Link metadata bond percolation

1 The model

Let us consider a network generated through the configurational model, with degree distribution p_k. Consider also a feature value for each network's link, and assume that these values are distributed according to a feature distribution p_F. In the following p_F will be assumed, without loss of generality, to be a discrete distribution.
The goal is to derive an analytical solution for the giant cluster size in the case that the edges with the highest feature value are removed first.
The key idea is that the edge occupation probability depends only on the feature distribution. Specifically, assuming that the feature distribution is independent of any other quantity of the network, the probability that an arbitrary edge is occupied when all the edges with features values $\geq F_0$ are removed is:

$$\phi\left(F_0\right) = \sum_{F=0}^{F_0-1} p_F \tag{1}$$

With this observation, the remaining work follows the typical bond percolation computations. Let u be the average probability that a node is not connected to the giant cluster through a specific edge. u is given by the sum of two contributions, i.e. one is given by the probability that an edge is unoccupied, and the other by the probability that it is occupied but the node at the other end of the edge is itself not a member of the giant cluster. This leads to the usual self-consistent equation:

$$u = 1 - \phi + \phi \cdot g_1\left(u\right) \tag{2}$$

where $g_1\left(z\right)$ is the generating function of the excess degree distribution, namely:

$$g_1\left(z\right) = \sum_{k=1}^{\infty} \frac{p_k \cdot k}{\langle k \rangle} \cdot z^{k-1} \tag{3}$$

Finally, the sought result is given by:

$$S = 1 - g_0\left(u\right) \tag{4}$$

with $g_0\left(u\right)$ the generating function of the degree distribution, namely:

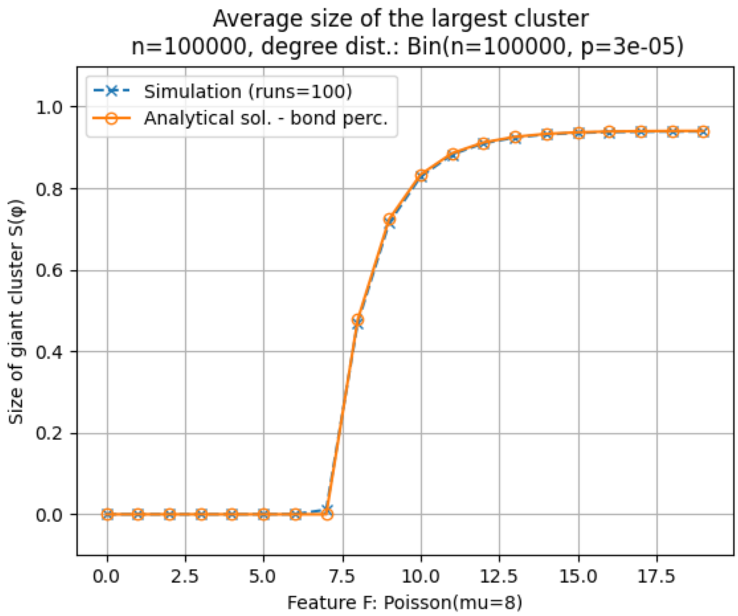
$$g_0\left(u\right) = \sum_{k=0}^{\infty} p_k \cdot z^k \tag{5}$$

2 Simulations

2.1 Erdős–Rényi network

Let us consider a network generated via the configurational model with binomial degree distribution, i.e. $p_k \sim Bin(n,p)$ with $\langle k \rangle = 3$. Moreover, let p_F be Poisson distributed with

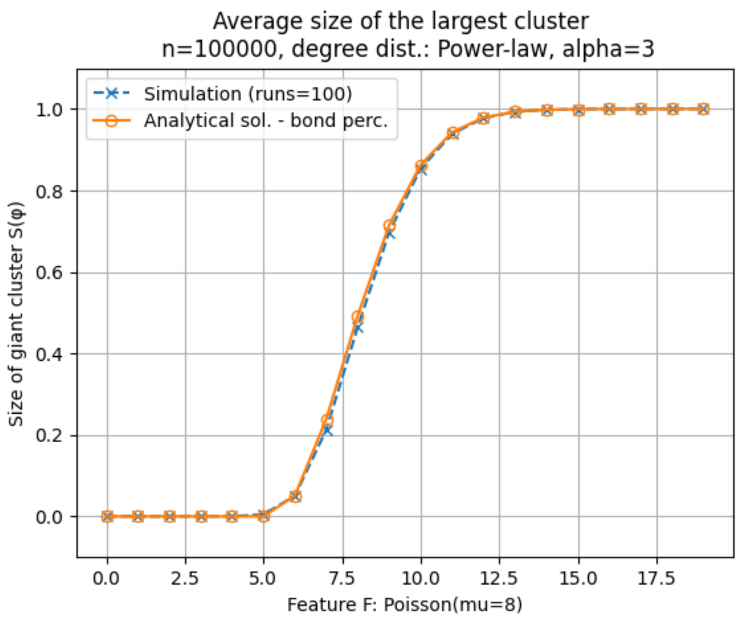
$\mu = 8$. Figure 1 compare the analytical solution with the simulation results. The simulation is performed on a network with 100K nodes and averaged over 100 runs.



Figur 1: ER network

2.2 Scale-Free network

Similarly to the Erdős–Rényi case, let us consider a network generated via the configurational model but with power-law degree distribution with $\alpha = 3$. Note that the degree is constrained in the range $(2, \sqrt{n})$. Moreover, let p_F be Poisson distributed with $\mu = 8$. Figure 2 compare the analytical solution with the simulation results. The simulation is performed on a network with 100K nodes and averaged over 100 runs.



Figur 2: SF network