A Portfolio Allocation Framework for Algorithmic Trading Strategies

Master Project

In cooperation with NAFORA SA

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Academic Year
2017/2018

Contents

Acnkowledgments

Introduction

In this thesis we address the challenge of portfolio allocation applied to a set of trading algorithms. The aim here is to allocate risk to many algorithmic trading strategies on a weekly basis. The problem is to be solved in two seprate parts: firstly address at any moment in time which strategies out of the many available to put into production and at last assign proper risk weights to these strategies. We will see that both the steps are to be addressed with care as none of these problem if solved alone can achieve satisfactory results. All the results will be compared with a proper benchmark that mimics the current non-systematic allocation strategy. If the procedure is will be implemented it will make the whole investing process completely systematic. The weekly allocation period is chosen as it fits at best the charachteristics of the market of interest and it avoids incurring in excessive transaction costs arousing from daily rebalancing of the portfolio that would erase any improvement given by the selection methodology.
The challenges that have been faced include the abundance of strategies and the well known issue that alpha in algorithmic trading strategies is not everlasting. There is a point in time at which any strategy will stop working and will necessarily be switched off, on the other hand, as a reaction to market changes, some strategies that in the past performed poorly might become alpha generators. Achieving optimal timing in putting into production and swithcing off the strategies represents a challenge but also an opportunity to substantially increase trading performance.This task is hard to perform in other ways than algorithmic selection because often what is selected might not look intuitive to trade at first sight. Inter-market relationships change through time. For example, if one is trading on the well known relationship between Gold and US Government Bond (which is expected to be stedily meaningful and potentially a good source of alpha), it might be that due to some specific event this correlation breaks down, changing all the underlying market-dynamics and making the algorithms unprofitable. Moreover, in some cases, a certain strategy might perform really well for years until somebody in the market strats exploiting it systematically and at high frequency bringing liquidity and margin for trading out of the scope of hedge funds. In such cases detecting a switching point in the performance of strategies is of crucial importance.

CONTROLLA CHE NON SIA UNA CAZZATA

Literature Review

Our Approach

The "schedule" we set at the beginning is to find firstly a satisfactory method to switch strategies on and off and then move to the part of weight allocation. We will consider the first step to be completed once the resulting selection allows for efficient trading of around a hundredth of strategies with at least half of the trading days with positive pnl.
To achieve this step a simple and robust feature-based approach has been used. We decided not to try to use any hard-core machine learning type approach to reduce the risk of overfitting and to fit the specificity of the problem. In fact, the abundance of strategies and the lack of a long samples would have made traning a machine learning method cumbersome and time-consuming.

QUI E' TUTTO DA VERIFICARE ED AGGIORNARE

Statistic	Mean	Median	Min	Max	IQR
Mean Return	-0.0002	-0.0002	-0.0032	0.0024	0.0002
Skew	-1.9393	-1.698	-34.2374	21.119	2.5754
Kurtosis	56.9226	25.5496	-5.4507	1202.95	37.851
Sharpe Ratio	-1.3653	-1.1725	-20.0072	15.547	1.4080
Sortino	-1.3514	-1.1808	-32.9069	40.322	1.3692

Table 1: Global strategy statistics.

Before getting into this part, relevant features are needed to give some predictive insight to our models. To this end we built several different features and evaluated their predictive power through a *Random Forest Tree* (details of this model will be provided later).
For what concerns assigning the weights we developed two different approaches, one of which is more computationally oriented and the other is more diversification-driven. The first approach is to train a genetic portfolio allocator that will select the best portfolio in-sample and then apply iy out of sample. The second approach is based on clustering, and aims at reducing as much as possible the realized variance of the portfolio.

The data

As mentioned before the dataset at hand consists of 13000 simulated strategies, based on mean-reversion. All these strategies are trading one futures against another one looking for relative mispricings. This number of strategies comes out of a simulation of all possible trading pairs among roughly 150 futures traded worldwide. The huge diversity among these strategies makes it hard to find a unique model to allocate risk among all of them. Diversification has to be applied not only to asset classes, but also taking into account type of algorithms, trading latency and underlying country or region of exposure.
The data spans through almost six years, from January 2012 to August 2017. To perform the studies, the final year has been dropped to be used as a final validation set (also referred to as production set). The first remaining chunk has been divided into train and test set.
We also decided to remove any strategy involved in swiss franc trading, to avoid our results to be biased by the famous drop that happened on the 15th of January 2015. We don't want to penalize or advantage any strategy that happened to be trading the swiss franc in either long or short side in that day because we believe that was a statistically unpredictable event. There is no guarantee that such an event could be forecasted only with information coming from strategy performance.

Some Descriptive Statistics

Here we want to give a taste of what our data looks like. We first run a code that computes sample statistics for all the strategies. The results are exposed in Table 1.

We can see how on average the mean return per strategy is negative. The sharpe ratio of course follows this pattern as well. On the other hand we notice how some sharpes are very high (e have peaks at around 15 on the wole history!). Here an important remark must be made, many

strategies with good performance seem really appealing, but for several reasons might not be tradable due to liquidity issues, regulation or asynchronization of quotes data.
Back to our statistical analysis, we notice how the skew and kurtosis reach extreme values, signaling that the returns of these strategies might not be normally distributed. To this end we conducted a Shapiro-Wilks normality test for each strategy, where the null Hypothesis of normality is challenged (for details on this procedure refer to the Appendix). The results are the following:

We can observe that for the majority of the cases the normality hypothesis is rejected. Some strategies survive the test, but a deeper analysis supports the idea that this is caused by a lack of data for these strategies.

Part 1: Strategy Selection

Problem Statement

We give here an additional re-statement of the problem we try to tackle here. On each monday we have to allocate risk on each of the given strategies by choosing which ones to put into production for the following week. In an ideal world we would switch on all the strategies that will perform well during the following week and vice-versa with the bad ones. Unfortunately this is quite an impossible task, and we just seek a "statistical edge" that allows us to profit from appropriate selection of strategies on the long run.

Building the Features

To be able to predict the performance of trading strategies we first need to build meaningful features that come out of a manipulation of the raw data. We start from simple performance metrics to advanced features computed on rolling windows. Here you can find a list with detailed information.

- *Hit Ratio:* This feature computes the percentage of days with positive PnL over a certain rolling window. The higher the Hit Ratio, we expect that the higher the probability of positive returns in the future.

- *Sharpe Ratio:*. This world-known measure comes as an evolution of the previous and is supposed to give some more information about the shape of the pnl line of a strategy. Intuition suggests that a strategy with high sharpe over long periods might continue providing gains in the foreseable future.

- *Robust Sharpe Ratio* This feature is supposed to be a robust version of the sharpe ratio, computed trying to avoid the distorsive effects of outliers and measurement errors. The formula is the following (given $\mathbf{r}$ of past returns):

$$Robust\_Sharpe = \frac{med(\mathbf{r})}{IQR(\mathbf{r})}$$

Where med stands for median and IQR stands for interquantile range. Hopefully this feature should allow to ignore the non-normality of the distribution of returns and give a robust measure of performance.

- *Exponetially Weighted Sharpe Ratio:*. This feature is an evolution of the simple sharpe ratio. It is computed as a roolling mean divided by a rolling standard deviation, calculated with exponential weighting. The rational between this choice is that an exponential sharpe should be able to capture faster changes in the evaluation of a performance of a strategy.

- *Performance Quantile:* This feature looks on a rolling window at the performance over a certain horizon. This past performance is averaged at a daily level and compared with the distribution of past returns. The are some interesting dynamics that this fature should capture. For example if a strategy that has been trading with very good performance over the last years suddenly stops being profitable, this feature will immediately advise to switch the strategy off. On the contrary, a strategy that has been performing poorly suddently records some good performance, resulting in a high position in the historical distribution and some risk being allocated in production.

- *Exponential Moving Average of PnL:* this feature is computed as the moving average over a certain period of the cumulative pnl line of a strategy weighted over history with exponential weighting. Given a time period T, a weight factor is computed as $k = \frac{2}{T+1}$ and the exponential moving average is computed as

$$EMA[i] = (pnl\_curve[i] - EMA[i-1])\,k + EMA[i-1]$$

  Hopefully this feature should rapidly capture switching point in the performance of a strategy by looking at the difference betweek the pnl curve and its exponential moving average. An alternative could be to look at the crossing between moving averages, at the risk of switching late, but removing a good amount of noise.

- *Tail Ratio:* This feature is computed as the ratio over a rolling window between the 95th and the absolute 5th percentile of the distribution of returns. The higher the tail ratio the more positively biased the distribution and the bigger the odds of getting positive weights by trading in the strategy. This feature has the really good characteristic of not being too sensitive to outliers allowing for a robust estimation of the strategy performance.

- *Sortino Ratio:* Computed as the Sharpe ratio, but considering only the volatility of negative returns.

- *Drawdown Mode:* This simple feature indicates whether a strategy is in drawdown or not. In other words it looks at the cumulative PnL of a given strategy and trades it when the current cumulative PnL is above the rolling max. More precisely, to give a bit more freedom in switching we allow the strategy to loose 2% from the previous max before being switched off, to eliminate the effect of noise.

Relevant Features

Once the features have been built we have to decide which ones give the more predictive power to solve out problem. Moreover we need to assess which rolling window is ideal for any feature to be able to forecast at best. The approach chosen at this stage is to use a *Random Forest* model to rank these features. The idea is to feed this model with all the possible features and let the algorithm select the best ones. To dig more in detail on how this process can be applied, a discussion of random forest trees is appropriate. A decision tree is a machine learning model that can predict quantitative and $\{0, 1\}$ outputs given a set of features. The model takes binary decisions based on the input features partitioning the sample into different "leaves" and assigns output values minimizing the impurity that is a measure of homogenety of the data (See the appendix for greater detail on ow the algorithm works). Their use in feature selection is abundant thanks to their simple approach abd their ability to model dependencies between features. If a tree, trained on some data, consistently splits based on the value of only one feature, it's a strong indication of importance of that feature. A Random forest uses the powerful concept of bootstrap on top of this model: it trains several trees, where any of these is trained only on a subset of the data sample and a subset of the features. The output is then the average split decision across all trees.

For our problem we even went further adapting this model to our specific dataset that has few datapoints (6 years of daily returns) for many different strategies. What we did is to use the powerful Python library *Scikit Learn* to train a random forest on each of the 13000 strategies at hand (only in our train sample). Once the tree is trained we retrieve the feature importances and we sum them up across all the strategies. Each tree will be feeded with all the features computed above with different rolling windows (in our case 30, 60, 90, 120, 180, 210, 250, 300 days). Before going to the results, two important steps must be taken. The first is to compute an output feature on which the tree can actually train on. We decided to use a binary output (0/1) that tells whether the strategy has a positive (1) or negative (0) returns over the following 20 trading days. We didn't limit the output to 5 trading days, even though it will be our final target, as the tree would have been subject to high noise, while the reliability of certain features should emerge on slightly longer terms.

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The last part to take care of before training the model is to clean the data. We normalized the data, dropped extreme values and dropped strategies that had too few trading days, as these would haven't let the tree train properly.

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Once the tree had been trained we recorded the seven most important features:

Once we agreed on the relevant features we started building a model to predict which strategies to put into production each week.

Switching Model

As opposed to a traditional machine learning model, we want something more simple, interpretable and faster. Following the results of our random forest tree classifier we decided to base our robust threshold on Sharpe Ratios, Exponential Moving Averages and Quantile Performance. We run different tests (in our in-sample period) to see which meaningful combination of features could come up with a proper switching model. It turned out that using only one feature was not enough as the data is really diverse and many strategies have very poor performance, forcing our method to somehow filter them out. We directed our endeavours towards finding a meaningful filter of strategies. What this filter has to do, is to look at the past performance of any signle strategy and set a threshold below which even if the current performance is good this strategy would not be switched on. The reason for this is that many strategies have some very short period where they work well due to specific market conditions that don't last for long. We luckily have a huge wealth of strategies and we can afford being strict in selecting strategies giving more strength to our method. After some tests and discussions we decided that a good filter is given by a rolling-sharpe looking back for a certain period where the strategy performed significantly well. In other words, every monday the historical x-days sharpe ratio for each strategy is computed and if we are able to find a period of x-days when a strategy performed sufficiently well in terms of sharpe we believe this is a strategy that can be switched on and off in the future. Once this filter is applied the remaining features are switched according...

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Part 2: Risk Allocation

Once we have a robust and trustable switching method, we can move our scope towards risk minimization, or in more precise terms, sharpe ratio maximization. We will build on top of the selected portfolio two different weights systems that will be benchmarked against a simple equally weighted portfolio and a Markowitz-like minimum variance portfolio (see appendix for building details). As it was for the switching problem, our aim is still to find the best out-of-sample portfolio for the following week (setting the new weights on monday) given information up to the previous friday.

Method 1

The first method we try to implement is a Genetic-Leaning portfolio allocator. The method is based on the idea of making the algorithm evolve to find an optimal allocation through extensive genetic mutation. .The approach is rather brute-force as it tries to test as many portfolios as possible until the optimal one is found. A more human-like analogy is the following: the algorithm acts as a boss letting many portfolio managers allocate risk according to their views. As time passes the boss will evaluate the portfolio managers based on specific performance measures (that are not only raw pnl) and kicks out the worst performing. At each stage he tries to replace the worst portfolio managers with completely new ones and with a set of managers that trade similarly to the best ons. Let's dig into the underlying methodology: on each monday we face the challenge of assigning weights (between 0 and 1) to the set of tradable strategies. The algorithm is initialized with a set of random portfolios $\mathbf{w} = (\mathbf{w_1} \dots \mathbf{w_N})$, where each $\mathbf{w_1}$ represents a feasible allocation of risk. The algorithm lets these portfolios trade over a certain window in the past and evaluates their performance. Once all of them have traded, the algorithm ranks them assigning a score given by a so-called *Fitness Function*, which takes many metrics into account to evaluate a portfolio. Then the algorithm kicks out the worst performing, and substitutes them with a new generation (details on this part will be explaine later).

DESCRITTO DA QUAL-CUNO, CITA ATRI-COLI

The procedure is repeated until an optimum is reached, or in other terms this optimizer is not able to find better portfolios. At this point the final portfolio will be an average of the best found portfolios.

The name Genetic comes from the idea that natural selection and evolution are applied to the set of portfolios. If a portfolio is just bad it will not survive the selection step, while if a portfolio is good it will be challenged with a muted version of itself that might represent an evolutional step. This kind of approach has pros and cons, let's first evaluate the positive aspects:

- The optimization carried in a way that is able to be conducted in a multidimentional space, in different local minima in parallel avoiding the risk of missing a global minimum. The mutation happens in a way that optimization is more refined in well performing areas, while it is also randomized to cover the whole space
- The algorithm is conceived in such a way that it serves really well our needs and requirements. Evaluating portfolios with the so called Fitness Function it allows to penalize portfolios that perform well but that give rise to the typical issues of portfolio optimization like instability of weights, poor diversification or meaningless negative weights. The optimization is already done without having to worry about any type of complex mathematical formulation to impose constraints.
- The approach requires very little parameters: the length of the lookback window and the weights to give to any performance measure used to assess portfolio performance.
- The algorithm might fully embrace the non-linearity of the problem and autonomously find relationships between strategies that other methods might not find.

On the other hand this method has some drawbacks:

- This brute-force algorithm requires an enormous computing power to span the whole space and rank all the portfolio. Needless to say, we will notice later that the more computing time is given to the algoithm the more the randomness in it is limited and the performance improves. We will dig later in this aspect.

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- As outlined above, there is some randomness, as most of the portfolios that are tested are just randomly generated, so there is little chance to find a precise optimum, but rather something that is quite close to it
- The algorithm looks backward and makes the assumption that the best performing combination in the past will still be the best for the next week.
- Even though there are few parameters to be set, the algorithm is quite sensitive to these values.

Implementation

Let's now address the issue of defining the fitness function, this function that we will indicate with $f_f : \mathbb{R}^N \to \mathbb{R}$ is a function that given a portfolio vector $\mathbf{w}$ returns a real number as a score. This function is the core of the whole algorithm, because it can evaluate a portfolio based on its performance but also based on how this respects our requirement. For example it might penalize a portfolio that assigns a lot of weight to few strategies, or a portfolio that changes too much compared to what was traded the previous week. Defining this function in the proper way takes more intuition than calculation, and requires to pay attention to a couple of details. Of course, the more complex the fitness function the more our taste can be satisfied, but also the more computational time is required.
We will evaluate the performance of the portfolio based on a mix of sharpe ratio and sortino ratio (achieved over a certain lookback period), somehow taking into account the diversification benefit of a portfolio allocation. We will also take into account how much a portfolio will be different from the previous one with a norm-1 penalty. So our fitness function will look like:

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$$f_f = \alpha_1 * sharpe(\mathbf{w}) + \alpha_2 * sortino(\mathbf{w}) + \alpha_3 * ||\mathbf{w} - \mathbf{w_{old}}||$$

Where $alpha_1$, $alpha_2$, and $alpha_3$ are weights on which no optimization will be carried to limit the computaional burden. $alpha_1$ and $alpha_3$ will be the same, while $alpha_3$ will be such that the influence on the portfolio is relevan but still not that big to prevent the portfolio form evolving. If a portfolio is really different to the one traded the previous week, requiring a complete rebalancing it has to be good enough to make us switch towards itself.

Qui metti tipo di randomizzazione/replacement

Metti dettagli per definire l'ottimo

Results

Appendix

Shapiro-Wilks normality test

As suggested by the name, the Shapiro-Wilks test checks if a sample is drawn from a normal distribution. More precisely, given a sample it tests H_0 (normality) versus the alternative hypothesis of non-normality. This test is ideal for our case as it doesn't require too much data to come to a conclusion. The test is non parametric and starts with sorting the data. Once the data is sorted, the test statistic can be computed:

$$W = \frac{\left(\sum_{i=1}^{N} a_i x_{(i)}\right)^2}{\sum_{i=1}^{N} (x_i - \bar{x})^2}$$

Each element has its specific meaning:

- $\bar{x}$ is the sample mean of the data.
- $x_{(i)}$ is the i-th order statistic.
- a_i are tabulated coefficients coming out of the distribution of order statistics of a normal standard distribution.

The larger the statistic the more "normal" the data. This comes from the idea that the test wants to measure the similatity of the ordered statistics to those of a standard normal distribution. The W statistic somehow measures the closedness of these two entities.

Minimum Variance Portfolio

Here we build the foundations of the Minimum Variance portfolio used as a benchmark to measure the relative performance of our weight assignment methods.

Firstly, we set the problem in rigorous terms: given a set of N tradable instruments (in our case trading strategies) we want to find the optimal trading vector $\mathbf{w} = (\mathbf{w_1} \dots \mathbf{w_N})$ that represents the composition of our portfolio. This composition will optimally be the one that minimizes the in-sample variance of the portfolio. The latter is measured as:

$$\sigma_\pi^2 = \frac{1}{2}\mathbf{w}^T \mathbf{\Sigma} \mathbf{w}$$

This optimization problem is usually solved under the constraint that the sum of the weights should be equal to one. We will solve the problem and then impose that the weights are also positive (it wouldn't make sense to trade strtegies with negative weights).
The lagrangean to solve to minimize the variance is the following:

$$\mathbf{L} = \frac{1}{2}\mathbf{w}^T \mathbf{\Sigma} \mathbf{w} - \lambda \left(\mathbf{1}^T \mathbf{w} - 1 \right)$$

Where $\mathbf{1}$ is a vector made up of ones.
We compute the first order conditions:

$$\frac{\partial \mathbf{L}}{\partial \mathbf{w}} = \mathbf{\Sigma w} - \lambda \mathbf{1} = 0 \qquad \frac{\partial \mathbf{L}}{\partial \lambda} = \mathbf{1}^T \mathbf{w} - 1 = 0$$

From the first F.O.C. we immediately find:

$$\mathbf{w} = \lambda \mathbf{\Sigma}^{-1} \mathbf{1}$$

We plug this result into the other F.O.C.:

$$\lambda \mathbf{1}^T \mathbf{\Sigma}^{-1} \mathbf{1} - 1 = 0 \Longrightarrow \lambda = \frac{1}{\mathbf{1}^T \mathbf{\Sigma}^{-1} \mathbf{1}}$$

Therefore getting a nice analytical closed-form solution for our minimum variance portfolio:

$$\mathbf{w} = \frac{\mathbf{\Sigma}^{-1} \mathbf{1}}{\mathbf{1}^T \mathbf{\Sigma}^{-1} \mathbf{1}}$$

The beauty of this formula comes with some drawbacks:

- $\mathbf{\Sigma}$ is often not precisely estimated due to the huge number of strategies and the little amount of samples to use to measure standard deviations and correlations. Moreover this matrix is not to invert leading to numerical errors. To partially address these issues we use a *LedoitWolf* covariance matrix whose construction is explained in the next chapter.
- This approach completely ignores transaction costs, leading to a fastly changing and unstable portfolio composition
- The model works making a basic assumption: in-sample correlations and variances will hold out-of-sample with very simila values. Unfortunately this is rarely the case in the real world, making this portfolio sub-optimal in terms of variance.

Ledoit Wolf Covariance Matrix

Random Forest Tree

As outlined before, the decision trees are an all-purpose machine learning algorithm able to be trained on extremely non-linear phenomena. The beauty of these algorithm lies in the simplicity of the underlying learning process, the data is split in "sectors" in a way that the highest "purity" is achieved. The Random forest algorithm adds robustness to this process. Let's first explore in detail the training process for a simple decision tree.

- Given an m-dimensional set of data with an output feature (we are in the case of supervised learning) examine all the possible splits on one feature.
- Evaluate each split based on the purity of the splitted areas. This is done trough the *Gini* impurity measure: $I_G = \sum_{i=1}^{N} p_i(1-p_1)$, where N is the number of labels/classes in the data. Ths measure indicates how often a randomly chosen element from the set would be incorrectly labeled if it was randomly labeled according to the distribution of labels in the subset. This comes clear with the fact that the tree assigns probability to labels.
- The purest split gives rise to a new node.
- From this split others are generated until the highest purity or the maximum number of splits is achieved.

As it might emerge from this brief explanation, decision trees tend to overfit the data, as they learn very complex non linear-features. This behaviour is described in the context of the bias-variance trade-off where decision trees stand more in favor of variance rather than bias. Random forest try to overcome this issue by averaging many trees.
Once we understood how a general decision tree is trained we can explore in depth the training of a random forest algorithm:

- Generate M different trees.
- Each tree is trained (as a normal decision tree) on a random subset of the features. This number is usually believed to be a fraction $\sqrt{n}/n$ where n is the number of features.
- The results from all the trees are averaged, that means that for each point the final label will be given by the average of all labels given by the different M trees.

This robust procedure is useful to train powerful regressors or classifiers, but might be used as well to measure the forecasting ability of the input features. If a feature has real predictive power, it will be used in many bootstrapped samples to produce splits in the data, therefore being used many times. Computing the number of times each feature is used to produce a split will give a ranking of feature importances.

Figures and Tables

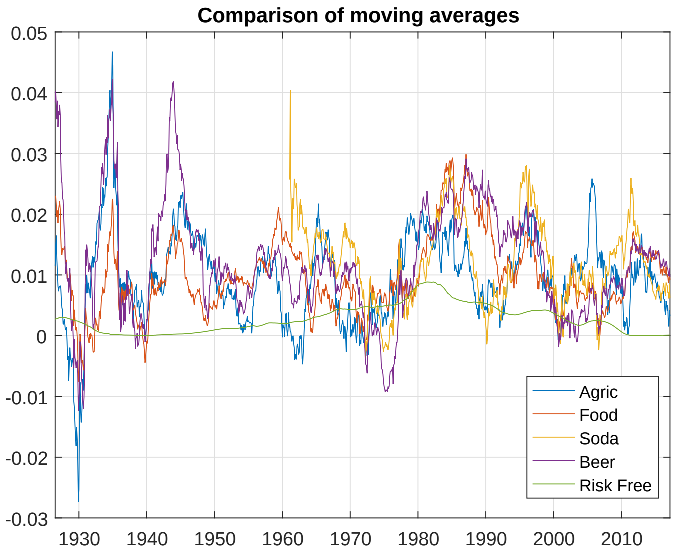

Figure 1: Rolling average of the returns for 4 industries and Rf

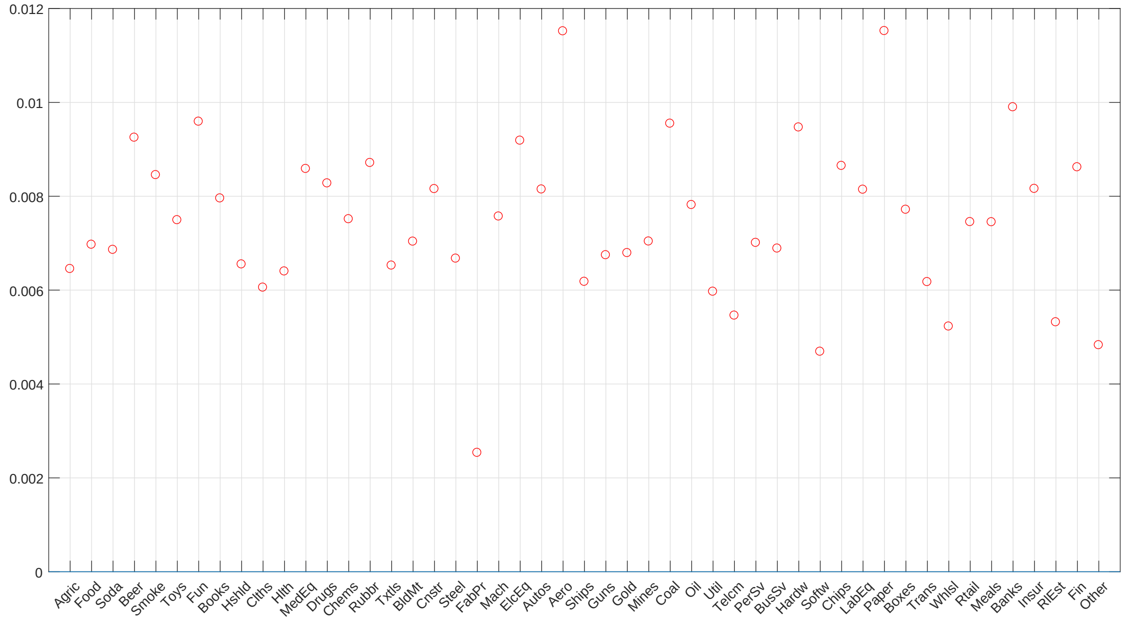

Figure 2: Arithmetic mean of simple returns

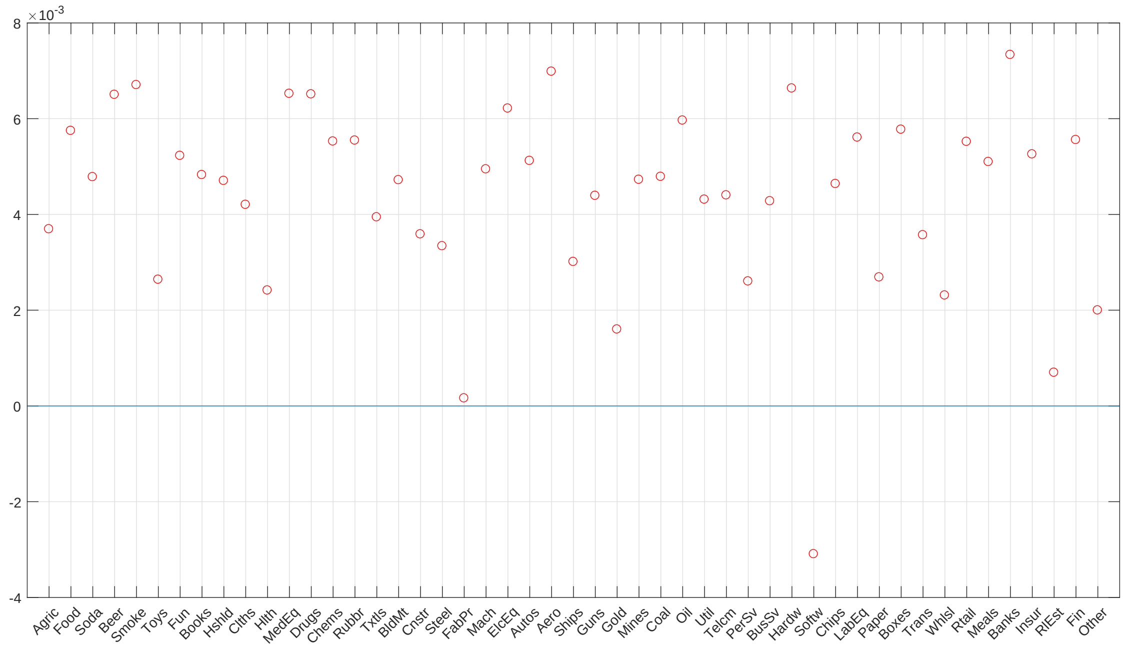

Figure 3: Geometric mean of simple returns

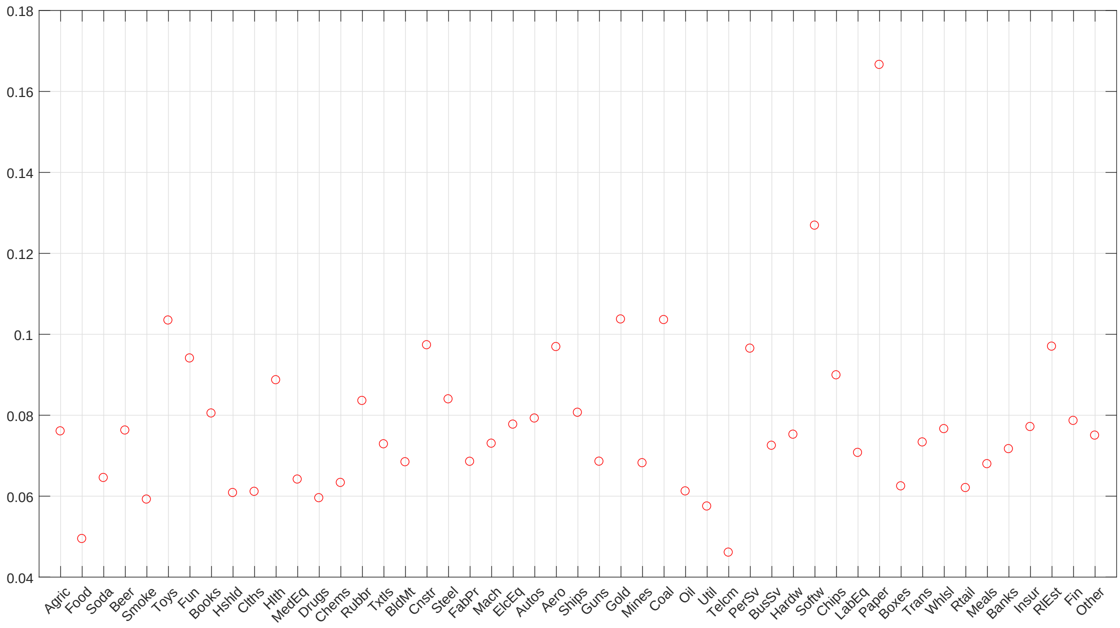

Figure 4: Standard deviations of simple returns

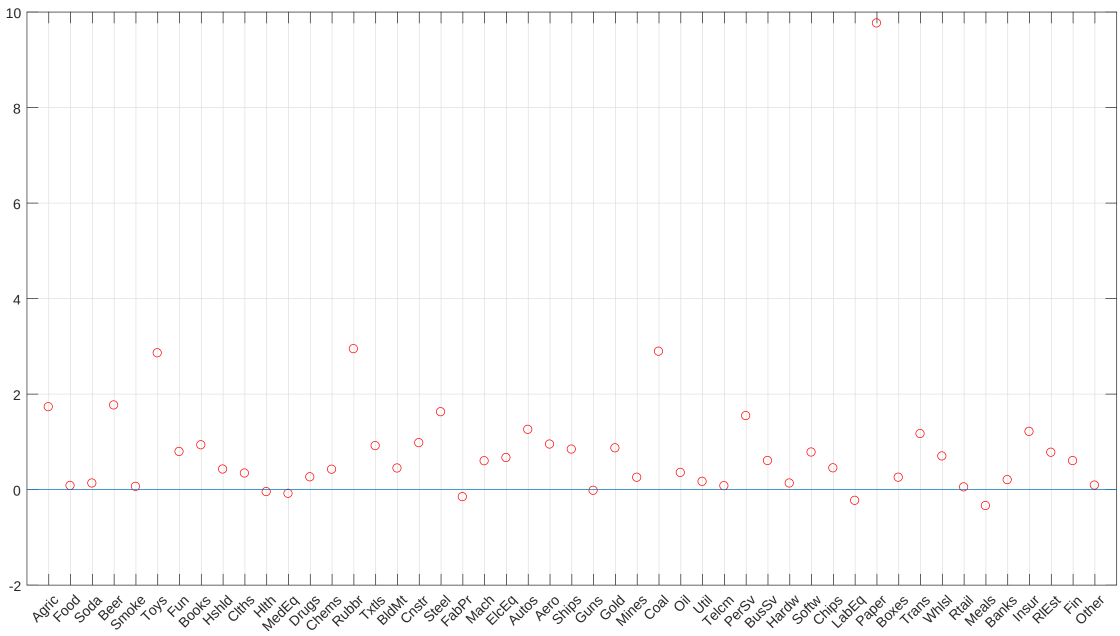

Figure 5: Skewness of simple returns

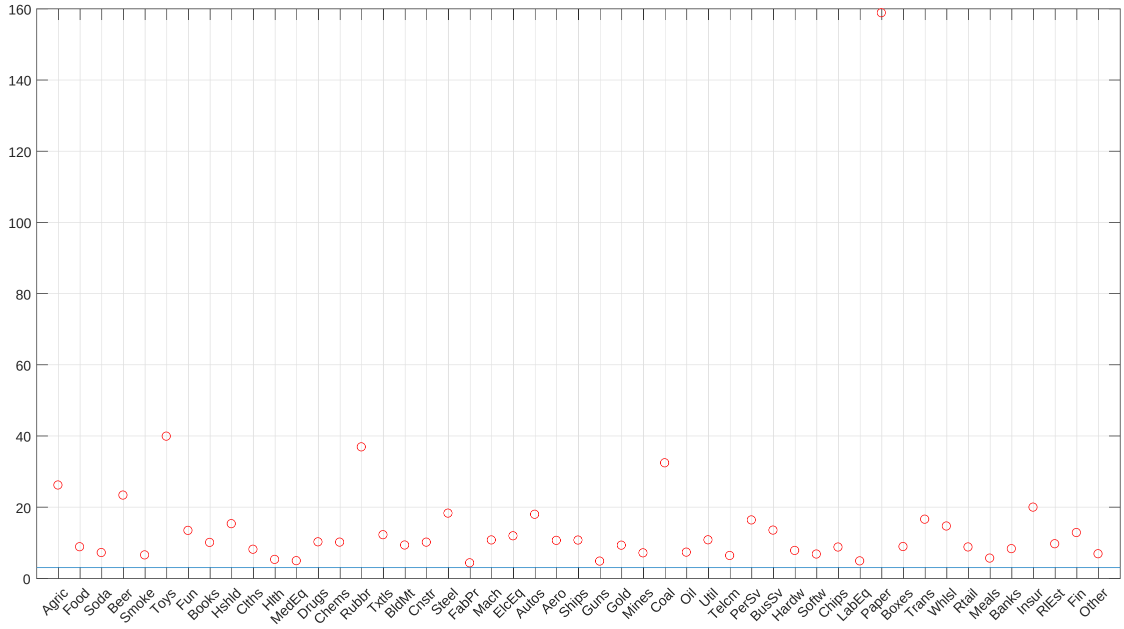

Figure 6: Kurtosis of simple returns

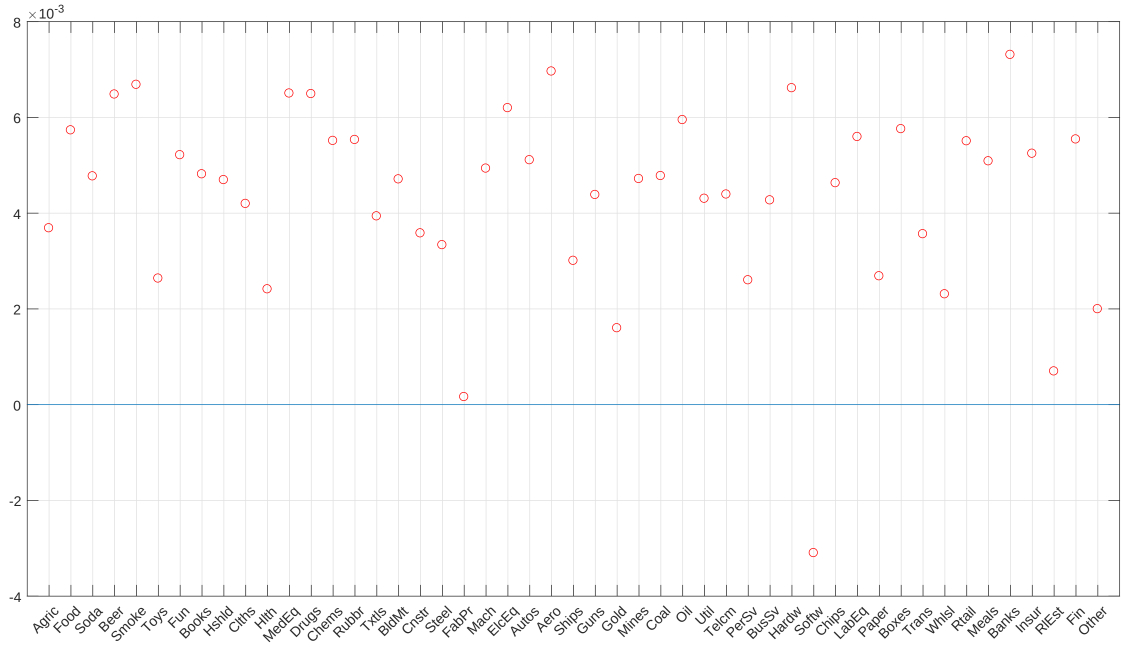

Figure 7: Arithmetic mean of log returns

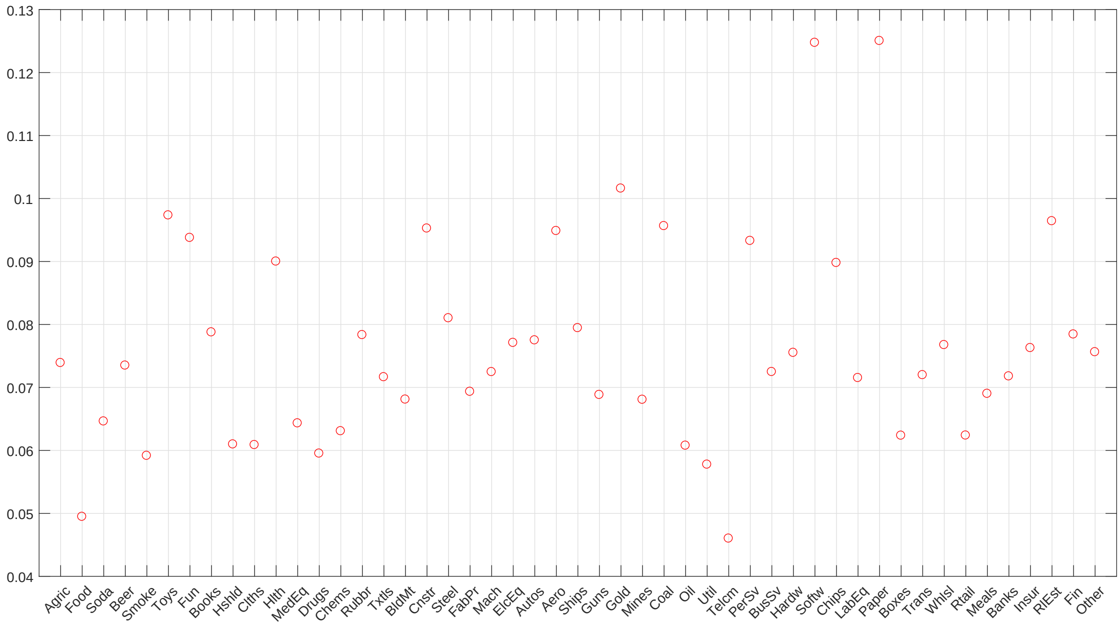

Figure 8: Standard deviation of log returns

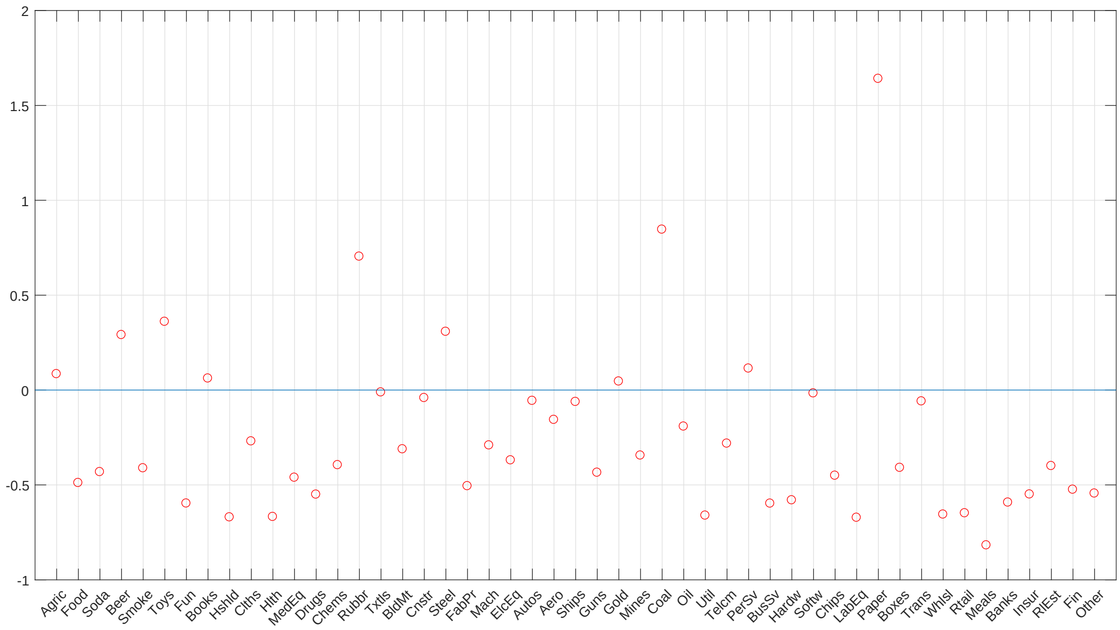

Figure 9: Skewness of log returns

Figure 10: Kurtosis of log returns

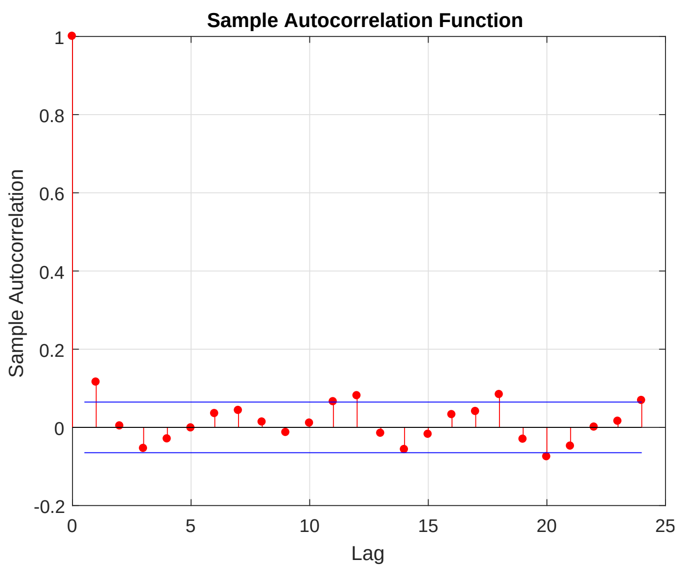

Figure 11: Autocorrelation function for industry 'Other'

Figure 12: Crosscorrelation function for two industries

Figure 13: Aligned returns following on the findings of Figure 12

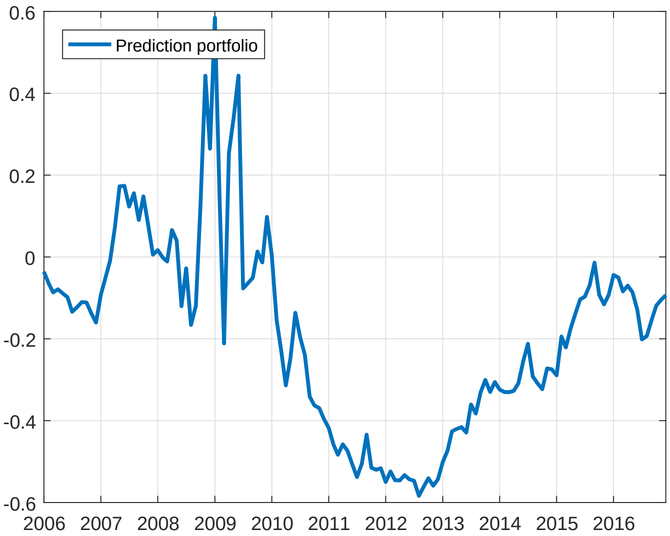

Figure 14: Trading strategy built on the prediction of future returns from other assets lagged returns

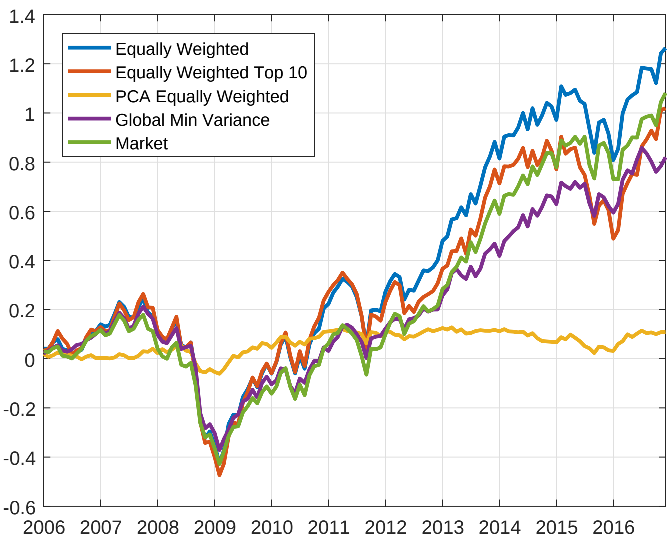

Figure 15: Portfolios Cumulative performance

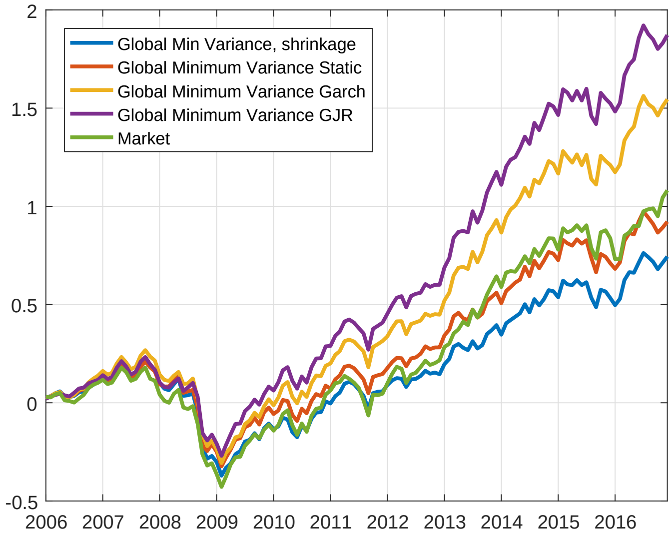

Figure 16: Portfolios Cumulative performance

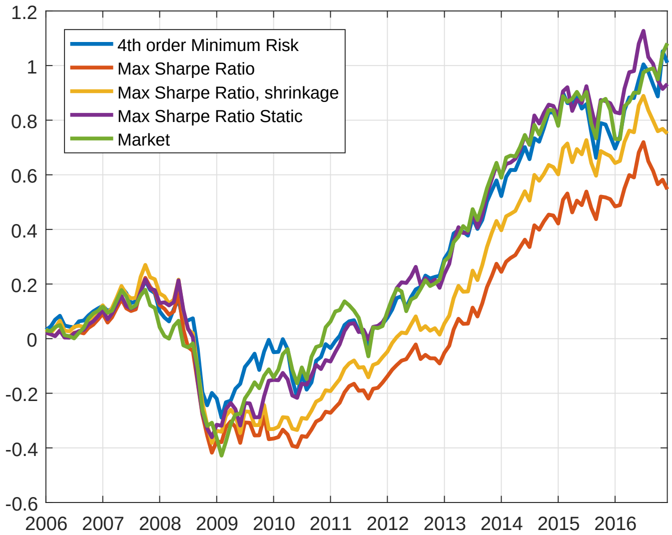

Figure 17: Portfolios Cumulative performance

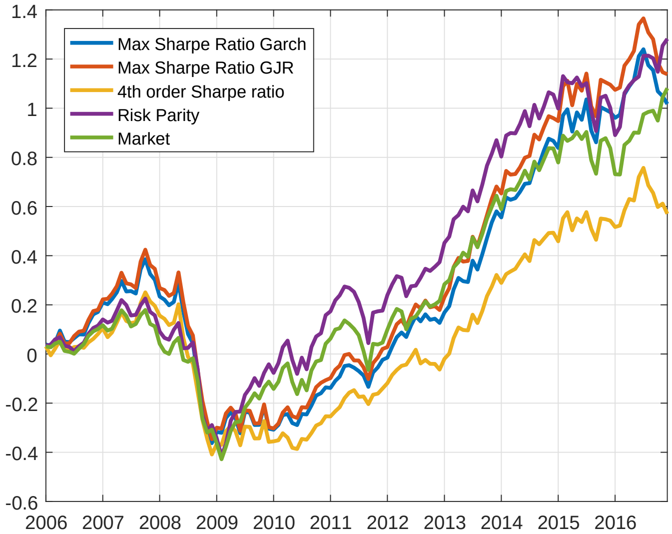

Figure 18: Portfolios Cumulative performance

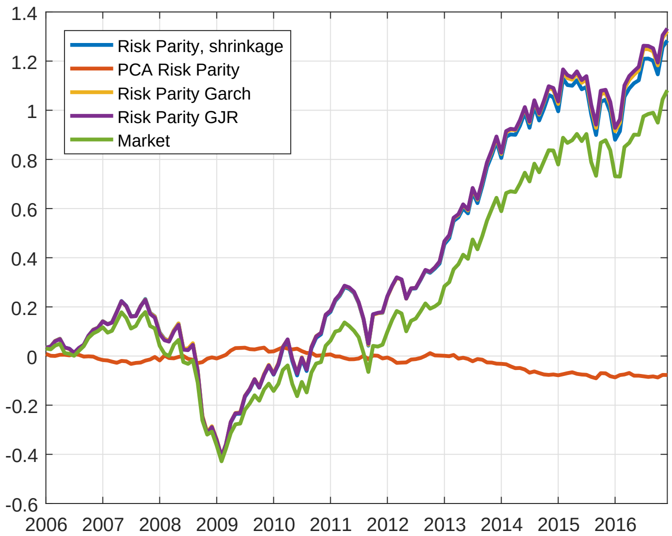

Figure 19: Portfolios Cumulative performance

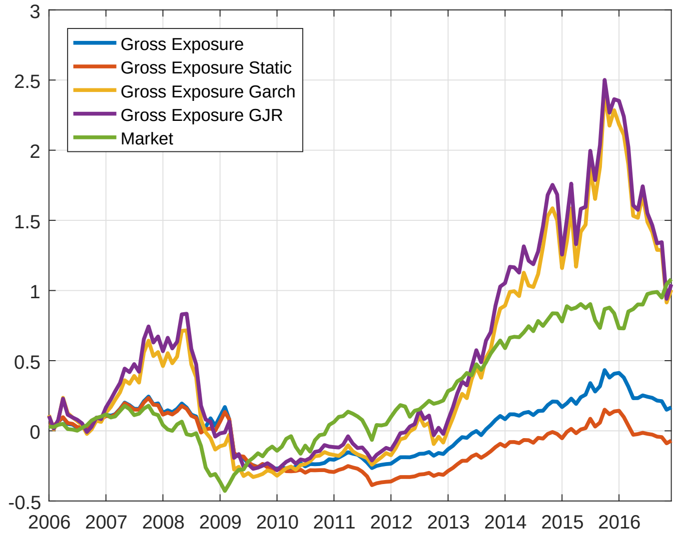

Figure 20: Portfolios Cumulative performance

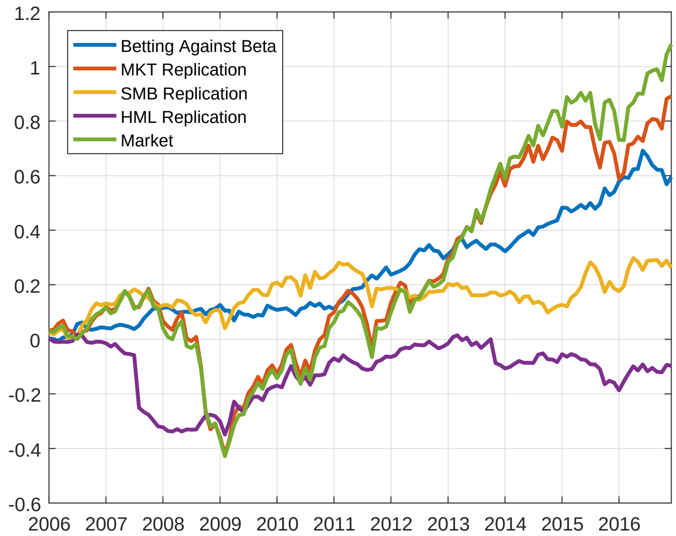

Figure 21: Portfolios Cumulative performance

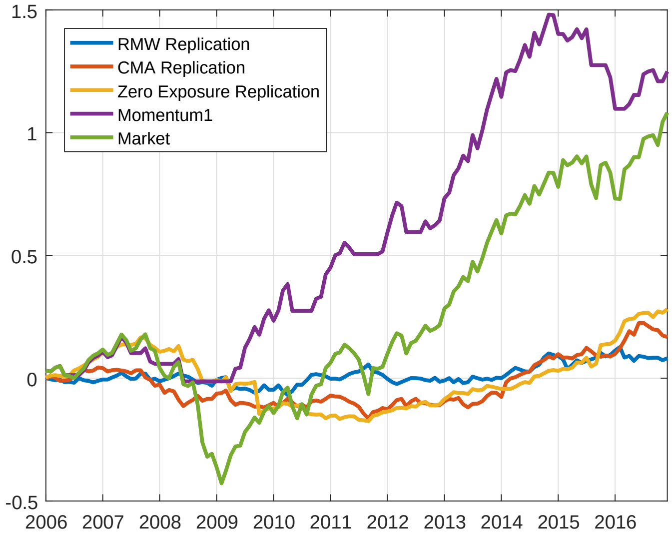

Figure 22: Portfolios Cumulative performance

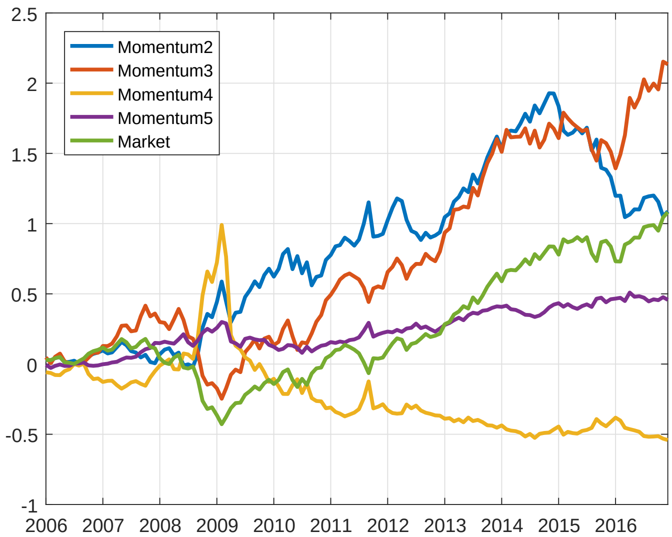

Figure 23: Portfolios Cumulative performance

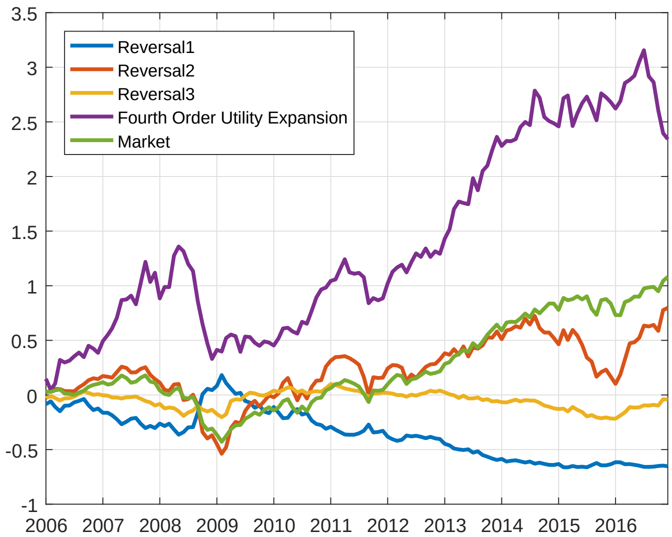

Figure 24: Portfolios Cumulative performance

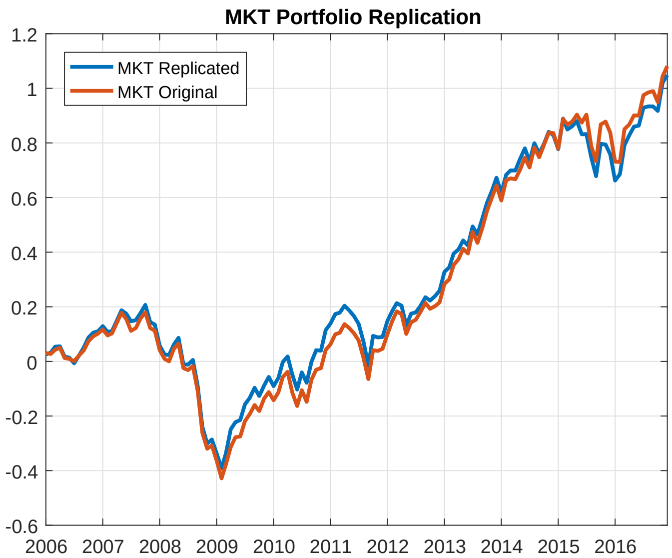

Figure 25: Market replication portfolio

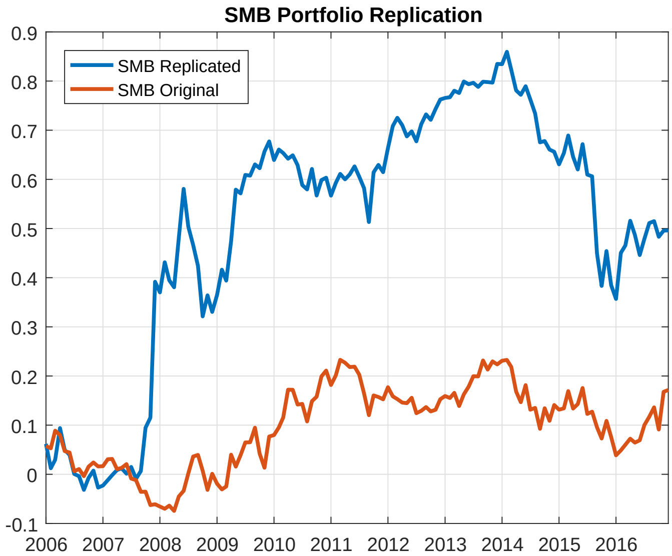

Figure 26: SMB factor replication portfolio

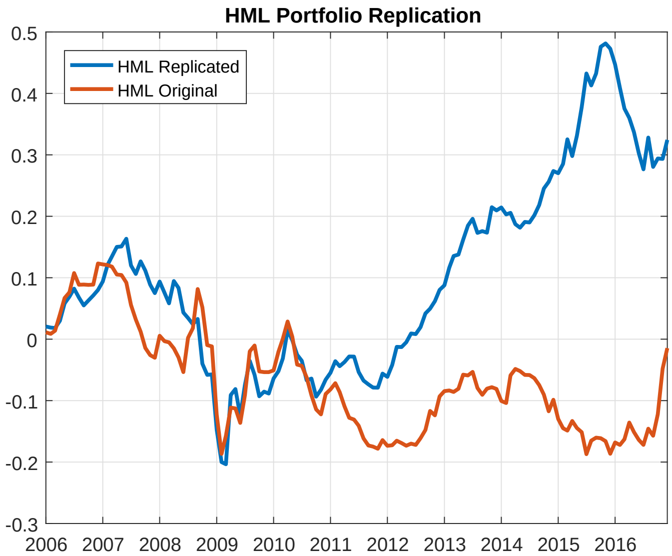

Figure 27: HML factor replication portfolio

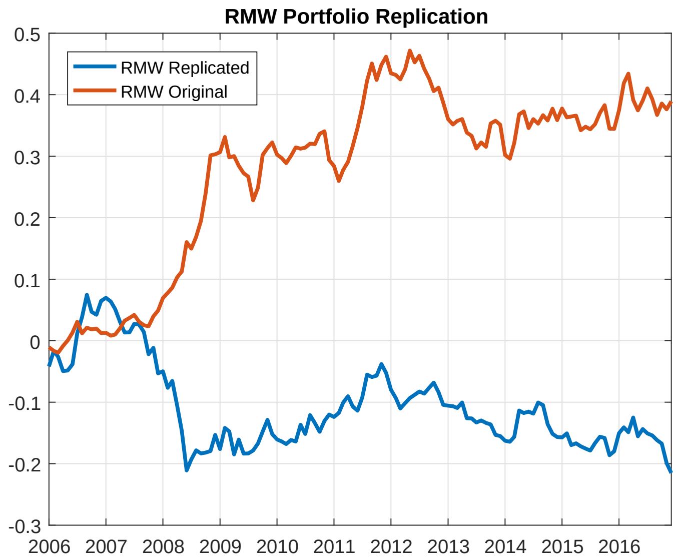

Figure 28: RMW factor replication portfolio

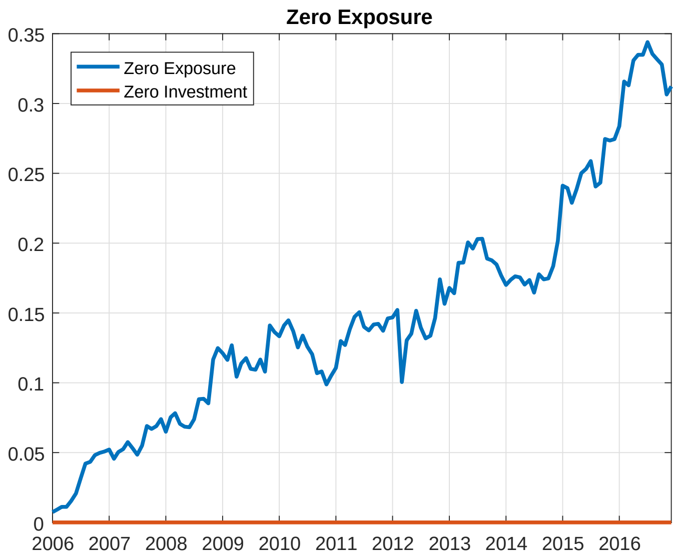

Figure 29: Zero exposure portfolio

	Means	**Geom. means**	**Std**	**Skewness**	**Kurtosis**
Agric	0.006451	0.003692	0.076022	1.7264	26.0881
Food	0.0069678	0.0057469	0.04943	0.077403	8.7436
Soda	0.0068588	0.0047807	0.064507	0.12859	7.1141
Beer	0.0092485	0.0065	0.076231	1.7631	23.2572
Smoke	0.0084513	0.0067035	0.05917	0.058214	6.4513
Toys	0.0074909	0.0026382	0.10341	2.8551	39.81
Fun	0.0095897	0.005225	0.094034	0.78908	13.3382
Books	0.0079547	0.0048241	0.080439	0.92906	9.9586
Hshld	0.0065486	0.0047023	0.060793	0.42088	15.2228
Clths	0.0060539	0.0042022	0.061084	0.3378	8.0408
Hlth	0.0063989	0.0024131	0.088659	-0.051356	5.1869
MedEq	0.0085827	0.0065211	0.064113	-0.087466	4.829
Drugs	0.0082751	0.0065093	0.05951	0.25836	10.1244
Chems	0.0075114	0.0055249	0.063286	0.41702	10.0373
Rubbr	0.0087101	0.0055448	0.083538	2.942	36.8028
Txtls	0.0065236	0.0039416	0.072826	0.91155	12.1285
BldMt	0.0070339	0.0047173	0.068393	0.44272	9.2266
Cnstr	0.0081537	0.0035854	0.0973	0.97382	10.0288
Steel	0.0066718	0.0033381	0.083934	1.6211	18.1863
FabPr	0.0025337	0.00015952	0.068503	-0.15752	4.2258
Mach	0.0075694	0.0049435	0.072975	0.59482	10.6562
ElcEq	0.0091851	0.0062135	0.077686	0.66236	11.8235
Autos	0.0081455	0.0051196	0.079195	1.2523	17.8642
Aero	0.011515	0.0069837	0.096852	0.94405	10.5309
Ships	0.0061779	0.0030093	0.080616	0.83866	10.6221
Guns	0.0067447	0.0043901	0.068531	-0.02446	4.7082
Gold	0.00679	0.0015985	0.10367	0.86529	9.1785
Mines	0.007037	0.0047254	0.068158	0.24924	7.0303
Coal	0.0095471	0.0047871	0.10352	2.8884	32.3239
Oil	0.0078142	0.0059628	0.061188	0.35	7.2215
Util	0.005967	0.0043107	0.05746	0.16329	10.6922
Telcm	0.0054578	0.0043998	0.046062	0.072809	6.2862
PerSv	0.0070072	0.0026022	0.096456	1.5402	16.2761
BusSv	0.0068864	0.0042772	0.072464	0.59962	13.4179
Hardw	0.0094668	0.0066327	0.075198	0.12849	7.706
Softw	0.0046885	-0.0030951	0.12684	0.7773	6.6936
Chips	0.0086475	0.0046374	0.08987	0.44638	8.6589
LabEq	0.0081403	0.0056075	0.070703	-0.23438	4.7707
Paper	0.01152	0.002686	0.16655	9.7625	158.7706
Boxes	0.007713	0.0057721	0.062435	0.24969	8.8003
Trans	0.0061721	0.0035677	0.073288	1.1642	16.4832
Whlsl	0.0052253	0.0023087	0.076574	0.69512	14.5684
Rtail	0.0074503	0.0055177	0.062017	0.046815	8.6814
Meals	0.0074474	0.0050969	0.067913	-0.34348	5.5525
Banks	0.0098968	0.0073329	0.071629	0.19938	8.2083
Insur	0.008158	0.0052558	0.077097	1.2091	19.8734
RlEst	0.0053157	0.00069534	0.096949	0.77275	9.5831
Fin	0.0086179	0.0055558	0.078608	0.59824	12.733
Other	0.0048273	0.0019969	0.07497	0.084923	6.7771

Table 2: Descriptive statistics for simple returns

	Means	**Std**	**Skewness**	**Kurtosis**
Agric	0.0036852	0.073884	0.084114	12.8522
Food	0.0057305	0.049454	-0.48958	8.9913
Soda	0.0047693	0.064603	-0.43209	6.9013
Beer	0.006479	0.073475	0.29016	11.8806
Smoke	0.0066812	0.059141	-0.41199	6.4635
Toys	0.0026347	0.097328	0.35994	13.0706
Fun	0.0052114	0.093754	-0.59771	10.6515
Books	0.0048125	0.07875	0.061096	8.0637
Hshld	0.0046913	0.060954	-0.67062	12.7042
Clths	0.0041934	0.060854	-0.2702	7.7474
Hlth	0.0024102	0.089995	-0.66845	6.4791
MedEq	0.0065	0.064299	-0.46166	5.1929
Drugs	0.0064882	0.059498	-0.5508	10.74
Chems	0.0055097	0.063053	-0.39554	9.8841
Rubbr	0.0055294	0.078319	0.7034	16.5828
Txtls	0.0039339	0.07163	-0.012223	8.8332
BldMt	0.0047062	0.068076	-0.31192	8.2602
Cnstr	0.003579	0.095244	-0.041824	7.8152
Steel	0.0033326	0.080992	0.30725	10.0841
FabPr	0.00015951	0.069312	-0.50631	4.776
Mach	0.0049314	0.072444	-0.29147	8.7644
ElcEq	0.0061942	0.07708	-0.37013	9.558
Autos	0.0051066	0.077482	-0.056654	10.5242
Aero	0.0069594	0.094849	-0.15696	8.5837
Ships	0.0030048	0.079424	-0.062392	8.0279
Guns	0.0043805	0.068813	-0.43537	5.4865
Gold	0.0015973	0.10159	0.045328	5.4733
Mines	0.0047143	0.068044	-0.34487	7.0841
Coal	0.0047757	0.095628	0.8456	12.103
Oil	0.0059451	0.060754	-0.1922	7.0472
Util	0.0043015	0.057741	-0.66145	11.1915
Telcm	0.0043901	0.045997	-0.28203	6.1893
PerSv	0.0025988	0.093276	0.11354	10.0438
BusSv	0.0042681	0.072456	-0.59819	12.1992
Hardw	0.0066108	0.075498	-0.58073	7.5959
Softw	-0.0030999	0.12473	-0.017286	4.9122
Chips	0.0046267	0.089773	-0.45116	7.6968
LabEq	0.0055918	0.071499	-0.67273	5.9033
Paper	0.0026824	0.12503	1.6402	38.3289
Boxes	0.0057555	0.062344	-0.40974	7.9998
Trans	0.0035614	0.071956	-0.059484	11.1324
Whlsl	0.0023061	0.076733	-0.65592	12.9507
Rtail	0.0055025	0.062366	-0.64911	8.9466
Meals	0.005084	0.068991	-0.81814	6.4506
Banks	0.0073061	0.071759	-0.59273	9.0526
Insur	0.0052421	0.076259	-0.54993	16.6633
RlEst	0.0006951	0.096406	-0.40021	10.0395
Fin	0.0055404	0.078418	-0.5249	10.2235
Other	0.0019949	0.075601	-0.54514	6.8812

Table 3: Descriptive statistics for log returns

	LBQ test 0011	**LBQ test 1111**	**LBQ test 0111**	**LBQ test 1011**
Agric	0.1104	0.05015	0.13289	0.13328
Food	0.01481	0.016721	0.018068	0.018377
Soda	0.011869	0.0035459	0.0082053	0.0078574
Beer	0.32141	0.27175	0.3593	0.35192
Smoke	0.5661	0.5055	0.54815	0.54554
Toys	0.10762	0.054437	0.099205	0.099504
Fun	0.0074542	0.0083365	0.018912	0.019745
Books	0.011315	0.012094	0.013773	0.013643
Hshld	0.051172	0.08593	0.065976	0.063399
Clths	0.028758	0.10535	0.09996	0.071099
Hlth	0.021069	0.019017	0.031656	0.03014
MedEq	0.073324	0.12766	0.071623	0.071772
Drugs	0.00090201	0.00068743	0.0017368	0.0018741
Chems	0.044676	0.091434	0.025703	0.028461
Rubbr	0.0094089	0.0057572	0.0043939	0.003261
Txtls	7.3601e-05	0.00043061	0.001966	0.0017128
BldMt	0.25476	0.12649	0.15971	0.15153
Cnstr	0.16701	0.25005	0.22858	0.21956
Steel	0.04418	0.046516	0.036244	0.036755
FabPr	0.074945	0.082679	0.10148	0.10085
Mach	0.0092048	0.0070863	0.0082487	0.0081885
ElcEq	0.070028	0.077363	0.23409	0.22434
Autos	0.032043	0.081924	0.068913	0.064514
Aero	0.12977	0.15091	0.12343	0.12278
Ships	0.062563	0.040418	0.060811	0.060735
Guns	0.13036	0.15322	0.11577	0.11364
Gold	0.0034218	0.0087277	0.0038824	0.0031296
Mines	0.095573	0.074076	0.089648	0.089897
Coal	0.045144	0.027249	0.030753	0.027179
Oil	0.070475	0.048442	0.075821	0.077253
Util	0.055193	0.28068	0.05493	0.055056
Telcm	0.028812	0.039995	0.0342	0.035074
PerSv	0.047925	0.041597	0.12499	0.11665
BusSv	0.068838	0.09558	0.096085	0.092729
Hardw	0.013399	0.0042322	0.012516	0.011815
Softw	0.12182	0.10934	0.14279	0.14092
Chips	0.10853	0.06103	0.10781	0.10766
LabEq	0.11618	0.058514	0.16087	0.16317
Paper	0.01778	0.024734	0.01148	0.011961
Boxes	0.3053	0.20869	0.27144	0.27251
Trans	0.24559	0.087564	0.19479	0.18867
Whlsl	0.0011148	0.00043479	0.0016481	0.0017519
Rtail	0.17088	0.11435	0.17956	0.17624
Meals	0.10961	0.051319	0.13571	0.13869
Banks	0.12351	0.064904	0.1789	0.17647
Insur	0.011405	0.021135	0.011148	0.011084
RlEst	0.00023626	0.20714	0.41059	0.12451
Fin	0.11136	0.18689	0.13345	0.13076
Other	0.035945	0.034803	0.03508	0.035272

Table 4: Ljung-Box Q-test results for GARCH(1,1), ARMA(1,1)-GARCH(1,1), MA(1)-GARCH(1,1), AR(1)-GARCH(1,1)

	LBQ test 0011	LBQ test 1111	LBQ test 0111	LBQ test 1011
Agric	0.039421	0.036418	0.042218	0.042313
Food	0.013906	0.010142	0.018223	0.018224
Soda	0.0029027	0.0053743	0.0031421	0.0029456
Beer	0.30174	0.17975	0.31056	0.30718
Smoke	0.51386	0.36286	0.5375	0.53665
Toys	0.054937	0.034883	0.072045	0.072265
Fun	0.0029348	0.0067887	0.0063381	0.0067774
Books	0.0051651	0.0022861	0.0030029	0.0030034
Hshld	0.044545	0.062484	0.025203	0.025087
Clths	0.028546	0.10342	0.11759	0.07888
Hlth	0.018809	0.054102	0.065517	0.062753
MedEq	0.054825	0.1129	0.062177	0.062375
Drugs	0.00031606	0.00034114	0.0010799	0.0011366
Chems	0.024431	0.083776	0.020976	0.0224
Rubbr	0.0080966	0.0024624	0.004583	0.0034302
Txtls	1.2843e-05	0.00067529	0.0006564	0.00036223
BldMt	0.17571	0.10278	0.10649	0.10163
Cnstr	0.15305	0.1604	0.20776	0.19992
Steel	0.034839	0.010547	0.0278	0.028283
FabPr	0.065105	0.035214	0.090309	0.090425
Mach	0.008474	0.0063764	0.0075272	0.0075049
ElcEq	0.035144	0.055132	0.095807	0.091474
Autos	0.030551	0.040942	0.065399	0.061591
Aero	0.1247	0.057056	0.082837	0.083712
Ships	0.046361	0.0314	0.026441	0.026594
Guns	0.12067	0.079138	0.15059	0.14641
Gold	0.0012546	0.0014173	0.0040359	0.0034253
Mines	0.056021	0.050101	0.069849	0.069985
Coal	0.043578	0.012065	0.013729	0.011334
Oil	0.029524	0.014026	0.032061	0.032759
Util	0.055071	0.27409	0.058799	0.058583
Telcm	0.0077346	0.032781	0.0094603	0.0096378
PerSv	0.013701	0.032114	0.032473	0.030597
BusSv	0.050663	0.067719	0.075358	0.073216
Hardw	0.0047234	0.00094676	0.0043712	0.0041121
Softw	0.11393	0.086783	0.13212	0.13024
Chips	0.057183	0.056914	0.065343	0.065176
LabEq	0.060175	0.03502	0.089422	0.091159
Paper	0.0090089	0.0083356	0.015513	0.015264
Boxes	0.20347	0.11359	0.21057	0.21773
Trans	0.13683	0.061348	0.067132	0.064935
Whlsl	0.00044143	1.6568e-05	0.00036033	0.00047552
Rtail	0.12178	0.053225	0.11435	0.11188
Meals	0.035484	0.020994	0.0201	0.021371
Banks	0.10032	0.021809	0.16532	0.16269
Insur	0.0075124	0.00096589	0.0020415	0.0018101
RlEst	0.00013798	0.12245	0.17755	0.071688
Fin	0.0372	0.19895	0.053751	0.051562
Other	0.034206	0.038543	0.040168	0.040301

Table 5: Ljung-Box Q-test results for GJR(1,1), ARMA(1,1)-GJR(1,1), MA(1)-GJR(1,1), AR(1)-GJR(1,1)

	Mean	**Volatility**	**Skewness**	**Kurtosis**	**VaR**
Momentum 1	0.088	0.095	-5.307	52.196	-0.118
Momentum 2	0.088	0.151	-1.110	52.698	-0.265
Momentum 3	0.127	0.166	-2.823	42.091	-0.239
Momentum 4	-0.034	0.229	-3.613	100.078	-0.294
Momentum 5	0.046	0.074	-13.603	91.678	-0.085
Gross Exposure	0.032	0.126	-23.789	164.331	-0.173
Gross Exposure Garch	0.106	0.257	-7.177	50.005	-0.468
Max Sharpe Ratio	0.060	0.149	-9.018	56.654	-0.290
Max Sharpe Ratio Shrinkage	0.072	0.149	-8.888	56.792	-0.298
Max Sharpe Ratio Garch	0.085	0.154	-7.645	51.549	-0.289
Global Min Variance	0.073	0.133	-16.183	97.303	-0.213
Global Min Variance Shrinkage	0.069	0.132	-15.162	90.010	-0.224
Global Min Variance Garch	0.103	0.130	-14.599	82.164	-0.198
PCA Equally Weighted	0.020	0.047	-1.340	46.312	-0.063
PCA Risk Parity	0.007	0.031	1.144	38.164	-0.045
Risk Parity	0.098	0.162	-9.126	70.808	-0.263
Risk Parity Shrinkage	0.098	0.164	-8.850	70.717	-0.265
Risk Parity Garch	0.099	0.161	-9.478	68.721	-0.258
Betting against Beta	0.053	0.048	0.141	39.849	-0.065
FOA	0.062	0.149	-8.922	56.330	-0.292
FOA2	0.137	0.189	1.203	48.945	-0.319
FOA3	0.084	0.148	-9.055	59.563	-0.246
CMA Replication	0.025	0.057	1.246	48.492	-0.093
HML Replication	0.006	0.106	-23.551	219.338	-0.114
MKT Replication	0.079	0.152	-8.537	73.559	-0.232
RMW Replication	0.018	0.047	-9.299	56.523	-0.065
SMB Replication	0.033	0.075	0.495	47.064	-0.101
Zero Exposure	0.034	0.069	-31.316	247.877	-0.109
GE Static	0.011	0.126	-23.790	164.354	-0.177
GMV Static	0.078	0.132	-16.133	98.460	-0.186
MSR Static	0.080	0.144	-8.668	65.150	-0.230
GE GJR	0.108	0.255	-7.378	45.003	-0.466
GMV GJR	0.114	0.126	-14.159	89.829	-0.190
MSR GJR	0.091	0.157	-8.156	50.704	-0.307
Risk Parity GJR	0.100	0.161	-9.596	70.022	-0.260
Reversal 1	-0.073	0.165	13.437	76.102	-0.250
Reversal 2	0.092	0.243	2.680	94.914	-0.334
Reversal 3	0.009	0.088	19.186	125.893	-0.118
Eq. Weighted	0.099	0.174	-7.130	70.906	-0.276
Eq. Weighted Top	0.091	0.192	-4.954	59.304	-0.320

Table 6: Descriptive statistics values for total returns

	Mean	**Volatility**	**Skewness**	**Kurtosis**	**VaR**
Momentum 1	16	10	14	12	10
Momentum 2	15	24	9	13	28
Momentum 3	2	33	11	3	21
Momentum 4	39	37	12	35	33
Momentum 5	28	7	31	31	5
Gross Exposure	31	14	38	37	12
Gross Exposure Garch	5	40	16	9	40
Max Sharpe Ratio	26	23	25	16	31
Max Sharpe Ratio Shrinkage	23	22	23	17	34
Max Sharpe Ratio Garch	17	26	18	11	30
Global Min Variance	22	18	36	33	17
Global Min Variance Shrinkage	24	17	34	30	18
Global Min Variance Garch	6	15	33	28	16
PCA Equally Weighted	33	3	10	5	2
PCA Risk Parity	37	1	6	1	1
Risk Parity	11	30	27	24	26
Risk Parity Shrinkage	10	31	22	23	27
Risk Parity Garch	8	28	29	21	24
Betting against Beta	27	4	8	2	3
FOA	25	21	24	14	32
FOA2	1	35	5	8	36
FOA3	18	20	26	19	22
CMA Replication	32	5	4	7	6
HML Replication	38	11	37	39	9
MKT Replication	20	25	20	26	20
RMW Replication	34	2	28	15	4
SMB Replication	30	8	7	6	7
Zero Exposure	29	6	40	40	8
GE Static	35	12	39	38	13
GMV Static	21	16	35	34	14
MSR Static	19	19	21	20	19
GE GJR	4	39	17	4	39
GMV GJR	3	13	32	29	15
MSR GJR	14	27	19	10	35
Risk Parity GJR	7	29	30	22	25
Reversal 1	40	32	2	27	23
Reversal 2	12	38	3	32	38
Reversal 3	36	9	1	36	11
Eq. Weighted	9	34	15	25	29
Eq. Weighted Top	13	36	13	18	37

Table 7: Descriptive statistics ranking for total returns

	Mean	**Volatility**	**Skewness**	**Kurtosis**	**VaR**
Momentum 1	0.078	0.095	-4.961	52.291	-0.129
Momentum 2	0.078	0.151	-0.717	52.605	-0.266
Momentum 3	0.118	0.166	-2.591	42.082	-0.240
Momentum 4	-0.044	0.229	-3.319	100.043	-0.295
Momentum 5	0.037	0.074	-13.021	91.221	-0.086
Gross Exposure	0.022	0.126	-23.708	164.460	-0.174
Gross Exposure Garch	0.097	0.257	-7.154	49.773	-0.468
Max Sharpe Ratio	0.051	0.149	-8.662	56.347	-0.290
Max Sharpe Ratio Shrinkage	0.062	0.149	-8.555	56.467	-0.298
Max Sharpe Ratio Garch	0.076	0.154	-7.387	51.402	-0.290
Global Min Variance	0.064	0.133	-15.749	96.425	-0.214
Global Min Variance Shrinkage	0.060	0.132	-14.742	89.276	-0.225
Global Min Variance Garch	0.094	0.130	-14.152	81.231	-0.203
PCA Equally Weighted	0.011	0.047	-0.351	46.753	-0.063
PCA Risk Parity	-0.002	0.030	2.106	39.284	-0.047
Risk Parity	0.088	0.162	-8.729	70.319	-0.263
Risk Parity Shrinkage	0.089	0.164	-8.461	70.258	-0.265
Risk Parity Garch	0.090	0.161	-9.085	68.211	-0.259
Betting against Beta	0.044	0.048	0.751	38.977	-0.067
FOA	0.052	0.149	-8.571	56.032	-0.293
FOA2	0.127	0.188	0.855	48.323	-0.321
FOA3	0.075	0.148	-8.641	59.082	-0.246
CMA Replication	0.016	0.057	1.909	49.420	-0.098
HML Replication	-0.003	0.107	-23.615	222.416	-0.115
MKT Replication	0.070	0.152	-8.117	73.077	-0.232
RMW Replication	0.008	0.047	-8.381	55.432	-0.066
SMB Replication	0.024	0.075	0.871	47.736	-0.114
Zero Exposure	0.025	0.068	-31.316	251.299	-0.110
GE Static	0.002	0.126	-23.752	164.944	-0.177
GMV Static	0.068	0.132	-15.690	97.621	-0.192
MSR Static	0.070	0.144	-8.255	64.710	-0.231
GE GJR	0.098	0.255	-7.363	44.808	-0.469
GMV GJR	0.104	0.126	-13.650	88.600	-0.193
MSR GJR	0.082	0.156	-7.903	50.554	-0.312
Risk Parity GJR	0.090	0.161	-9.198	69.492	-0.261
Reversal 1	-0.083	0.166	13.522	76.754	-0.250
Reversal 2	0.082	0.243	2.991	95.122	-0.334
Reversal 3	-0.000	0.088	19.548	127.457	-0.119
Eq. Weighted	0.089	0.175	-6.873	70.714	-0.276
Eq. Weighted Top	0.082	0.192	-4.745	59.277	-0.320

Table 8: Descriptive statistics values for excess returns

	Mean	Volatility	Skewness	Kurtosis	VaR
Momentum 1	16	10	14	12	11
Momentum 2	15	24	10	13	28
Momentum 3	2	33	11	3	21
Momentum 4	39	37	12	35	33
Momentum 5	28	7	31	31	5
Gross Exposure	31	13	38	37	12
Gross Exposure Garch	5	40	16	9	39
Max Sharpe Ratio	26	23	27	16	31
Max Sharpe Ratio Shrinkage	23	22	24	17	34
Max Sharpe Ratio Garch	17	26	18	11	30
Global Min Variance	22	18	36	33	17
Global Min Variance Shrinkage	24	17	34	30	18
Global Min Variance Garch	6	15	33	28	16
PCA Equally Weighted	33	3	9	5	2
PCA Risk Parity	37	1	4	2	1
Risk Parity	11	30	28	24	26
Risk Parity Shrinkage	10	31	23	23	27
Risk Parity Garch	8	28	29	21	24
Betting against Beta	27	4	8	1	4
FOA	25	21	25	15	32
FOA2	1	35	7	7	37
FOA3	18	20	26	18	22
CMA Replication	32	5	5	8	6
HML Replication	38	11	37	39	9
MKT Replication	20	25	20	26	20
RMW Replication	34	2	22	14	3
SMB Replication	30	8	6	6	8
Zero Exposure	29	6	40	40	7
GE Static	35	12	39	38	13
GMV Static	21	16	35	34	14
MSR Static	19	19	21	20	19
GE GJR	4	39	17	4	40
GMV GJR	3	14	32	29	15
MSR GJR	14	27	19	10	35
Risk Parity GJR	7	29	30	22	25
Reversal 1	40	32	2	27	23
Reversal 2	12	38	3	32	38
Reversal 3	36	9	1	36	10
Eq. Weighted	9	34	15	25	29
Eq. Weighted Top	13	36	13	19	36

Table 9: Descriptive statistics ranking for excess returns

	Sharpe Ratio	Semi Vol.	Sortino Ratio	Inf. Ratio	Jensen Alpha	Treynor Ratio	App. Ratio
Momentum 1	0.827	0.069	1.141	-0.008	0.075	2.124	0.019
Momentum 2	0.518	0.110	0.714	-0.006	0.082	-2.001	0.013
Momentum 3	0.710	0.120	0.982	0.187	0.103	0.633	0.015
Momentum 4	-0.192	0.160	-0.274	-0.420	-0.025	0.180	-0.003
Momentum 5	0.502	0.057	0.652	-0.248	0.039	-1.281	0.013
Gross Exposure	0.177	0.101	0.220	-0.275	0.030	-0.226	0.006
GE Garch	0.377	0.193	0.501	0.058	0.095	3.500	0.009
MSR	0.341	0.114	0.445	-0.142	0.043	0.527	0.007
MSR Shrinkage	0.418	0.114	0.547	-0.087	0.054	0.635	0.009
MSR Garch	0.492	0.117	0.650	-0.019	0.066	0.606	0.010
GMV	0.479	0.104	0.609	-0.085	0.056	0.687	0.010
GMV Shrinkage	0.451	0.104	0.575	-0.106	0.052	0.661	0.010
GMV Garch	0.720	0.102	0.915	0.074	0.088	1.206	0.016
PCA Eq. Weighted	0.224	0.033	0.318	-0.447	0.008	0.377	0.004
PCA Risk Parity	-0.067	0.021	-0.096	-0.530	-0.002	-0.610	-0.002
Risk Parity	0.546	0.123	0.717	0.042	0.078	0.657	0.012
Risk Parity Shrink	0.541	0.125	0.712	0.044	0.078	0.641	0.011
Risk Parity Garch	0.558	0.123	0.730	0.049	0.079	0.666	0.012
Betting against Beta	0.909	0.034	1.281	-0.222	0.046	-1.745	0.023
FOA	0.352	0.114	0.460	-0.136	0.045	0.550	0.007
FOA2	0.678	0.132	0.968	0.203	0.123	2.140	0.016
FOA3	0.505	0.113	0.663	-0.025	0.068	0.849	0.011
CMA Replication	0.275	0.040	0.390	-0.408	0.013	0.426	0.005
HML Replication	-0.031	0.082	-0.041	-0.477	-0.011	-0.037	-0.002
MKT Replication	0.458	0.115	0.605	-0.050	0.058	0.487	0.009
RMW Replication	0.176	0.035	0.231	-0.450	0.008	-3.967	0.004
SMB Replication	0.321	0.052	0.461	-0.350	0.018	0.344	0.006
Zero Exposure	0.366	0.057	0.441	-0.337	0.023	0.937	0.008
GE Static	0.013	0.101	0.016	-0.382	0.007	-0.025	0.001
GMV Static	0.518	0.104	0.661	-0.059	0.062	0.838	0.011
MSR Static	0.489	0.109	0.648	-0.047	0.060	0.552	0.010
GE GJR	0.385	0.193	0.510	0.063	0.096	3.222	0.009
GMV GJR	0.826	0.098	1.067	0.131	0.099	1.432	0.019
MSR GJR	0.521	0.119	0.686	0.009	0.070	0.574	0.011
Risk Parity GJR	0.561	0.123	0.734	0.052	0.079	0.642	0.012
Reversal 1	-0.499	0.103	-0.799	-0.695	-0.076	0.940	-0.011
Reversal 2	0.340	0.170	0.484	0.010	0.064	0.349	0.006
Reversal 3	-0.001	0.053	-0.002	-0.482	-0.006	-0.002	-0.002
Eq. Weighted	0.511	0.131	0.683	0.235	-0.001	0.079	-0.000
Eq. Weighted Top	0.427	0.143	0.575	0.037	-0.015	0.068	-0.007

Table 10: Risk adjusted and model based risk measures values

	Sharpe Ratio	Semi Vol.	Sortino Ratio	Inf. Ratio	Jensen Alpha	Treynor Ratio	App. Ratio
Momentum 1	2	10	2	16	12	4	2
Momentum 2	13	21	10	15	7	39	7
Momentum 3	5	29	4	3	2	18	6
Momentum 4	39	37	39	34	39	29	38
Momentum 5	16	9	16	28	26	37	8
GE	33	14	34	29	27	35	29
GE Garch	25	40	25	7	5	1	22
MSR	28	25	29	26	25	23	26
MSR Shrinkage	23	23	23	23	21	17	23
MSR Garch	17	27	17	17	15	19	16
GMV	19	19	19	22	20	11	17
GMV Shrink	21	18	22	24	22	13	19
GMV Garch	4	15	6	5	6	6	4
PCA Eq. Weighted	32	2	32	35	32	26	32
PCA Risk Parity	38	1	38	39	35	36	36
Risk Parity	9	32	9	11	11	14	11
Risk Parity Shrinkage	10	33	11	10	10	16	12
Risk Parity Garch	8	30	8	9	8	12	9
Betting against Beta	1	3	1	27	23	38	1
FOA	27	24	28	25	24	22	25
FOA2	6	35	5	2	1	3	5
FOA3	15	22	14	18	14	9	14
CMA Replication	31	5	31	33	30	25	30
HML Replication	37	11	37	37	37	34	37
MKT Replication	20	26	20	20	19	24	20
RMW Replication	34	4	33	36	31	40	31
SMB Replication	30	6	27	31	29	28	28
Zero Exposure	26	8	30	30	28	8	24
GE Static	35	13	35	32	33	33	33
GMV Static	12	17	15	21	17	10	13
MSR Static	18	20	18	19	18	21	18
GE GJR	24	39	24	6	4	2	21
GMV GJR	3	12	3	4	3	5	3
MSR GJR	11	28	12	14	13	20	15
Risk Parity GJR	7	31	7	8	9	15	10
Reversal 1	40	16	40	40	40	7	40
Reversal 2	29	38	26	13	16	27	27
Reversal 3	36	7	36	38	36	32	35
Eq. Weighted	14	34	13	1	34	30	34
Eq. Weighted Top	22	36	21	12	38	31	39

Table 11: Risk adjusted and model based risk measures ranking

	Intercept	r_m	R^2_{adj}
Equally Weighted	0.000	1.130	0.958
Equally Weighted Top 10	-0.001	1.216	0.919
PCA Equally Weighted	-0.001	0.217	0.486
Global Min Variance	-0.000	0.822	0.880
Global Min Variance, shrinkage	-0.000	0.820	0.883
Global Minimum Variance Static	0.000	0.814	0.870
Global Minimum Variance Garch	0.002	0.814	0.895
Global Minimum Variance GJR	0.003	0.712	0.808
4th order Minimum Risk	0.001	0.859	0.777
Max Sharpe Ratio	0.000	0.612	0.381
Max Sharpe Ratio, shrinkage	0.001	0.628	0.404
Max Sharpe Ratio Static	0.002	0.627	0.431
Max Sharpe Ratio Garch	0.002	0.663	0.421
Max Sharpe Ratio GJR	0.002	0.595	0.366
4th order Sharpe ratio	0.000	0.614	0.387
Risk Parity	0.000	1.055	0.976
Risk Parity, shrinkage	0.000	1.070	0.975
PCA Risk Parity	-0.001	0.081	0.156
Risk Parity Garch	0.001	1.050	0.977
Risk Parity GJR	0.000	0.937	0.860
Gross Exposure	0.002	-0.002	-0.008
Gross Exposure Static	-0.000	0.039	-0.005
Gross Exposure Garch	0.005	0.425	0.056
Gross Exposure GJR	0.007	0.391	0.072
Betting Against Beta	0.004	-0.002	-0.008
MKT Replication	-0.001	0.986	0.963
SMB Replication	0.000	0.261	0.275
HML Replication	-0.001	0.185	0.062
RMW Replication	0.001	-0.052	0.021
CMA Replication	0.001	0.098	0.060
Zero Exposure Replication	0.001	0.187	0.166
Momentum1	0.004	0.392	0.387
Momentum2	0.008	-0.217	0.040
Momentum3	0.004	0.961	0.767
Momentum4	0.005	-1.357	0.807
Momentum5	0.004	-0.198	0.160
Reversal1	-0.001	-0.946	0.747
Reversal2	-0.002	1.406	0.769
Reversal3	-0.002	0.230	0.148
Fourth Order Utility Expansion	0.006	0.685	0.300

Table 12: Risk factor exposures in the CAPM model

	Intercept	r_m	**SMB**	**HML**	R^2_{adj}
Equally Weighted	0.000	1.081	0.204	0.046	0.966
Equally Weighted Top 10	-0.001	1.169	0.169	0.074	0.924
PCA Equally Weighted	-0.001	0.215	0.053	-0.045	0.492
Global Min Variance	-0.000	0.855	-0.108	-0.063	0.884
Global Min Variance, shrinkage	-0.000	0.861	-0.101	-0.107	0.890
Global Minimum Variance Static	0.000	0.850	-0.101	-0.084	0.875
Global Minimum Variance Garch	0.002	0.857	-0.144	-0.080	0.903
Global Minimum Variance GJR	0.003	0.719	-0.058	0.026	0.806
4th order Minimum Risk	0.000	0.887	-0.039	-0.105	0.778
Max Sharpe Ratio	-0.000	0.765	-0.410	-0.375	0.468
Max Sharpe Ratio, shrinkage	0.001	0.774	-0.374	-0.377	0.486
Max Sharpe Ratio Static	0.001	0.765	-0.243	-0.467	0.526
Max Sharpe Ratio Garch	0.002	0.829	-0.397	-0.456	0.523
Max Sharpe Ratio GJR	0.002	0.722	-0.347	-0.305	0.424
4th order Sharpe ratio	-0.000	0.760	-0.392	-0.361	0.467
Risk Parity	0.000	1.027	0.124	0.021	0.979
Risk Parity, shrinkage	0.000	1.038	0.140	0.025	0.979
PCA Risk Parity	-0.001	0.083	0.032	-0.044	0.165
Risk Parity Garch	0.001	1.021	0.133	0.018	0.981
Risk Parity GJR	0.001	0.902	0.108	0.072	0.863
Gross Exposure	0.001	0.168	-0.195	-0.682	0.231
Gross Exposure Static	-0.001	0.202	-0.164	-0.677	0.226
Gross Exposure Garch	0.004	0.777	-0.651	-1.161	0.248
Gross Exposure GJR	0.006	0.603	-0.307	-0.786	0.184
Betting Against Beta	0.004	0.069	-0.284	-0.081	0.203
MKT Replication	-0.001	0.955	0.134	0.022	0.967
SMB Replication	0.000	0.224	0.206	-0.015	0.307
HML Replication	-0.001	0.086	0.122	0.391	0.164
RMW Replication	0.001	0.011	-0.189	-0.134	0.171
CMA Replication	0.001	0.123	-0.076	-0.051	0.062
Zero Exposure Replication	0.001	0.221	-0.118	-0.060	0.177
Momentum1	0.004	0.400	-0.021	-0.025	0.378
Momentum2	0.008	-0.199	-0.042	-0.049	0.026
Momentum3	0.004	0.945	0.124	-0.040	0.767
Momentum4	0.005	-1.271	-0.364	-0.075	0.819
Momentum5	0.004	-0.163	-0.120	-0.057	0.167
Reversal1	-0.000	-1.014	-0.099	0.451	0.804
Reversal2	-0.002	1.245	0.296	0.531	0.813
Reversal3	-0.001	0.115	0.099	0.491	0.393
Fourth Order Utility Expansion	0.006	0.807	-0.096	-0.535	0.356

Table 13: Risk factor exposures in the Fama-French 3-factor model

	Intercept	r_m	**SMB**	**HML**	**WML**	R^2_{adj}
Equally Weighted	0.001	1.058	0.214	-0.011	-0.113	0.971
Equally Weighted Top 10	-0.000	1.133	0.185	-0.013	-0.171	0.933
PCA Equally Weighted	-0.000	0.208	0.056	-0.063	-0.035	0.495
GMV	-0.000	0.860	-0.110	-0.051	0.024	0.883
GMV, shrinkage	-0.001	0.872	-0.106	-0.079	0.056	0.891
GMV Static	0.000	0.853	-0.103	-0.076	0.016	0.874
GMV Garch	0.002	0.879	-0.154	-0.027	0.105	0.911
GMV GJR	0.003	0.728	-0.062	0.049	0.046	0.807
4th order Minimum Risk	-0.000	0.913	-0.050	-0.042	0.123	0.785
Max Sharpe Ratio	-0.001	0.790	-0.421	-0.314	0.121	0.472
Max Sharpe Ratio, shrinkage	0.000	0.808	-0.389	-0.297	0.159	0.496
Max Sharpe Ratio Static	0.001	0.793	-0.256	-0.400	0.132	0.533
Max Sharpe Ratio Garch	0.001	0.872	-0.416	-0.352	0.206	0.542
Max Sharpe Ratio GJR	0.001	0.766	-0.366	-0.200	0.208	0.444
4th order Sharpe ratio	-0.001	0.785	-0.403	-0.300	0.120	0.471
Risk Parity	0.001	1.012	0.131	-0.015	-0.072	0.982
Risk Parity, shrinkage	0.001	1.021	0.147	-0.015	-0.079	0.981
PCA Risk Parity	-0.001	0.076	0.035	-0.061	-0.034	0.175
Risk Parity Garch	0.001	1.008	0.138	-0.012	-0.059	0.982
Risk Parity GJR	0.001	0.899	0.109	0.066	-0.014	0.862
Gross Exposure	-0.001	0.292	-0.249	-0.385	0.590	0.502
Gross Exposure Static	-0.003	0.322	-0.216	-0.388	0.571	0.481
Gross Exposure Garch	-0.000	0.996	-0.747	-0.634	1.044	0.452
Gross Exposure GJR	0.002	0.795	-0.391	-0.323	0.917	0.417
Betting Against Beta	0.003	0.088	-0.293	-0.035	0.091	0.242
MKT Replication	-0.000	0.933	0.144	-0.031	-0.105	0.973
SMB Replication	0.000	0.224	0.206	-0.014	0.001	0.301
HML Replication	-0.001	0.079	0.125	0.375	-0.031	0.159
RMW Replication	0.001	0.025	-0.195	-0.099	0.069	0.192
CMA Replication	0.000	0.142	-0.084	-0.004	0.093	0.089
Zero Exposure Replication	0.000	0.238	-0.125	-0.020	0.081	0.188
Momentum1	0.004	0.420	-0.030	0.022	0.092	0.385
Momentum2	0.007	-0.161	-0.059	0.043	0.183	0.037
Momentum3	0.003	0.956	0.119	-0.012	0.056	0.767
Momentum4	0.003	-1.181	-0.404	0.143	0.432	0.863
Momentum5	0.003	-0.112	-0.143	0.066	0.244	0.300
Reversal1	0.001	-1.054	-0.081	0.356	-0.187	0.819
Reversal2	0.001	1.129	0.346	0.252	-0.553	0.878
Reversal3	0.001	0.038	0.132	0.304	-0.370	0.610
F4th Utility Expansion	0.004	0.887	-0.130	-0.343	0.380	0.403

Table 14: Risk factor exposures in the Carhart model

	Intercept	r_m	**SMB**	**HML**	**RMW**	**CMA**	R^2_{adj}
Equally Weighted	0.010	0.685	0.446	-0.855	-0.511	-1.211	0.346
Equally Weighted Top 10	0.010	0.687	0.532	-0.939	-0.632	-0.834	0.330
PCA Equally Weighted	0.001	0.156	0.038	-0.124	-0.102	-0.081	0.121
GMV	0.008	0.273	0.257	-0.702	-0.287	-0.828	0.195
GMV, shrinkage	0.007	0.270	0.201	-0.749	-0.249	-0.704	0.192
GMV Static	0.008	0.281	0.214	-0.684	-0.208	-0.883	0.178
GMV Garch	0.011	0.231	0.240	-0.703	-0.255	-1.101	0.180
GMV GJR	0.010	0.249	0.289	-0.626	-0.242	-1.323	0.221
4th order Minimum Risk	0.008	0.345	0.140	-0.747	-0.004	-1.009	0.161
MSR	0.008	-0.144	-0.187	-0.959	0.176	-1.119	0.068
MSR, shrink	0.008	-0.093	-0.163	-0.905	0.089	-0.861	0.060
MSR Static	0.009	0.046	-0.269	-0.817	0.105	-1.410	0.068
MSR Garch	0.010	-0.116	-0.227	-1.054	0.066	-0.476	0.087
MSR GJR	0.009	-0.122	-0.088	-0.963	-0.013	-0.082	0.074
4th order Sharpe ratio	0.008	-0.126	-0.175	-0.950	0.170	-1.162	0.068
Risk Parity	0.010	0.580	0.397	-0.824	-0.439	-1.420	0.323
Risk Parity, shrinkage	0.010	0.600	0.406	-0.833	-0.454	-1.375	0.329
PCA Risk Parity	-0.000	0.077	-0.004	-0.025	-0.067	0.010	0.025
Risk Parity Garch	0.011	0.579	0.388	-0.840	-0.429	-1.399	0.328
Risk Parity GJR	0.010	0.479	0.399	-0.828	-0.399	-1.273	0.313
Gross Exposure	0.002	-0.209	-0.730	-0.447	0.371	0.980	0.228
Gross Exposure Static	0.000	-0.170	-0.637	-0.461	0.071	1.403	0.203
Gross Exposure Garch	0.011	-0.351	-0.705	-0.711	-0.589	1.007	0.098
Gross Exposure GJR	0.010	-0.095	-0.495	-0.628	-0.350	1.910	0.050
Betting Against Beta	0.004	-0.235	-0.043	0.007	0.097	-0.277	0.162
MKT Replication	0.009	0.547	0.392	-0.802	-0.503	-1.367	0.343
SMB Replication	0.001	0.303	-0.018	-0.262	0.085	1.345	0.175
HML Replication	0.002	0.228	0.370	0.219	0.208	-4.243	0.207
RMW Replication	0.001	-0.206	-0.146	-0.091	0.136	-0.436	0.191
CMA Replication	0.002	-0.048	-0.100	-0.205	0.320	-0.788	0.040
Zero Exp. Replication	0.003	-0.078	-0.027	-0.410	0.094	0.598	0.054
Momentum1	0.009	0.105	0.047	-0.484	0.116	-1.454	0.100
Momentum2	0.007	-0.209	-0.299	-0.085	0.713	-1.157	0.023
Momentum3	0.011	0.549	0.256	-0.764	-0.230	-0.081	0.223
Momentum4	-0.007	-0.942	-0.621	0.972	0.950	1.394	0.343
Momentum5	0.002	-0.196	-0.183	0.104	0.360	0.657	0.125
Reversal1	-0.009	-0.545	-0.028	0.782	0.810	-0.721	0.221
Reversal2	0.009	0.881	0.938	-0.937	-0.423	-0.952	0.375
Reversal3	0.000	0.168	0.455	-0.078	0.193	-0.837	0.368
4th Utility Expansion	0.008	0.290	-0.147	-0.650	-0.640	5.907	0.091

Table 15: Risk factor exposures in the Fama-French 5-factor model