Forecasting Financial Market: Homework 1

Zixin Huang

February 6, 2020

Question 1

a

I used the time series data of the USD/Euro Exchange rate and the S&P 500 from 2015-01-21 to 2019-01-21.

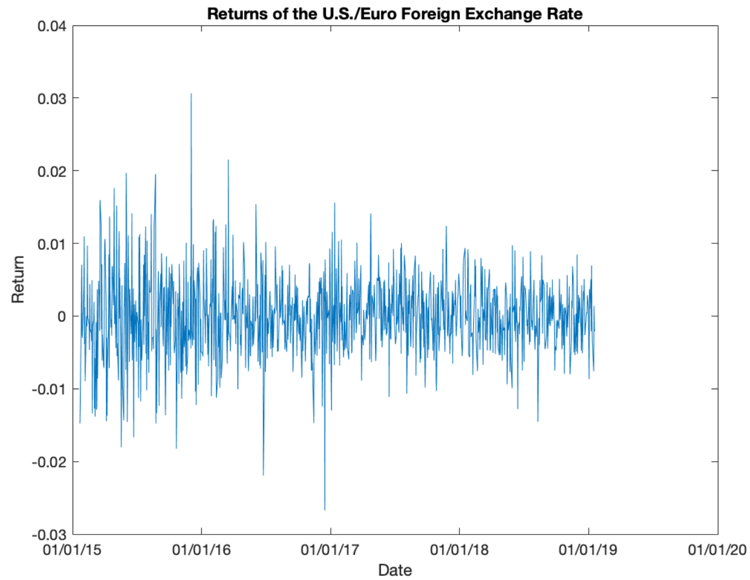

Figure 1: Continuously Compounded Returns of the USD/Euro Exchange Rate

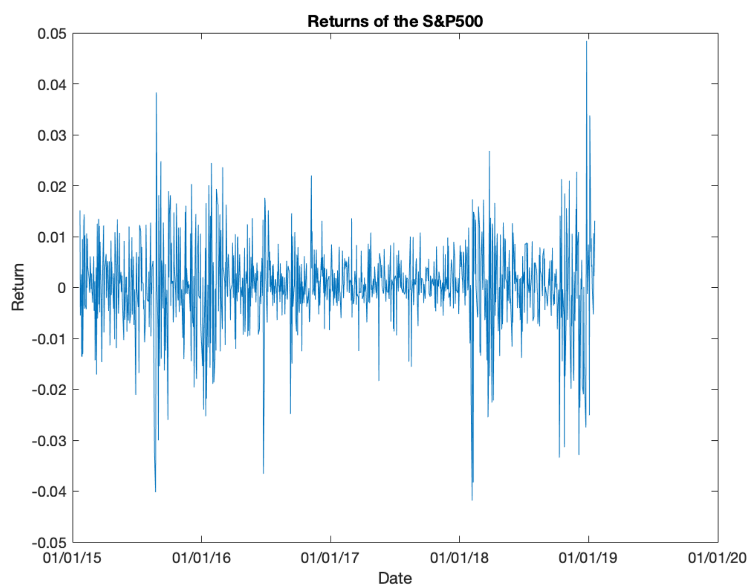

Figure 2: Continuously Compounded Returns of the S&P500

b

Table 1: Summary Statistics

Asset	mean	median	maximum	minimum	STD	skewness	kurtosis
DEXUSEU	-1.9370×10^{-5}	-2.5341×10^{-4}	0.0306	-0.0267	0.0056	0.1047	5.0468
SP500	2.7163×10^{-4}	3.4225×10^{-4}	0.0484	-0.0418	0.0087	-0.4698	7.0125

Table 2: Jarque-Bera Test

Asset	JB test statistic	JB p-value	JB test result(H)
DEXUSEU	176.2116	0.001	1
SP500	711.8687	0.001	1

```
function [meanX, medianX, maxX, minX, stdX, skeX, kurX, JBHX, JBPX,JBSTAX]
    = sum_stats(X)

% this function perform summary stats on a time series data
% input: X (a time series data)
% output: mean, median, maximum, minimum, standard deviation, skewness,
% kurtosis, Jarque-Bera test statistics, Jarque-Bera p-value

meanX = mean(X); % compute the mean of X
medianX = median(X); % compute the median of X
maxX = max(X); % compute the maximum of X
minX = min(X); % compute the minimum of X
stdX = std(X); % compute the standard deviation of X
skeX = skewness(X); % compute the skewness of X
kurX = kurtosis(X); % compute the kurtosis of X
[JBHX,JBPX,JBSTAX] = jbtest(X); % compute the JB test stats on X

end
```

Exercise 2

Question a

Table 3: Coefficients of the OLS model

Parameters	Estimated Value	SE	t-statistics
β_0	-6.8978×10^{-6}	0.00017771	-0.038815
β_1	-0.031506	0.020556	-1.5327

R-squared: 0.00236. Adjusted R-Squared: 0.00136.

Question b

Table 4: T Test at 5% Significance Level

hypothesis	t-statistic	critical value	result
$H_0 : \beta_1 = 1$	-50.1791	1.96	Reject H_0

We are 95% confident to reject the hypothesis that return of the USD/Euro exchange rate move along with the return of the S&P 500 a day before.

Question c

Table 5: Coefficients of the OLS model

Parameters	Estimated Value	SE	t-statistics
β_0	-8.7705×10^{-6}	0.00017789	-0.049302
β_1	-0.027624	0.020593	-1.3414
β_2	0.0077265	0.020581	0.37542
β_3	0.030052	0.020566	1.4612

R-squared: 0.00429, Adjusted R-Squared: 0.00127

Question d

Table 6: χ^2 Test at 5% Significance Level

hypothesis	χ^2-statistic	critical value	result
$H_0 : \beta_1 = \beta_2 = \beta_3 = 0$	4.2588	7.8147	Do not reject H_0

We are of 95% confidence to say that we do not reject the null hypothesis that $\beta_1 = \beta_2 = \beta_3 = 0$. In other words, we do not reject the hypothesis that return of the USD/Euro exchange rate does not affect by return of the SP500 with neither 1, 2, or 3 lags.

Question e

Table 7: χ^2 Test at 5% Significance Level

hypothesis	χ^2-statistic	critical value	result
$H_0 : \beta_1 = \beta_2 = \beta_3$	4.1805	5.9915	Do not reject H_0

We are of 95% confidence to say that we do not reject the null hypothesis that $\beta_1 = \beta_2 = \beta_3$. In other words, we do not reject the hypothesis that the dependence of return of USD/Euro exchange rate on return of SP500 with 1, 2, and 3 lags are the same.

Question 3

a

The theoretical unconditional mean of the AR(1) process can be compute as:
$\mu = E(Y_t) = E(\Phi_0 + \Phi_1 Y_{t-1} + \epsilon_t) = \Phi_0 + \Phi_1 E(Y_{t-1}) = \Phi_0 + \Phi_1 \mu$
So $\mu = \frac{\Phi_0}{1-\Phi_1}$.
The MATLAB function I used to generate AR(1) process:

```
function Y = AR1(theta, T)

% this function estimate data from AR1

% input: theta, contains [phi0, phi1, sig2eps]; T (number of days in our
% sample)
% output: Y_t (a T*1 vector of simulated data)

% compute the theoretical unconditional mean
mu = theta(1)/(1-theta(2));

Y = [mu zeros(1,T-1)];

for t = 2: T
    eps = sqrt(theta(3)) *randn();
    Y(t) = theta(1) + theta(2) * Y(t-1) + eps;


end



end
```

b

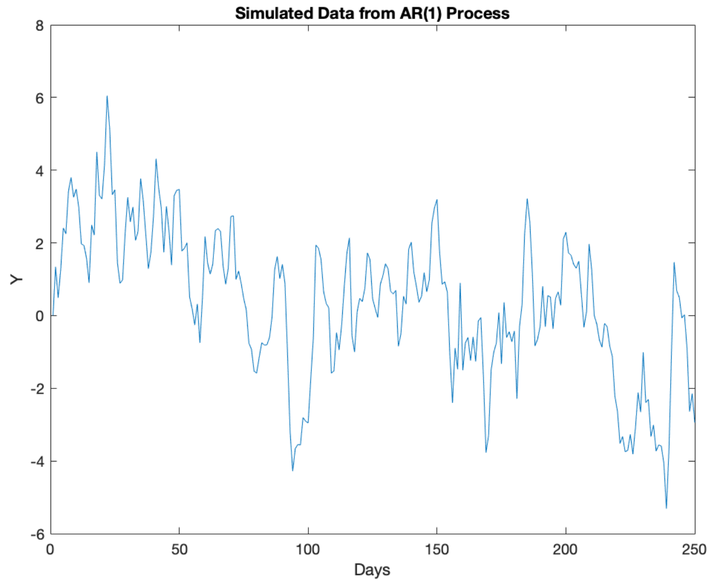

Figure 3: Simulated AR(1) Process with Normally Distributed Innovations

c

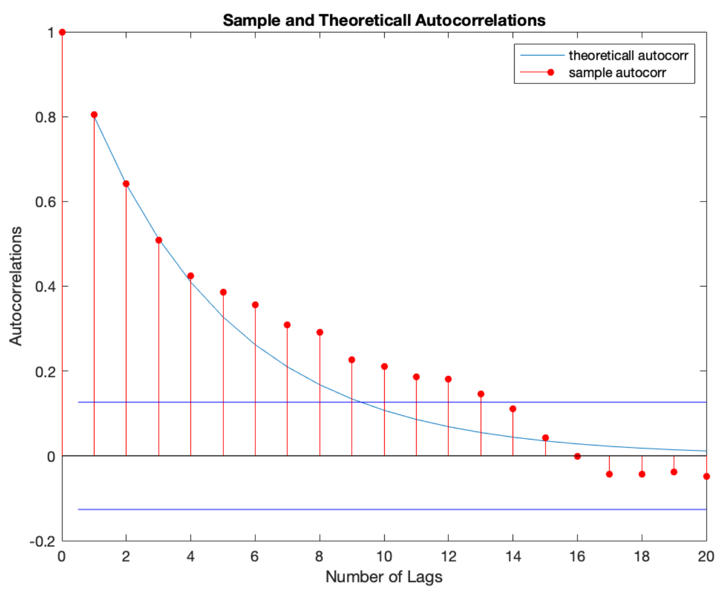

Figure 4: Sample Autocorrelations and Theoretical Autocorrelations of the Simulated Data

Theoretically:
$\gamma_0 = Var(Y_t) = Var(\Phi_0 + \Phi_1 Y_{t-1} + \epsilon_t) = \Phi_1^2 Var(Y_{t-1}) + Var(\epsilon_t) = \Phi_1^2 \gamma_0 + \sigma_\epsilon^2$
So $\gamma_0 = \frac{\sigma_\epsilon^2}{1-\Phi_1^2}$

$\gamma_j = \frac{\sigma_\epsilon^2}{1-\Phi_1^2}\Phi_1^j = \gamma_0\Phi_1^j$

That is $\rho_j = \frac{\gamma_j}{\gamma_0} = \Phi_1^j$.

To compute sample autocorrelations:

$\rho_j = \frac{\frac{1}{T-j}\sum_{t=j+1}^{T}(y_t-\bar{y})(y_{t-j}-\bar{y})}{\frac{1}{T}\sum_{t=1}^{T}(y_t-\bar{y})^2}$.

```
% Question 3

% a

T = 250;
phi0 = 0;
phi1 = 0.8;
sig2eps = 1;
theta = [phi0,phi1,sig2eps];

Y = AR1(theta, T);

seed = 3005;
rng(seed,'twister');

% b
% plot Y_t
figure(1)
plot(1:T,Y);
xlabel('Days')
ylabel('Y')
title('Simulated Data from AR(1) Process')


% c

% theoretical autocorrelations
trho = zeros(1,20);
for i = 1:20
   trho(i) = phi1^i;

end




% plot
figure(2)
plot(1:20,trho);
hold on
autocorr(Y,'Numlags',20)
hold off
xlabel('Number of Lags')
ylabel('Autocorrelations')
title('Sample and Theoretical1 Autocorrelations')
legend('theoreticall autocorr','sample autocorr')
```

Question 4

a

```
function X = AR1_uni(theta,L, U, T)

% this function simulated the AR1 process with a uniform distributed
% innovation

% input: theta, a vecrtor contains (phi0, phi1); L and U are bondary of the
% uniform distribution; T is the length of the simulated data
% output: X, the simulated AR1 process

X = zeros(T,1);

for t = 2:T
    eps = (U-L).*rand + L; % generate uni-distributed innovation
    X(t) = theta(1)+ theta(2)* X(t-1)+eps;

end

end
```

b

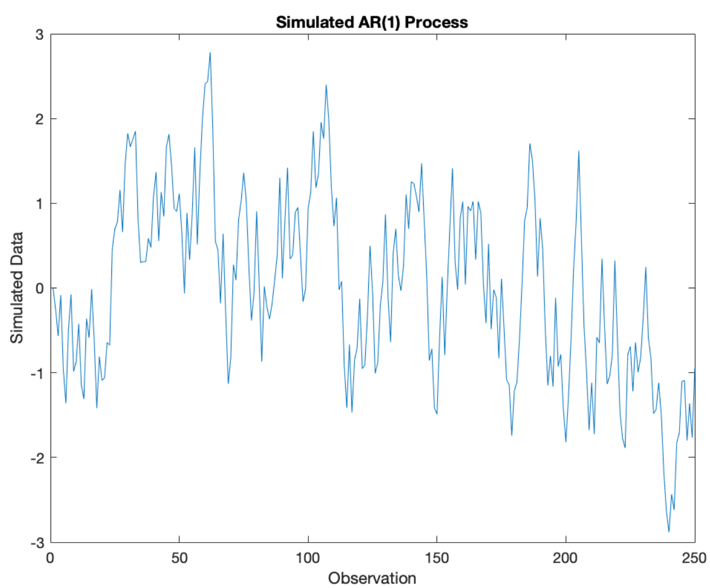

Figure 5: Simulated AR(1) Process with Uniformly Distributed innovations

c

$E(X_t) = \mu = \frac{\phi_0}{1-\phi_1} + \frac{U+L}{2(1-\phi_1)}$

d

Table 8: Sample and Theoretical mean of the AR(1) Process

Parameters	**Sample Mean**	$E(X_t)$
$(0, 0.8, -1, 1)$	-7.8386×10^{-4}	0
$(0, 0.8, -2, 1)$	-2.4967	-2.5
$(1, 0.8, -5, 2)$	-2.5076	-2.5

From the table we can see that when $T = 1000,000$ the sample mean is very close to the unconditional expectation for each of these three scenarios.