f periodică $\Leftrightarrow f(x+T)=f(x)$

Serii trigonometrice Fourier

Coeficienții Fourier

$$\boxed{a_n=\frac{2}{b-a}\int_a^b f(x)\cos\frac{2n\pi x}{b-a}\,dx} \qquad \boxed{b_n=\frac{2}{b-a}\int_a^b f(x)\sin\frac{2n\pi x}{b-a}\,dx}$$

Coef. Fourier pt fct periodică:

$$\boxed{a_n=\frac{2}{T}\int_a^{a+T} f(x)\cos(n\omega x)\,dx,\ n\in\mathbb{N}} \qquad \boxed{b_n=\frac{2}{T}\int_a^{a+T} f(x)\cos(n\omega x)\,dx,\ n\in\mathbb{N}^*}$$

$\omega=\frac{2\pi}{T}$

Seria Fourier atașată fct $\boxed{f(x)\sim\frac{a_0}{2}+\sum_{n=1}^{\infty}\left(a_n\cos\frac{2n\pi x}{b-a}+b_n\sin\frac{2n\pi x}{b-a}\right)}$

Seria Fourier atașată fct $\boxed{f(x)\sim\frac{a_0}{2}+\sum_{n=1}^{\infty}\left(a_n\cos(n\omega x)+b_n\sin(n\omega x)\right)}$

$x\in(a,a+T)$

Propr: 1) $f,g:[a,b]\to\mathbb{R}$ fct cont

$f(a)=f(b)$

$g(a)=g(b)$

-dacă cele 2 fct au aceiași coef Fourier $\Rightarrow$ coincid

2) egalitatea lui P.L

$f:[a,b]\to\mathbb{R}$, integrabilă, cu $f(a)=f(b)$ at. $\boxed{\frac{a_0^2}{2}+\sum_{n=1}^{\infty}(a_n^2+b_n^2)=\frac{2}{b-a}\int_a^b f^2(x)\,dx}$

3) inegalitatea lui B.

-coef F. ai fct integ. $f:[a,b]\to\mathbb{R}$ verifică ineg:

$$\boxed{\frac{a_0^2}{2}+\sum_{n=1}^{\infty}(a_n^2+b_n^2)\le\frac{2}{b-a}\int_a^b f^2(x)\,dx}$$

→ $f:[0,l]\to\mathbb{R}$ fct integrabilă și $\tilde{f}$ prelungire a sa.

$\tilde{f}_p$ prelungire prin paritate a lui f la int. simetric $[-l,l]$

$$\tilde{f}_p(x)=\begin{cases}f(x), & x\in[0,l]\\ f(-x), & x\in[-l,0]\end{cases}$$

$\tilde{f}_i$ prelungire prin imparitate a lui f la int. simetric $[-l,l]$

$$\tilde{f}_i(x)=\begin{cases}f(x), & x\in[0,l]\\ -f(-x), & x\in[-l,0]\end{cases}$$

Seria Fourier de cosinusuri

$$\boxed{f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega x)}$$

$$\boxed{a_n = \frac{4}{T}\int_0^l f(x)\cos(n\omega x)\,dx}$$

$\omega = \frac{2\pi}{T}$, $T = 2l$, $n \in \mathbb{N}$

Seria Fourier de sinusuri

$$\boxed{f(x) \sim \sum_{n=1}^{\infty} b_n \sin(n\omega x)}$$

$$\boxed{b_n = \frac{4}{T}\int_0^l f(x)\sin(n\omega x)}$$

$\omega = \frac{2\pi}{T}$, $T = 2l$, $n \in \mathbb{N}^*$

Fie $\bar{f}$ prelungirea funcției f prin periodicitate

$f: [a,b] \to \mathbb{R}$

$\bar{f}: \mathbb{R} \to \mathbb{R}$, $\boxed{\bar{f}(x) = f(x - k(b-a))}$, $\forall x \in [a + k(b-a), b + k(b-a)]$

T Dirichlet

$f: [a,b] \to \mathbb{R}$. Dacă:
1) f integrabilă pe $[a,b]$
2) f are un nr finit de disc de sp I pe $[a,b]$
3) f are derivate laterale finite pe $[a,b]$

$\Rightarrow$ seria Fourier atașată fct, e simplu converg.

Suma seriei e: $s: [a,b] \to \mathbb{R}$

$$s(x) = \begin{cases} f(x), & x \text{ pct de continuitate al lui } f \\ \frac{f(a+0) + f(b-0)}{2}, & x = a \text{ sau } x = b \\ \frac{f(x-0) + f(x+0)}{2}, & x \text{ pct de discont.} \end{cases}$$

$$\boxed{s(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\frac{2n\pi x}{b-a} + b_n \sin\frac{2n\pi x}{b-a}\right), x \in [a,b]}$$

$$\boxed{\bar{s}(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\frac{2n\pi x}{b-a} + b_n \sin\frac{2n\pi x}{b-a}\right), x \in \mathbb{R}}$$

T condiție suficientă pt conv uniformă

$f: [a,b] \to \mathbb{R}$. Dacă:
1) f derivabilă pe $[a,b]$
2) f' integrabilă pe $[a,b]$
3) $f(a) = f(b)$

$\Rightarrow$ fct, e dezvoltabilă în serie Fourier pe $[a,b]$

$\Rightarrow$ seria Fourier atașată lui f converge uniform pe $\mathbb{R}$ la $\bar{f}$:

$$\boxed{\bar{f}(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\frac{2n\pi x}{b-a} + b_n \sin\frac{2n\pi x}{b-a}\right)}$$

Spații metrice

M - mulț nevidă

Metrică (distanță) pe M orice fct $d: M \times M \to \mathbb{R}$ care satisface:

$$\begin{cases} M_1 & d(x,y)=0 \iff x=y \\ M_2 & d(x,y)=d(y,x),\ \forall x,y \in M \\ M_3 & d(x,z) \le d(x,y)+d(y,z),\ \forall x,y,z \in M \end{cases}$$

(M,d) - spațiu metric. În orice sp. metric:

$$\begin{cases} 1) \ |d(x,z)-d(y,z)| \le d(x,y),\ \forall x,y,z \in M \\ 2) \ |d(x,y)-d(x_0,y_0)| \le d(x,x_0)+d(y,y_0),\ \forall x,y,x_0,y_0 \in M \end{cases}$$

Mulțimile:

$D_d(x_0,r)=\{x \in M \mid d(x,x_0)<r\}$ - disc deschis

$D_d[x_0,r]=\{x \in M \mid d(x,x_0) \le r\}$ - disc închis

$S_d(x_0,r)=\{x \in M \mid d(x,x_0)=r\}$ - sferă

Numerele:

$d(x,A)=\inf\{d(x,y) \mid y \in A\}$ dist de la pct x la mulț A

$d(A,B)=\inf\{d(x,y) \mid x \in A, y \in B\}$ dist dintre mulț A și B

$\delta(A)=\sup\{d(x,y) \mid x,y \in A\}$ diametrul mulț. A

Șirul $(x_n)_{n\in\mathbb{N}}$ convergent $\iff \forall \varepsilon>0, \exists n_\varepsilon \in \mathbb{N}$ a.î. $\forall n \ge n_\varepsilon$, $d(x_n,l)<\varepsilon$

unde $\lim\limits_{n\to\infty} x_n = l$

Șirul $(x_n)_{n\in\mathbb{N}}$ fundamental (Cauchy) $\iff \forall \varepsilon>0, \exists n_\varepsilon \in \mathbb{N}$ a.î. $\forall n \ge n_\varepsilon, \forall p \in \mathbb{N}$, $d(x_{n+p},x_n)<\varepsilon$

(M,d) spațiu metric complet $\iff \forall$ șir fundamental din $M \in \mathcal{C}$

Limita și continuitatea fct de mai multe variabile

LIMITE

Pt a arăta că o fct. nu are limită putem folosi:

- Crit. Heine
- limite relative la o mulțime
- limite iterate

Crit Heine

Dacă $\exists$ 2 șiruri conv. din D a.î.

$$\left.\begin{array}{l} \exists x_{n_1} \in D, \lim\limits_{n\to\infty} x_{n_1} = a \quad \text{a.î. } \lim\limits_{n\to\infty} (f(x_{n_1})) = l_1 \\ \exists x_{n_2} \in D, \lim\limits_{n\to\infty} x_{n_2} = a \quad \text{a.î. } \lim\limits_{n\to\infty} (f(x_{n_2})) = l_2 \end{array}\right\} \xrightarrow[\text{Heine}]{\text{crit.}} f. \text{ nu are lim în } a$$

Limite relative la o mulțime

Ex: $f(x,y) = \dfrac{x^2-2y^2}{3x^2+y^2}, (x,y) \neq (0,0)$

Fie $A_m = \{(x,y) \in \mathbb{R}^2 \mid y = mx, m \in \mathbb{R}^*\}$

$$\lim_{\substack{(x,y)\to(0,0)\\(x,y)\in A_m}} f(x,y) = \lim_{\substack{(x,y)\to(0,0)\\ y=mx}} \frac{x^2-2y^2}{3x^2+y^2} = \lim_{x\to 0} \frac{x^2-2m^2x^2}{3x^2+m^2x^2} = \frac{1-2m^2}{3+m^2}$$

lim relativă depinde de $m \Rightarrow f.$ nu are lim în $(0,0)$

Limite iterate

Fie $f: A \subset \mathbb{R}^2 \to \mathbb{R}$, $X_0 = (a_1, a_2)$

$l_{12} = \lim\limits_{y\to a_2} \left(\lim\limits_{x\to a_1} f(x,y)\right)$

$l_{12} = \lim\limits_{x\to a_1} \left(\lim\limits_{y\to a_2} f(x,y)\right)$

1) $l_{12} \neq l_{21} \Rightarrow \nexists \lim\limits_{(x,y)\to(a_1,a_2)} f(x,y)$

2) $l_{12} = l_{21}$ nu știm (trebuie calculat)

3) $\nexists l_{12}, \exists l_{21} =$ limita globală

Limite după o direcție

Ex: $f: \mathbb{R}^2 \setminus \{(0,0)\} \to \mathbb{R}$, $f(x,y) = \dfrac{x^2 y}{x^4+y^2}$, $h = \{h_1, h_2\} \in \mathbb{R}^2 \setminus \{(0,0)\}$

$A_h = \{(x,y) = (0,0) + t(h_1,h_2), t \in \mathbb{R}\} \cap A$

$$\lim_{\substack{(x,y)\to(0,0)\\(x,y)\in A_h}} f(x,y) = \lim_{t\to 0} f(th_1, th_2) = \lim_{t\to 0} \frac{t^2h_1^2 \cdot th_2}{t^4h_1^4 + t^2h_2^2} = 0$$

$\Rightarrow f.$ are limită-n origine după orice direcție $h \in A_h$

Crit. majorării

Ex: $\lim_{(x,y)\to(0,0)} \frac{y\cdot \sin^2(2x)}{x^2+3y^2} = l_1$

$l_1 = \lim_{(x,y)\to(0,0)} \frac{y\cdot \sin^2(2x)}{(x^2+3y^2)\,4x^2}\cdot 4x^2 = 1\cdot 0 = 0$

$\left|\frac{4yx^2}{x^2+3y^2}\right| = |4y|\cdot \underbrace{\frac{x^2}{x^2+3y^2}}_{<1} < |4y| \xrightarrow[(x,y)\to(0,0)]{} 0$

$\xRightarrow[\text{maj}]{c} \lim_{(x,y)\to(0,0)} \frac{4yx^2}{x^2+3y^2} = 0$

Limite parțiale

$f: A\subset\mathbb{R}^2\to\mathbb{R},\ x_0=(a,b)$

$l_1 = \lim_{x\to a} f(x,b)$ $\quad l_2 = \lim_{y\to b} f(a,y)$

1) $l_1 \neq l_2 \Rightarrow \nexists \lim_{(x,y)\to(a,b)} f(x,y)$

2) $l_1 = l_2 \Rightarrow$ nu rezultă $\exists$ limitei

CONTINUITATE

Continuitate → cont parțială

Necont parțială → nu e cont

Conț parț. → nu rez. cont globală (trebuie studiat)

↓

limite parțiale

Ex: $f(x,y) = \begin{cases} \frac{xy}{x^2+y^2}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$

Cont parț: $\lim_{x\to 0} f(x,0) = 0 = f(0,0) \Rightarrow$ f. cont parț. în rap. cu var x

$\lim_{x\to 0} f(0,y) = 0 = f(0,0) \Rightarrow$ —‖— —‖— —‖— y

Fie $(x_n,y_n) = \left(-\frac{1}{n}, \frac{1}{n}\right) \xrightarrow[n\to\infty]{} (0,0)$

$\Rightarrow \lim_{n\to\infty} \left(-\frac{1}{n}, \frac{1}{n}\right) = -\frac{1}{2}$

$(x_n', y_n') = \left(\frac{1}{n}, \frac{2}{n}\right) \xrightarrow[n\to\infty]{} (0,0)$

$\Rightarrow \lim_{n\to\infty} f\left(\frac{1}{n}, \frac{2}{n}\right) = \frac{2}{5} \xRightarrow[\text{Heine}]{c} \nexists \lim_{(x,y)\to(0,0)} f(x,y) \Rightarrow$ f. nu e cont în (0,0)

Calcul diferențial în $\mathbb{R}^p$

Derivate parțiale de ord I.

$f : A \subset \mathbb{R}^2 \to \mathbb{R}$, $x_0 = (a,b) \in \mathring{A}$

$$\boxed{\frac{\partial f}{\partial x}(a,b) = \lim_{x \to a} \frac{f(x-b) - f(a,b)}{x-a}} \qquad \boxed{\frac{\partial f}{\partial y}(a,b) = \lim_{y \to b} \frac{f(a,y) - f(a,b)}{y-b}}$$

Derivata după o direcție

Fct. f derivabilă-n $x_0 = (a,b)$ după direcția h $\Leftrightarrow$

$$\boxed{\frac{\partial f}{\partial h}(x_0) = \frac{\partial f}{\partial x}(x_0) \cdot h_1 + \frac{\partial f}{\partial y}(x_0) \cdot h_2} \quad \text{sau} \quad \boxed{\frac{\partial f}{\partial h}(x_0) = \lim_{t \to 0} \frac{f(x_0 + th) - f(x_0)}{t}}$$

Gradientul câmpului **scalar** f e câmpul **vectorial**:

$$\boxed{\text{grad } f = \frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y} j + \frac{\partial f}{\partial z} k \quad (\mathbb{R}^3)}$$

Divergența câmpului **vectorial** F e câmpul **scalar**:

$$\boxed{\text{div } F = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}}, \text{ unde } F = P\bar{i} + Q\bar{j} + R\bar{k}$$

(div $=0 \Rightarrow$ câmp solenoidal)

Rotorul câmpului **vectorial** F e câmpul **vectorial**:

$$\boxed{\text{rot } F = \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right) i + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right) j + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) k}$$

(rot $=0 \Rightarrow$ câmp irotațional)

grad $f = \nabla f$ (produs)

div $F = \nabla \cdot F$ (produs scalar)

rot $F = \nabla \times F$ (produs vectorial)

$$= \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix}$$

← **Operatorul nabla**

$$\nabla \square = \frac{\partial \square}{\partial x} i + \frac{\partial \square}{\partial y} j + \frac{\partial \square}{\partial z} k$$

$$f(x,y,z) \Rightarrow \nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

Omog. lui Euler

$f(x,y) = \ldots$. Dacă $\underline{f(t \cdot x, t \cdot y) = t^\alpha f(x,y)} \Rightarrow \begin{cases} f \text{ omogenă în sens Euler} \\ \underline{r = \alpha} \end{cases}$

$$\xrightarrow[\text{Euler}]{f} \boxed{x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = r \cdot f(x,y)}$$

Matricea Jacobi

$F(x) = (f_1(x), f_2(x), \dots, f_q(x))$

$x = (x_1, \dots, x_p)$

$$J_F = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \dots & \frac{\partial f_1}{\partial x_p} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \dots & \frac{\partial f_2}{\partial x_p} \\ \vdots & \vdots & & \vdots \\ \frac{\partial f_q}{\partial x_1} & \frac{\partial f_q}{\partial x_2} & \dots & \frac{\partial f_q}{\partial x_p} \end{pmatrix}$$

Diferențiala de ord. I în x_0

$$\boxed{d_{x_0} f = \frac{\partial f}{\partial x}(x_0)\,dx + \frac{\partial f}{\partial y}(x_0)\,dy}$$

Diferențiala totală

$$\boxed{\frac{\partial f}{\partial x}\,dx + \frac{\partial f}{\partial y}\,dy}$$

$f(x,y)$ e diferențiabilă în x_0 dacă:

1) $\exists$ derivate parțiale de ord I în x_0
2) $\exists\, \omega : \mathbb{R}^n \to \mathbb{R}$, continuă, nulă-n x_0 a.î.:

$$\boxed{f(x,y) = f(x_0) + \frac{\partial f}{\partial x}(x_0)(x-x_0) + \frac{\partial f}{\partial y}(x_0)(y-y_0) + \|(x,y)-(x_0)\| \cdot \omega(x,y)}$$

Dacă ω cont. în $x_0 \Rightarrow f$. diferențiabilă-n x_0

Diferențiale de ordin superior

$f(x,y,z) = \dots$

$$d^2_{x_0} f = \frac{\partial^2 f}{\partial x^2}(x_0)dx^2 + \frac{\partial^2 f}{\partial y^2}(x_0)dy^2 + \frac{\partial^2 f}{\partial z^2}(x_0)dz^2 + 2\frac{\partial^2 f}{\partial x \partial y}(x_0)dx dy +$$
$$+ 2\frac{\partial^2 f}{\partial y \partial z}(x_0)dy dz + 2\frac{\partial^2 f}{\partial z \partial x}(x_0)dz dx$$

diferențiala de ord. 2 în x_0

$$H_f = \begin{vmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} & \frac{\partial^2 f}{\partial x \partial z} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} & \frac{\partial^2 f}{\partial y \partial z} \\ \frac{\partial^2 f}{\partial z \partial x} & \frac{\partial^2 f}{\partial z \partial y} & \frac{\partial^2 f}{\partial z^2} \end{vmatrix}$$

Derivate de ordin superior

$$\frac{\partial^2 f}{\partial x^2}(a,b) = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right)(a,b) = \lim_{x \to a} \frac{\frac{\partial f}{\partial x}(x,b) - \frac{\partial f}{\partial x}(a,b)}{x-a}$$

$$\frac{\partial^2 f}{\partial x \partial y}(a,b) = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)(a,b) = \lim_{x \to a} \frac{\frac{\partial f}{\partial y}(x,b) - \frac{\partial f}{\partial x}(a,b)}{x-a}$$

Derivare funcții compuse

$F : A \subset \mathbb{R}^p \to B \subset \mathbb{R}^q$

I. $p = q = 1$ $\quad g'(x) = f'(u(x)) \cdot u'(x)$

II. $p=1, q=2$ $\quad \frac{dh}{dx} = \frac{\partial g}{\partial u}\frac{du}{dx} + \frac{\partial g}{\partial v}\frac{dv}{dx}$

III. $p=2, q=1$ $\quad \frac{\partial h}{\partial x} = \frac{dg}{du}\frac{\partial u}{\partial x}, \quad \frac{\partial h}{\partial y} = \frac{dg}{du}\frac{\partial u}{\partial y}$

IV. $p=q=2$ $\quad \frac{\partial h}{\partial x} = \frac{\partial g}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial g}{\partial v}\frac{\partial v}{\partial x}, \quad \frac{\partial h}{\partial y} = \frac{\partial g}{\partial u}\cdot\frac{\partial u}{\partial y} + \frac{\partial g}{\partial v}\frac{\partial v}{\partial y}$

Taylor

$f : A \subset \mathbb{R}^3 \to \mathbb{R}, x_0 = (a,b)$

$$f(x,y) = f(a,b) + \frac{1}{1!}\left[\frac{\partial f}{\partial x}(x_0)(x-a) + \frac{\partial f}{\partial y}(x_0)(y-b)\right] + \frac{1}{2!}\left[\frac{\partial^2 f}{\partial x^2}(x_0)(x-a)^2 + \right.$$
$$\left. + 2\frac{\partial^2 f}{\partial x \partial y}(x_0)(x-a)(y-b) + \frac{\partial^2 f}{\partial y^2}(x_0)(y-b)^2\right]$$

Formula lui Leibniz

$$(u\cdot v)^{(n)} = u^{(n)}\cdot v + C_n^1 u^{(n-1)} v' + \dots + C_n^{n-1}\cdot u'\cdot v^{(n-1)} + C_n^n u\cdot v^{(n)}$$

$$(\sin x)^{(n)} = \sin\left(x + n\frac{\pi}{2}\right)$$

$$(\cos x)^{(n)} = \cos\left(x + n\frac{\pi}{2}\right)$$

Laplacianul câmpului scalar

$$\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

Identit. lui Euler de ord. 2

$$x^2\frac{\partial^2 f}{\partial x^2} + 2xy\frac{\partial^2 f}{\partial x\partial y} + y^2\frac{\partial^2 f}{\partial y^2} = r(r-1)\cdot f$$

Extreme locale ale fct. de mai multe variabile

$$\begin{cases} \dfrac{\partial f}{\partial x} = 0 \\ \dfrac{\partial f}{\partial y} = 0 \\ \dots \end{cases}$$

egalăm și aflăm x, y, ...

$$H = \begin{pmatrix} \dfrac{\partial^2 f}{\partial x^2} & \dfrac{\partial^2 f}{\partial x \partial y} & \dfrac{\partial^2 f}{\partial x \partial z} \\ \dfrac{\partial^2 f}{\partial y \partial x} & \dfrac{\partial^2 f}{\partial y^2} & \dfrac{\partial^2 f}{\partial y \partial z} \\ \dfrac{\partial^2 f}{\partial z \partial x} & \dfrac{\partial^2 f}{\partial z \partial y} & \dfrac{\partial^2 f}{\partial z^2} \end{pmatrix}$$

H_f(valorile aflate)

$\Delta_1 \dots > 0$ $\Delta_2 \dots > 0$ $\Delta_3 \dots > 0$ pct de min local	$\Delta_1 < 0$ $\Delta_2 > 0$ $\Delta_3 < 0$ pct de max local	dacă semnele alternează => nu are pct de extrem

Metoda lui Lagrange (cu condiție...)

$f(x, y, z) = \dots$

$L(x, y, z, \lambda) = f(x, y, z) + \lambda \cdot$ condiție

$$\begin{cases} \dfrac{\partial L}{\partial x} = 0 \\ \dfrac{\partial L}{\partial y} = 0 \\ \dots \\ \dfrac{\partial L}{\partial \lambda} = 0 \end{cases}$$

Aflăm x, y, z, λ

$$\underset{\text{înlocuim}}{L(x, y, z, \lambda)} = \mathcal{L}(x, y, z) = \underbrace{f(x, y, z) + \lambda \cdot \text{cond}}_{\text{înlocuim}}.$$

$$d^2_{(x,y,z)} \mathcal{L} = \frac{\partial^2 \mathcal{L}}{\partial x^2}(x,y,z)dx^2 + \frac{\partial^2 \mathcal{L}}{\partial y^2}(x,y,z)dy^2 + \frac{\partial^2 \mathcal{L}}{\partial z^2}(x,y,z)dz^2 + 2\frac{\partial^2 f}{\partial x \partial y}(x,y,z)\,dx\,dy + 2\frac{\partial^2 f}{\partial y \partial z}(x,y,z)\,dy\,dz + \frac{2\partial^2 f}{\partial z \partial x}(x,y,z)\,dz\,dx$$

Calculăm => $d^2_{(x,y,z)} \mathcal{L} = \dots$

Diferențiem relația de leg.

ex: cond: $x+2y+3z=6$

Diferențiere: $dx+2dy+3dz=0$

★ Scoatem dx și înlocuim în $d^2_{(x,y,z)}\mathcal{L}$

★ Formăm pătrat perf. pt a ne da seama

★ cum este def. [negativ $\Rightarrow (x,y,z)$ pct de max condiționat
pozitiv $\Rightarrow (x,y,z)$ pct de minim condiționat

Funcții speciale Euler

$\int_a^b f(x)dx$ improprie dacă [$a=-\infty$ sau $b=\infty$
$f(x)$ nemărg [într-un pct $c\in(a,b)$ sau în a sau b

Integrale improprie cu param definită prin $\Gamma:(0,\infty)\to\mathbb{R}$

$$\Gamma(a)=\int_0^\infty x^{a-1}e^{-x}dx \quad \text{funcția gama}$$

Propr. fct. Γ

$$\Gamma(a+1)=a\Gamma(a),\ a\in(0,\infty)$$

$$\Gamma(n+1)=n!\quad(n\in\mathbb{N})$$

$$\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$$

$$\Gamma\left(n+\frac{1}{2}\right)=\frac{(2n-1)!!}{2^n}\sqrt{\pi}\quad(n\in\mathbb{N})$$

$$\beta(a,b)=\int_0^1 x^{a-1}(1-x)^{b-1}dx \quad \text{funcția beta}$$
$a,b>0$

Propr fct. β

$$\beta(a,b)=\beta(b,a)$$

$$\beta(a,b)=\frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

$$\beta(a,1-a)=\frac{\pi}{\sin\pi a},\ a\in(0,1)$$

$$\Gamma(a)\Gamma(1-a)=\frac{\pi}{\sin\pi a},\ a\in(0,1)$$

$$\int_0^{\frac{\pi}{2}}\sin^{2p-1}x\cdot\cos^{2q-1}x\,dx=\frac{1}{2}\beta(p,q)$$

Obs: Pt o fct $f: A \subset \mathbb{R}^2 \to \mathbb{R}$, $f \in C^2(A)$, cu $x_0 \in A$ pct critic

$$H_f = \begin{array}{cc} r & s \\ \left(\begin{array}{cc} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} \end{array}\right) & \\ & t \end{array}$$

① $\left.\begin{array}{l} s^2 - rt < 0 \\ r > 0 \end{array}\right\}$ x_0 pct de minim local

② $\left.\begin{array}{l} s^2 - rt < 0 \\ r < 0 \end{array}\right\}$ x_0 pct de maxim local

③ $s^2 - rt > 0 \Rightarrow x_0$ nu e pct de extrem

④ $s^2 - rt = 0 \Rightarrow$ nu se poate afirma nimic