# DEMCEM: a Matlab/C++ package for the computation of singular integrals arising in Galerkin EM SIE formulations

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**Abstract**

Direct Evaluation Method in Computational ElectroMagnetics DEMCEM is a package in both Matlab and C++ for the accurate and efficient evaluation of singular integrals arising in Galerkin EM surface integral equation formulations over triangular tessellations. All aspects of the algorithms implemented in this package have been described in the articles found in references [1-4].

## 1 Introduction

The main building blocks in EM SIE formulations are the (electric) *Maxwell single layer potential*

$$\boldsymbol{\mathcal{L}}(\mathbf{f})(\mathbf{r}) := ik\boldsymbol{\mathcal{S}}(\mathbf{f})(\mathbf{r}) - \frac{1}{ik}\nabla\mathcal{S}(\nabla_s' \cdot \mathbf{f})(\mathbf{r}) \tag{1}$$

and the (electric) *Maxwell double layer potential*

$$\boldsymbol{\mathcal{K}}(\mathbf{f})(\mathbf{r}) := \nabla \times \boldsymbol{\mathcal{S}}(\mathbf{f})(\mathbf{r}) \tag{2}$$

where

$$\boldsymbol{\mathcal{S}}(\mathbf{f})(\mathbf{r}) := \int\limits_{\Gamma} G(\mathbf{r},\mathbf{r}')\mathbf{f}(\mathbf{r}')dS' \tag{3}$$

and

$$\mathcal{S}(f)(\mathbf{r}) := \int\limits_{\Gamma} G(\mathbf{r},\mathbf{r}')f(\mathbf{r}')dS' \tag{4}$$

are the associated single layer (acoustic) potentials. Also, $\mathbf{r} \notin \Gamma$ and $G(\mathbf{r},\mathbf{r}') = e^{-ik|\mathbf{r}-\mathbf{r}'|}/4\pi|\mathbf{r}-\mathbf{r}'|$ is the homogeneous Green's function.

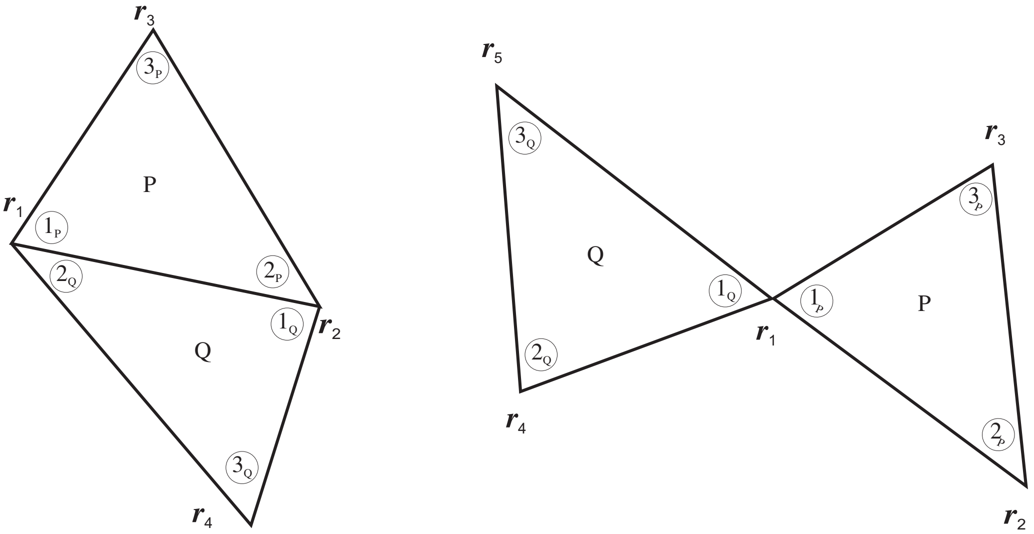
Following a Galerkin discretization method over planar triangular tessellations, we encounter the evaluation of the following singular integrals:

$$(I^l_{\boldsymbol{\mathcal{L}}})_{m,n} := ik\int_{E_P} \mathbf{f}_m \cdot \int_{E_Q} G\,\mathbf{f}_n\,dS'dS + \frac{1}{ik}\int_{E_P} \nabla_s \cdot \mathbf{f}_m \int_{E_Q} G\,\nabla_s' \cdot \mathbf{f}_n\,dS'dS \tag{5}$$

and

$$(I^p_{\boldsymbol{\mathcal{K}}})_{m,n} := \int_{E_P} \mathbf{f}_m \cdot \int_{E_Q} \nabla G\, \times \mathbf{f}_n\,dS'dS \tag{6}$$

where $l \in \{\mathrm{ST},\mathrm{EA},\mathrm{VA}\}$ and $p \in \{\mathrm{EA},\mathrm{VA}\}$ stand for the cases of coincident (or self-term, ST), edge adjacent (EA) and vertex adjacent (VA) observation and source triangles, $E_P$ and $E_Q$, respectively.




(a) Edge-adjacent case. (b) Vertex-adjacent case.

Figure 1: Orientation of the triangular elements in DEMCEM.

## 2 Weakly Singular Integrals

DIRECT_WS_{ST,EA,VA}_RWG subfolders contain codes for the evaluation of the weakly singular integrals $(I^l_{\mathcal{L}})_{m,n}$. The basic output parameters for all three cases are given as follows:

$$
\begin{aligned}
I_{\mathrm{DE}}(1) &\to (I^l_{\mathcal{L}})_{1,1} \\
I_{\mathrm{DE}}(2) &\to (I^l_{\mathcal{L}})_{1,2} \\
I_{\mathrm{DE}}(3) &\to (I^l_{\mathcal{L}})_{1,3} \\
I_{\mathrm{DE}}(4) &\to (I^l_{\mathcal{L}})_{2,1} \\
I_{\mathrm{DE}}(5) &\to (I^l_{\mathcal{L}})_{2,2} \\
I_{\mathrm{DE}}(6) &\to (I^l_{\mathcal{L}})_{2,3} \\
I_{\mathrm{DE}}(7) &\to (I^l_{\mathcal{L}})_{3,1} \\
I_{\mathrm{DE}}(8) &\to (I^l_{\mathcal{L}})_{3,2} \\
I_{\mathrm{DE}}(9) &\to (I^l_{\mathcal{L}})_{3,3}
\end{aligned}
$$

omitting the constant $1/4\pi$ in homogeneous Green's function $G$! Basis and testing functions are the traditional RWG. `EXAMPLE_WS_{ST,EA,VA}_RWG.m` scripts are used as examples to call the main codes:

- Self-term

  `I_DE = DIRECT_WS_ST_RWG(r1,r2,r3,Np_1D)`

- Edge adjacent

  `I_DE = DIRECT_WS_EA_RWG(r1,r2,r3,r4,Np_theta,Np_psi)`

- Vertex adjacent

  `I_DE = DIRECT_WS_VA_RWG(r1,r2,r3,r4,r5,Np_theta_p,Np_theta_q,Np_psi)`

Note that the nodes of the associated triangles, especially for the EA and VA cases, follow the orientation depicted in Fig.1 (a) and (b), respectively. Wavenumber, denoted as $k_o$ is declared as a global variable. Moreover, it is better to keep the same order for 1-D quadratures in EA and VA cases, i.e., `Np_theta=Np_psi` and `Np_theta_p=Np_theta_q=Np_psi`. Detailed comments regarding the input and output parameters are also provided in each separate main function.

## 3 Strongly Singular Integrals

DIRECT_SS_{EA,VA}_{RWG,nxRWG} subfolders contain codes for the evaluation of the strongly singular integrals $(I^p_{\mathcal{K}})_{m,n}$ with RWG and nxRWG testing functions. In all cases, RWG basis functions are considered. Also, EA and VA stand for the edge adjacent and vertex adjacent triangles (coincident integrals' case is identically zero).

The basic output parameters for both cases are given as follows:

$$
\begin{aligned}
I_{\mathrm{DE}}(1) &\rightarrow (I^p_{\mathcal{K}})_{1,1}\\
I_{\mathrm{DE}}(2) &\rightarrow (I^p_{\mathcal{K}})_{1,2}\\
I_{\mathrm{DE}}(3) &\rightarrow (I^p_{\mathcal{K}})_{1,3}\\
I_{\mathrm{DE}}(4) &\rightarrow (I^p_{\mathcal{K}})_{2,1}\\
I_{\mathrm{DE}}(5) &\rightarrow (I^p_{\mathcal{K}})_{2,2}\\
I_{\mathrm{DE}}(6) &\rightarrow (I^p_{\mathcal{K}})_{2,3}\\
I_{\mathrm{DE}}(7) &\rightarrow (I^p_{\mathcal{K}})_{3,1}\\
I_{\mathrm{DE}}(8) &\rightarrow (I^p_{\mathcal{K}})_{3,2}\\
I_{\mathrm{DE}}(9) &\rightarrow (I^p_{\mathcal{K}})_{3,3}
\end{aligned}
$$

Note again that the constant $1/4\pi$ in homogeneous Green's function $G$ is omitted!

EXAMPLE_SS_{ST,EA,VA}_{RWG,nxRWG}.m scripts are used as examples to call the main codes:

- Edge adjacent with RWG tesing functions

  `I_DE = DIRECT_SS_EA_RWG(r1,r2,r3,r4,Np_theta,Np_psi)`

- Edge adjacent with nxRWG tesing functions

  `I_DE = DIRECT_SS_EA_nxRWG(r1,r2,r3,r4,Np_theta,Np_psi)`

- Vertex adjacent with RWG tesing functions

  `I_DE = DIRECT_SS_VA_RWG(r1,r2,r3,r4,r5,Np_theta_p,Np_theta_q,Np_psi)`

- Vertex adjacent with nxRWG tesing functions

  `I_DE = DIRECT_SS_VA_nxRWG(r1,r2,r3,r4,r5,Np_theta_p,Np_theta_q,Np_psi)`

Detailed comments regarding the input and output parameters are also provided in the Section 2 as well as in each separate main function.

## 4 Implementation in C

DEMCEM package in C follows the same philosophy as the Matlab version presented above, but typically runs ≈15-30 faster, depending on the case.

## 5 Matlab plugins

This package also includes source code for compiled Matlab plugins (MEX files) to provide a fast Matlab interface to this functionality. Given a C compiler, simply run the provided script

`mexDEMCEM_build`

in Matlab. This will compile all functions with `mex` prefix. Note that the input arguments are different in this case (include weights and points of the 1-D GL quadratures)!

## Licence



## References

[1] A. G. Polimeridis and T. V. Yioultsis, "On the direct evaluation of weakly singular integrals in Galerkin mixed potential integral equation formulations", *IEEE Trans. Antennas Propag.*, vol. 56, no. 9, pp. 3011-3019, Sep. 2008.

[2] A. G. Polimeridis and J. R. Mosig, "Complete semi-analytical treatment of weakly singular integrals on planar triangles via the direct evaluation method", *Int. J. Numerical Methods Eng.*, vol. 83, pp. 1625-1650, 2010.

[3] A. G. Polimeridis, J. M. Tamayo, J. M. Rius, and J. R. Mosig, "Fast and accurate computation of hyper-singular integrals in Galerkin surface integral equation formulations via the direct evaluation method", *IEEE Trans. Antennas Propag.*, vol. 59, no. 6, pp. 2329-2340, Jun. 2011.

[4] A. G. Polimeridis and J. R. Mosig, "On the direct evaluation of surface integral equation impedance matrix elements involving point singularities", *IEEE Antennas Wireless Propag. Lett.*, vol. 10, pp. 599-602, 2011.