

## Summer School in Quantitative Fisheries Stock Assessment

### Day 4: An introduction to Statistical Catch-at-Age models

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Alessandro Mannini, Danai Mantopoulou Palouka



Food and Agriculture  
Organization of the  
United Nations



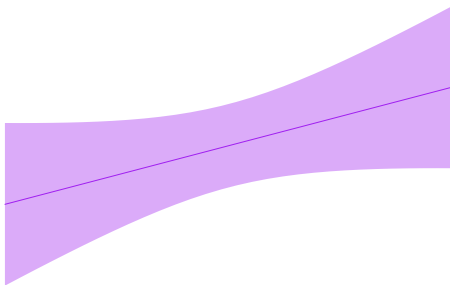
General Fisheries Commission  
for the Mediterranean  
Commission générale des pêches  
pour la Méditerranée



European  
Commission

# Goal

Build a fully statistical model to estimate the parameters of the stock assessment, including uncertainty of estimates



# Cohort

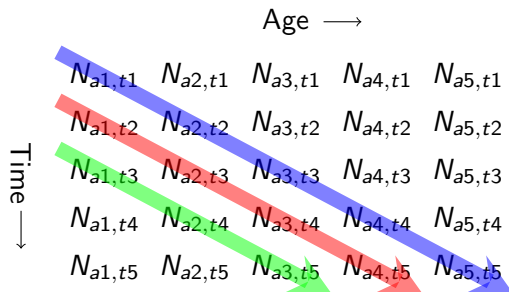
Cohort progression

	Age →				
	$N_{a1,t1}$	$N_{a2,t1}$	$N_{a3,t1}$	$N_{a4,t1}$	$N_{a5,t1}$
	$N_{a1,t2}$	$N_{a2,t2}$	$N_{a3,t2}$	$N_{a4,t2}$	$N_{a5,t2}$
	$N_{a1,t3}$	$N_{a2,t3}$	$N_{a3,t3}$	$N_{a4,t3}$	$N_{a5,t3}$
	$N_{a1,t4}$	$N_{a2,t4}$	$N_{a3,t4}$	$N_{a4,t4}$	$N_{a5,t4}$
Time ↓	$N_{a1,t5}$	$N_{a2,t5}$	$N_{a3,t5}$	$N_{a4,t5}$	$N_{a5,t5}$

Yesterday with VPA & XSA we worked backwards through the cohort from the terminal value

# Cohort

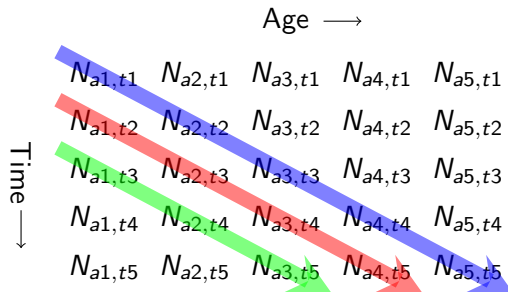
Cohort progression



Today we will work forwards

# Cohort

## Cohort progression



$$N_{a+1,t+1} = N_{a,t} e^{-(F_{a,t} + M_{a,t})}$$

# Starting a cohort

How do we start the cohorts?

		Age →			
Time ↓	$N_{a1,t1}$	$N_{a2,t1}$	$N_{a3,t1}$	$N_{a4,t1}$	$N_{a5,t1}$
	$N_{a1,t2}$	$N_{a2,t2}$	$N_{a3,t2}$	$N_{a4,t2}$	$N_{a5,t2}$
	$N_{a1,t3}$	$N_{a2,t3}$	$N_{a3,t3}$	$N_{a4,t3}$	$N_{a5,t3}$
	$N_{a1,t4}$	$N_{a2,t4}$	$N_{a3,t4}$	$N_{a4,t4}$	$N_{a5,t4}$
	$N_{a1,t5}$	$N_{a2,t5}$	$N_{a3,t5}$	$N_{a4,t5}$	$N_{a5,t5}$

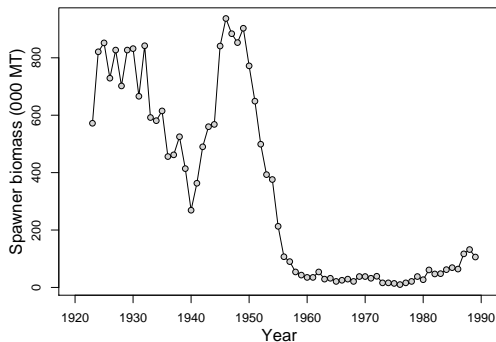
# Starting a cohort

Need to relate youngest fish  $N_{a1,t}$  to all adult fish  $N_{t-a1}$ , that is, we need to relate the number of new individuals to the population (spawners, spawning potential) when they were spawned.

That's called recruitment and in age-structured populations, it's where compensation enters.

# Historical interlude

East Anglia herring collapse: recruitment overfishing



Enter: Cushing, Ricker, Beverton, and Holt



# Theory of recruitment predecessor

## THE BALANCE OF ANIMAL POPULATIONS

BY A. J. NICHOLSON, D.Sc.

*(Division of Economic Entomology, Commonwealth Council  
for Scientific and Industrial Research, Canberra, Australia.)*

*(With eleven Figures in the Text.)*



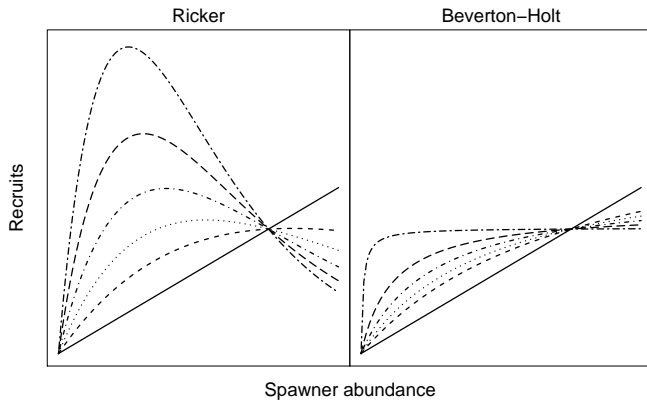
Alexander  
Nicholson

- Main thesis: competition as the agent of population regulation

# Theory of recruitment

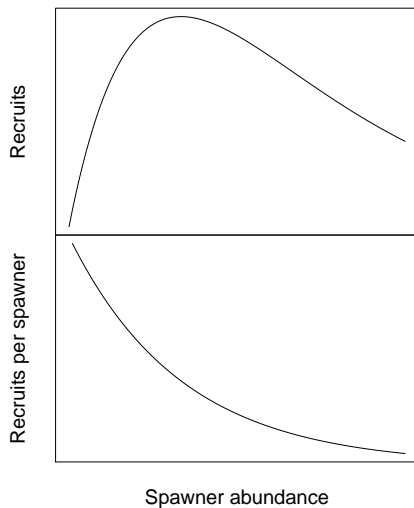
Population regulation:  $R = \alpha Sf(S)$

Where  $f(S)$  is a density-dependent function



# Theory of recruitment

All have compensation at their core



# Theory of recruitment

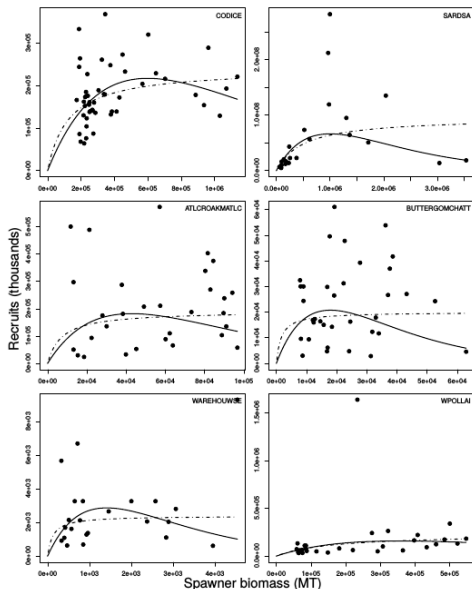
By specifying a stock-recruitment relationship, we allow our theoretical model to replenish itself

Population described by

$$N_{a+1,t+1} = N_{a,t} e^{-(F_{a,t} + M_{a,t})}$$

$$N_{a1,t} = a S_{t-a1} f(S_{t-a1})$$

In reality, we have ...



The 'good'

The 'bad'

The 'ugly'

# Starting a cohort

		Age $\longrightarrow$				
Time $\longrightarrow$	$N_{a1,t1}$	$N_{a2,t1}$	$N_{a3,t1}$	$N_{a4,t1}$	$N_{a5,t1}$	
	$N_{a1,t2}$	$N_{a2,t2}$	$N_{a3,t2}$	$N_{a4,t2}$	$N_{a5,t2}$	
	$N_{a1,t3}$	$N_{a2,t3}$	$N_{a3,t3}$	$N_{a4,t3}$	$N_{a5,t3}$	
	$N_{a1,t4}$	$N_{a2,t4}$	$N_{a3,t4}$	$N_{a4,t4}$	$N_{a5,t4}$	
	$N_{a1,t5}$	$N_{a2,t5}$	$N_{a3,t5}$	$N_{a4,t5}$	$N_{a5,t5}$	

Here 5 parameters to be estimated<sup>1</sup>

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<sup>1</sup>Or a smooth function, e.g., in a4a

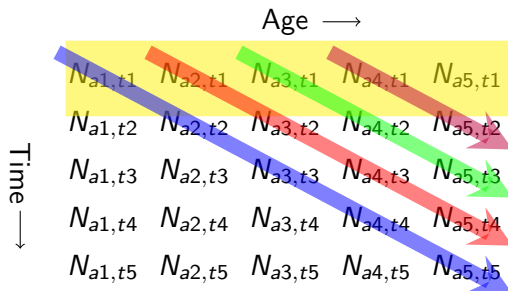
## Year one

We also need to start all ages in the first year

		Age $\longrightarrow$				
Time $\longrightarrow$		$N_{a1,t1}$	$N_{a2,t1}$	$N_{a3,t1}$	$N_{a4,t1}$	$N_{a5,t1}$
		$N_{a1,t2}$	$N_{a2,t2}$	$N_{a3,t2}$	$N_{a4,t2}$	$N_{a5,t2}$
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		$N_{a1,t5}$	$N_{a2,t5}$	$N_{a3,t5}$	$N_{a4,t5}$	$N_{a5,t5}$

# Year one

We also need to start all ages in the first year



Here 4 parameters to be estimated



# How about Fishing mortality at age?

Time ↓	Age				
	$F_{a1,t1}$	$F_{a2,t1}$	$F_{a3,t1}$	$F_{a4,t1}$	$F_{a5,t1}$
	$F_{a1,t2}$	$F_{a2,t2}$	$F_{a3,t2}$	$F_{a4,t2}$	$F_{a5,t2}$
	$F_{a1,t3}$	$F_{a2,t3}$	$F_{a3,t3}$	$F_{a4,t3}$	$F_{a5,t3}$
	$F_{a1,t4}$	$F_{a2,t4}$	$F_{a3,t4}$	$F_{a4,t4}$	$F_{a5,t4}$
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## How about Fishing mortality at age?

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	$F_{a1,t5}$	$F_{a2,t5}$	$F_{a3,t5}$	$F_{a4,t5}$	$F_{a5,t5}$

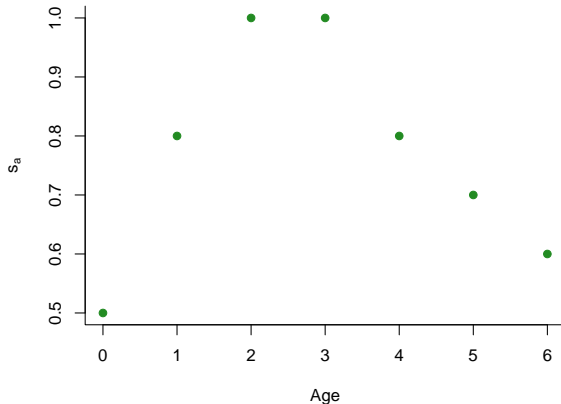
If all are free requires 25 parameters here (one for each age in each year)  
that's a lot of parameters!

## How about $F$ ?

Need to conserve degrees of freedom in the model but also need the  $\mathbf{F}$  matrix. One approach is a separable model

Model the age effects assumed constant across time

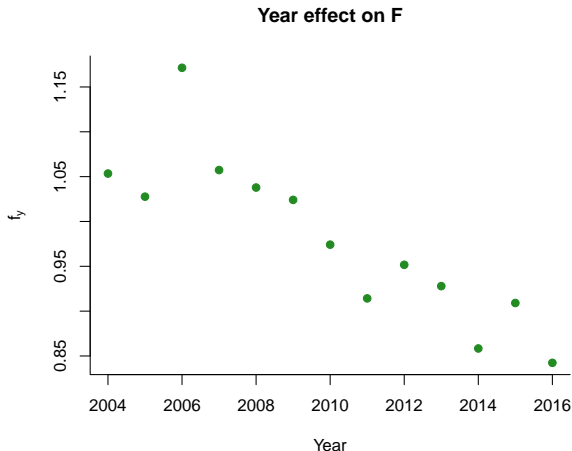
Age effect of  $F$  (population selection pattern)



## How about $F$ ?

Need to conserve degrees of freedom in the model but also need the  $\mathbf{F}$  matrix. One approach is a separable model

Model year effects assumed constant across ages



## How about $F$ ?

Need to conserve degrees of freedom in the model but also need the  $\mathbf{F}$  matrix. One approach is a separable model

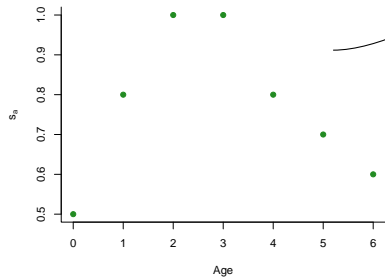
$$\mathbf{F} = \begin{pmatrix} s_{a1} & s_{a2} & s_{a3} & s_{a4} & s_{a5} \\ s_{a1} & s_{a2} & s_{a3} & s_{a4} & s_{a5} \\ s_{a1} & s_{a2} & s_{a3} & s_{a4} & s_{a5} \\ s_{a1} & s_{a2} & s_{a3} & s_{a4} & s_{a5} \\ s_{a1} & s_{a2} & s_{a3} & s_{a4} & s_{a5} \end{pmatrix} \star \begin{pmatrix} f_{t1} & f_{t1} & f_{t1} & f_{t1} & f_{t1} \\ f_{t2} & f_{t2} & f_{t2} & f_{t2} & f_{t2} \\ f_{t3} & f_{t3} & f_{t3} & f_{t3} & f_{t3} \\ f_{t4} & f_{t4} & f_{t4} & f_{t4} & f_{t4} \\ f_{t5} & f_{t5} & f_{t5} & f_{t5} & f_{t5} \end{pmatrix}$$

## How about $F$ ?

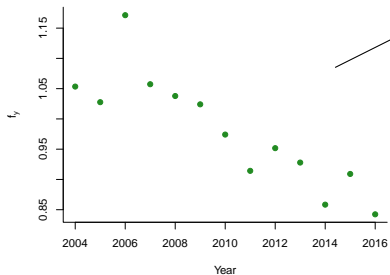
Need to conserve degrees of freedom in the model but also need the  $\mathbf{F}$  matrix. One approach is a separable model

$$\mathbf{F} = \begin{pmatrix} s_{a1}f_{t1} & s_{a2}f_{t1} & s_{a3}f_{t1} & s_{a4}f_{t1} & s_{a5}f_{t1} \\ s_{a1}f_{t2} & s_{a2}f_{t2} & s_{a3}f_{t2} & s_{a4}f_{t2} & s_{a5}f_{t2} \\ s_{a1}f_{t3} & s_{a2}f_{t3} & s_{a3}f_{t3} & s_{a4}f_{t3} & s_{a5}f_{t3} \\ s_{a1}f_{t4} & s_{a2}f_{t4} & s_{a3}f_{t4} & s_{a4}f_{t4} & s_{a5}f_{t4} \\ s_{a1}f_{t5} & s_{a2}f_{t5} & s_{a3}f_{t5} & s_{a4}f_{t5} & s_{a5}f_{t5} \end{pmatrix}$$

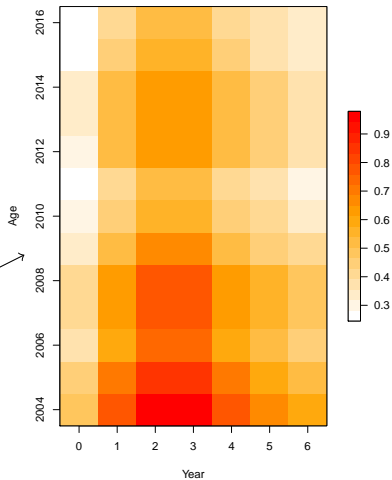
Age effect of F (population selection pattern)



Year effect on F



Seperable  $F_{a,y}$



# Separable model

So the parameters we have in the model are:

- $N_{1,t=1\dots T}$  number of recruits
- $N_{a=2\dots A,t=1}$  numbers in first year
- $s_{a=1\dots A}$  exploitation pattern (age effects)
- $f_{t=1\dots T}$  fishing mortality level (year effects)



# Predictions

From this set of parameters we can predict  $\hat{\mathbf{F}}$  and  $\hat{\mathbf{N}}$  from which we can predict catch

$$\hat{C}_{a,t} = \frac{\hat{F}_{a,t}}{\hat{F}_{a,t} + M_{a,t}} \left( 1 - e^{-(\hat{F}_{a,t} + M_{a,t})} \right) \hat{N}_{a,t}$$

Or, with the addition of catchability, predict a survey index

$$\hat{I}_{a,t} = q_{a,t} \hat{N}_{a,t}$$

# Estimation

We observe  $C_{a,t}$ ,  $I_{a,t}$  so in principle we can estimate the parameters by maximising the likelihood of the parameters given the data

$$L(\mathbf{N}_{a=1}, \mathbf{N}_{t=1}, \mathbf{s}, \mathbf{f}, \mathbf{q}, \boldsymbol{\sigma} | \mathbf{C}, \mathbf{I}) = \prod_{a=1}^A \prod_{t=1}^T (\mathcal{N}(\ln(I_{a,t}) - \ln(\hat{I}_{a,t}), \sigma_I^2) \\ \times \mathcal{N}(\ln(C_{a,t}) - \ln(\hat{C}_{a,t}), \sigma_C^2))$$

We maximise the (log) likelihood to estimate the best fitting parameters of the model given the data!

Can use less parameters in SCA by using smoothers (a4a)

intro to splines