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**Q1** Describe the Joux's three-party Diffie-Hellman protocol

**Q2** What is the (min) order of the following multiplicative groups defined by the chosen parameters p and g:  
[hint: 593, 1187, 3559 primes, with  $1187 = 2 \times 593 + 1$  and  $3559 = 593 \times 2 \times 3 + 1 \rightarrow$  with the exception of (2), no need to do any modular exponentiation! – **NOTE: EXPLAIN YOUR ANSWER otherwise answer is not valid**]

1)  $g^x \bmod p$ , with  $g=1186$ ,  $p=1187$

2)  $g^x \bmod p$ , with  $g=7$ ,  $p=1187$

3)  $g^x \bmod p$ , with  $g=9$ ,  $p=1187$

4)  $g^x \bmod p$ , with  $g=64$ ,  $p=3599$

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**Q3** Describe the Pedersen Verifiable Secret sharing

**Q4** Being  $e: G \times G \rightarrow G_t$  a bilinear map, and  $g$  a generator of  $G$ , simplify the expression:

$$e(g^a g^b, g^c g^d) / e(g^{ac}, g^{bd})$$

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**E1.** Consider the Elliptic curve  $y^2 = x^3 - 2x - 3$  defined over the modular integer field  $\mathbb{Z}_5$ .

**A.** find all the points  $EC(\mathbb{Z}_5)$  and state what is the order of the corresponding group

$$\begin{aligned} P &= (x_1, y_1) \\ Q &= (x_2, y_2) \\ R &= P + Q = (x_3, y_3) \\ x_3 &= \lambda^2 - x_1 - x_2 \\ y_3 &= \lambda(x_1 - x_3) - y_1 \\ \lambda &= \begin{cases} \frac{y_2 - y_1}{x_2 - x_1} & P \neq Q \\ \frac{3x_1^2 + a}{2y_1} & P = Q \end{cases} \end{aligned}$$

**B.** Compute [4](1,1)

[HELP: possibly useful mnemonic hints reported here on the right;

MUST-DO: show step-by-step detailed computations; try to minimize the number of EC additions]

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**E2 – Secret Sharing and Threshold RSA.** A group of four parties forms a (3,4) secret sharing scheme. Using ordinary arithmetic (no moduli) parties  $P_i=\{1,2,3,4\}$  are dealt with shares  $\sigma_i$  of a key SK, which is the private key of an RSA scheme with modulus  $n=91$  and public key  $PK=5$ .

**A.** Using shares  $\sigma_1=34$ ,  $\sigma_2=41$ ,  $\sigma_4=61$ , reconstruct the secret key SK and verify that it is the correct one for the considered RSA public key PK;

**B.** If the attacker only knows shares  $\sigma_1$  and  $\sigma_4$ , and does not yet know share  $\sigma_2$ , what's his/her advantage in terms of chances to guess the secret key versus a pure random guess? *[ignore here the further info that SK is an RSA secret key]*

**C.** Using the Shoup construction, show step by step how the three parties P1, P2 and P4 can distributively compute a threshold RSA signature for the message  $m=15$ , and verify that the result is correct by directly computing the signature using the SK value computed before.

*[NOTE: if you prefer, to simplify computation instead of using the full L! you can use the minimum value that permits to make all lambda coefficients integer] [HINT:  $15^{-1} \bmod 91 = 85$ ]*