THEORETICAL COMPUTER SCIENCE TUTORING (1)

Maurizio Fiusco



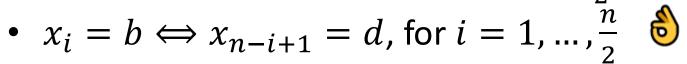
Let L be the set of strings $s = \langle x_1 x_2 \dots x_n \rangle$ of even length such that:



•
$$x_i \in \{a, b\}$$
, for $i = 1, ..., \frac{n}{2}$

•
$$x_i \in \{c, d\}$$
, for $i = \frac{n}{2} + 1, ..., n$

•
$$x_i = a \Leftrightarrow x_{n-i+1} = c$$
, for $i = 1, \dots, \frac{n}{2}$





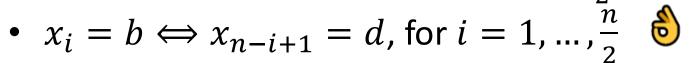
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a a b b d d c c	а	а	b	b	d	d	С	С
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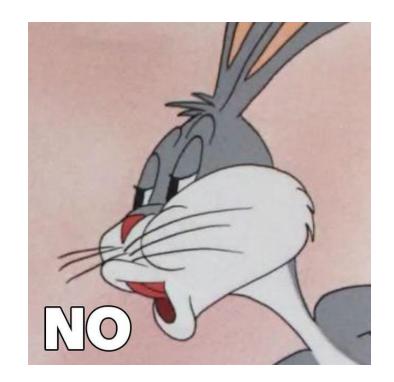


Let L be the set of strings $s = \langle x_1 x_2 \dots x_n \rangle$ of even length such that: \P



- $x_i \in \{a, b\}$, for $i = 1, ..., \frac{n}{2}$
- $x_i \in \{c, d\}$, for $i = \frac{n}{2} + 1, ..., n$
- $x_i = a \Leftrightarrow x_{n-i+1} = c$, for $i = 1, ..., \frac{n}{2}$
- $x_i = b \iff x_{n-i+1} = d$, for $i = 1, \dots, \frac{n}{2}$

a b a d c



Let L be the set of strings $s = \langle x_1 x_2 \dots x_n \rangle$ of even length such that:



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, for $i = 1, \dots, \frac{\overline{n}}{2}$



а	а	b	b	а	d	d	d	С	С
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Let L be the set of strings $s = \langle x_1 x_2 \dots x_n \rangle$ of even length such that:

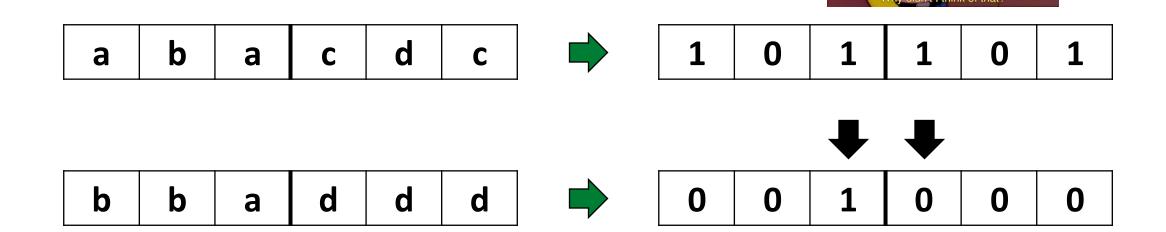
- $x_i \in \{a, b\}$, for $i = 1, ..., \frac{n}{2}$
- $x_i \in \{c, d\}$, for $i = \frac{n}{2} + 1, ..., n$
- $x_i = a \Leftrightarrow x_{n-i+1} = c$, for $i = 1, ..., \frac{n}{2}$
- $x_i = b \Leftrightarrow x_{n-i+1} = d$, for $i = 1, ..., \frac{n}{2}$

Define a deterministic Turing Machine that accepts all and only the words contained in \boldsymbol{L}



Let $s = \langle x_1 x_2 \dots x_n \rangle \in \{a, b, c, d\}^n$ e $\sigma = \langle y_1 y_2 \dots y_n \rangle \in \{0, 1\}^n$ the binary string associated with s according to the following rules:

- $y_i = 0 \Leftrightarrow x_i = a \ \lor x_i = c$, per $1 \le i \le n$
- $y_i = 1 \Leftrightarrow x_i = b \lor x_i = d$, per $1 \le i \le n$





I just have to check if the string is palindrome

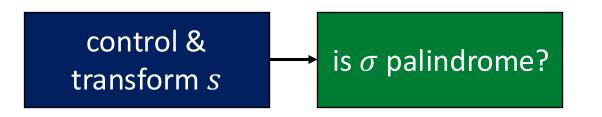
Let $s = \langle x_1 x_2 \dots x_n \rangle \in \{a, b, c, d\}^n$ e $\sigma = \langle y_1 y_2 \dots y_n \rangle \in \{0, 1\}^n$ the binary string associated with s according to the following rules:

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is σ palindrome?

2 possible ways:

- Transform the string and check if it is a palindrome using the appropriate Turing machine
- Modify the Turing machine that checks if a string is palindrome



Modified TM

$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$

 q_0



	_	а	а	b	d	C	С					

$$\langle q_0, a, \square, q_a, right \rangle$$

similar if I find *b*

$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$



								_	 		
		а	b	d	С	C					

$$\langle q_a, a, a, q_a, right \rangle$$

$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$



									_	 	
		а	b	d	С	С					

$$\langle q_a, b, b, q_a, right \rangle$$

$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$



		а	b	d	С	C					

$$\langle q_a, d, d, q_a, right \rangle$$

$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$



		а	b	d	O	C					

$$\langle q_a, c, c, q_a, right \rangle$$

$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$



						•					
		а	b	d	С	С					

$$\langle q_a, c, c, q_a, right \rangle$$

$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$



									_		
		а	þ	d	C	С					

$$\langle q_a, \Box, \Box, q_c, left \rangle$$

$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$

 q_c



		а	b	d	С	С					

$$\langle q_c, c, \Box, q_{left}, left \rangle$$

$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$



		-								
		а	b	d	C					

$$\langle q_{left}, c, c, q_{left}, left \rangle$$

$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$



								_			
		а	b	d	С						

$$\langle q_{left}, d, d, q_{left}, left \rangle$$

$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$



<u> </u>									_		
		а	b	d	С						

$$\langle q_{left}, b, b, q_{left}, left \rangle$$

$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$



		а	b	d	С						
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$$\langle q_{left}, a, a, q_{left}, left \rangle$$

$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$





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		а	b	d	С							

$$\langle q_{left}, \Box, \Box, q_0, right \rangle$$

$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$

 q_0



		а	b	d	С					

$$\langle q_0, a, \square, q_a, right \rangle$$

$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$

 q_a



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$$\langle q_a, b, b, q_a, right \rangle$$

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$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$

 q_0



		b	d						

$$\langle q_0, b, \square, q_b, right \rangle$$

$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$

 q_b



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			þ						

$$\langle q_b, d, d, q_b, right \rangle$$

similar if I find a, b or c

$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$

 q_b



			d						

$$\langle q_b, \Box, \Box, q_d, left \rangle$$

$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$

 q_d



			d						

$$\langle q_d, d, \square, q_{left}, left \rangle$$

$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$





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$$\langle q_{ind}, \Box, \Box, q_0, right \rangle$$

$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$

 q_0



$$\langle q_0, \square, \square, q_{acc}, stop \rangle$$

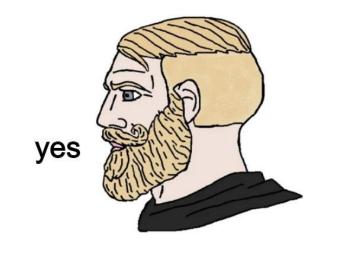
$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$





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$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$

 q_0



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			d	b	а	С	d	d				

$$\langle q_0, d, d, q_{rej}, stop \rangle$$

$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$





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$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$

 q_0



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$$\langle q_0, c, c, q_{rej}, stop \rangle$$

$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$





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$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$

 q_c



			b	a	С	d	d			

$$\langle q_c, d, d, q_{rej}, stop \rangle$$

similar if I find a, b or \square

$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$





<u> </u>											
				b	а	С	d	d			



$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$

 q_d



			b	а	С	d	С			

$$\langle q_c, c, c, q_{rej}, stop \rangle$$

similar if I find a, b or \square

$$\Sigma = \{a, b, c, d, \square\}$$

$$Q = \{q_0, q_a, q_b, q_c, q_d, q_{left}, q_{acc}, q_{rej}\}$$





				b	a	C	d	C			
			<u> </u>								



Problem 2.3 from EsMacchineTuring.pdf (uniroma2.it)

Design a Turing machine that computes the two functions described below:

•
$$f(n,k) = \left[\frac{n}{k}\right]$$

•
$$f(n,k) = \left\lceil \frac{n}{k} \right\rceil$$

• $g(n,k) = \left\lceil \frac{n}{k} \right\rceil$



n

k

f(n,k)

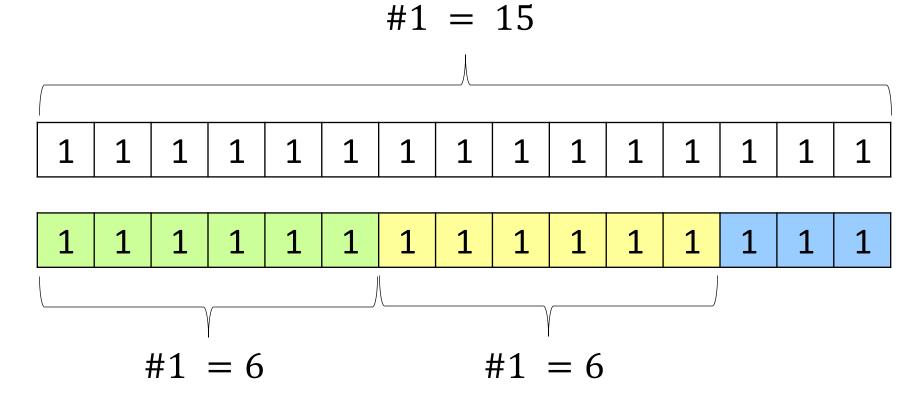
Problem 2.3 from EsMacchineTuring.pdf (uniroma2.it)

Es.

$$n = 15, k = 6$$

 $f(15,6) = 3$
 $g(15,6) = 2$





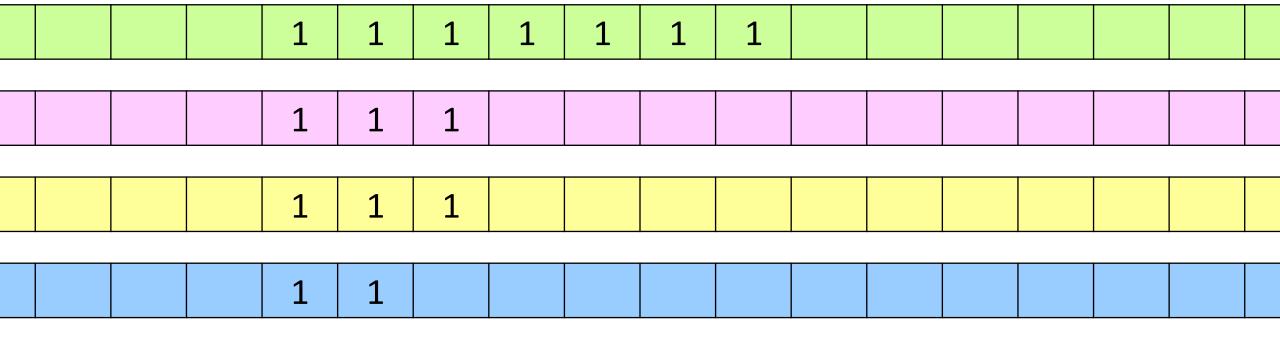
$$\Sigma = \{1, \square\}, Q = \{q_0, q_1, q_2, q_f\}$$

Es.

$$n = 1111111, k = 111$$

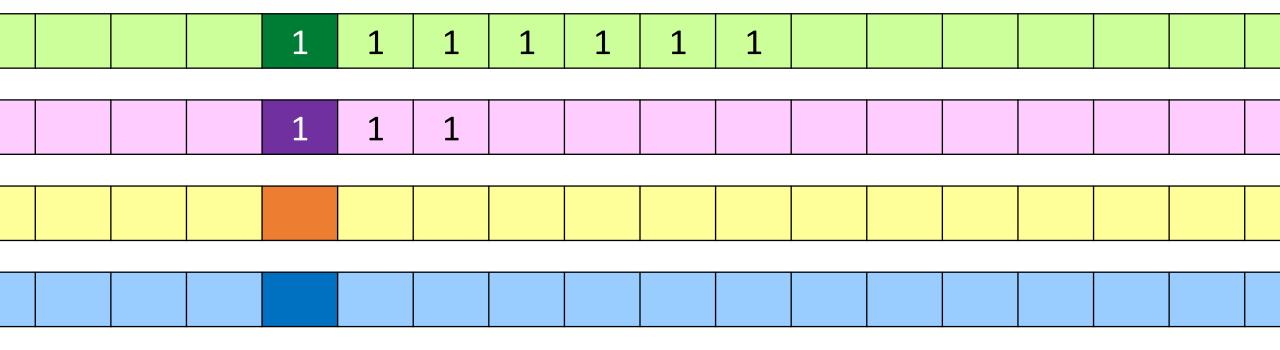
 $f(1111111,111) = 111, g(1111111,111) = 11$

 q_f



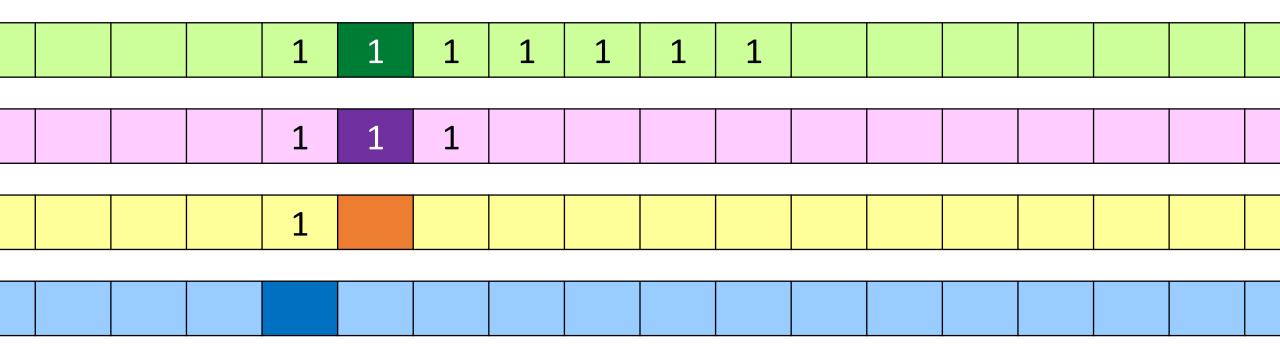
$$\Sigma = \{1, \square\}, Q = \{q_0, q_1, q_2, q_f\}$$

$$\langle q_0, (1,1,\Box,\Box), (1,1,1,\Box), q_1, (r,r,r,s) \rangle$$



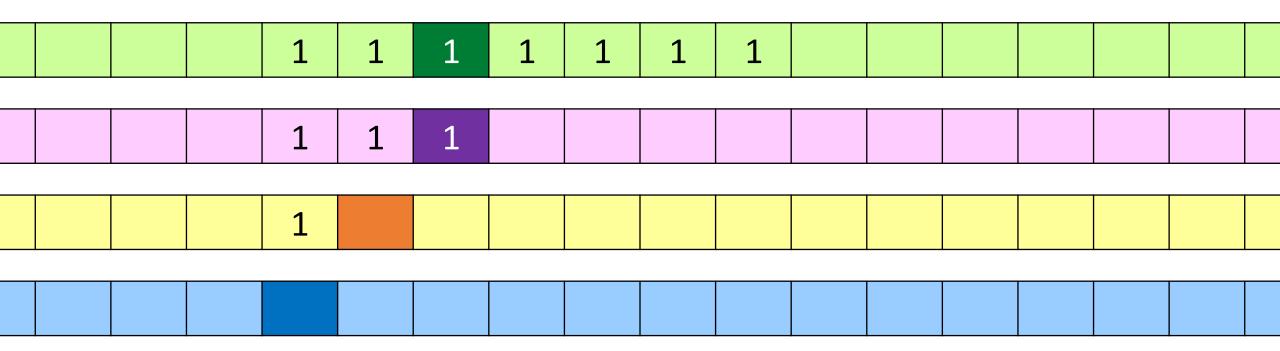
$$\Sigma = \{1, \square\}, Q = \{q_0, q_1, q_2, q_f\}$$

$$\langle q_1, (1,1,\Box,\Box), (1,1,\Box,\Box), q_1, (r,r,s,s) \rangle$$



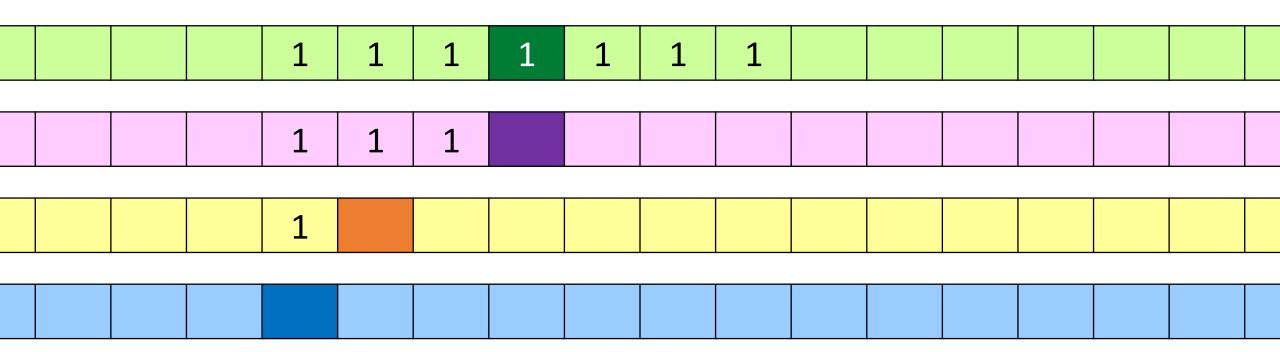
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$$\langle q_1, (1,1,\Box,\Box), (1,1,\Box,\Box), q_1, (r,r,s,s) \rangle$$



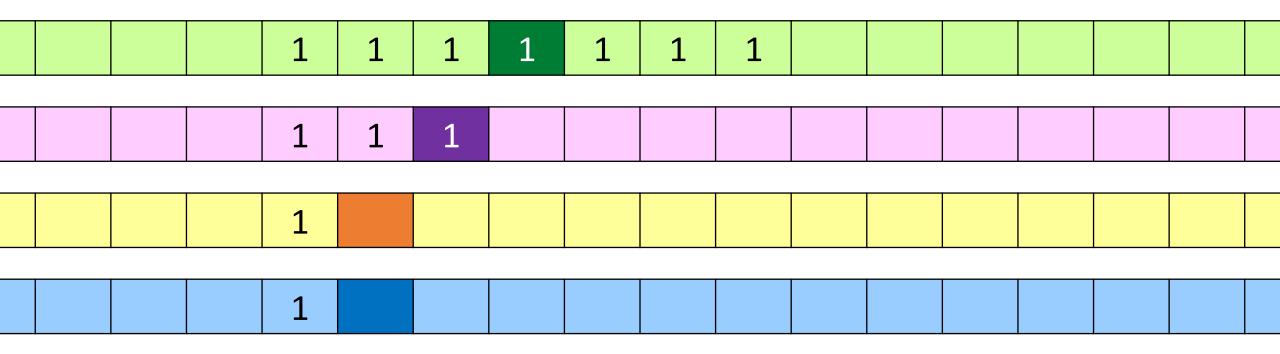
$$\Sigma = \{1, \square\}, Q = \{q_0, q_1, q_2, q_f\}$$

$$\langle q_1, (1, \square, \square, \square), (1, \square, \square, 1), q_2, (s, l, s, r) \rangle$$



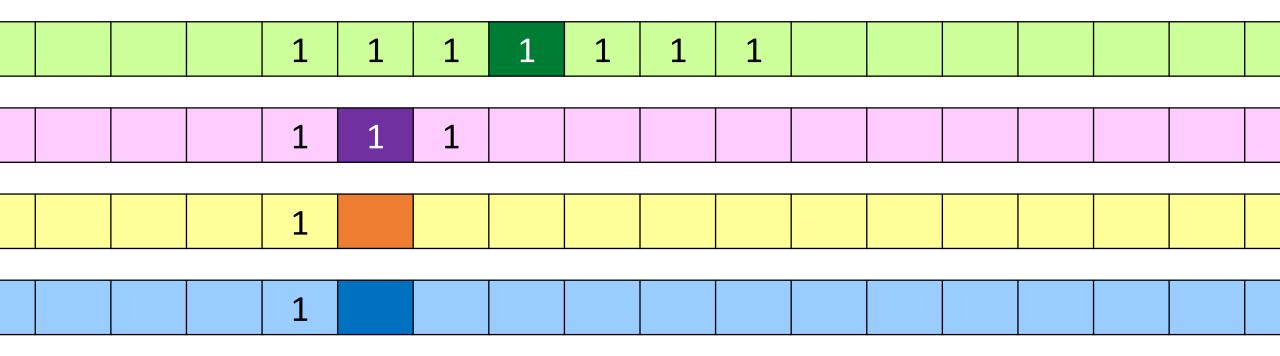
$$\Sigma = \{1, \square\}, Q = \{q_0, q_1, q_2, q_f\}$$

$$\langle q_2, (1,1,\Box,\Box), (1,1,\Box,\Box), q_2, (s,l,s,s) \rangle$$



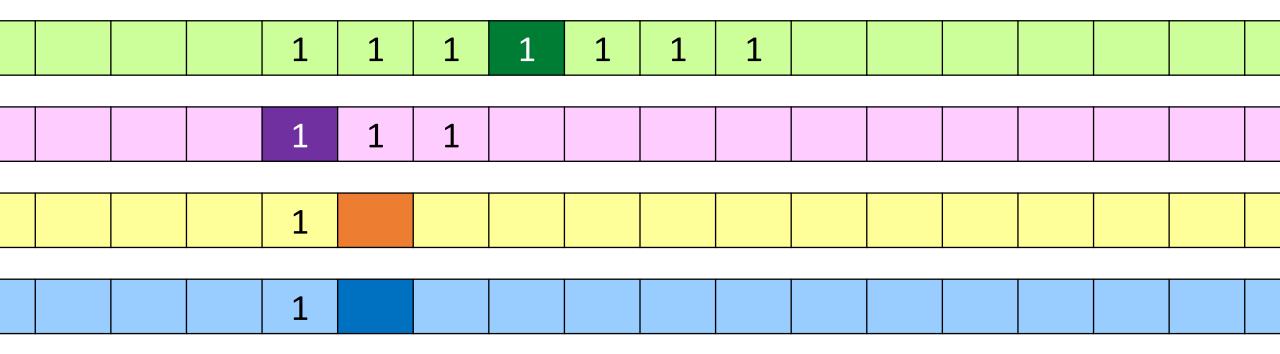
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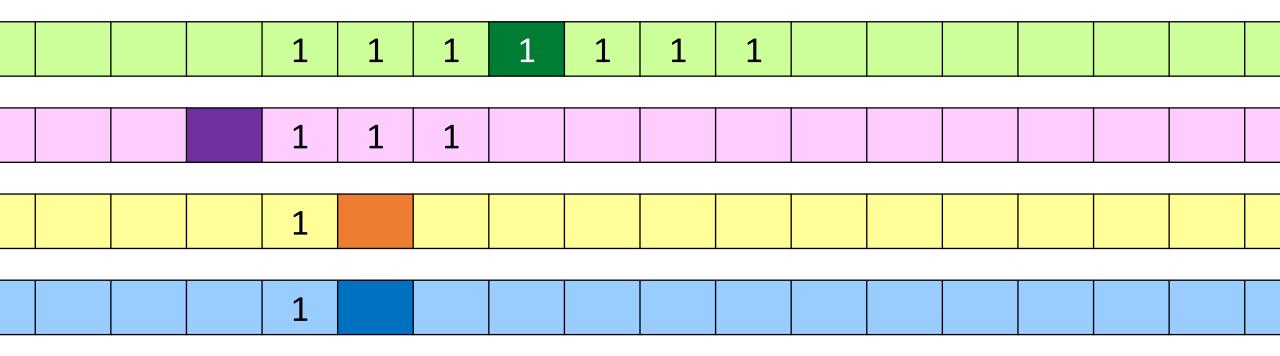
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$$\langle q_2, (1,1,\Box,\Box), (1,1,\Box,\Box), q_2, (s,l,s,s) \rangle$$



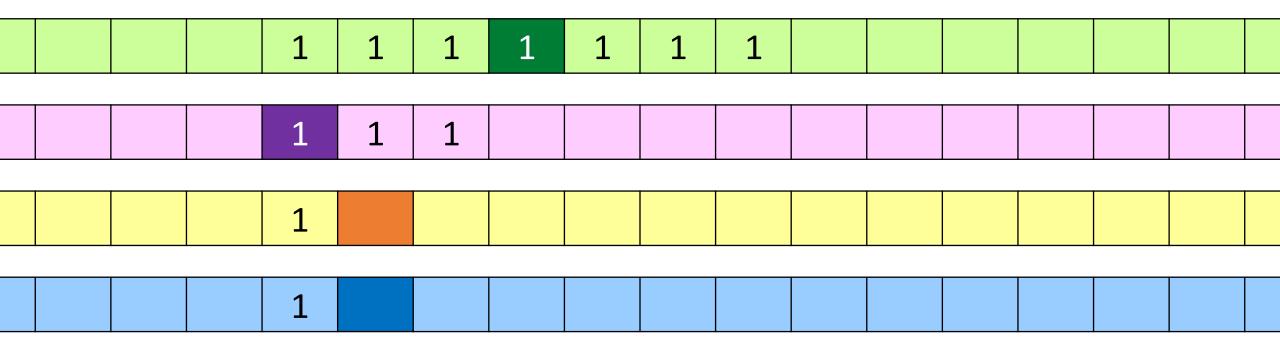
$$\Sigma = \{1, \square\}, Q = \{q_0, q_1, q_2, q_f\}$$

$$\langle q_2, (1, \square, \square, \square), (1, \square, \square, \square), q_0, (s, r, s, s) \rangle$$



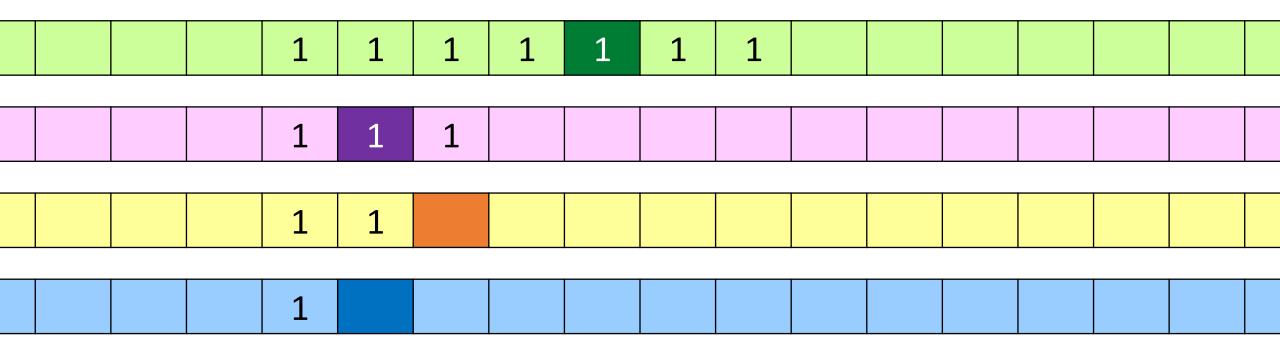
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$$\langle q_0, (1,1,\Box,\Box), (1,1,1,\Box), q_1, (r,r,r,s) \rangle$$



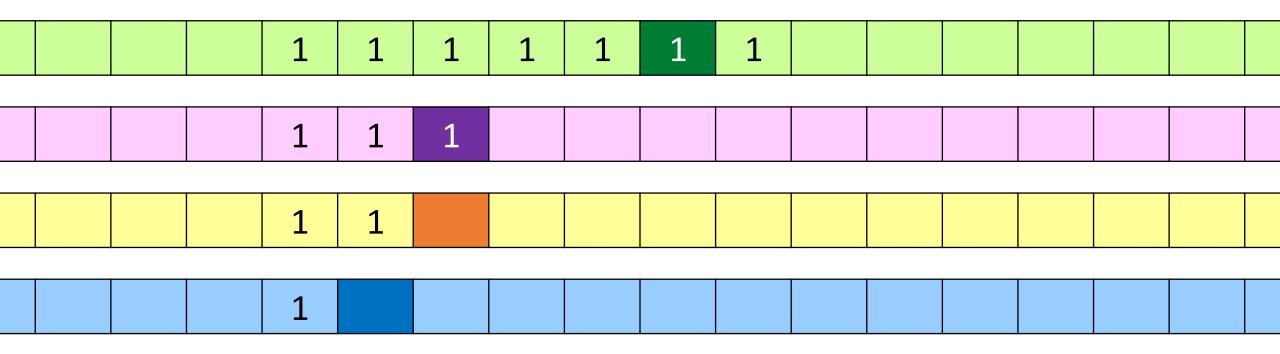
$$\Sigma = \{1, \square\}, Q = \{q_0, q_1, q_2, q_f\}$$

$$\langle q_1, (1,1,\Box,\Box), (1,1,\Box,\Box), q_1, (r,r,s,s) \rangle$$



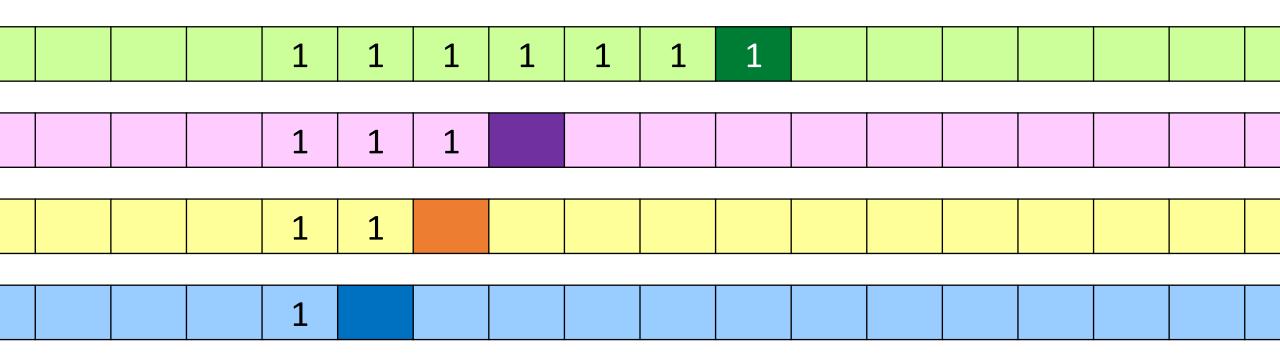
$$\Sigma = \{1, \square\}, Q = \{q_0, q_1, q_2, q_f\}$$

$$\langle q_1, (1,1,\Box,\Box), (1,1,\Box,\Box), q_1, (r,r,s,s) \rangle$$



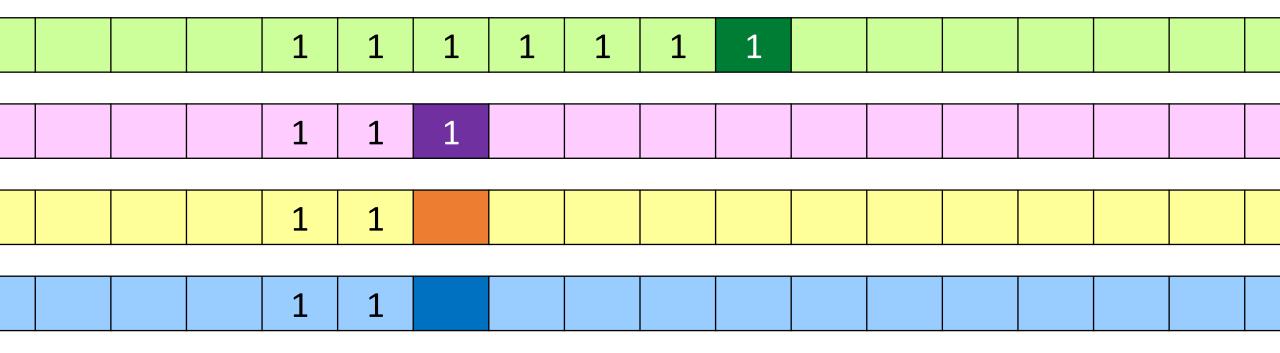
$$\Sigma = \{1, \square\}, Q = \{q_0, q_1, q_2, q_f\}$$

$$\langle q_1, (1, \square, \square, \square), (1, \square, \square, 1), q_2, (s, l, s, r) \rangle$$



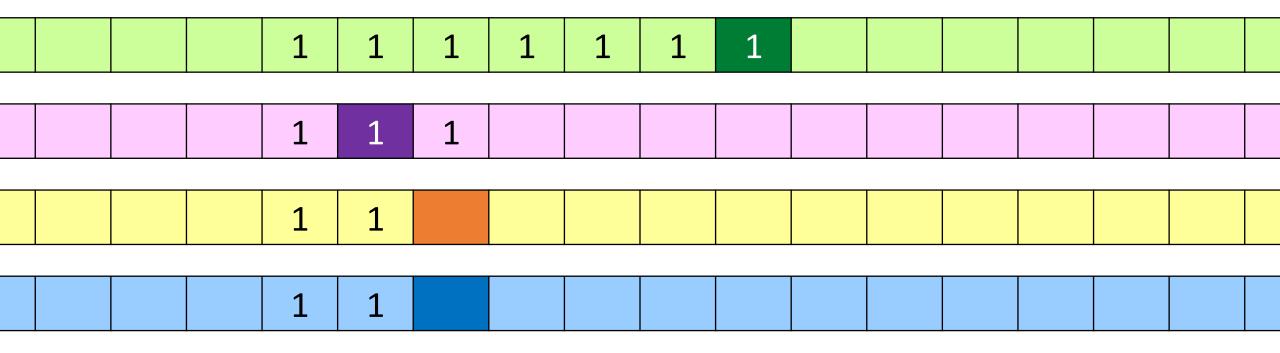
$$\Sigma = \{1, \square\}, Q = \{q_0, q_1, q_2, q_f\}$$

$$\langle q_2, (1,1,\Box,\Box), (1,1,\Box,\Box), q_2, (s,l,s,s) \rangle$$



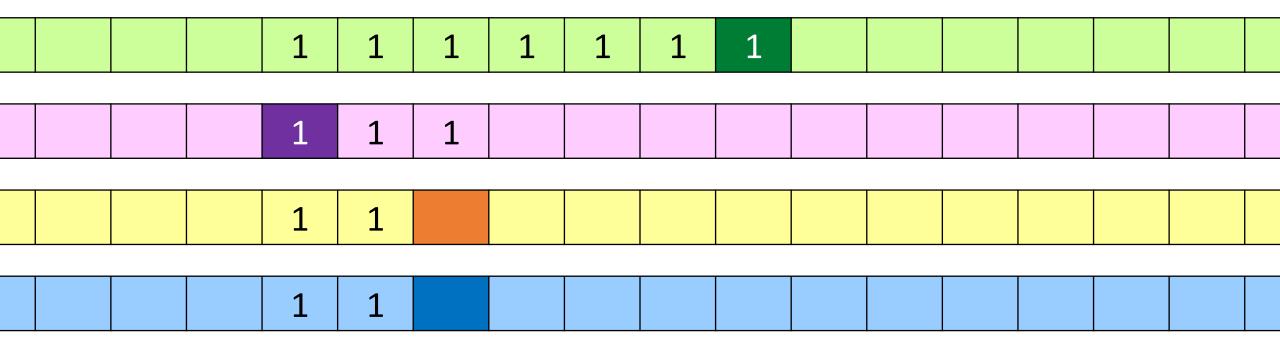
$$\Sigma = \{1, \square\}, Q = \{q_0, q_1, q_2, q_f\}$$

$$\langle q_2, (1,1,\Box,\Box), (1,1,\Box,\Box), q_2, (s,l,s,s) \rangle$$



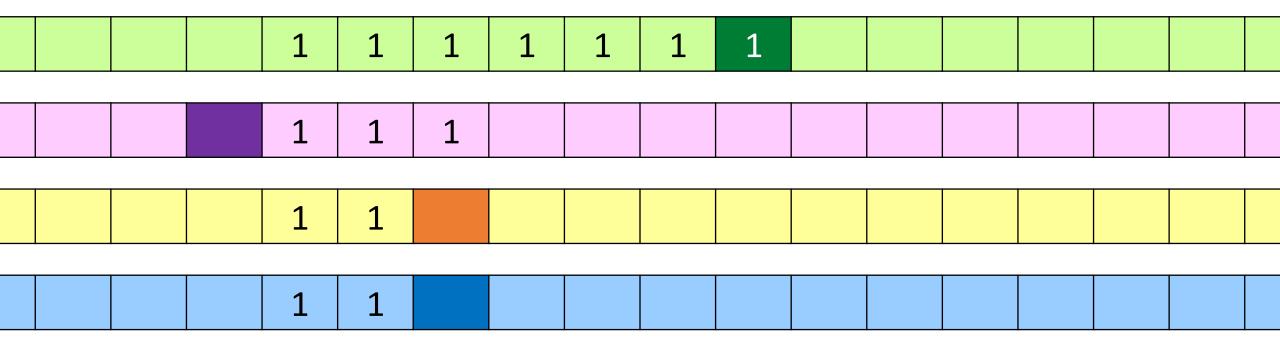
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$$\langle q_2, (1,1,\Box,\Box), (1,1,\Box,\Box), q_2, (s,l,s,s) \rangle$$



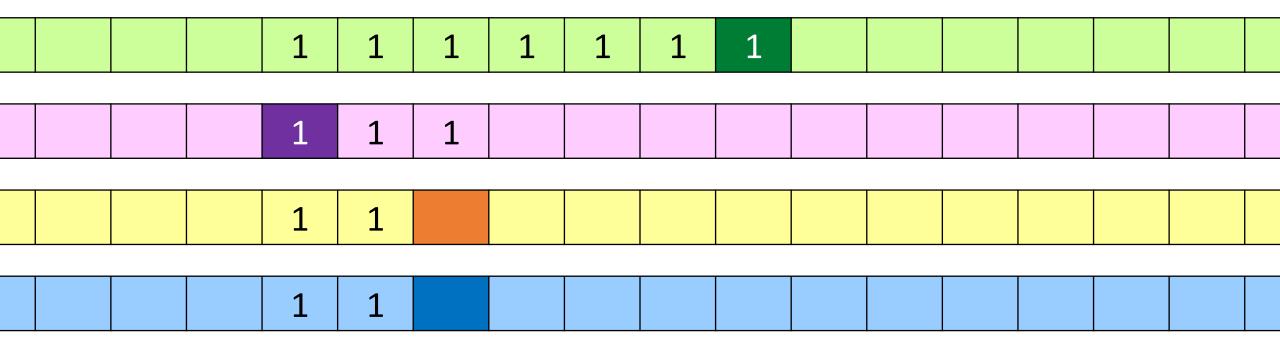
$$\Sigma = \{1, \square\}, Q = \{q_0, q_1, q_2, q_f\}$$

$$\langle q_2, (1, \square, \square, \square), (1, \square, \square, \square), q_0, (s, r, s, s) \rangle$$



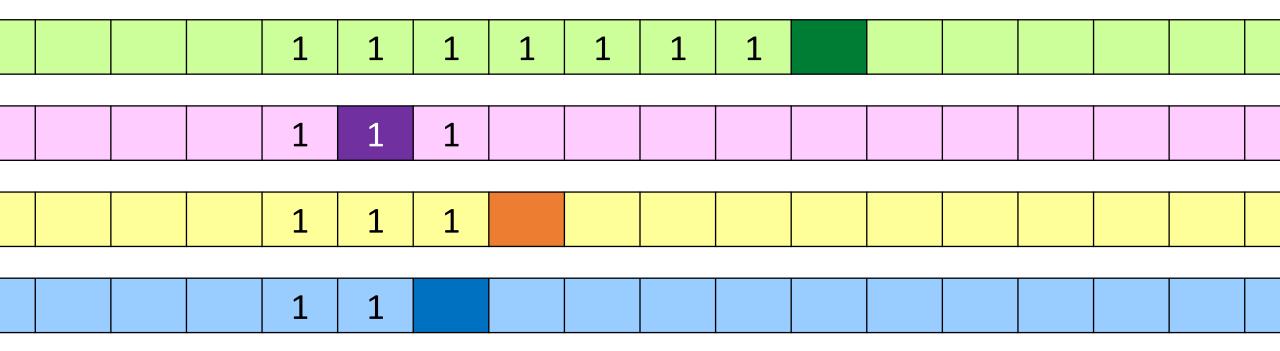
$$\Sigma = \{1, \square\}, Q = \{q_0, q_1, q_2, q_f\}$$

$$\langle q_0, (1,1,\Box,\Box), (1,1,1,\Box), q_1, (r,r,r,s) \rangle$$



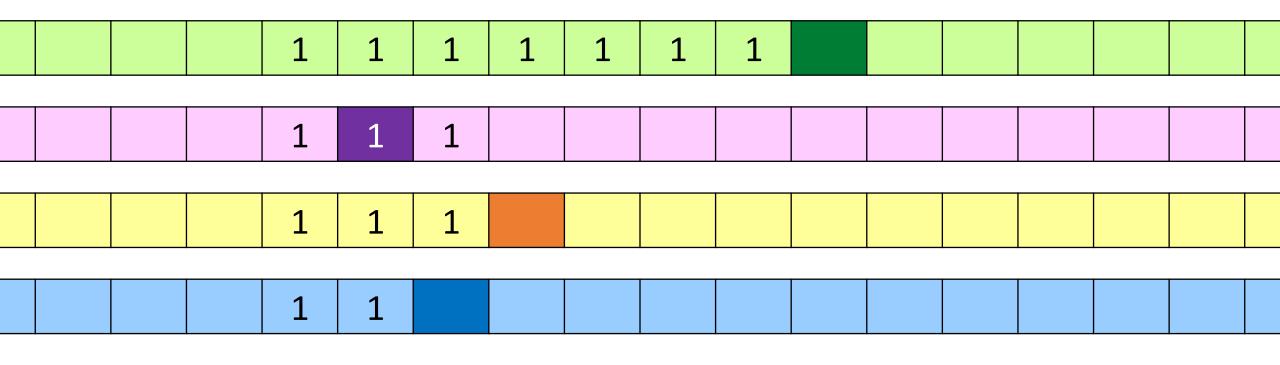
$$\Sigma = \{1, \square\}, Q = \{q_0, q_1, q_2, q_f\}$$

$$\langle q_1, (\Box, 1, \Box, \Box), (\Box, 1, \Box, \Box), q_f, (s, s, s, s) \rangle$$



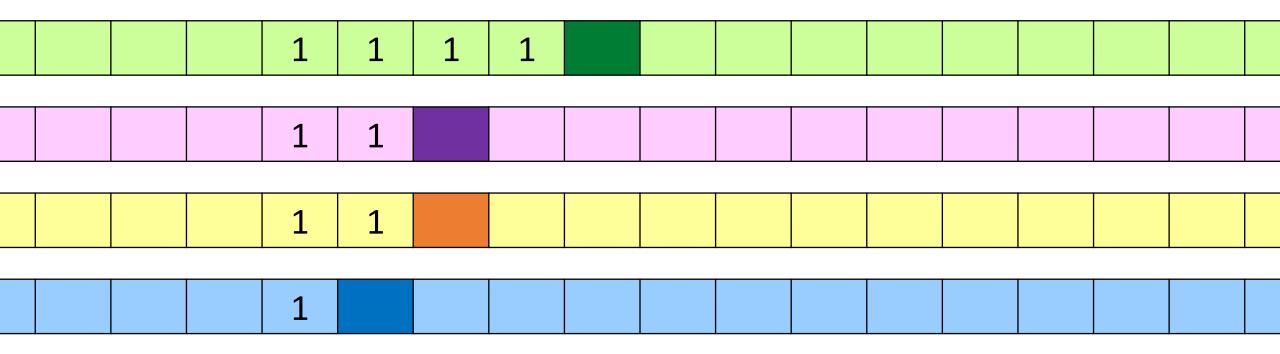
$$\Sigma = \{1, \square\}, Q = \{q_0, q_1, q_2, q_f\}$$

 q_f



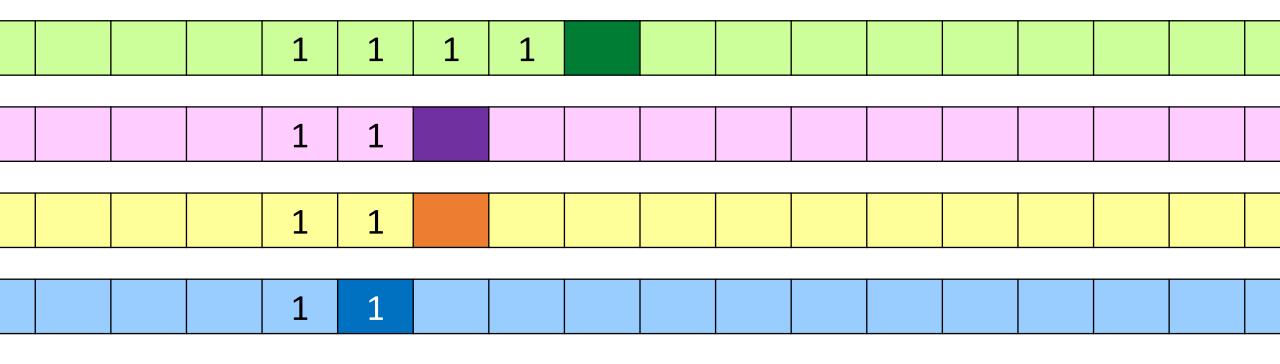
$$\Sigma = \{1, \square\}, Q = \{q_0, q_1, q_2, q_f\}$$

$$\langle q_1, (\Box, \Box, \Box, \Box), (\Box, \Box, \Box, 1), q_f, (s, s, s, s) \rangle$$



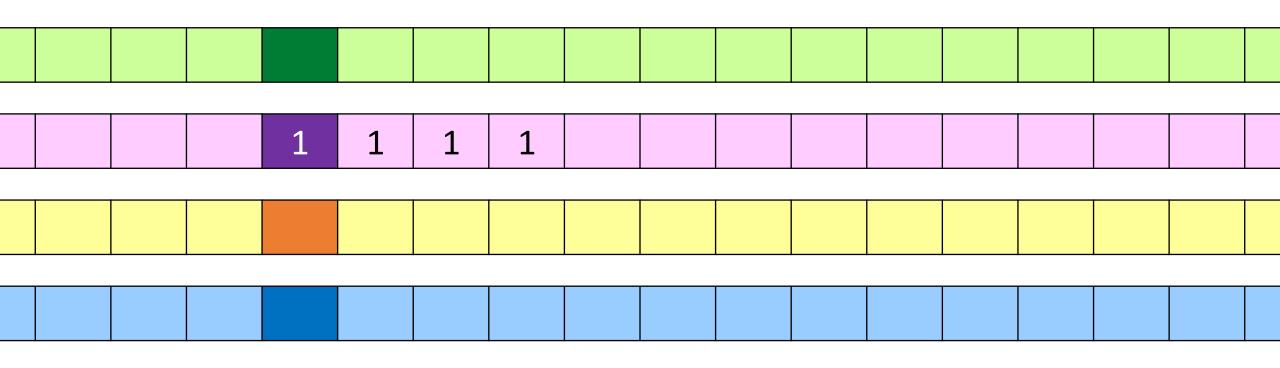
$$\Sigma = \{1, \square\}, Q = \{q_0, q_1, q_2, q_f\}$$

 q_f



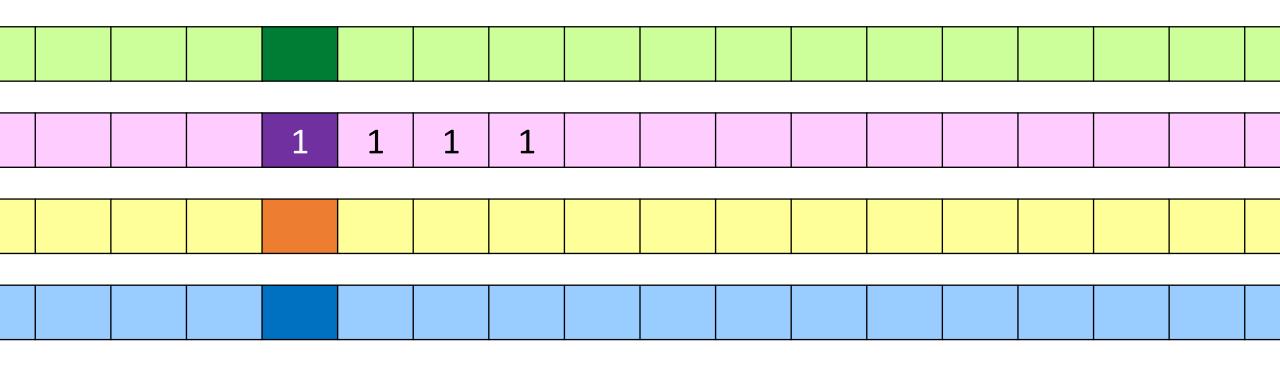
$$\Sigma = \{1, \square\}, Q = \{q_0, q_1, q_2, q_f\}$$

$$\langle q_0, (\square, 1, \square, \square), (\square, 1, \square, \square), q_f, (s, s, s, s) \rangle$$



$$\Sigma = \{1, \square\}, Q = \{q_0, q_1, q_2, q_f\}$$

 q_f



Let k be a constant in \mathbb{N} , and let NT_k be a non-deterministic Turing machine with a degree of non-determinism equal to k. Define a non-deterministic Turing machine NT_2 with a degree of non-determinism equal to 2 that is equivalent to NT_k