 + x_2 + x_3 + s_2 = 1$$

		x_1	x_2	x_3	s_1	s_2
	0	-3	-2	5	0	0
s_1	4	4	-2	2	1	0
s_2	1	$\boxed{2}$	1	1	0	1

	0	$\cancel{-3}$	-2	5	0	0
s_1	4	$\cancel{4}$	-2	2	1	0
s_2	1/2	$\boxed{1}$	1/2	1/2	0	1/2

$R_1 \to 4R_2 - R_1$

$R_0 \to 3R_2 + R_0$

	3/2	0	-1/2	13/2	0	3/2
s_1	-2	0	4	0	1	2
x_1	1	2	$\boxed{1}$	1	0	1

$R_1 \to 4R_2 - R_1$

$R_0 \to \frac{1}{2}R_2 + R_0$

	2	1	0	7	0	2
s_1	6	8	0	4	-1	2
x_2	1	2	1	1	0	1

$$\begin{pmatrix} 0 \\ 1 \\ 0 \\ 6 \\ 0 \end{pmatrix} \qquad z = 2$$

$$\max \quad 4x_1 + x_2 + 5x_3$$

$$\begin{aligned} -x_1 + x_2 &\le 1 \\ 2x_2 - x_3 &\le 2 \\ x_1 + x_3 &\le 1 \\ x_1, x_2, x_3 &\ge 0 \end{aligned}$$

$$\min \; -4x_1 - x_2 - 5x_3$$

$$\begin{aligned}
-x_1 + x_2 + s_1 &= 1 \\
2x_2 - x_3 + s_2 &= 2 \\
x_1 + x_3 + s_3 &= 1
\end{aligned}$$

	x_1	x_2	x_3	s_1	s_2	s_3	
0	-4	-1	-5	0	0	0	
1	-1	1	0	1	0	0	s_1
2	0	2	-1	0	1	0	s_2
1	1	0	1	0	0	1	s_3

$R_0 \to 5R_3 + R_0$

$R_2 \to R_3 + R_2$

$\longrightarrow$

5	1	-1	0	0	0	5	
1	-1	$\boxed{1}$	0	1	0	0	s_1
3	1	2	0	0	1	1	s_2
1	1	0	1	0	0	1	x_3

$R_0 \to R_1 + R_0$

$R_2 \to -2R_1 + R_2$

6	0	0	0	1	0	5	
1	-1	1	0		0	0	x_2
1	3	0	0	-2	1	1	s_2
1	1	0	1	0	0	1	x_3

$z = 6$

$$x = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\min \quad 3x_1 + x_2 - 2x_3 - x_4$$

$$2x_1 + x_2 - x_3 + 3x_4 \leq 8$$
$$-x_1 + 2x_2 - 2x_3 + 2x_4 \leq 4$$
$$x_1 + x_3 \leq 10$$

$$\min \quad 3x_1 + x_2 - 2x_3 - x_4 + S_2 + S_3$$

$$2x_1 + x_2 - x_3 + 3x_4 + S_1 = 8$$
$$-x_1 + 2x_2 - 2x_3 + 2x_4 + S_2 = 4$$
$$x_1 + x_3 + S_3 = 10$$

		x_1	x_2	x_3	x_4	S_1	S_2	S_3
	0	3	1	-2	-1	0	0	0
S_1	8	2	1	-1	3	1	0	0
S_2	4	-1	2	-2	2	0	1	0
	10	1	0	$\boxed{1}$	0	0	0	1

$R_0 \to 2R_3 + R_0$

$R_1 \to R_3 + R_1$

$R_2 \to 2R_1 + R_1$

20	5	1	0	-1	0	0	2	
18	3	1	0	3	1	0	1	S_1
24	1	2	0	2	0	1	2	S_2
10	1	0	1	0	0	0	1	x_3

20	5	1	0	-1	0	0	2	
6	1	1/3	0	1	1/3	0	1/3	x_4
24	1	2	0	2	0	1	2	S_2
10	1	0	1	0	0	0	1	x_3

$R_0 \to R_1 + R_0$

$R_2 \to -2R_1 + R_2$

26	6	4/3	0	0	1/3	0	7/3
6	1	1/3	0	1	1/3	0	1/3
12	-1	4/3	0	0	-2/3	1	4/3
10	1	0	1	0	0	0	1

$z = 26$

$$x = \begin{pmatrix} 0 \\ 0 \\ 10 \\ 6 \\ 0 \\ 12 \\ 0 \end{pmatrix}$$

SIMPLESSO A 2 FASI

$$\min \quad 6x_1 + x_2 + 3x_3$$

$$10x_1 - 2x_2 + 5x_3 \geq 15$$

$$x_1 - x_2 + 3x_3 \geq 6$$

$$\underbrace{x_1, x_2, x_3 \geq 0}$$

min $6x_1 + x_2 + 3x_3$

$10x_1 - 2x_2 + 5x_3 - S_1 + a_1 = 15$

$x_1 - x_2 + 3x_3 - S_2 + a_2 = 6$

$\Downarrow$ art.

min $a_1 + a_2$

$10x_1 - 2x_2 + 5x_3 - S_1 + a_1 = 15$

$x_1 - x_2 + 3x_3 - S_2 + a_2 = 6$

	x_1	x_2	x_3	S_1	S_2	a_1	a_2
0	0	0	0	0	0	1	1
15	10	-2	5	-1	0	1	0
6	1	-1	3	0	-1	0	1

$R_0 \to R_0 - R_1$

$R_0 \to R_0 - R_2$

-21	$\boxed{-11}$	3	-8	1	1	0	0
15	$\boxed{10}$	-2	5	-1	0	1	0
6	1	-1	3	0	-1	0	1

-21	-11	3	-8	1	1	0	0
$3/2$	1	$-1/5$	$\frac{1}{2}$	$-\frac{1}{10}$	0	$\frac{1}{10}$	0
6	1	-1	3	0	-1	0	1

$R_0 \to 11R_1 + R_0$

$R_2 \to -R_1 + R_2$

						a_1	a_2
$-9/2$	0	$4/5$	$-\frac{5}{2}$	$-\frac{1}{10}$	1	$\frac{11}{10}$	0
$3/2$	1	$-1/5$	$\frac{1}{2}$	$-\frac{1}{10}$	0	$\frac{1}{10}$	0
$9/2$	0	$-\frac{4}{5}$	$\boxed{\frac{5}{2}}$	$\frac{1}{10}$	-1	$-\frac{1}{10}$	1

$R_0 \to R_2 + R_0$

$R_2 \to \frac{2}{5} R_2$

$R_1 \to -\frac{1}{2} R_2 + R_1$

	x_1	x_2	x_3	s_1	s_2		
0	0	0	0	0	0	1	1
$3/5$	1	$-\frac{1}{25}$	0	$-\frac{6}{50}$	$\frac{1}{5}$	$\frac{6}{50}$	$-\frac{1}{5}$
$-9/5$	0	$-\frac{8}{25}$	1	$\frac{1}{25}$	$-\frac{2}{5}$	$-\frac{1}{25}$	$\frac{2}{5}$

$+\frac{4}{25} - \frac{5}{25} = -\frac{1}{25}$

	x_1	x_2	x_3	s_1	s_2
0	6	1	3	0	0
$3/5$	1	$-\frac{1}{25}$	0	$-6/50$	$\frac{1}{5}$
$-9/5$	0	$-\frac{8}{25}$	1	$1/22$	$-\frac{2}{5}$

$$\min \quad 7x_1 + 2x_2 - 5x_3 - x_4$$

$$4x_1 + 3x_2 \qquad + 2x_4 \geq 2$$

$$-5x_1 - 3x_2 + x_3 - x_4 \leq 1$$

$$x_1, x_2, x_3, x_4 \geq 0$$

$$\min \quad 7x_1 + 2x_2 - 5x_3 - x_4$$

$$4x_1 + 3x_2 \qquad + 2x_4 - S_1 + a_1 = 2$$

$$-5x_1 - 3x_2 + x_3 - x_4 \qquad + S_2 \qquad = 1$$

I $\quad \min \quad a_1$

$$4x_1 + 3x_2 + 2x_4 - S_1 + a_1 = 2$$

$$-5x_1 - 3x_2 + x_3 - x_4 + S_2 = 1$$

		x_1	x_2	x_3	x_4	s_1	s_2	a_1
	0	0	0	0	0	0	0	1
a_1	2	4	3	0	2	-1	0	1
s_2	1	-5	-3	1	-1	0	1	0

$R_0 \to R_0 - R_1$

-2	-4	-3	0	-2	1	0	0	
2	$\boxed{4}$	3	0	2	-1	0	1	$\Rightarrow x_1$
1	-5	-3	1	-1	0	1	0	

$R_0 \to 4R_1 + R_0$

$R_2 \to 5R_1 + R_2$

	0	0	0	0	0	0	0	~~a_1~~
x_1	$\frac{1}{2}$	1	$\frac{3}{4}$	0	$\frac{1}{2}$	$-\frac{1}{4}$	0	
s_2	$7/2$	0	$\frac{3}{4}$	1	$\frac{3}{2}$	$-\frac{5}{4}$	1	

0	7	2	-5	-1	0	0	
$1/2$	1	$\frac{3}{4}$	0	$1/2$	$\frac{1}{4}$	0	x_1
$7/2$	0	$\frac{3}{4}$	1	$3/2$	$-5/4$	1	x_3

$R_0 \to 5R_2 + R_0$

$35/2$	7	$27/4$	0	$13/2$	$-25/4$	5
$1/2$	1	$3/4$	0	$1/2$	$-1/4$	0
$7/2$	0	$3/4$	1	$3/2$	$-5/4$	1

	x_1	x_2	x_3	s_1	s_2
0	6	1	3	0	0
$3/5$	1	$-\frac{1}{25}$	0	$-3/25$	$\frac{1}{5}$
$+9/5$	0	$-\frac{8}{25}$	1	$1/25$	$-\frac{2}{5}$

$R_0 \to -6R_1 + R_0$

$R_0 \to -3R_2 + R_0$

9 $\cancel{45}$ $-\cancel{\frac{\cancel{}}{5}}$	0	$\frac{\cancel{55}^{11}}{\cancel{25}_{5}}$	0	$+\frac{\cancel{15}^{3}}{\cancel{25}_{5}}$	0
$3/5$	1	$-\frac{1}{25}$	0	$-3/25$	$\frac{1}{5}$
$+9/5$	0	$-\frac{8}{25}$	1	$\frac{1}{25}$	$-\frac{2}{5}$

$z = 9$

$x = \begin{pmatrix} 3/5 \\ 0 \\ 9/5 \end{pmatrix}$

min $2x_1 + x_2 + 4x_3$ min $a_1 + a_2$

$x_1 + x_2 + 2x_3 + a_1 = 3$ $\Longrightarrow$

$2x_1 + x_2 + 3x_3 + a_2 = 5$

$x_1, x_2, x_3, a_1, a_2 \geq 0$

		x_1	x_2	x_3	a_1	a_2
	0	0	0	0	1	1
a_1	3	1	1	2	1	0
a_2	5	2	1	3	0	1

$R_0 \to R_0 - R_1 - R_2$

$R_0 \to 5R_1 + R_0$

$R_2 \to -3R_1 + R_2$

			x_3		
-8	-3	-2	-5	0	0
3	1	1	2	1	0
5	2	1	3	0	1

$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{5}{2}$	0	
$3/2$	$1/2$	$1/2$	1	$1/2$	0	x_3
$\frac{1}{2}$	$\boxed{\frac{1}{2}}$	$-\frac{1}{2}$	0	$-\frac{3}{2}$	1	x_1

$R_0 \to \frac{1}{2} R_2 + R_0$

$R_1 \to -\frac{1}{2} R_2 + R_1$

0	0	0	0	~~1~~	~~1~~
1	0	1	1	~~1/2~~	~~0~~
1	1	-1	0	~~-3~~	~~2~~

	x_1	x_2	x_3	
0	2	1	4	
1	0	1	1	x_3
1	1	-1	0	x_1

$R_0 \to -4R_1 + R_0$

-6	0	-1	0	
1	0	1	1	x_2
1	1	-1	0	x_1

$$
\begin{array}{r|rrr}
-6 & 0 & -1 & 0 \\
\hline
1 & 0 & 1 & 1 \\
1 & 1 & -1 & 0
\end{array}
$$

$R_0 \to R_1 + R_0$

$R_2 \to R_1 + R_2$

$$
\begin{array}{r|rrrl}
-5 & 0 & 0 & 1 & \\
\hline
1 & 0 & 1 & 1 & x_2 \\
2 & 1 & 0 & 1 & x_1
\end{array}
$$

$$z^* = 5 \qquad x = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$

$$
\begin{aligned}
-\min\ & -x_1 - x_2 - x_3 \\
& -x_1 - 2x_2 + x_3 + x_4 + a_1 = 5 \\
& 4x_2 + x_3 + 2x_4 + a_2 = 2 \\
& 3x_2 - 2x_4 + s_1 = 6
\end{aligned}
$$

$$\min\ a_1 + a_2$$

$$
\begin{array}{r|rrrrrrr}
 & x_1 & x_2 & x_3 & x_4 & s_1 & a_1 & a_2 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
\hline
5 & -1 & -2 & 1 & 1 & 0 & 1 & 0 \\
2 & 0 & 4 & 1 & 2 & 0 & 0 & 1 \\
6 & 0 & 3 & 0 & -2 & 1 & 0 & 0
\end{array}
$$

$R_0 \to R_0 - R_1 - R_2$

$$
\begin{array}{r|rrrrrrr}
-7 & 1 & -2 & -2 & -3 & 0 & 0 & 0 \\
\hline
5 & -1 & -2 & 1 & 1 & 0 & 1 & 0 \\
2 & 0 & 4 & 1 & 2 & 0 & 0 & 1 \\
6 & 0 & 3 & 0 & -2 & 1 & 0 & 0
\end{array}
$$

-4	1	4	-1/2	0	0	0	3/2	
4	-1	-4	1/2	0	0	1	-1/2	
1	0	2	$\boxed{1/2}$	1	0	0	1/2	
8	0	7	1	0	1	0	1	

$R_0 \to 3R_2 + R_0$

$R_1 \to -R_2 + R_1$

$R_3 \to 2R_2 + R_3$

-3	1	6	0	1	0	0	2
3	-1	-6	0	-1	0	1	-1
2	0	4	1	2	0	0	1
6	0	5	0	-1	1	0	1/2

$R_0 \to \frac{1}{2}R_2 + R_0$

$R_1 \to -\frac{1}{2}R_2 + R_1$

$R_3 \to -R_2 + R_3$

$\max\ 4x_1 + 3x_2 - x_3 + 2x_4 \longrightarrow -\min\ -4x_1 - 3x_2 + x_3 - 2x_4$

$-x_1 + 2x_2 + x_3 - x_4 - S_1 + a_1 = 2$

$-2x_1 + x_2 - x_3 + 5x_4 - S_2 + a_2 = +3$

$\min\ a_1 + a_2$

	x_1	x_2	x_3	x_4	S_1	S_2	a_1	a_2	
0	0	0	0	0	0	0	1	1	
2	-1	2	1	-1	-1	0	1	0	a_1
3	-2	1	-1	5	0	-1	0	1	a_2

$R_0 \to R_0 - R_1 - R_2$

-5	3	-3	0	-4	1	1	0	0	
2	-1	2	1	-1	-1	0	1	0	
3	-2	1	-1	5	0	-1	0	1	x_4

$R_0 \to 4R_2 + R_0$

$R_1 \to R_2 + R_1$

$-13/5$	$7/5$	$-\frac{11}{5}$	$-4/5$	0	1	$1/5$	0	$4/5$	
$13/5$	$-7/5$	$\boxed{\frac{11}{5}}$	$4/5$	0	-1	$-1/5$	1	$1/5$	x_2
$3/5$	$-2/5$	$1/5$	$-1/5$	1	0	$-1/5$	0	$1/5$	x_4

0	0	0	0	0	0	0	~~1~~	~~1~~
$\frac{13}{11}$	$-\frac{7}{11}$	1	$\frac{4}{11}$	0	$-\frac{5}{11}$	$-\frac{1}{11}$	~~$\frac{5}{11}$~~	~~$\frac{1}{11}$~~
$\frac{4}{11}$	$-\frac{3}{11}$	0	$-\frac{3}{11}$	1	$\frac{1}{11}$	$-\frac{2}{11}$	~~$\frac{1}{11}$~~	~~$\frac{2}{11}$~~

$R_0 \to \frac{11}{5}R_1 + R_0$

$R_2 \to -\frac{1}{5}R_1 + R_2$

0	$\boxed{-4}$	-3	$+1$	-2	0	0
$\frac{13}{11}$	$-\frac{7}{11}$	1	$\frac{4}{11}$	0	$-\frac{5}{11}$	$-\frac{1}{11}$
$\frac{4}{11}$	$-\frac{3}{11}$	0	$\frac{3}{11}$	1	$\frac{1}{11}$	$-\frac{2}{11}$

SOL. ILLIMITATA

$$\max \; 7x_1 + x_2$$
$$4x_1 - 2x_2 + 2x_3 \leq 4$$
$$2x_1 + x_2 + x_3 \leq 1$$
$$x_1, x_2, x_3 \geq 0$$

$$x^* = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 6 \\ 0 \end{pmatrix} \text{ è ottimo?}$$

$\downarrow$ STANDARD.

$$\max \; 7x_1 + x_2$$
$$4x_1 - 2x_2 + 2x_3 + S_1 = 4$$
$$2x_1 + x_2 + x_3 + S_2 = 1$$

$\hookrightarrow$ duale

$$\min \; 4y_1 + y_2$$
$$4y_1 + 2y_2 \leq 7$$
$$-2y_1 + y_2 \leq 1$$
$$2y_1 + y_2 \leq 0$$
$$y_1 \leq 0$$
$$+y_2 \leq 0$$
$$y_1, y_2 \in \mathbb{R}$$

$$\langle sd, x \rangle = 0 \qquad x^* = \begin{pmatrix} 0 \\ >0 \\ 0 \\ >0 \\ 0 \end{pmatrix}$$

$$\begin{cases} 1 + 2y_1 - y_2 = 0 \\ y_1 = 0 \end{cases} \rightarrow \begin{cases} y_2 = 1 \\ y_1 = 0 \end{cases}$$

verifico ammissibilità

$2 \leq 7$ ok.

$1 \leq 1$ ok

$1 \leq 0$ NO

la sol x^* non è ottima

$$x^* \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\begin{aligned} \min \quad & x_1 + 2x_2 - x_3 \\ & -x_1 + x_2 \leq 1 \\ & 2x_2 - x_3 \leq 2 \\ & x_1 + x_3 \leq 1 \end{aligned}$$

$\downarrow$ std.

$$\begin{aligned} \min \quad & x_1 + 2x_2 - x_3 \\ & -x_1 + x_2 + S_1 = 1 \\ & 2x_2 - x_3 + S_2 = 2 \\ & x_1 + x_3 + S_3 = 1 \end{aligned}$$

duale

$$\max \quad y_1 + 2y_2 + y_3$$

$$\begin{aligned} -y_1 \qquad\quad + y_3 &\le 1 \\ y_1 + 2y_2 \qquad\quad &\le 2 \\ -y_2 + y_3 &\le -1 \end{aligned}$$

$y_1 \le 0$

$y_2 \le 0$

$y_3 \le 0$

$$x^* = \begin{pmatrix} 0 \\ >0 \\ >0 \\ 0 \\ >0 \\ 0 \end{pmatrix}$$

$< sd, x > = 0$

$$\begin{cases} 2 - y_1 + 2y_2 = 0 \\ -1 + y_2 - y_3 = 0 \\ y_2 = 0 \end{cases}$$

$$\begin{cases} y_1 = 2 \\ y_2 = 0 \\ y_3 = -1 \end{cases}$$

x^* non è ottima

$$\min\ 5x_1 + 3x_2 - 2x_3 - x_4$$
$$2x_1 + x_2 - x_3 + 3x_4 \leq 8$$
$$-x_1 + 2x_2 - 2x_3 + 2x_4 \leq 4$$
$$x_1 + x_3 \leq 10$$

$$x^* = \begin{pmatrix} 0 \\ 0 \\ 10 \\ 6 \\ 0 \\ 12 \\ 0 \end{pmatrix}$$

$\downarrow$ std

$$\min\ 5x_1 + 3x_2 - 2x_3 - x_4$$
$$2x_1 + x_2 - x_3 + 3x_4 + s_1 = 8$$
$$-x_1 + 2x_2 - 2x_3 + 2x_4 + s_2 = 4$$
$$x_1 + x_3 + s_3 = 10$$

$\downarrow$ duale

$$\max\ 8y_1 + 4y_2 + 10y_3$$
$$2y_1 - y_2 + y_3 \leq 5$$
$$y_1 + 2y_2 \leq 3$$
$$-y_1 - 2y_2 + y_3 \leq -2$$
$$3y_1 + 2y_2 \leq -1$$

$$y_1 \leq 0$$

$$y_2 \leq 0$$

$$y_3 \leq 0$$

$$\langle Sd, x \rangle = 0 \qquad x^* \quad \begin{pmatrix} 0 \\ 0 \\ >0 \\ >0 \\ 0 \\ >0 \\ 0 \end{pmatrix}$$

$$\begin{cases} -2 + y_1 + 2y_2 - y_3 = 0 \\ 1 - 3y_1 - 2y_2 = 0 \\ y_2 = 0 \end{cases}$$

$$\begin{cases} y_1 = -\frac{1}{3} \\ y_2 = 0 \\ -2 - \frac{1}{3} = y_3 = -\frac{7}{3} \end{cases}$$

verifico ammiss.

$$-\frac{2}{3} \cdot \frac{7}{3} = -\frac{\cancel{9}}{\cancel{3}}3 = -3 \leq 5 \quad \text{ok.}$$

$$-\frac{1}{3} \leq 3 \quad \text{ok}$$

$$\frac{1}{3} - \frac{7}{3} = -\frac{6}{3} = -2 \leq -2 \quad \text{ok}$$

$-\cancel{z}^{1} \leq -1$ ok

$\cancel{z} 1$ $\rightarrow x^*$ è ottima

$y_1, y_2, y_3 \leq 0$ ok.

$$\min \; -ax_1 + bx_2$$
$$-x_1 + 3x_2 \leq 2$$
$$x_1 - 2x_2 \leq 2$$
$$x_1 + 2x_2 \leq 4$$
$$x_1, x_2 \geq 0$$

trovare range di valori per cui il PL amm. sol. ottima con x_1 e x_2 in base

x_1, x_2 in base $\rightarrow$ x_1 e $x_2 > 0$

Troviamo le condizioni per cui x_1 e x_2 sono in basi ottime

duale $\Longrightarrow$

$$\max \; 2y_1 + 2y_2 + 4y_3$$
$$-y_1 + y_2 + y_3 \leq -a$$
$$3y_1 - 2y_2 + 2y_3 \leq b$$
$$y_1, y_2, y_3 \leq 0$$

$x_1 > 0 \rightarrow sd_1 = 0$

$x_2 > 0 \rightarrow sd_2 = 0$

$$\begin{cases} -y_1 + y_2 + y_3 = -a \\ 3y_1 - 2y_2 + 2y_3 = b \end{cases}$$

$$\begin{cases} y_1 = a + y_2 + y_3 \\ 3a + 3y_2 + 3y_3 - 2y_2 + 2y_3 = b \end{cases} \rightarrow \begin{cases} y_1 = a + y_2 + y_3 \\ y_2 = b - 3a - 5y_3 \end{cases}$$

$y_1 = -2a + b - 4y_3 \leq 0$

$y_2 = b - 3a - 5y_3 \leq 0$

$$\frac{-2a+b}{\cancel{4}} \leq y_3 \leq 0 \qquad \longrightarrow \qquad 2a \geq b$$

$$\frac{-3a+b}{\cancel{5}} \leq y_3 \leq 0 \qquad \qquad 3a \geq b$$

fissare un valore per a e b nel range trovato e trovare la soluzione ottima del problema duale corrispondente

Ipotizziamo a e $b > 0$

$b \leq 2a$ soddisfatto per $a = b = 1$

il prob diventa

$$\min \; -x_1 + x_2$$
$$-x_1 + 3x_2 \leq 2$$
$$x_1 - 2x_2 \leq 2$$
$$x_1 + 2x_2 \leq 4$$

$\Downarrow$ std

$$\min \; -x_1 + x_2$$
$$-x_1 + 3x_2 + s_1 = 2$$
$$x_1 - 2x_2 + s_2 = 2$$
$$x_1 + 2x_2 + s_3 = 4$$

per trovare sol duale possiamo leggere il tableau utilizzando metodo del simplesso primale

0	-1	+1	0	0	0		→ Soluzione duale
2	-1	3	1	0	0	s_1	
2	$\boxed{1}$	-2	0	1	0	s_2	
4	1	2	0	0	1	s_3	

$R_0 \to R_2 + R_0$

$R_1 \to R_2 + R_1$

$R_3 \to -R_2 + R_3$

2	0	-1	0	1	0	
4	0	1	1	0	0	s_1
2	1	-2	0	1	0	x_1
2	0	$\boxed{4}$	0	-1	1	x_2

$R_0 \to R_3 + R_0$

$R_1 \to -R_3 + R_1$

$R_2 \to 2R_3 + R_2$

5/2	0	0	0	3/4	1/4
7/2	0	0	1	5/4	-1/4
3	1	0	0	1/2	1/2
1/2	0	1	0	-1/4	1/4

$z = -\frac{5}{2}$

Sol ottima del duale

$y_1 = 0 \qquad y_2 = -\frac{3}{4} \qquad y_3 = -\frac{1}{4}$

$$\begin{cases} 5y_1 + y_2 = 1 \\ -4y_1 - y_2 = 6 \\ 13y_1 + 5y_2 = -7 \\ y_1 + y_2 = 5 \end{cases}$$

$$\begin{cases} y_1 = 5 - y_2 \\ 25 - 5y_2 + y_2 = 1 \\ -20 + 4y_2 - y_2 = 6 \\ 65 - y_2 + 5y_2 = -7 \end{cases}$$

$$\begin{cases} y_1 + 2y_2 = 5 \\ y_1 - y_2 = -2 \end{cases} \rightarrow \begin{cases} -2 + y_2 + 2y_2 = 5 \\ y_1 = -2 + y_2 \end{cases}$$

$$\begin{cases} 3y_2 = \frac{7}{3} \\ y_1 = \frac{1}{3} \end{cases} \qquad \begin{cases} y_1 = 1/3 \\ y_2 = 7/3 \end{cases}$$

$$\min \quad 5x_1 - 2x_2$$

$$x_1 + x_2 \le 10$$

$$2x_1 - x_2 \ge 8$$

$$x_1, x_2 \ge 0$$

$$\xrightarrow{\text{std}}$$

$$\min \quad 5x_1 - 2x_2$$

$$x_1 + x_2 + S_1 = 10$$

$$2x_1 - x_2 - S_2 = 8$$

$$x_1, x_2 \ge 0$$

duale

$$\max \quad 10y_1 + 8y_2$$

$$\rightarrow \quad y_1 + 2y_2 \le 5$$

$$\rightarrow \quad y_1 - y_2 \le -2$$

$$y_1 \le 0$$

$$-y_2 \le 0$$

$$y_1 \in \mathbb{R} \quad e \quad y_2 \in \mathbb{R}$$

$$y^{(0)} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$y_1^{(0)} + 2y_2^{(0)} = 4 \le 5 \qquad x_1^{(0)} = 0$$

$$y_1 - y_2 = -2 \le -2 \qquad x_2^{(0)} \ne 0$$

$$\min \quad a_1 + a_2$$

$$x_2 + S_1 + a_1 = 10$$

$$-x_2 - S_2 + a_2 = 8$$

	x_2	s_1	s_2	a_1	a_2
	0	0	0	1	1
10	1	1	0	1	0
8	-1	0	-1	0	1

-18	0	-1	1	0	0
10	1	$\boxed{1}$	0	1	0
8	-1	0	-1	0	1

$R_0 \to R_1 + R_0$

				a_1	a_2
-8	1	0	1	1	0
10	1	1	0	1	0
8	-1	0	-1	0	1

$$\min \quad 2x_1 + x_2 - x_3$$

$$x_1 + 3x_2 - 2x_3 \le 8$$

$$2x_1 - x_2 + 4x_3 \ge 6$$

$$x_1 \in \mathbb{R}, \; x_2, x_3 \ge 0$$

$\tilde{x} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$ è SBA ? no perché # vinc. $\le$ # comp in base

$\exists$ SBA con x_1 e x_2 in base ?

$$x^* = \begin{pmatrix} >0 \\ >0 \\ 0 \end{pmatrix}$$

$\rightarrow$ standardizzo $\qquad x_1 = x_1^+ - x_1^- \ge 0$

$$\min \quad 2x_1^+ - 2x_1^- + x_2 - x_3$$

$$x_1^+ - x_1^- + 3x_2 - 2x_3 + S_1 = 8$$

$$2x_1^+ - 2x_1^- - x_2 + 4x_3 - S_2 = 6$$

$$x_1^+, x_2, x_3 \ge 0 \qquad x_1^- \le 0$$

$$x^* = \begin{pmatrix} >0 \\ 0 \\ >0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{array}{l} x_1^+ \\ x_1^- \\ x_2 \\ x_3 \\ s_1 \\ s_2 \end{array}$$

$$\begin{cases} x_1^+ + 3x_2 = 8 \\ 2x_1^+ - x_2 = 6 \end{cases} \qquad \begin{cases} x_1^+ + 6x_1^+ - 18 = 8 \\ x_2 = 2x_1^+ - 6 \end{cases}$$

$$\begin{cases} x_1^+ = \frac{26}{7} \\ x^2 = \frac{52-42}{7} = \frac{10}{7} \end{cases}$$

$\exists$ SBA con x_1 e x_2 in base

ed è $x^1 = \begin{pmatrix} 26/7 \\ 0 \\ 10/7 \\ 0 \end{pmatrix}$

$\exists$ SBA OTTIMA con x_1 e x_2 in base ?

$$\min \; 2x_1^- - 2x_1^- + x_2 - x_3$$

$$x_1^+ - x_1^- + 3x_2 - 2x_3 + S_1 = 8$$

$$2x_1^+ - 2x_1^- - x_2 + 4x_3 - S_2 = 6$$

$$x_1^+, x_2, x_3 \geq 0 \qquad x_1^- \leq 0$$

$$x = \begin{array}{c} \begin{pmatrix} >0 \\ 0 \\ 0 \\ >0 \\ 0 \\ 0 \end{pmatrix} \end{array} \begin{array}{l} x_1^+ \\ x_1^- \\ x_2 \\ x_3 \\ s_1 \\ s_2 \end{array} \qquad \begin{cases} x_1^+ - 2x_3 = 8 \\ 2x_1^+ + 4x_3 = 6 \end{cases}$$

$$\rightarrow \begin{cases} x_1^+ = 8 + 2x_3 \\ x_3 = -5/4 \end{cases} \qquad \begin{cases} x_1^+ = 11/2 \\ x_3 = -5/4 \end{cases}$$

NON AMMISSIB.

$x_3 \geq 0$

$$\min \; x_1 - x_2 \qquad y^{(0)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$2x_1 - x_2 \leq 6$$

$$x_1 + x_2 \geq 1$$

$$\tilde{x}_2 = -x_2 \geq 0$$

$$x_1 \geq 0 \quad x_2 \leq 0$$

std

$$\min \; x_1 + \tilde{x}_2$$

$$2x_1 + \tilde{x}_2 + S_1 = 6$$

$$x_1 - \tilde{x}_2 - S_2 = 1$$

$$x_1, \tilde{x}_2, S_1, S_2 \geq 0$$

duale

$$\max \; 6y_1 + y_2$$

$$2y_1 + y_2 \leq 1$$

$$y_1 - y_2 \leq 1$$

$$y_1 \leq 0$$

$$-y_2 \leq 0$$

$$y_1, y_2 \in \mathbb{R}$$

$$\begin{cases} Sd_1: 1 - 2y_1 - y_2 = 0 \\ Sd_2: 1 - y_1 + y_2 = 0 \\ Sd_3: y_1 = 0 \\ Sd_4: -y_2 = 0 \end{cases} \quad \Bigg| \quad y^0 \begin{cases} Sd_1: 1 = 0 \\ Sd_2: 1 = 0 \\ Sd_3: 0 = 0 \\ Sd_4: 0 = 0 \end{cases} \Rightarrow \begin{bmatrix} x_1 = 0 \\ x_2 = 0 \\ S_1 > 0 \\ S_2 > 0 \end{bmatrix}$$

$$\min \quad a_1$$

$$S_1 = 6$$

$$-S_2 + a_1 = 1$$

$$\begin{array}{c|ccc} 0 & 0 & 0 & 1 \\ \hline 6 & 1 & 0 & 0 \\ 1 & 0 & -1 & 1 \end{array}$$

$R_0 \to -R_2 + R_0$

$$\begin{array}{c|ccc} -1 & 0 & 1 & 0 \\ \hline 6 & 1 & 0 & 0 \\ 1 & 0 & -1 & 1 \end{array}$$

Stop $\to$ f.o. $\neq 0$ cerco altra sol.

$\to y^{(1)} = y^{(0)} + \theta \pi^*$

$\Rightarrow$ duale ristretto

$$\max \; 6\pi_1 + 1\pi_2$$

$$\pi_1 \leq 0$$

$$-\pi_2 \leq 0$$

$$\pi_2 \leq 1$$

$\pi_1 = 0$

$\pi_2 = 1$

$$\pi^* = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$y^{(1)} = y^0 + \theta \pi^* = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \theta \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ \theta \end{pmatrix}$$

$$\begin{cases} \theta \leq 1 \\ -\theta \leq 1 \\ 0 \leq 0 \\ \theta \geq 0 \end{cases} \to \theta = 1 \qquad y_1 = y^0 + \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$1 \leq 1$

$-1 \leq 1$

$0 \leq 0$

$-1 \leq 0$

y^1 amm.

$$\begin{cases} Sd_1 = 0 \\ Sd_2 = 2 \\ Sd_3 = 0 \\ Sd_4 = 1 \end{cases}$$

$$\begin{cases} x_1 > 0 \\ x_2 = 0 \\ S_1 > 0 \\ S_L = 0 \end{cases}$$

prim ristr.

min 0

$2x_1 + S_1 = 6$

$x_1 \qquad = 1$

0	0	0
6	2	1
1	1	0

$\to$ f.o. è 0

$\Downarrow$

y^1 è ottima

$$y^1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$x^* = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\max\ 3x_1 + x_2 + 8x_3 + 10x_4$$

$$3x_1 + x_2 + 4x_3 \leq 2$$

$$4x_1 + 3x_2 + 3x_3 + 6x_4 \leq 5$$

$$x_1, x_2, x_3, x_4 \geq 0$$

$$\min\ -3x_1 - x_2 - 8x_3 - 10x_4$$

$$3x_1 + x_2 + 4x_3 + s_1 = 2$$

$$4x_1 + 3x_2 + 3x_3 + 6x_4 + s_2 = 5$$

$$x_1, x_2, x_3, x_4, s_1, s_2 \geq 0$$

duale

$$\max\ 2y_1 + 5y$$

$$3y_1 + 4y_2 \leq -3$$

$$y_1 + 3y_2 \leq -1$$

$$4y_1 + 3y_2 \leq -8$$

$$6y_2 \leq -10$$

$$y_1 \leq 0$$

$$y_2 \leq 0$$

$$y_1, y_2 \in \mathbb{R}$$

$y^{(0)} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$ non amm.

ne trovo una amm.

$y^{(0)} = \begin{pmatrix} -1 \\ -2 \end{pmatrix}$

$$\Rightarrow \begin{cases} Sd_1: & -3 - 3y_1 - 4y_2 = 0 \\ Sd_2: & -1 - y_1 - 3y_2 = 0 \\ Sd_3: & -8 - 4y_1 - 3y_2 = 0 \\ Sd_4: & -10 - 6y_1 = 0 \\ Sd_5: & y_1 = 0 \\ Sd_6: & y_2 = 0 \end{cases} \Bigg|_{y^{(0)}} \rightarrow \begin{cases} -\cancel{3} + \cancel{3} + 8 = 0 \\ 6 = 0 \\ 2 = 0 \\ 2 = 1 \\ 1 = 0 \\ 2 = 0 \end{cases} \quad x^{+} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\max \; -4x_1 + 3x_2 - x_3$$

$$x_1 + 3x_2 \geq 10$$

$$x_1 - x_2 + 4x_3 \geq 8$$

$$x_1 \geq 0, \quad x_2 \leq 0 \quad x_3 \geq 0$$

$$y^{(0)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\tilde{x}_2 = -x_2 \geq 0$$

std.

$$\min \; 4x_1 + 3\tilde{x}_2 + x_3$$

$$x_1 - 3\tilde{x}_2 \quad - s_1 \quad = 10$$

$$x_1 + \tilde{x}_2 + 4x_3 \quad - s_2 = 8$$

$$x_1 \geq 0, \; \tilde{x}_2 \geq 0, \; x_3 \geq 0$$

duale

$$\max \; 10y_1 + 8y_2$$

$$y_1 + y_2 \leq 4$$

$$-3y_1 + y_2 \leq 3$$

$$4y_2 \leq 1$$

$$-y_1 \leq 0$$

$$-y_2 \leq 0$$

$$y_1, y_2 \in \mathbb{R}$$

$y^{(0)}$ amm.

$$\begin{cases} Sd_1: & 4 - \cancel{y_1} - \cancel{y_2} = 0 & x_1 = 0 \\ Sd_2: & 3 + 3\cancel{y_1} - \cancel{y_2} = 0 & x_2 = 0 \\ Sd_3: & 1 - \cancel{4y_2} = 0 & x_3 = 0 \\ Sd_4: & y_1 = 0 \rightarrow s_1 > 0 & \\ Sd_5: & y_2 = 0 \rightarrow s_2 > 0 & \end{cases}$$

$$x^* = \begin{pmatrix} 0 \\ 0 \\ 0 \\ >0 \\ >0 \end{pmatrix}$$

primale ristretto

$$\min \; a_1 + a_2$$
$$a_1 \quad - s_1 \quad = 10$$
$$+ a_2 \quad - s_2 \quad = 8$$
$$a_1, a_2, s_1, s_2 \geq 0$$

0	0	0	1	1
10	-1	0	1	0
8	0	-1	0	1

$R_0 \to R_0 - R_1 - R_2$

-18	1	1	0	0
10	-1	0	1	0
8	0	-1	0	1

f.o. $\neq 0 \to$ trovo $y^{(1)}$:

$$y^{(1)} = y^{(0)} + \theta \pi^*$$

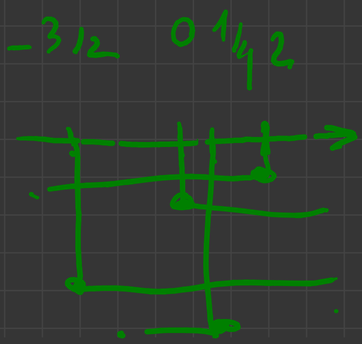
duale ristretto

$$\max \; 10\pi_1 + 8\pi_2$$
$$\pi_1 \leq 1$$
$$\pi_2 \leq 1$$
$$-\pi_1 \leq 0$$
$$-\pi_2 \leq 0$$

$$\pi^* = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$y^{(1)} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \theta \\ \theta \end{pmatrix} = \begin{pmatrix} \theta \\ \theta \end{pmatrix} \to$$ lo sostituisco nel duale

$$\begin{cases} 2\theta \leq 4 \\ -2\theta \leq 3 \\ 4\theta \leq 1 \\ -\theta \leq 0 \end{cases}$$



$$\theta = 1/4$$

$$y^{(1)} = \begin{pmatrix} 1/4 \\ 1/4 \end{pmatrix}$$

$$\begin{cases} \frac{2}{4} \leq 4 & \text{OK.} \\ -\frac{2}{4} \leq 3 & \text{OK} \\ 1 \leq 1 & \text{OK} \\ -\frac{1}{4} \leq 0 & \text{OK} \end{cases}$$

$y^{(1)}$ ammiss.

$$\begin{cases} sd_1 = 4 - y_1 - y_2 = 0 \\ sd_2 = 3 + 3y_1 - y_2 = 0 \\ sd_3 = 1 - 4y_2 = 0 \\ sd_4 = y_1 = 0 \\ sd_5 = y_2 = 0 \end{cases}$$

$$\begin{cases} 4 - 1/2 = 0 \\ 3 + 1/2 = 0 \\ \cancel{1 - 1} = 0 \\ 1/4 = 0 \\ 1/4 = 0 \end{cases}$$

$$x^* = \begin{bmatrix} 0 \\ 0 \\ > 0 \\ 0 \\ 0 \end{bmatrix}$$

min a_1

$a_1 = 10$

$x_3 = 2$

		x_3	a_1
	0	0	1
	10	1	0
	2	0	1

$\rightarrow$

-10	0	0
10	1	0
2	0	1

f.o. $\neq 0 \rightarrow y^1$ non ottima $\rightarrow$ cerco y^2

$$y^2 = \begin{pmatrix} 1/4 \\ 1/4 \end{pmatrix} + \theta \pi^*$$

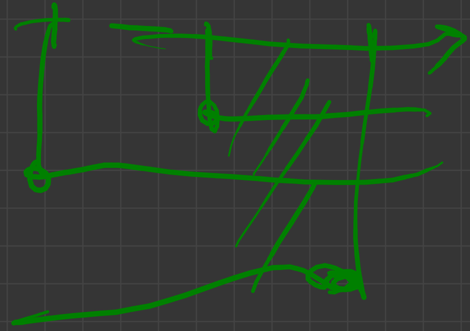
duale ristr. max $10\pi_1 + 2\pi_2$

$\pi_1 \leq 1$

$\pi_2 \leq 0$

$$\pi^* \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\begin{cases} \frac{1}{4}\theta + \frac{1}{4} \le 4 \\ -\frac{3}{4} - 3\theta + \frac{1}{4} \le 3 \\ \frac{4}{4} \le 1 \quad \checkmark \\ -\frac{1}{4} - \theta \le 0 \\ -\frac{1}{4} \le 0 \quad \checkmark \end{cases} \qquad \begin{cases} \theta \le \frac{7}{2} \\ \theta \ge -\frac{7}{6} \\ \theta \ge -\frac{1}{4} \end{cases}$$



$\theta = 7/2$

$$y^2 = \begin{pmatrix} 15/4 \\ 1/4 \end{pmatrix}$$ admiss.? yes.

$$\begin{cases} 4 \le 4 & \text{ok.} \\ -3 \cdot \frac{15}{4} + \frac{1}{4} \le 3 & \text{ok} \\ 1 \le 1 & \text{ok} \\ -\frac{15}{4} \le 0 & \text{ok} \\ -\frac{1}{4} \le 0 & \text{ok} \end{cases}$$

$$\begin{cases} Sd_1 : 4 - y_1 - y_2 = 0 \\ Sd_2 : 3 + 3y_1 - y_2 = 0 \\ Sd_3 : 1 - 4y_2 = 0 \\ Sd_4 : y_1 = 0 \\ Sd_5 : y_2 = 0 \end{cases} \rightarrow \begin{cases} 4 - \frac{\cancel{14}\,7}{\cancel{4}\,2} = 0 \\ 3 + \frac{\cancel{44}\,11}{\cancel{4}\,1} = 14 \ne 0 \\ 1 - 1 = 0 \\ 15/4 \ne 0 \\ 1/4 \ne 0 \end{cases}$$

$$\max \quad 3x_1 - 2x_2 + x_3$$

$$5x_1 + x_2 - x_3 \leq 10$$

$$-x_1 + x_2 + 2x_3 \geq 2$$

$$x_1 \geq 0, \quad x_2 \leq 0, \quad x_3 \in \mathbb{R}$$

$$\tilde{x}_2 = -x_2 \geq 0$$

std

$$-\min \quad -3x_1 - 2\tilde{x}_2 - x_3^+ + x_3^-$$

$$5x_1 - \tilde{x}_2 - x_3^+ + x_3^- + s_1 = 10$$

$$-x_1 - \tilde{x}_2 + 2x_3^+ - 2x_3^- - s_2 = 2$$

$$x_1 \geq 0 \quad \tilde{x}_2 \geq 0 \quad x_3^+ \geq 0 \quad x_3^- \geq 0$$

$$x^* = \begin{array}{cc} \begin{pmatrix} \geq 0 \\ 0 \\ \geq 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{array}{c} x_1 \\ \tilde{x}_2 \\ x_3^+ \\ x_3^- \\ s_1 \\ s_2 \end{array} \end{array}$$

$$\max \quad 10y_1 + 2y_2$$

$$5y_1 - y_2 \leq -3$$

$$-y_1 - y_2 \leq -2$$

$$-y_1 + 2y_2 \leq -1$$

$$y_1 - 2y_2 \leq 1$$

$$y_1 \leq 0$$

$$-y_2 \leq 0$$

$$y_1, y_2 \in \mathbb{R}$$

$$\begin{cases} sd_1 : 5y_1 - y_2 = -3 \\ sd_3 : -y_1 + 2y_2 = -1 \end{cases} \rightarrow \begin{cases} y_1 = 2y_2 + 1 \\ 10y_2 \quad - y_2 = -8 \end{cases}$$

$$\begin{cases} y_1 = -\frac{7}{9} \\ \cancel{9}y_2 = -\frac{8}{9} \end{cases}$$

$\exists$ SBA OTTIMA con x_1 e x_2 non in base?

Del tipo $x^{(1)} = \begin{bmatrix} 0 \\ 0 \\ >0 \\ 0 \\ >0 \\ 0 \end{bmatrix}$ $x^{(2)} = \begin{bmatrix} 0 \\ 0 \\ >0 \\ 0 \\ 0 \\ >0 \end{bmatrix}$ $x^{(3)} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ >0 \\ >0 \end{bmatrix}$

$$x^{(1)} \begin{cases} -x_3^+ + S_1 = 10 \\ x_3^+ = 1 \end{cases} \quad \begin{cases} S_1 = 11 \\ x_3^+ = 1 \end{cases} \rightarrow \text{ammiss.}$$

$$\begin{cases} -\cancel{y_1} + 2y_2 = -\frac{1}{2} \\ y_1 = 0 \end{cases} \rightarrow \text{no ottima}$$

$$x^{(2)} \begin{cases} -x_3^+ = 10 \\ -S_2 = 2 \end{cases} \rightarrow \begin{cases} x_3^+ = -10 \\ S_2 = -2 \end{cases} \rightarrow \text{no amm.}$$

$\Rightarrow \nexists$ SBA OTTIMA con x_1 e x_2 non in base