Link metadata bond percolation

1 The model

Let us consider a network generated through the configurational model, with degree distribution p_k. Consider also a feature value for each network's link, and assume that these values are distributed according to a feature distribution p_F. In the following p_F will be assumed, without loss of generality, to be a discrete distribution.
The goal is to derive an analytical solution for the giant cluster size in the case that the edges with the highest feature value are removed first during the percolation process.
In other words, all the edges with feature value greater or equal than the threshold value F_0 are occupied with probability 0 (removed from the network), while all the other edges are present in the network.
This is translated mathematically into the following edge occupation probability:

$$\phi_F = \begin{cases} 0 \text{ if } F \geq F_0 \\ 1 \text{ otherwise} \end{cases} \tag{1}$$

Note that the p_F and ϕ_F are uncorrelated from every other quantity of the network.

Let u be the average probability that a node is not connected to the giant cluster through a specific edge. u is given by the sum of two contributions, i.e. the probability that an edge is unoccupied, and the probability that it is occupied but the node at the other end of the edge is itself not a member of the giant cluster. Hence, the total probability is: $1-\phi_F+\phi_F u^k$.
Averaging over the value of k, distributed according to the excess degree distribution q_k, and the value of F, distributed according to p_F, leads to the following self-consistent equation:

$$u = \sum_{F=0}^{\infty}\sum_{k=0}^{\infty} p_F q_k \left(1-\phi_F+\phi_F u^k\right) = 1-\left(\sum_{F=0}^{F_0-1} p_F\right)+\left(\sum_{F=0}^{F_0-1} p_F\right) g_1(u) \tag{2}$$

where $g_1(z)$ is the generating function of the excess degree distribution, namely:

$$g_1(z) = \sum_{k=0}^{\infty} q_k z^k = \sum_{k=1}^{\infty} \frac{p_k \cdot k}{\langle k \rangle} \cdot z^{k-1} \tag{3}$$

The probability that a node does not belong to the giant cluster is u^k. Thus, the average over the degree distribution p_k is equal to $1-S$, and the sought result is given:

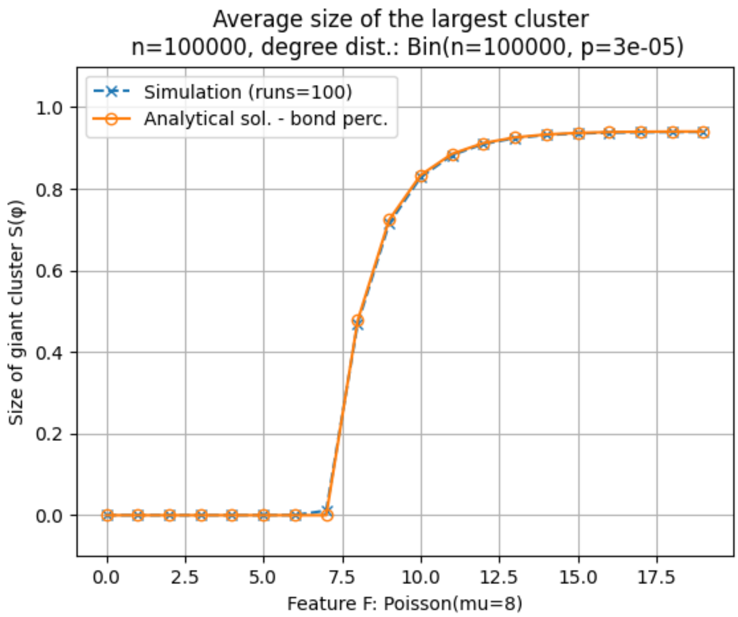
$$S = 1-\sum_{k=0}^{\infty} p_k u^k = 1-g_0(u) \tag{4}$$

where $g_0(z)$ is the generating function of the degree distribution.

2 Simulations

2.1 Erdős–Rényi network

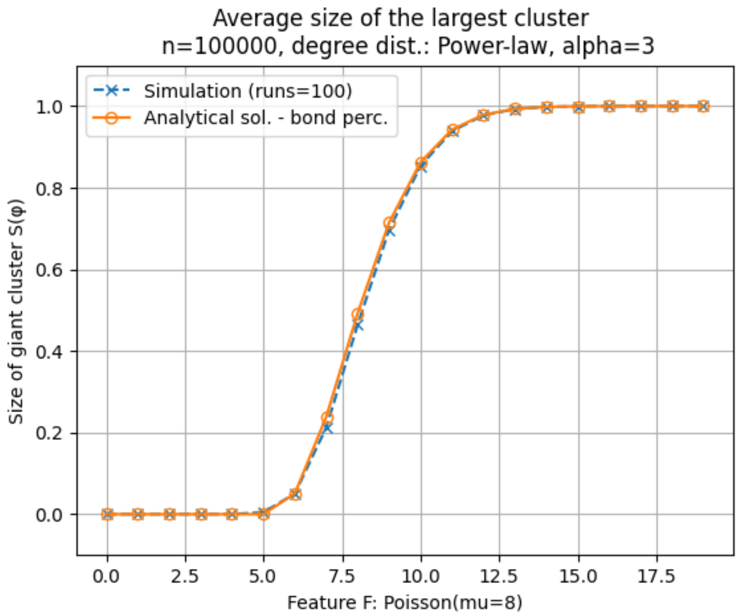
Let us consider a network generated via the configurational model with binomial degree distribution, i.e. $p_k \sim Bin(n,p)$ with $\langle k \rangle = 3$. Moreover, let p_F be Poisson distributed with $\mu = 8$. Figure 1 compare the analytical solution with the simulation results. The simulation is performed on a network with 100K nodes and averaged over 100 runs.



Figur 1: ER network

2.2 Scale-Free network

Similarly to the Erdős–Rényi case, let us consider a network generated via the configurational model but with power-law degree distribution with $\alpha = 3$. Note that the degree is constrained in the range $(2, \sqrt{n})$. Moreover, let p_F be Poisson distributed with $\mu = 8$. Figure 2 compare the analytical solution with the simulation results. The simulation is performed on a network with 100K nodes and averaged over 100 runs.



Figur 2: SF network