

Statistical Rethinking Winter 2020 – Homework Week 8

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1. Revisit the Reed frog survival data, `data(reedfrogs)`, and add the predation and size treatment variables to the varying intercepts model. Consider models with either predictor alone, both predictors, as well as a model including their interaction. What do you infer about the causal influence of these predictor variables? Also focus on the inferred variation across tanks (the σ across tanks). Explain why it changes as it does across models with different predictors included.

```
## R code 13.1
library(rethinking)
data(reedfrogs)
d <- reedfrogs
str(d)

## 'data.frame': 48 obs. of 5 variables:
## $ density : int 10 10 10 10 10 10 10 10 10 10 ...
## $ pred : Factor w/ 2 levels "no","pred": 1 1 1 1 1 1 1 1 2 2 ...
## $ size : Factor w/ 2 levels "big","small": 1 1 1 1 2 2 2 2 1 1 ...
## $ surv : int 9 10 7 10 9 9 10 9 4 9 ...
## $ propsurv: num 0.9 1 0.7 1 0.9 0.9 1 0.9 0.4 0.9 ...

## R code 13.2
# make the tank cluster variable
d$tank <- 1:nrow(d)

dat <- list(
  S = d$surv,
  N = d$density,
  tank = d$tank )

## R code 13.3
m13.2 <- ulam(
  alist(
    S ~ dbinom( N , p ) ,
    logit(p) <- a[tank] ,
    a[tank] ~ dnorm( a_bar , sigma ) ,
    a_bar ~ dnorm( 0 , 1.5 ) ,
    sigma ~ dexp( 1 )
  ), data=dat , chains=4 , log_lik=TRUE )

##
## SAMPLING FOR MODEL '71890fa4702bb6983de3ea7367f0b982' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 1.7e-05 seconds
```

```

## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0.17 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: Iteration: 1 / 1000 [ 0%] (Warmup)
## Chain 1: Iteration: 100 / 1000 [ 10%] (Warmup)
## Chain 1: Iteration: 200 / 1000 [ 20%] (Warmup)
## Chain 1: Iteration: 300 / 1000 [ 30%] (Warmup)
## Chain 1: Iteration: 400 / 1000 [ 40%] (Warmup)
## Chain 1: Iteration: 500 / 1000 [ 50%] (Warmup)
## Chain 1: Iteration: 501 / 1000 [ 50%] (Sampling)
## Chain 1: Iteration: 600 / 1000 [ 60%] (Sampling)
## Chain 1: Iteration: 700 / 1000 [ 70%] (Sampling)
## Chain 1: Iteration: 800 / 1000 [ 80%] (Sampling)
## Chain 1: Iteration: 900 / 1000 [ 90%] (Sampling)
## Chain 1: Iteration: 1000 / 1000 [100%] (Sampling)
## Chain 1:
## Chain 1: Elapsed Time: 0.122431 seconds (Warm-up)
## Chain 1: 0.084026 seconds (Sampling)
## Chain 1: 0.206457 seconds (Total)
## Chain 1:
##
## SAMPLING FOR MODEL '71890fa4702bb6983de3ea7367f0b982' NOW (CHAIN 2).
## Chain 2:
## Chain 2: Gradient evaluation took 1.3e-05 seconds
## Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0.13 seconds.
## Chain 2: Adjust your expectations accordingly!
## Chain 2:
## Chain 2:
## Chain 2: Iteration: 1 / 1000 [ 0%] (Warmup)
## Chain 2: Iteration: 100 / 1000 [ 10%] (Warmup)
## Chain 2: Iteration: 200 / 1000 [ 20%] (Warmup)
## Chain 2: Iteration: 300 / 1000 [ 30%] (Warmup)
## Chain 2: Iteration: 400 / 1000 [ 40%] (Warmup)
## Chain 2: Iteration: 500 / 1000 [ 50%] (Warmup)
## Chain 2: Iteration: 501 / 1000 [ 50%] (Sampling)
## Chain 2: Iteration: 600 / 1000 [ 60%] (Sampling)
## Chain 2: Iteration: 700 / 1000 [ 70%] (Sampling)
## Chain 2: Iteration: 800 / 1000 [ 80%] (Sampling)
## Chain 2: Iteration: 900 / 1000 [ 90%] (Sampling)
## Chain 2: Iteration: 1000 / 1000 [100%] (Sampling)
## Chain 2:
## Chain 2: Elapsed Time: 0.12234 seconds (Warm-up)
## Chain 2: 0.096127 seconds (Sampling)
## Chain 2: 0.218467 seconds (Total)
## Chain 2:
##
## SAMPLING FOR MODEL '71890fa4702bb6983de3ea7367f0b982' NOW (CHAIN 3).
## Chain 3:
## Chain 3: Gradient evaluation took 2.6e-05 seconds
## Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0.26 seconds.
## Chain 3: Adjust your expectations accordingly!
## Chain 3:
## Chain 3:

```

```

## Chain 3: Iteration: 1 / 1000 [ 0%] (Warmup)
## Chain 3: Iteration: 100 / 1000 [ 10%] (Warmup)
## Chain 3: Iteration: 200 / 1000 [ 20%] (Warmup)
## Chain 3: Iteration: 300 / 1000 [ 30%] (Warmup)
## Chain 3: Iteration: 400 / 1000 [ 40%] (Warmup)
## Chain 3: Iteration: 500 / 1000 [ 50%] (Warmup)
## Chain 3: Iteration: 501 / 1000 [ 50%] (Sampling)
## Chain 3: Iteration: 600 / 1000 [ 60%] (Sampling)
## Chain 3: Iteration: 700 / 1000 [ 70%] (Sampling)
## Chain 3: Iteration: 800 / 1000 [ 80%] (Sampling)
## Chain 3: Iteration: 900 / 1000 [ 90%] (Sampling)
## Chain 3: Iteration: 1000 / 1000 [100%] (Sampling)
## Chain 3:
## Chain 3: Elapsed Time: 0.133197 seconds (Warm-up)
## Chain 3: 0.091208 seconds (Sampling)
## Chain 3: 0.224405 seconds (Total)
## Chain 3:
##
## SAMPLING FOR MODEL '71890fa4702bb6983de3ea7367f0b982' NOW (CHAIN 4).
## Chain 4:
## Chain 4: Gradient evaluation took 1.5e-05 seconds
## Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 0.15 seconds.
## Chain 4: Adjust your expectations accordingly!
## Chain 4:
## Chain 4:
## Chain 4: Iteration: 1 / 1000 [ 0%] (Warmup)
## Chain 4: Iteration: 100 / 1000 [ 10%] (Warmup)
## Chain 4: Iteration: 200 / 1000 [ 20%] (Warmup)
## Chain 4: Iteration: 300 / 1000 [ 30%] (Warmup)
## Chain 4: Iteration: 400 / 1000 [ 40%] (Warmup)
## Chain 4: Iteration: 500 / 1000 [ 50%] (Warmup)
## Chain 4: Iteration: 501 / 1000 [ 50%] (Sampling)
## Chain 4: Iteration: 600 / 1000 [ 60%] (Sampling)
## Chain 4: Iteration: 700 / 1000 [ 70%] (Sampling)
## Chain 4: Iteration: 800 / 1000 [ 80%] (Sampling)
## Chain 4: Iteration: 900 / 1000 [ 90%] (Sampling)
## Chain 4: Iteration: 1000 / 1000 [100%] (Sampling)
## Chain 4:
## Chain 4: Elapsed Time: 0.124564 seconds (Warm-up)
## Chain 4: 0.095202 seconds (Sampling)
## Chain 4: 0.219766 seconds (Total)
## Chain 4:

```

```
precis(m13.2,depth=2)
```

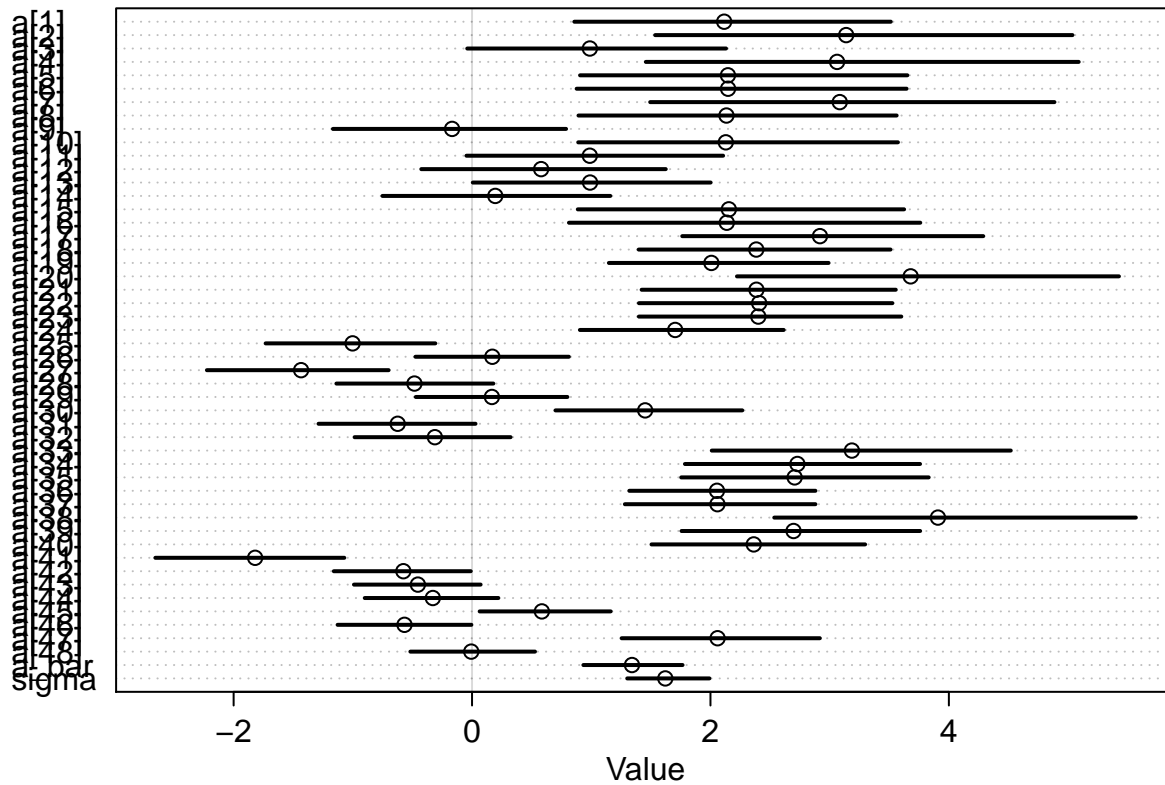
##	mean	sd	5.5%	94.5%	n_eff	Rhat4
## a[1]	2.115122847	0.8398981	0.857892892	3.512957149	3056.442	0.9993920
## a[2]	3.138466138	1.1065433	1.536172247	5.036937643	2722.737	0.9986273
## a[3]	0.988242717	0.6867407	-0.040375035	2.130859952	4248.389	0.9991561
## a[4]	3.062251633	1.1360042	1.457518806	5.089307021	2759.494	0.9995964
## a[5]	2.146578540	0.8722962	0.906196730	3.651871927	3178.951	0.9994894
## a[6]	2.147672219	0.8952787	0.877446778	3.645754779	2941.665	1.0001744
## a[7]	3.084690511	1.0874902	1.494405018	4.885928803	2938.976	0.9991030
## a[8]	2.134146908	0.8450553	0.892754419	3.562274727	3033.580	0.9983117
## a[9]	-0.167276818	0.6151297	-1.167171716	0.789060996	4948.317	0.9988916

```

## a[10] 2.128677574 0.8632049 0.890620802 3.572907949 3285.325 0.9998057
## a[11] 0.987892684 0.6584056 -0.046913614 2.106404432 3651.571 0.9984481
## a[12] 0.579687659 0.6484930 -0.426061409 1.625142427 4162.673 0.9990619
## a[13] 0.990035596 0.6411046 0.006782159 2.001629241 3640.357 0.9983695
## a[14] 0.194967355 0.6075352 -0.751702523 1.160066825 4031.964 0.9985246
## a[15] 2.154313928 0.8684499 0.887030894 3.624636117 3459.322 0.9990015
## a[16] 2.136958998 0.8951364 0.812999106 3.760357148 3400.058 0.9993232
## a[17] 2.918714293 0.7823758 1.763250498 4.288533244 3076.362 0.9987088
## a[18] 2.384474836 0.6616888 1.397810758 3.510825869 3317.004 0.9992560
## a[19] 2.007898528 0.5697471 1.147715764 2.991213004 3557.347 0.9984009
## a[20] 3.679385362 1.0188432 2.222375161 5.426811732 2585.877 1.0006706
## a[21] 2.385706262 0.6641442 1.422582024 3.554762531 2894.999 1.0002310
## a[22] 2.408766794 0.6857817 1.399946034 3.526410694 3863.009 0.9997541
## a[23] 2.401426245 0.7075897 1.399541120 3.601051780 2442.532 0.9986584
## a[24] 1.704143209 0.5435488 0.904567239 2.614719924 3626.379 0.9992437
## a[25] -1.002069321 0.4433324 -1.731660039 -0.307802834 3699.645 0.9992243
## a[26] 0.170261164 0.4007728 -0.474836896 0.812419011 4066.719 0.9984106
## a[27] -1.434073554 0.4843659 -2.224380133 -0.699658269 4217.933 0.9986142
## a[28] -0.483224017 0.4173271 -1.137938223 0.178948445 3546.328 0.9990067
## a[29] 0.166671375 0.3968838 -0.471957331 0.799952122 2933.864 0.9995088
## a[30] 1.452208204 0.4931355 0.699935572 2.267301918 3561.456 0.9988510
## a[31] -0.623799834 0.4214171 -1.287779908 0.029223328 3568.817 0.9988978
## a[32] -0.311853868 0.4084567 -0.985482437 0.323576659 5759.424 0.9982658
## a[33] 3.186301666 0.7807966 2.010941100 4.519357936 3253.297 0.9987662
## a[34] 2.729396409 0.6408275 1.785726738 3.758512812 3483.787 0.9984452
## a[35] 2.706077070 0.6551232 1.754781619 3.830852290 3377.362 0.9996396
## a[36] 2.054767241 0.4955220 1.319262425 2.880823591 4197.856 0.9991924
## a[37] 2.058643992 0.5017407 1.281953674 2.880523478 3777.327 0.9989297
## a[38] 3.908242670 0.9853944 2.535220730 5.568893986 2284.603 1.0013492
## a[39] 2.697032380 0.6189988 1.757189690 3.759281491 3489.319 0.9995314
## a[40] 2.363241395 0.5683026 1.505339550 3.297606700 3613.463 1.0004142
## a[41] -1.819580135 0.4957972 -2.656686742 -1.071321377 3949.467 0.9990584
## a[42] -0.575999009 0.3646484 -1.159676738 -0.010677005 4734.524 0.9987881
## a[43] -0.454368239 0.3416899 -0.990500633 0.072129935 4117.058 0.9988911
## a[44] -0.328807902 0.3521192 -0.900217829 0.220926620 3984.824 0.9985939
## a[45] 0.585575012 0.3393372 0.063689000 1.163291252 3130.555 0.9987379
## a[46] -0.566773682 0.3453234 -1.128003171 -0.005123592 4548.805 0.9982715
## a[47] 2.059737869 0.5163811 1.254885941 2.918050335 3621.654 0.9999033
## a[48] -0.006513156 0.3319615 -0.517557038 0.528370909 4085.824 0.9986917
## a_bar 1.341648391 0.2612297 0.933777258 1.766031299 2963.259 0.9994110
## sigma 1.621238373 0.2119349 1.301685957 1.991311407 1763.923 1.0022267

```

```
plot(precis(m13.2,depth=2))
```



A model with both pred and size:

```
d$pred_idx <- ifelse(d$pred=='no',1,2)
d$size_idx <- ifelse(d$size=='small',1,2)

dat <- list(
  S = d$surv,
  N = d$density,
  tank = d$tank,
  pred = d$pred_idx,
  my_size = d$size_idx
)

## R code 13.3
m_both <- ulam(
  alist(
    S ~ dbinom( N , p ) ,
    logit(p) <- a[tank] + a_size[my_size]+ a_pred[pred],
    a[tank] ~ dnorm( a_bar , sigma ) ,
    a_size[my_size] ~ dnorm( a_bar_size , sigma_size ) ,
    a_pred[pred] ~ dnorm( a_bar_pred , sigma_pred ) ,
    a_bar ~ dnorm( 0 , 1.5 ) ,
    a_bar_size ~ dnorm( 0 , 1.5 ) ,
    a_bar_pred ~ dnorm( 0 , 1.5 ) ,
    sigma ~ dexp( 1 ) ,
    sigma_size ~ dexp( 1 ) ,
    sigma_pred ~ dexp( 1 )
  ), data=dat , chains=4 , log_lik=TRUE )
```

##

```

## SAMPLING FOR MODEL '2d1461f663ad59fd2211e0c98e87fbe4' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 2.3e-05 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0.23 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: Iteration: 1 / 1000 [ 0%] (Warmup)
## Chain 1: Iteration: 100 / 1000 [ 10%] (Warmup)
## Chain 1: Iteration: 200 / 1000 [ 20%] (Warmup)
## Chain 1: Iteration: 300 / 1000 [ 30%] (Warmup)
## Chain 1: Iteration: 400 / 1000 [ 40%] (Warmup)
## Chain 1: Iteration: 500 / 1000 [ 50%] (Warmup)
## Chain 1: Iteration: 501 / 1000 [ 50%] (Sampling)
## Chain 1: Iteration: 600 / 1000 [ 60%] (Sampling)
## Chain 1: Iteration: 700 / 1000 [ 70%] (Sampling)
## Chain 1: Iteration: 800 / 1000 [ 80%] (Sampling)
## Chain 1: Iteration: 900 / 1000 [ 90%] (Sampling)
## Chain 1: Iteration: 1000 / 1000 [100%] (Sampling)
## Chain 1:
## Chain 1: Elapsed Time: 0.534882 seconds (Warm-up)
## Chain 1: 0.739985 seconds (Sampling)
## Chain 1: 1.27487 seconds (Total)
## Chain 1:
##
## SAMPLING FOR MODEL '2d1461f663ad59fd2211e0c98e87fbe4' NOW (CHAIN 2).
## Chain 2:
## Chain 2: Gradient evaluation took 2.2e-05 seconds
## Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0.22 seconds.
## Chain 2: Adjust your expectations accordingly!
## Chain 2:
## Chain 2:
## Chain 2: Iteration: 1 / 1000 [ 0%] (Warmup)
## Chain 2: Iteration: 100 / 1000 [ 10%] (Warmup)
## Chain 2: Iteration: 200 / 1000 [ 20%] (Warmup)
## Chain 2: Iteration: 300 / 1000 [ 30%] (Warmup)
## Chain 2: Iteration: 400 / 1000 [ 40%] (Warmup)
## Chain 2: Iteration: 500 / 1000 [ 50%] (Warmup)
## Chain 2: Iteration: 501 / 1000 [ 50%] (Sampling)
## Chain 2: Iteration: 600 / 1000 [ 60%] (Sampling)
## Chain 2: Iteration: 700 / 1000 [ 70%] (Sampling)
## Chain 2: Iteration: 800 / 1000 [ 80%] (Sampling)
## Chain 2: Iteration: 900 / 1000 [ 90%] (Sampling)
## Chain 2: Iteration: 1000 / 1000 [100%] (Sampling)
## Chain 2:
## Chain 2: Elapsed Time: 0.533042 seconds (Warm-up)
## Chain 2: 0.549781 seconds (Sampling)
## Chain 2: 1.08282 seconds (Total)
## Chain 2:
##
## SAMPLING FOR MODEL '2d1461f663ad59fd2211e0c98e87fbe4' NOW (CHAIN 3).
## Chain 3:
## Chain 3: Gradient evaluation took 1.4e-05 seconds
## Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0.14 seconds.

```

```

## Chain 3: Adjust your expectations accordingly!
## Chain 3:
## Chain 3:
## Chain 3: Iteration: 1 / 1000 [ 0%] (Warmup)
## Chain 3: Iteration: 100 / 1000 [ 10%] (Warmup)
## Chain 3: Iteration: 200 / 1000 [ 20%] (Warmup)
## Chain 3: Iteration: 300 / 1000 [ 30%] (Warmup)
## Chain 3: Iteration: 400 / 1000 [ 40%] (Warmup)
## Chain 3: Iteration: 500 / 1000 [ 50%] (Warmup)
## Chain 3: Iteration: 501 / 1000 [ 50%] (Sampling)
## Chain 3: Iteration: 600 / 1000 [ 60%] (Sampling)
## Chain 3: Iteration: 700 / 1000 [ 70%] (Sampling)
## Chain 3: Iteration: 800 / 1000 [ 80%] (Sampling)
## Chain 3: Iteration: 900 / 1000 [ 90%] (Sampling)
## Chain 3: Iteration: 1000 / 1000 [100%] (Sampling)
## Chain 3:
## Chain 3: Elapsed Time: 0.513655 seconds (Warm-up)
## Chain 3: 0.290173 seconds (Sampling)
## Chain 3: 0.803828 seconds (Total)
## Chain 3:
##
## SAMPLING FOR MODEL '2d1461f663ad59fd2211e0c98e87fbe4' NOW (CHAIN 4).
## Chain 4:
## Chain 4: Gradient evaluation took 1.4e-05 seconds
## Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 0.14 seconds.
## Chain 4: Adjust your expectations accordingly!
## Chain 4:
## Chain 4:
## Chain 4: Iteration: 1 / 1000 [ 0%] (Warmup)
## Chain 4: Iteration: 100 / 1000 [ 10%] (Warmup)
## Chain 4: Iteration: 200 / 1000 [ 20%] (Warmup)
## Chain 4: Iteration: 300 / 1000 [ 30%] (Warmup)
## Chain 4: Iteration: 400 / 1000 [ 40%] (Warmup)
## Chain 4: Iteration: 500 / 1000 [ 50%] (Warmup)
## Chain 4: Iteration: 501 / 1000 [ 50%] (Sampling)
## Chain 4: Iteration: 600 / 1000 [ 60%] (Sampling)
## Chain 4: Iteration: 700 / 1000 [ 70%] (Sampling)
## Chain 4: Iteration: 800 / 1000 [ 80%] (Sampling)
## Chain 4: Iteration: 900 / 1000 [ 90%] (Sampling)
## Chain 4: Iteration: 1000 / 1000 [100%] (Sampling)
## Chain 4:
## Chain 4: Elapsed Time: 0.508394 seconds (Warm-up)
## Chain 4: 0.398419 seconds (Sampling)
## Chain 4: 0.906813 seconds (Total)
## Chain 4:

## Warning: There were 14 divergent transitions after warmup. See
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup
## to find out why this is a problem and how to eliminate them.

## Warning: Examine the pairs() plot to diagnose sampling problems

## Warning: The largest R-hat is 1.19, indicating chains have not mixed.
## Running the chains for more iterations may help. See
## http://mc-stan.org/misc/warnings.html#r-hat

```

```
## Warning: Bulk Effective Samples Size (ESS) is too low, indicating posterior means and medians may be
## Running the chains for more iterations may help. See
## http://mc-stan.org/misc/warnings.html#bulk-ess
```

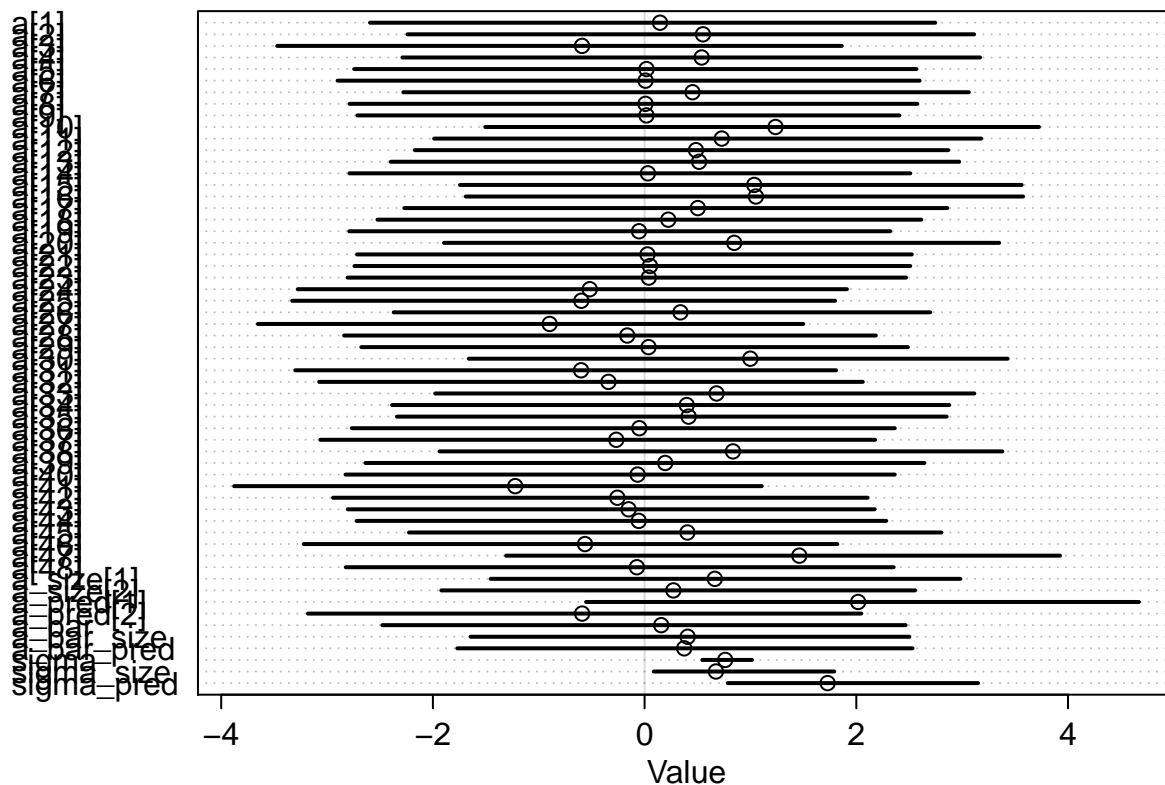
```
## Warning: Tail Effective Samples Size (ESS) is too low, indicating posterior variances and tail quant
## Running the chains for more iterations may help. See
## http://mc-stan.org/misc/warnings.html#tail-ess
```

```
precis(m_both,depth=2)
```

##		mean	sd	5.5%	94.5%	n_eff	Rhat4
##	a[1]	0.145892876	1.6825374	-2.59434999	2.746002	30.63839	1.062990
##	a[2]	0.551513736	1.6554739	-2.24003467	3.113042	30.48408	1.064341
##	a[3]	-0.590864032	1.6248021	-3.47080704	1.862791	29.27160	1.073414
##	a[4]	0.539008735	1.6525680	-2.28857503	3.169396	30.61891	1.068684
##	a[5]	0.017993139	1.6491463	-2.74402707	2.568751	29.76022	1.066475
##	a[6]	0.009150115	1.6821193	-2.90039731	2.598679	31.09423	1.067715
##	a[7]	0.452733064	1.6816704	-2.28007096	3.063387	30.10433	1.068032
##	a[8]	0.008232611	1.6500399	-2.78681798	2.576628	28.93716	1.071015
##	a[9]	0.017511550	1.5891805	-2.71421433	2.406931	28.32396	1.072365
##	a[10]	1.236771107	1.6262534	-1.50497428	3.725512	28.83169	1.067380
##	a[11]	0.728929710	1.6192963	-1.98826740	3.180201	28.78866	1.073152
##	a[12]	0.485165425	1.5771337	-2.17102692	2.868836	27.98438	1.074850
##	a[13]	0.514805530	1.6455792	-2.39791640	2.972588	27.97028	1.071042
##	a[14]	0.030916958	1.6181152	-2.78887318	2.510572	28.68322	1.076324
##	a[15]	1.035695866	1.6478742	-1.74440817	3.562316	28.64223	1.073379
##	a[16]	1.051162905	1.6338102	-1.68952819	3.576212	28.62100	1.074176
##	a[17]	0.502590171	1.6087813	-2.26984527	2.859945	28.80138	1.070938
##	a[18]	0.223561967	1.6048322	-2.52397111	2.613800	28.76388	1.074625
##	a[19]	-0.052802269	1.5877126	-2.79023247	2.320519	28.39205	1.077980
##	a[20]	0.846825556	1.6392233	-1.89564709	3.350125	29.29618	1.073764
##	a[21]	0.027223867	1.6273036	-2.71653880	2.524497	28.38565	1.074557
##	a[22]	0.050582422	1.6197287	-2.74132788	2.509953	28.62388	1.076160
##	a[23]	0.039708170	1.6373388	-2.80643924	2.469705	28.84811	1.071594
##	a[24]	-0.518177353	1.5889414	-3.27682476	1.911693	28.45767	1.076072
##	a[25]	-0.597811858	1.5739997	-3.32946280	1.800119	27.80984	1.077613
##	a[26]	0.338808561	1.5685471	-2.37036520	2.698107	27.37901	1.074862
##	a[27]	-0.897090808	1.5723516	-3.65368787	1.499643	28.39935	1.075717
##	a[28]	-0.163869722	1.5712522	-2.83867374	2.184583	27.28997	1.077596
##	a[29]	0.037181734	1.5787530	-2.67678867	2.487040	26.75406	1.078660
##	a[30]	0.998688197	1.5903763	-1.66072646	3.430123	27.09566	1.074388
##	a[31]	-0.600251522	1.5661651	-3.30115672	1.808766	27.34999	1.078896
##	a[32]	-0.341459497	1.5687160	-3.07468489	2.061179	26.95566	1.076262
##	a[33]	0.679524744	1.6168533	-1.97884689	3.115749	27.80704	1.076752
##	a[34]	0.398224827	1.6279259	-2.38597742	2.878304	28.11950	1.073375
##	a[35]	0.415891135	1.6149365	-2.33990385	2.853824	28.42106	1.074625
##	a[36]	-0.049947398	1.6026916	-2.76686998	2.363981	27.83714	1.077914
##	a[37]	-0.267824480	1.6070770	-3.06439396	2.177252	28.34321	1.072311
##	a[38]	0.834814051	1.6794519	-1.93573400	3.378936	29.64064	1.067186
##	a[39]	0.194659003	1.6307407	-2.63723554	2.643505	28.21701	1.076987
##	a[40]	-0.066313672	1.6107001	-2.82564936	2.363595	28.07472	1.077600
##	a[41]	-1.222408119	1.5519231	-3.87782352	1.107302	28.00894	1.080546
##	a[42]	-0.257586148	1.5544561	-2.94335387	2.107305	27.16850	1.079440
##	a[43]	-0.151273256	1.5506498	-2.80454237	2.173202	26.91796	1.079498
##	a[44]	-0.055319414	1.5529500	-2.72005146	2.283163	27.24657	1.078401


```
## a[45]      0.404634286 1.5638733 -2.22616191 2.804393 26.63719 1.076520
## a[46]     -0.563025340 1.5525784 -3.22000370 1.820314 26.77994 1.077766
## a[47]      1.461583165 1.6079660 -1.30857141 3.924557 26.86210 1.074627
## a[48]     -0.073059570 1.5648351 -2.82335500 2.353386 26.64067 1.079257
## a_size[1]  0.663192069 1.3763260 -1.45411487 2.983607 74.96433 1.052719
## a_size[2]  0.271917853 1.3744069 -1.92044948 2.555859 79.84688 1.050977
## a_pred[1]  2.017257383 1.5685384 -0.55299727 4.670037 67.73174 1.026990
## a_pred[2] -0.589982297 1.5538818 -3.18017023 2.047811 68.69364 1.027727
## a_bar      0.157110585 1.5207910 -2.47760212 2.466032 25.61396 1.082799
## a_bar_size 0.406700911 1.2998272 -1.64473467 2.501497 88.82864 1.042396
## a_bar_pred 0.374281328 1.3518256 -1.76958240 2.531476 125.51800 1.014797
## sigma      0.761421129 0.1452575 0.54552839 1.013913 324.13446 1.009533
## sigma_size 0.673531635 0.5861636 0.08688906 1.792304 479.04428 1.005278
## sigma_pred 1.728920274 0.7855133 0.78540594 3.150237 528.35337 1.008909
```

```
plot(precis(m_both,depth=2))
```



A model with just pred:

```
m_pred <- ulam(
  alist(
    S ~ dbinom( N , p ) ,
    logit(p) <- a[tank] + a_pred[pred],
    a[tank] ~ dnorm( a_bar , sigma ) ,
    a_pred[pred] ~ dnorm( a_bar_pred , sigma_pred ) ,
    a_bar ~ dnorm( 0 , 1.5 ) ,
    a_bar_pred ~ dnorm( 0 , 1.5 ) ,
    sigma ~ dexp( 1 ) ,
    sigma_pred ~ dexp( 1 )
  ), data=dat , chains=4 , log_lik=TRUE )
```

```

##
## SAMPLING FOR MODEL 'fe4f5d6a7170dc68deddacdfcfb5ed92' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 2.4e-05 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0.24 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: Iteration: 1 / 1000 [ 0%] (Warmup)
## Chain 1: Iteration: 100 / 1000 [ 10%] (Warmup)
## Chain 1: Iteration: 200 / 1000 [ 20%] (Warmup)
## Chain 1: Iteration: 300 / 1000 [ 30%] (Warmup)
## Chain 1: Iteration: 400 / 1000 [ 40%] (Warmup)
## Chain 1: Iteration: 500 / 1000 [ 50%] (Warmup)
## Chain 1: Iteration: 501 / 1000 [ 50%] (Sampling)
## Chain 1: Iteration: 600 / 1000 [ 60%] (Sampling)
## Chain 1: Iteration: 700 / 1000 [ 70%] (Sampling)
## Chain 1: Iteration: 800 / 1000 [ 80%] (Sampling)
## Chain 1: Iteration: 900 / 1000 [ 90%] (Sampling)
## Chain 1: Iteration: 1000 / 1000 [100%] (Sampling)
## Chain 1:
## Chain 1: Elapsed Time: 0.171345 seconds (Warm-up)
## Chain 1: 0.170026 seconds (Sampling)
## Chain 1: 0.341371 seconds (Total)
## Chain 1:
##
## SAMPLING FOR MODEL 'fe4f5d6a7170dc68deddacdfcfb5ed92' NOW (CHAIN 2).
## Chain 2:
## Chain 2: Gradient evaluation took 1.5e-05 seconds
## Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0.15 seconds.
## Chain 2: Adjust your expectations accordingly!
## Chain 2:
## Chain 2:
## Chain 2: Iteration: 1 / 1000 [ 0%] (Warmup)
## Chain 2: Iteration: 100 / 1000 [ 10%] (Warmup)
## Chain 2: Iteration: 200 / 1000 [ 20%] (Warmup)
## Chain 2: Iteration: 300 / 1000 [ 30%] (Warmup)
## Chain 2: Iteration: 400 / 1000 [ 40%] (Warmup)
## Chain 2: Iteration: 500 / 1000 [ 50%] (Warmup)
## Chain 2: Iteration: 501 / 1000 [ 50%] (Sampling)
## Chain 2: Iteration: 600 / 1000 [ 60%] (Sampling)
## Chain 2: Iteration: 700 / 1000 [ 70%] (Sampling)
## Chain 2: Iteration: 800 / 1000 [ 80%] (Sampling)
## Chain 2: Iteration: 900 / 1000 [ 90%] (Sampling)
## Chain 2: Iteration: 1000 / 1000 [100%] (Sampling)
## Chain 2:
## Chain 2: Elapsed Time: 0.201156 seconds (Warm-up)
## Chain 2: 0.181654 seconds (Sampling)
## Chain 2: 0.38281 seconds (Total)
## Chain 2:
##
## SAMPLING FOR MODEL 'fe4f5d6a7170dc68deddacdfcfb5ed92' NOW (CHAIN 3).
## Chain 3:
## Chain 3: Gradient evaluation took 1.4e-05 seconds

```

```

## Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0.14 seconds.
## Chain 3: Adjust your expectations accordingly!
## Chain 3:
## Chain 3:
## Chain 3: Iteration: 1 / 1000 [ 0%] (Warmup)
## Chain 3: Iteration: 100 / 1000 [ 10%] (Warmup)
## Chain 3: Iteration: 200 / 1000 [ 20%] (Warmup)
## Chain 3: Iteration: 300 / 1000 [ 30%] (Warmup)
## Chain 3: Iteration: 400 / 1000 [ 40%] (Warmup)
## Chain 3: Iteration: 500 / 1000 [ 50%] (Warmup)
## Chain 3: Iteration: 501 / 1000 [ 50%] (Sampling)
## Chain 3: Iteration: 600 / 1000 [ 60%] (Sampling)
## Chain 3: Iteration: 700 / 1000 [ 70%] (Sampling)
## Chain 3: Iteration: 800 / 1000 [ 80%] (Sampling)
## Chain 3: Iteration: 900 / 1000 [ 90%] (Sampling)
## Chain 3: Iteration: 1000 / 1000 [100%] (Sampling)
## Chain 3:
## Chain 3: Elapsed Time: 0.239282 seconds (Warm-up)
## Chain 3: 0.17056 seconds (Sampling)
## Chain 3: 0.409842 seconds (Total)
## Chain 3:
##
## SAMPLING FOR MODEL 'fe4f5d6a7170dc68deddacfcb5ed92' NOW (CHAIN 4).
## Chain 4:
## Chain 4: Gradient evaluation took 1.3e-05 seconds
## Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 0.13 seconds.
## Chain 4: Adjust your expectations accordingly!
## Chain 4:
## Chain 4:
## Chain 4: Iteration: 1 / 1000 [ 0%] (Warmup)
## Chain 4: Iteration: 100 / 1000 [ 10%] (Warmup)
## Chain 4: Iteration: 200 / 1000 [ 20%] (Warmup)
## Chain 4: Iteration: 300 / 1000 [ 30%] (Warmup)
## Chain 4: Iteration: 400 / 1000 [ 40%] (Warmup)
## Chain 4: Iteration: 500 / 1000 [ 50%] (Warmup)
## Chain 4: Iteration: 501 / 1000 [ 50%] (Sampling)
## Chain 4: Iteration: 600 / 1000 [ 60%] (Sampling)
## Chain 4: Iteration: 700 / 1000 [ 70%] (Sampling)
## Chain 4: Iteration: 800 / 1000 [ 80%] (Sampling)
## Chain 4: Iteration: 900 / 1000 [ 90%] (Sampling)
## Chain 4: Iteration: 1000 / 1000 [100%] (Sampling)
## Chain 4:
## Chain 4: Elapsed Time: 0.175763 seconds (Warm-up)
## Chain 4: 0.168405 seconds (Sampling)
## Chain 4: 0.344168 seconds (Total)
## Chain 4:

## Warning: The largest R-hat is 1.16, indicating chains have not mixed.
## Running the chains for more iterations may help. See
## http://mc-stan.org/misc/warnings.html#r-hat

## Warning: Bulk Effective Samples Size (ESS) is too low, indicating posterior means and medians may be
## Running the chains for more iterations may help. See
## http://mc-stan.org/misc/warnings.html#bulk-ess

```

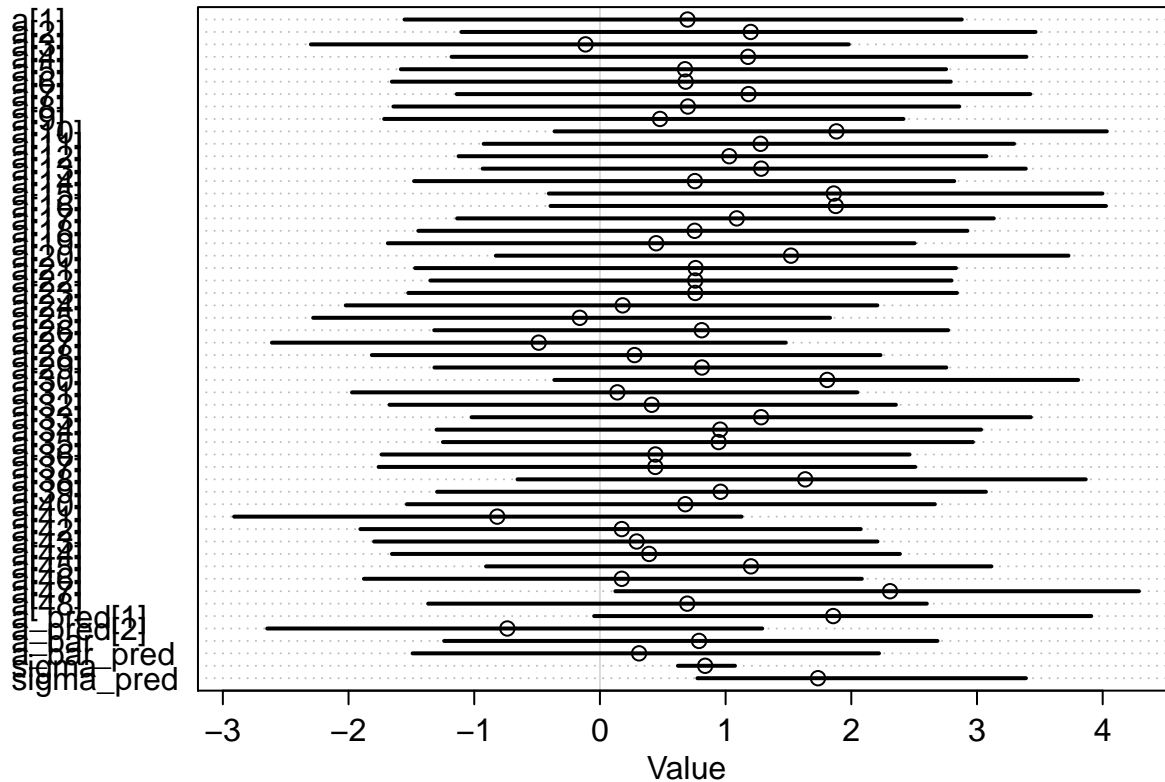
```
## Warning: Tail Effective Samples Size (ESS) is too low, indicating posterior variances and tail quantiles
## Running the chains for more iterations may help. See
## http://mc-stan.org/misc/warnings.html#tail-ess
```

```
precis(m_pred,depth=2)
```

##	mean	sd	5.5%	94.5%	n_eff	Rhat4
## a[1]	0.6966002	1.3979107	-1.55464617	2.878949	29.86046	1.126336
## a[2]	1.1993860	1.4198451	-1.10257731	3.466015	32.01674	1.119509
## a[3]	-0.1146747	1.3608991	-2.29969898	1.980055	30.28175	1.125996
## a[4]	1.1781474	1.4293268	-1.18037438	3.393160	32.58665	1.111071
## a[5]	0.6770012	1.3765621	-1.58415204	2.754118	31.27289	1.117535
## a[6]	0.6822234	1.3903487	-1.65585534	2.791507	31.12928	1.116242
## a[7]	1.1821274	1.4240789	-1.13925386	3.423732	31.71935	1.125865
## a[8]	0.6993361	1.3976218	-1.64319034	2.859018	31.53851	1.127123
## a[9]	0.4779350	1.3194276	-1.71673295	2.414895	28.41715	1.132097
## a[10]	1.8816864	1.3580994	-0.36103935	4.033033	30.09392	1.122985
## a[11]	1.2781916	1.3257751	-0.92505401	3.295034	30.64798	1.119565
## a[12]	1.0289155	1.3370601	-1.12480941	3.075084	28.56703	1.130809
## a[13]	1.2830262	1.3355101	-0.93356440	3.388334	28.13851	1.132812
## a[14]	0.7553735	1.3230952	-1.47882812	2.819116	28.68733	1.135241
## a[15]	1.8603614	1.3681369	-0.40641785	3.998041	29.07088	1.129724
## a[16]	1.8763286	1.3655470	-0.39346953	4.026012	29.70031	1.127324
## a[17]	1.0884154	1.3748321	-1.13610655	3.134023	30.04140	1.122325
## a[18]	0.7536140	1.3589158	-1.44495469	2.924146	29.19479	1.128480
## a[19]	0.4474720	1.3280665	-1.68755941	2.503843	27.26944	1.141008
## a[20]	1.5200743	1.3977334	-0.82832131	3.727952	29.71681	1.118512
## a[21]	0.7613155	1.3405018	-1.47040055	2.832968	29.32411	1.124769
## a[22]	0.7580220	1.3425289	-1.34938859	2.797464	29.65027	1.126477
## a[23]	0.7581717	1.3566400	-1.52528405	2.840631	27.20839	1.139964
## a[24]	0.1809274	1.3222394	-2.02233022	2.207311	27.47162	1.138360
## a[25]	-0.1603550	1.2721257	-2.28170221	1.830707	27.41011	1.137874
## a[26]	0.8097975	1.2766380	-1.31865367	2.770472	26.57455	1.143820
## a[27]	-0.4870290	1.2977844	-2.60981198	1.479287	27.60568	1.136588
## a[28]	0.2753193	1.2818586	-1.81503381	2.232166	26.94095	1.142366
## a[29]	0.8115477	1.2725780	-1.31696679	2.756083	26.30308	1.146640
## a[30]	1.8083989	1.2988231	-0.36300864	3.804940	28.08987	1.132549
## a[31]	0.1386370	1.2624628	-1.97286646	2.048031	26.76906	1.146483
## a[32]	0.4121643	1.2731697	-1.67411473	2.354836	26.58142	1.148233
## a[33]	1.2826593	1.3861115	-1.02144885	3.428507	28.25700	1.132476
## a[34]	0.9566395	1.3542170	-1.29968334	3.032336	29.63271	1.130335
## a[35]	0.9442139	1.3297674	-1.24920309	2.967652	29.62875	1.124409
## a[36]	0.4417537	1.3283785	-1.73709588	2.463779	26.58944	1.145507
## a[37]	0.4404986	1.3189382	-1.76087868	2.508398	27.39763	1.140963
## a[38]	1.6333829	1.3974760	-0.65578523	3.865286	29.92023	1.123322
## a[39]	0.9597748	1.3505393	-1.29567672	3.071221	28.88371	1.131414
## a[40]	0.6791249	1.3163642	-1.53623918	2.664992	27.61796	1.133825
## a[41]	-0.8169276	1.2792105	-2.91026933	1.126843	27.24967	1.143069
## a[42]	0.1737708	1.2586635	-1.90587406	2.075140	25.82154	1.151515
## a[43]	0.2921278	1.2665692	-1.79691534	2.208324	26.66611	1.144008
## a[44]	0.3914261	1.2663711	-1.65495910	2.386785	26.26572	1.148603
## a[45]	1.2014452	1.2743252	-0.90611355	3.115018	26.90093	1.144446
## a[46]	0.1736252	1.2679603	-1.87776295	2.083919	27.04467	1.140509
## a[47]	2.3082907	1.3011341	0.12272406	4.288305	27.99876	1.138567
## a[48]	0.6945129	1.2605241	-1.36672072	2.601022	26.85153	1.145751

```
## a_pred[1] 1.8559991 1.2367030 -0.04631841 3.909132 24.45907 1.157626
## a_pred[2] -0.7366445 1.2256373 -2.64718946 1.291171 24.85984 1.157872
## a_bar 0.7885133 1.2184505 -1.23950735 2.685609 24.21109 1.162798
## a_bar_pred 0.3115042 1.1954430 -1.49039829 2.220285 78.47994 1.055486
## sigma 0.8371962 0.1453617 0.62053735 1.074722 495.03801 1.003737
## sigma_pred 1.7348911 0.8912621 0.77559896 3.389425 612.73538 1.003383
```

```
plot(precis(m_pred,depth=2))
```



A model with just size:

```
m_size <- ulam(
  alist(
    S ~ dbinom( N , p ) ,
    logit(p) <- a[tank] + a_size[my_size],
    a[tank] ~ dnorm( a_bar , sigma ) ,
    a_size[my_size] ~ dnorm( a_bar_size , sigma_size ) ,
    a_bar ~ dnorm( 0 , 1.5 ) ,
    a_bar_size ~ dnorm( 0 , 1.5 ) ,
    sigma ~ dexp( 1 ) ,
    sigma_size ~ dexp( 1 )
  ), data=dat , chains=4 , log_lik=TRUE )
```

```
##
## SAMPLING FOR MODEL 'ee417900e62ab646404de962f5fb72eb' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 2.6e-05 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0.26 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
```

```

## Chain 1:
## Chain 1: Iteration: 1 / 1000 [ 0%] (Warmup)
## Chain 1: Iteration: 100 / 1000 [ 10%] (Warmup)
## Chain 1: Iteration: 200 / 1000 [ 20%] (Warmup)
## Chain 1: Iteration: 300 / 1000 [ 30%] (Warmup)
## Chain 1: Iteration: 400 / 1000 [ 40%] (Warmup)
## Chain 1: Iteration: 500 / 1000 [ 50%] (Warmup)
## Chain 1: Iteration: 501 / 1000 [ 50%] (Sampling)
## Chain 1: Iteration: 600 / 1000 [ 60%] (Sampling)
## Chain 1: Iteration: 700 / 1000 [ 70%] (Sampling)
## Chain 1: Iteration: 800 / 1000 [ 80%] (Sampling)
## Chain 1: Iteration: 900 / 1000 [ 90%] (Sampling)
## Chain 1: Iteration: 1000 / 1000 [100%] (Sampling)
## Chain 1:
## Chain 1: Elapsed Time: 0.300033 seconds (Warm-up)
## Chain 1: 0.159593 seconds (Sampling)
## Chain 1: 0.459626 seconds (Total)
## Chain 1:
##
## SAMPLING FOR MODEL 'ee417900e62ab646404de962f5fb72eb' NOW (CHAIN 2).
## Chain 2:
## Chain 2: Gradient evaluation took 1.3e-05 seconds
## Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0.13 seconds.
## Chain 2: Adjust your expectations accordingly!
## Chain 2:
## Chain 2:
## Chain 2: Iteration: 1 / 1000 [ 0%] (Warmup)
## Chain 2: Iteration: 100 / 1000 [ 10%] (Warmup)
## Chain 2: Iteration: 200 / 1000 [ 20%] (Warmup)
## Chain 2: Iteration: 300 / 1000 [ 30%] (Warmup)
## Chain 2: Iteration: 400 / 1000 [ 40%] (Warmup)
## Chain 2: Iteration: 500 / 1000 [ 50%] (Warmup)
## Chain 2: Iteration: 501 / 1000 [ 50%] (Sampling)
## Chain 2: Iteration: 600 / 1000 [ 60%] (Sampling)
## Chain 2: Iteration: 700 / 1000 [ 70%] (Sampling)
## Chain 2: Iteration: 800 / 1000 [ 80%] (Sampling)
## Chain 2: Iteration: 900 / 1000 [ 90%] (Sampling)
## Chain 2: Iteration: 1000 / 1000 [100%] (Sampling)
## Chain 2:
## Chain 2: Elapsed Time: 0.325828 seconds (Warm-up)
## Chain 2: 0.168434 seconds (Sampling)
## Chain 2: 0.494262 seconds (Total)
## Chain 2:
##
## SAMPLING FOR MODEL 'ee417900e62ab646404de962f5fb72eb' NOW (CHAIN 3).
## Chain 3:
## Chain 3: Gradient evaluation took 1.3e-05 seconds
## Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0.13 seconds.
## Chain 3: Adjust your expectations accordingly!
## Chain 3:
## Chain 3:
## Chain 3: Iteration: 1 / 1000 [ 0%] (Warmup)
## Chain 3: Iteration: 100 / 1000 [ 10%] (Warmup)
## Chain 3: Iteration: 200 / 1000 [ 20%] (Warmup)

```

```

## Chain 3: Iteration: 300 / 1000 [ 30%] (Warmup)
## Chain 3: Iteration: 400 / 1000 [ 40%] (Warmup)
## Chain 3: Iteration: 500 / 1000 [ 50%] (Warmup)
## Chain 3: Iteration: 501 / 1000 [ 50%] (Sampling)
## Chain 3: Iteration: 600 / 1000 [ 60%] (Sampling)
## Chain 3: Iteration: 700 / 1000 [ 70%] (Sampling)
## Chain 3: Iteration: 800 / 1000 [ 80%] (Sampling)
## Chain 3: Iteration: 900 / 1000 [ 90%] (Sampling)
## Chain 3: Iteration: 1000 / 1000 [100%] (Sampling)
## Chain 3:
## Chain 3: Elapsed Time: 0.357041 seconds (Warm-up)
## Chain 3: 0.31777 seconds (Sampling)
## Chain 3: 0.674811 seconds (Total)
## Chain 3:
##
## SAMPLING FOR MODEL 'ee417900e62ab646404de962f5fb72eb' NOW (CHAIN 4).
## Chain 4:
## Chain 4: Gradient evaluation took 1.3e-05 seconds
## Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 0.13 seconds.
## Chain 4: Adjust your expectations accordingly!
## Chain 4:
## Chain 4:
## Chain 4: Iteration: 1 / 1000 [ 0%] (Warmup)
## Chain 4: Iteration: 100 / 1000 [ 10%] (Warmup)
## Chain 4: Iteration: 200 / 1000 [ 20%] (Warmup)
## Chain 4: Iteration: 300 / 1000 [ 30%] (Warmup)
## Chain 4: Iteration: 400 / 1000 [ 40%] (Warmup)
## Chain 4: Iteration: 500 / 1000 [ 50%] (Warmup)
## Chain 4: Iteration: 501 / 1000 [ 50%] (Sampling)
## Chain 4: Iteration: 600 / 1000 [ 60%] (Sampling)
## Chain 4: Iteration: 700 / 1000 [ 70%] (Sampling)
## Chain 4: Iteration: 800 / 1000 [ 80%] (Sampling)
## Chain 4: Iteration: 900 / 1000 [ 90%] (Sampling)
## Chain 4: Iteration: 1000 / 1000 [100%] (Sampling)
## Chain 4:
## Chain 4: Elapsed Time: 0.458789 seconds (Warm-up)
## Chain 4: 1.08926 seconds (Sampling)
## Chain 4: 1.54805 seconds (Total)
## Chain 4:

## Warning: There were 65 divergent transitions after warmup. See
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup
## to find out why this is a problem and how to eliminate them.

## Warning: There were 1 transitions after warmup that exceeded the maximum treedepth. Increase max_treedepth. See
## http://mc-stan.org/misc/warnings.html#maximum-treedepth-exceeded

## Warning: Examine the pairs() plot to diagnose sampling problems

## Warning: The largest R-hat is 1.08, indicating chains have not mixed.
## Running the chains for more iterations may help. See
## http://mc-stan.org/misc/warnings.html#r-hat

## Warning: Bulk Effective Samples Size (ESS) is too low, indicating posterior means and medians may be biased.
## Running the chains for more iterations may help. See
## http://mc-stan.org/misc/warnings.html#bulk-ess

```

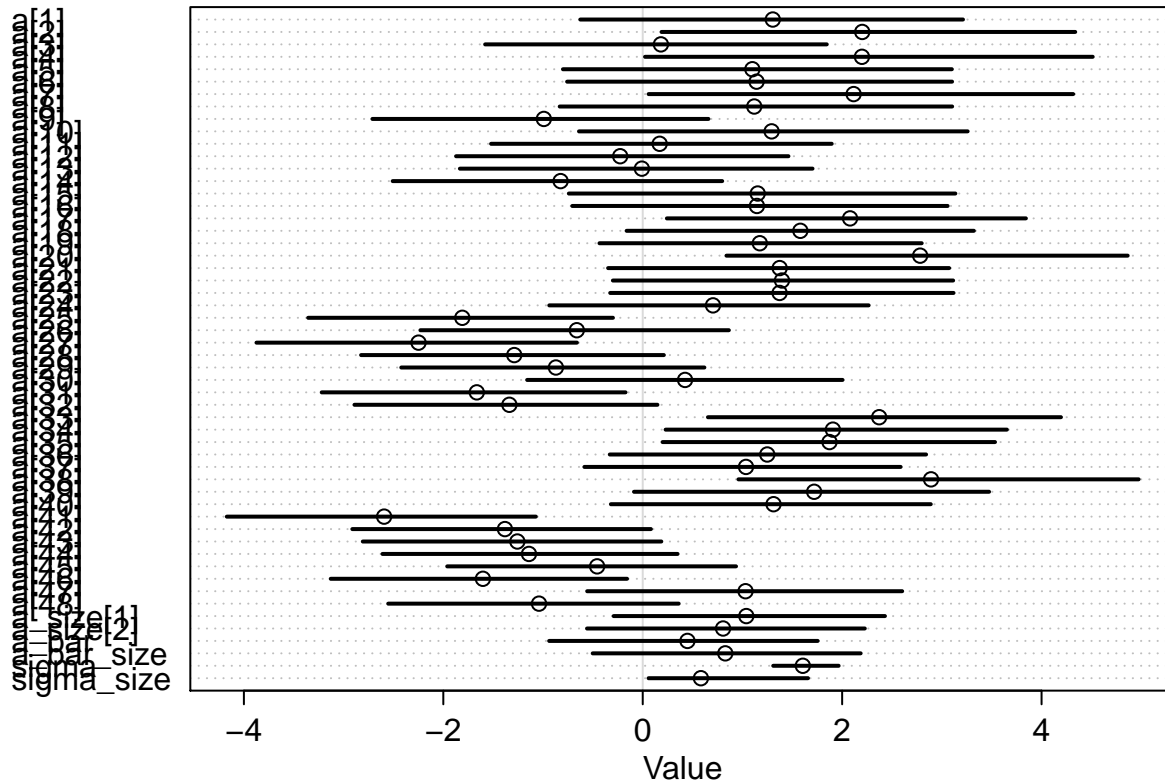
```
## Warning: Tail Effective Samples Size (ESS) is too low, indicating posterior variances and tail quantiles
## Running the chains for more iterations may help. See
## http://mc-stan.org/misc/warnings.html#tail-ess
```

```
precis(m_size,depth=2)
```

##	mean	sd	5.5%	94.5%	n_eff	Rhat4
## a[1]	1.304343320	1.1958965	-0.62614483	3.2122067	134.60487	1.032579
## a[2]	2.201338279	1.3023125	0.18834405	4.3378193	151.22038	1.042529
## a[3]	0.182325837	1.0871612	-1.58068775	1.8459923	109.50188	1.040861
## a[4]	2.198385426	1.3762847	0.02305542	4.5132113	154.60621	1.029743
## a[5]	1.099505053	1.2230799	-0.80015303	3.1006425	131.53781	1.040993
## a[6]	1.141781470	1.2029770	-0.75837622	3.1027397	133.28349	1.036549
## a[7]	2.115064353	1.3573668	0.05887981	4.3171722	189.92208	1.029772
## a[8]	1.118386338	1.2234149	-0.83287064	3.1036085	126.01929	1.037320
## a[9]	-0.991750662	1.0605116	-2.71140728	0.6589554	104.57103	1.044750
## a[10]	1.292721481	1.1931496	-0.63946468	3.2588095	127.03400	1.038201
## a[11]	0.169960124	1.0813459	-1.52111452	1.8956427	100.92596	1.048161
## a[12]	-0.225916555	1.0493747	-1.87183302	1.4617634	82.72342	1.055828
## a[13]	-0.008798088	1.1041207	-1.83384047	1.7031212	108.91559	1.047072
## a[14]	-0.823658688	1.0556082	-2.50800622	0.7971514	102.24989	1.050389
## a[15]	1.152940378	1.1922477	-0.74139388	3.1357409	117.78329	1.042020
## a[16]	1.145146797	1.1759271	-0.70696020	3.0581632	150.58641	1.026594
## a[17]	2.079463545	1.1461021	0.24210810	3.8429570	110.70034	1.043003
## a[18]	1.580822656	1.0895973	-0.16182701	3.3213278	99.46122	1.050858
## a[19]	1.174071478	0.9992791	-0.43397211	2.7980938	90.74391	1.048253
## a[20]	2.782265397	1.2704507	0.83963453	4.8639634	129.51834	1.045751
## a[21]	1.373479408	1.0562155	-0.34753275	3.0732585	96.52165	1.058916
## a[22]	1.396482517	1.0870303	-0.29918835	3.1134891	118.13229	1.039985
## a[23]	1.372688511	1.0667621	-0.32646090	3.1174555	105.24981	1.045413
## a[24]	0.704670981	1.0080500	-0.93731843	2.2660553	85.33340	1.061338
## a[25]	-1.809494204	0.9523025	-3.35544297	-0.2984165	80.84200	1.057958
## a[26]	-0.661235860	0.9432694	-2.22996672	0.8641485	78.97819	1.061147
## a[27]	-2.248597124	0.9928043	-3.87417579	-0.6596707	85.48749	1.058921
## a[28]	-1.288481577	0.9434523	-2.82472847	0.2120235	77.20308	1.060012
## a[29]	-0.870703436	0.9388586	-2.42169339	0.6186033	82.69768	1.064627
## a[30]	0.424556421	0.9845432	-1.15932771	2.0039222	82.47862	1.055337
## a[31]	-1.664253642	0.9448206	-3.21979308	-0.1725687	81.69311	1.058381
## a[32]	-1.337576449	0.9491107	-2.89146524	0.1476786	78.00521	1.063540
## a[33]	2.371166813	1.1249186	0.65398114	4.1947025	111.20477	1.042979
## a[34]	1.907044844	1.0790029	0.22917763	3.6524144	101.15074	1.049063
## a[35]	1.873185764	1.0543368	0.19906845	3.5334078	97.22649	1.048925
## a[36]	1.248876420	0.9834276	-0.33084370	2.8420712	83.40065	1.063163
## a[37]	1.035699667	0.9951968	-0.58544005	2.5867962	91.28967	1.055065
## a[38]	2.891528772	1.2374744	0.95986518	4.9736188	144.49317	1.035782
## a[39]	1.719113428	1.0782531	-0.08766357	3.4739415	99.66345	1.050800
## a[40]	1.311762598	1.0157272	-0.31952213	2.8890446	92.82859	1.052347
## a[41]	-2.593685048	0.9662063	-4.17146729	-1.0718722	82.36267	1.057606
## a[42]	-1.382077862	0.9172532	-2.91284706	0.0822549	73.09649	1.065248
## a[43]	-1.257443247	0.9272011	-2.80781577	0.1867493	79.31793	1.060759
## a[44]	-1.140821862	0.9127055	-2.61029787	0.3495998	75.64764	1.063942
## a[45]	-0.457440207	0.9286512	-1.96102240	0.9359055	81.35436	1.059439
## a[46]	-1.602843942	0.9227675	-3.12955879	-0.1573228	72.40774	1.067545
## a[47]	1.030965803	0.9894109	-0.55725455	2.6022999	83.59596	1.054974
## a[48]	-1.040159145	0.9179116	-2.55284796	0.3606801	78.80351	1.067651


```
## a_size[1] 1.038745980 0.8614009 -0.29191113 2.4295665 66.89926 1.076146
## a_size[2] 0.805957135 0.8573151 -0.55902122 2.2273904 66.37206 1.072417
## a_bar 0.448593667 0.8387281 -0.93867401 1.7548313 66.90063 1.076158
## a_bar_size 0.828082155 0.8620677 -0.50206746 2.1858228 86.79494 1.052481
## sigma 1.605495703 0.2091830 1.30868008 1.9650674 732.15971 1.001702
## sigma_size 0.583574836 0.6127002 0.05834743 1.6578444 518.95600 1.005602
```

```
plot(precis(m_size,depth=2))
```



I am not able to add an interaction between `pred` and `size`, I had a look at page 255 and following but R code 8.24 is about `water` and `shed` being both ordered variables while in this case `pred` is not ordered while `size` can be considered ordered.

2. In 1980, a typical Bengali woman could have 5 or more children in her lifetime. By the year 2000, a typical Bengali woman had only 2 or 3. You're going to look at a historical set of data, when contraception was widely available but many families chose not to use it. These data reside in `data(bangladesh)` and come from the 1988 Bangladesh Fertility Survey. Each row is one of 1934 women. There are six variables, but you can focus on two of them for this practice problem: (1) `district` : ID number of administrative district each woman resided in (2) `use.contraception` : An indicator (0/1) of whether the woman was using contraception. . . Now there are 60 values, contiguous integers 1 to 60. Now, focus on predicting `use.contraception`, clustered by `district_id`. Fit both (1) a traditional fixed-effects model that uses an index variable for `district` and

- (2) a multilevel model with varying intercepts for `district`. Plot the predicted proportions of women in each district using contraception, for both the fixed-effects model and the varying-effects model. That is, make a plot in which `district ID` is on the horizontal axis and expected proportion using contraception is on the vertical. Make one plot for each model, or layer them on the same plot, as you prefer. How do the models disagree? Can you explain the pattern of disagreement? In particular, can you explain the most extreme cases of disagreement, both why they happen where they do and why the models reach different inferences?

```

data(bangladesh)
d<-bangladesh
d$district_id <- as.integer(as.factor(d$district))

district_id <- c(unique(d$district_id))
use_c <- c()
district_size <- c()

for( did in district_id ){
  use_c <- c(use_c,sum(d[d$district_id==did,'use.contraception']))
  district_size <- c(district_size,length(d[d$district_id==did,'use.contraception']))
}

```

R code 13.2

```

dat <- list(
  S = use_c,
  N = district_size,
  district = district_id )

# approximate posterior
m1 <- ulam(
  alist(
    S ~ dbinom( N , p ) ,
    logit(p) <- a[district] ,
    a[district] ~ dnorm( 0 , 1.5 )
  ), data=dat , chains=4 , log_lik=TRUE )

```

```

##
## SAMPLING FOR MODEL '709fe398a220a468e86afba1fb2a21bd' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 2.3e-05 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0.23 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: Iteration:   1 / 1000 [  0%] (Warmup)
## Chain 1: Iteration: 100 / 1000 [ 10%] (Warmup)
## Chain 1: Iteration: 200 / 1000 [ 20%] (Warmup)
## Chain 1: Iteration: 300 / 1000 [ 30%] (Warmup)
## Chain 1: Iteration: 400 / 1000 [ 40%] (Warmup)
## Chain 1: Iteration: 500 / 1000 [ 50%] (Warmup)
## Chain 1: Iteration: 501 / 1000 [ 50%] (Sampling)
## Chain 1: Iteration: 600 / 1000 [ 60%] (Sampling)
## Chain 1: Iteration: 700 / 1000 [ 70%] (Sampling)
## Chain 1: Iteration: 800 / 1000 [ 80%] (Sampling)
## Chain 1: Iteration: 900 / 1000 [ 90%] (Sampling)
## Chain 1: Iteration: 1000 / 1000 [100%] (Sampling)
## Chain 1:
## Chain 1: Elapsed Time: 0.100693 seconds (Warm-up)
## Chain 1:                   0.089061 seconds (Sampling)
## Chain 1:                   0.189754 seconds (Total)
## Chain 1:

```

```

##
## SAMPLING FOR MODEL '709fe398a220a468e86afba1fb2a21bd' NOW (CHAIN 2).
## Chain 2:
## Chain 2: Gradient evaluation took 1.3e-05 seconds
## Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0.13 seconds.
## Chain 2: Adjust your expectations accordingly!
## Chain 2:
## Chain 2:
## Chain 2: Iteration: 1 / 1000 [ 0%] (Warmup)
## Chain 2: Iteration: 100 / 1000 [ 10%] (Warmup)
## Chain 2: Iteration: 200 / 1000 [ 20%] (Warmup)
## Chain 2: Iteration: 300 / 1000 [ 30%] (Warmup)
## Chain 2: Iteration: 400 / 1000 [ 40%] (Warmup)
## Chain 2: Iteration: 500 / 1000 [ 50%] (Warmup)
## Chain 2: Iteration: 501 / 1000 [ 50%] (Sampling)
## Chain 2: Iteration: 600 / 1000 [ 60%] (Sampling)
## Chain 2: Iteration: 700 / 1000 [ 70%] (Sampling)
## Chain 2: Iteration: 800 / 1000 [ 80%] (Sampling)
## Chain 2: Iteration: 900 / 1000 [ 90%] (Sampling)
## Chain 2: Iteration: 1000 / 1000 [100%] (Sampling)
## Chain 2:
## Chain 2: Elapsed Time: 0.103188 seconds (Warm-up)
## Chain 2: 0.089186 seconds (Sampling)
## Chain 2: 0.192374 seconds (Total)
## Chain 2:
##
## SAMPLING FOR MODEL '709fe398a220a468e86afba1fb2a21bd' NOW (CHAIN 3).
## Chain 3:
## Chain 3: Gradient evaluation took 1.2e-05 seconds
## Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0.12 seconds.
## Chain 3: Adjust your expectations accordingly!
## Chain 3:
## Chain 3:
## Chain 3: Iteration: 1 / 1000 [ 0%] (Warmup)
## Chain 3: Iteration: 100 / 1000 [ 10%] (Warmup)
## Chain 3: Iteration: 200 / 1000 [ 20%] (Warmup)
## Chain 3: Iteration: 300 / 1000 [ 30%] (Warmup)
## Chain 3: Iteration: 400 / 1000 [ 40%] (Warmup)
## Chain 3: Iteration: 500 / 1000 [ 50%] (Warmup)
## Chain 3: Iteration: 501 / 1000 [ 50%] (Sampling)
## Chain 3: Iteration: 600 / 1000 [ 60%] (Sampling)
## Chain 3: Iteration: 700 / 1000 [ 70%] (Sampling)
## Chain 3: Iteration: 800 / 1000 [ 80%] (Sampling)
## Chain 3: Iteration: 900 / 1000 [ 90%] (Sampling)
## Chain 3: Iteration: 1000 / 1000 [100%] (Sampling)
## Chain 3:
## Chain 3: Elapsed Time: 0.100248 seconds (Warm-up)
## Chain 3: 0.091519 seconds (Sampling)
## Chain 3: 0.191767 seconds (Total)
## Chain 3:
##
## SAMPLING FOR MODEL '709fe398a220a468e86afba1fb2a21bd' NOW (CHAIN 4).
## Chain 4:
## Chain 4: Gradient evaluation took 1.2e-05 seconds

```

```

## Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 0.12 seconds.
## Chain 4: Adjust your expectations accordingly!
## Chain 4:
## Chain 4:
## Chain 4: Iteration: 1 / 1000 [ 0%] (Warmup)
## Chain 4: Iteration: 100 / 1000 [ 10%] (Warmup)
## Chain 4: Iteration: 200 / 1000 [ 20%] (Warmup)
## Chain 4: Iteration: 300 / 1000 [ 30%] (Warmup)
## Chain 4: Iteration: 400 / 1000 [ 40%] (Warmup)
## Chain 4: Iteration: 500 / 1000 [ 50%] (Warmup)
## Chain 4: Iteration: 501 / 1000 [ 50%] (Sampling)
## Chain 4: Iteration: 600 / 1000 [ 60%] (Sampling)
## Chain 4: Iteration: 700 / 1000 [ 70%] (Sampling)
## Chain 4: Iteration: 800 / 1000 [ 80%] (Sampling)
## Chain 4: Iteration: 900 / 1000 [ 90%] (Sampling)
## Chain 4: Iteration: 1000 / 1000 [100%] (Sampling)
## Chain 4:
## Chain 4: Elapsed Time: 0.10315 seconds (Warm-up)
## Chain 4: 0.093959 seconds (Sampling)
## Chain 4: 0.197109 seconds (Total)
## Chain 4:

```

```
precis(m1,depth=2)
```

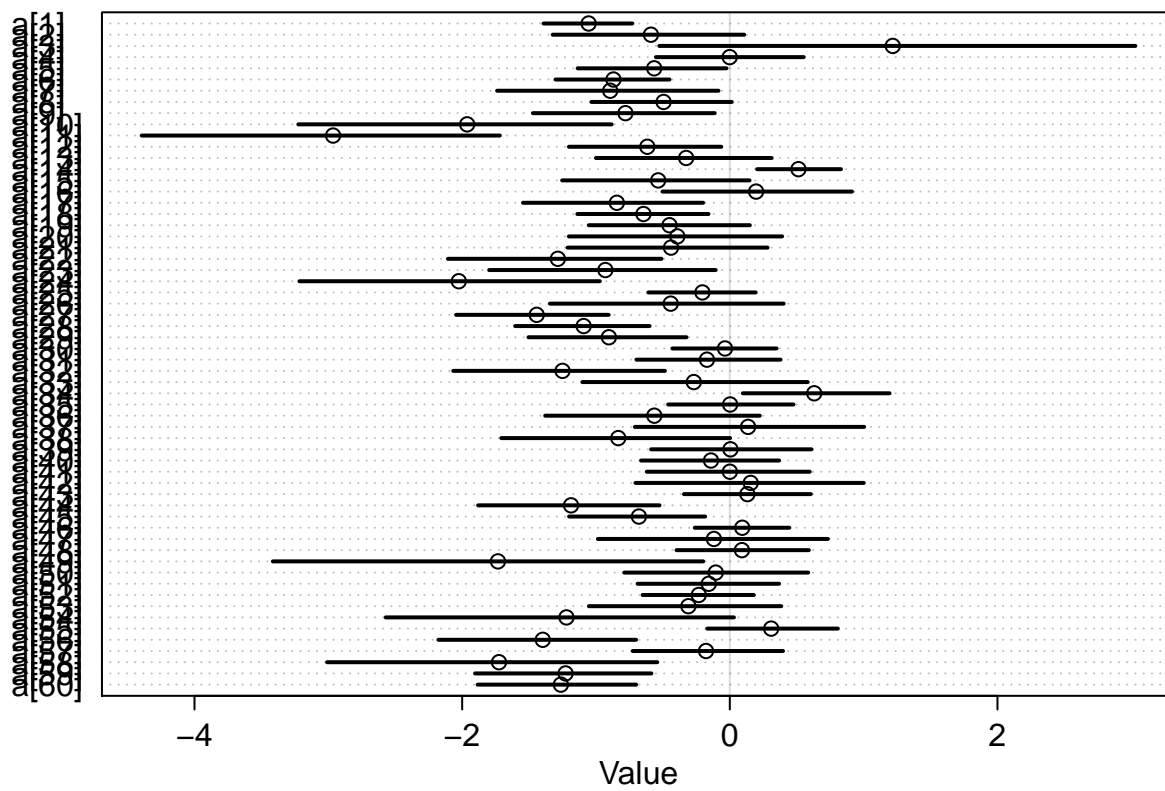
##	mean	sd	5.5%	94.5%	n_eff	Rhat4
## a[1]	-1.0548425931	0.2138895	-1.39195590	-0.728060332	3711.375	0.9994615
## a[2]	-0.5893226111	0.4419932	-1.32382414	0.107855198	4135.500	0.9991980
## a[3]	1.2177922595	1.1278124	-0.52648984	3.030631609	4768.306	0.9985022
## a[4]	-0.0008725889	0.3442745	-0.55234472	0.552437694	3566.055	0.9986034
## a[5]	-0.5650049786	0.3359372	-1.13685637	-0.024267853	2832.798	0.9993396
## a[6]	-0.8694667931	0.2671782	-1.30299126	-0.450497080	4819.484	0.9981877
## a[7]	-0.8937205014	0.5073182	-1.73861687	-0.084283116	3823.447	0.9994216
## a[8]	-0.4947852446	0.3281858	-1.03404201	0.015172857	4037.181	0.9986057
## a[9]	-0.7796785915	0.4333554	-1.47160555	-0.112473570	3532.816	0.9990159
## a[10]	-1.9625048628	0.7321373	-3.22442800	-0.883047292	3394.917	0.9984647
## a[11]	-2.9645108358	0.8492128	-4.39473767	-1.718059188	4073.705	0.9985178
## a[12]	-0.6162828107	0.3604222	-1.20221552	-0.063414658	6471.813	0.9987276
## a[13]	-0.3261875281	0.4102518	-1.00032288	0.313086686	4484.920	0.9981788
## a[14]	0.5129470800	0.1952474	0.20416375	0.831169501	4772.947	0.9984068
## a[15]	-0.5351811652	0.4348243	-1.25334150	0.147709441	4587.342	0.9994541
## a[16]	0.1967213236	0.4450413	-0.50341979	0.913454143	4444.348	0.9990182
## a[17]	-0.8440480744	0.4158606	-1.54534154	-0.199482584	4641.625	0.9987199
## a[18]	-0.6453427543	0.3085782	-1.13939075	-0.159026270	5182.675	0.9985462
## a[19]	-0.4506289475	0.3769549	-1.05403374	0.149861215	5179.249	0.9983800
## a[20]	-0.3922374090	0.5128086	-1.20262414	0.391200557	4605.524	0.9994258
## a[21]	-0.4393701623	0.4716272	-1.21240856	0.282387652	6602.060	0.9988859
## a[22]	-1.2850837635	0.5089363	-2.10657571	-0.510098899	4058.207	0.9985865
## a[23]	-0.9297682505	0.5487580	-1.79829606	-0.105372751	4148.759	0.9985445
## a[24]	-2.0249928189	0.7276461	-3.21609629	-0.970900716	4521.413	0.9988117
## a[25]	-0.2045510692	0.2480512	-0.60925601	0.193401464	4356.998	0.9984747
## a[26]	-0.4416144713	0.5597838	-1.34453632	0.402719894	5368.883	0.9982268
## a[27]	-1.4429596425	0.3612788	-2.04488308	-0.905003627	3044.131	0.9987260
## a[28]	-1.0922579919	0.3200337	-1.60456690	-0.599318422	5028.092	0.9982969
## a[29]	-0.9034394836	0.3832192	-1.50447943	-0.323801635	3865.881	0.9986047
## a[30]	-0.0359593846	0.2454400	-0.42971422	0.349775680	4939.402	0.9985233

```

## a[31] -0.1706384054 0.3425942 -0.69786269 0.379563835 5046.269 0.9988628
## a[32] -1.2501146241 0.4826829 -2.06651219 -0.484274701 4193.624 0.9983404
## a[33] -0.2688762320 0.5439063 -1.10208429 0.582516764 4354.878 0.9989640
## a[34] 0.6309403755 0.3442492 0.09766687 1.193778951 3929.724 0.9981682
## a[35] 0.0025274685 0.2898846 -0.46103888 0.475595079 3675.066 0.9988808
## a[36] -0.5639950357 0.4947712 -1.37958214 0.224337722 5704.253 0.9984714
## a[37] 0.1364020795 0.5458283 -0.70959625 1.003952125 5360.368 0.9987546
## a[38] -0.8333361192 0.5488578 -1.70776688 0.002094669 4231.023 0.9985986
## a[39] 0.0048907553 0.3800993 -0.58788010 0.610513204 3823.342 0.9992339
## a[40] -0.1406677464 0.3199628 -0.66300179 0.368537895 5228.353 0.9984645
## a[41] 0.0004016109 0.3854345 -0.62073385 0.597542895 4050.229 0.9982206
## a[42] 0.1561816991 0.5489456 -0.70514130 1.000915600 3664.526 0.9989291
## a[43] 0.1313601376 0.2927323 -0.34236309 0.605670718 4031.247 1.0000512
## a[44] -1.1862227406 0.4224920 -1.88089433 -0.525129739 3433.818 0.9984237
## a[45] -0.6792566437 0.3233596 -1.20097783 -0.184237057 4356.596 0.9984160
## a[46] 0.0931033064 0.2233119 -0.26205000 0.445183450 5565.970 0.9988095
## a[47] -0.1187596129 0.5305883 -0.98615643 0.733300871 4425.628 0.9983109
## a[48] 0.0905619779 0.3120960 -0.39809552 0.590575576 4894.220 0.9988007
## a[49] -1.7325773721 1.0293144 -3.41468483 -0.198587739 4140.468 0.9993561
## a[50] -0.1047557213 0.4324779 -0.78895233 0.585995054 5083.771 0.9990259
## a[51] -0.1571976892 0.3316959 -0.68932570 0.368708665 4418.676 0.9983406
## a[52] -0.2321519271 0.2578882 -0.65140710 0.179314743 6276.565 0.9986063
## a[53] -0.3094356954 0.4506031 -1.05309493 0.384690325 6207.186 0.9986813
## a[54] -1.2205134071 0.8120757 -2.57130889 0.031079089 4780.519 0.9990695
## a[55] 0.3092441614 0.3095728 -0.16901252 0.807872702 4199.245 0.9984992
## a[56] -1.3976356851 0.4700220 -2.17767022 -0.699784793 3715.173 0.9989585
## a[57] -0.1778652751 0.3472446 -0.72516870 0.398021904 4672.542 0.9984533
## a[58] -1.7247110139 0.7807272 -3.00970133 -0.541882457 4096.047 0.9987086
## a[59] -1.2271966526 0.4070474 -1.90384693 -0.585735867 3901.030 0.9989593
## a[60] -1.2622202038 0.3784201 -1.88443878 -0.699484456 4726.972 0.9988880

```

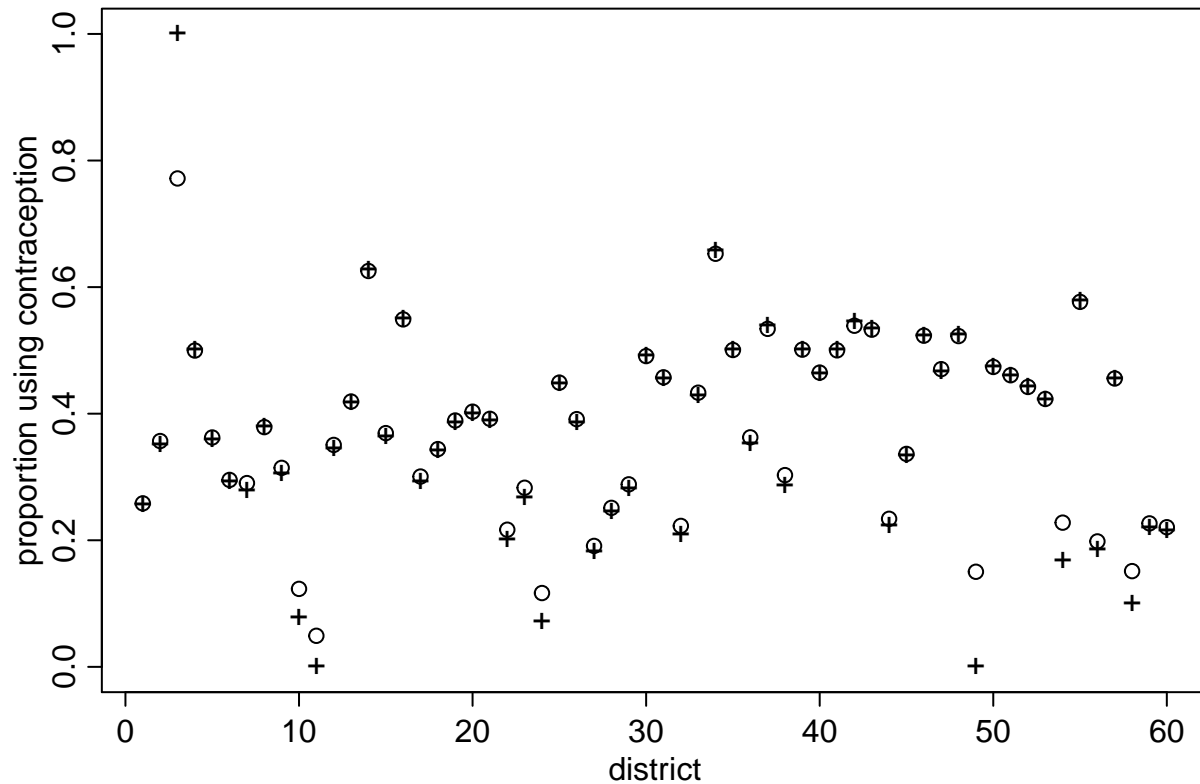
```
plot(precis(m1,depth=2))
```



```
post1 <- extract.samples(m1)
```

```
plot(logistic(apply(post1$a,2, mean)),ylim=c(0,1),main='non varying intercepts, o predicted, + empirical')
points(use_c/district_size,pch='+')
```

non varying intercepts, o predicted, + empirical



R code 13.2

approximate posterior

```
m2 <- ulam(
  alist(
    S ~ dbinom( N , p ) ,
    logit(p) <- a[district] ,
    a[district] ~ dnorm( a_bar , sigma ) ,
    a_bar ~ dnorm(0,1.5),
    sigma ~ dexp(1)
  ), data=dat , chains=4 , log_lik=TRUE )
```

##

SAMPLING FOR MODEL '04b21da6ff803f3e0af99f13859c3fa8' NOW (CHAIN 1).

Chain 1:

Chain 1: Gradient evaluation took 7.7e-05 seconds

Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0.77 seconds.

Chain 1: Adjust your expectations accordingly!

Chain 1:

Chain 1:

Chain 1: Iteration: 1 / 1000 [0%] (Warmup)

Chain 1: Iteration: 100 / 1000 [10%] (Warmup)

Chain 1: Iteration: 200 / 1000 [20%] (Warmup)

Chain 1: Iteration: 300 / 1000 [30%] (Warmup)

Chain 1: Iteration: 400 / 1000 [40%] (Warmup)

Chain 1: Iteration: 500 / 1000 [50%] (Warmup)

Chain 1: Iteration: 501 / 1000 [50%] (Sampling)

```

## Chain 1: Iteration: 600 / 1000 [ 60%] (Sampling)
## Chain 1: Iteration: 700 / 1000 [ 70%] (Sampling)
## Chain 1: Iteration: 800 / 1000 [ 80%] (Sampling)
## Chain 1: Iteration: 900 / 1000 [ 90%] (Sampling)
## Chain 1: Iteration: 1000 / 1000 [100%] (Sampling)
## Chain 1:
## Chain 1: Elapsed Time: 0.154356 seconds (Warm-up)
## Chain 1: 0.136006 seconds (Sampling)
## Chain 1: 0.290362 seconds (Total)
## Chain 1:
##
## SAMPLING FOR MODEL '04b21da6ff803f3e0af99f13859c3fa8' NOW (CHAIN 2).
## Chain 2:
## Chain 2: Gradient evaluation took 1.6e-05 seconds
## Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0.16 seconds.
## Chain 2: Adjust your expectations accordingly!
## Chain 2:
## Chain 2:
## Chain 2: Iteration: 1 / 1000 [ 0%] (Warmup)
## Chain 2: Iteration: 100 / 1000 [ 10%] (Warmup)
## Chain 2: Iteration: 200 / 1000 [ 20%] (Warmup)
## Chain 2: Iteration: 300 / 1000 [ 30%] (Warmup)
## Chain 2: Iteration: 400 / 1000 [ 40%] (Warmup)
## Chain 2: Iteration: 500 / 1000 [ 50%] (Warmup)
## Chain 2: Iteration: 501 / 1000 [ 50%] (Sampling)
## Chain 2: Iteration: 600 / 1000 [ 60%] (Sampling)
## Chain 2: Iteration: 700 / 1000 [ 70%] (Sampling)
## Chain 2: Iteration: 800 / 1000 [ 80%] (Sampling)
## Chain 2: Iteration: 900 / 1000 [ 90%] (Sampling)
## Chain 2: Iteration: 1000 / 1000 [100%] (Sampling)
## Chain 2:
## Chain 2: Elapsed Time: 0.162841 seconds (Warm-up)
## Chain 2: 0.095812 seconds (Sampling)
## Chain 2: 0.258653 seconds (Total)
## Chain 2:
##
## SAMPLING FOR MODEL '04b21da6ff803f3e0af99f13859c3fa8' NOW (CHAIN 3).
## Chain 3:
## Chain 3: Gradient evaluation took 1.5e-05 seconds
## Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0.15 seconds.
## Chain 3: Adjust your expectations accordingly!
## Chain 3:
## Chain 3:
## Chain 3: Iteration: 1 / 1000 [ 0%] (Warmup)
## Chain 3: Iteration: 100 / 1000 [ 10%] (Warmup)
## Chain 3: Iteration: 200 / 1000 [ 20%] (Warmup)
## Chain 3: Iteration: 300 / 1000 [ 30%] (Warmup)
## Chain 3: Iteration: 400 / 1000 [ 40%] (Warmup)
## Chain 3: Iteration: 500 / 1000 [ 50%] (Warmup)
## Chain 3: Iteration: 501 / 1000 [ 50%] (Sampling)
## Chain 3: Iteration: 600 / 1000 [ 60%] (Sampling)
## Chain 3: Iteration: 700 / 1000 [ 70%] (Sampling)
## Chain 3: Iteration: 800 / 1000 [ 80%] (Sampling)
## Chain 3: Iteration: 900 / 1000 [ 90%] (Sampling)

```



```

## Chain 3: Iteration: 1000 / 1000 [100%] (Sampling)
## Chain 3:
## Chain 3: Elapsed Time: 0.162528 seconds (Warm-up)
## Chain 3: 0.122012 seconds (Sampling)
## Chain 3: 0.28454 seconds (Total)
## Chain 3:
##
## SAMPLING FOR MODEL '04b21da6ff803f3e0af99f13859c3fa8' NOW (CHAIN 4).
## Chain 4:
## Chain 4: Gradient evaluation took 3.6e-05 seconds
## Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 0.36 seconds.
## Chain 4: Adjust your expectations accordingly!
## Chain 4:
## Chain 4:
## Chain 4: Iteration: 1 / 1000 [ 0%] (Warmup)
## Chain 4: Iteration: 100 / 1000 [ 10%] (Warmup)
## Chain 4: Iteration: 200 / 1000 [ 20%] (Warmup)
## Chain 4: Iteration: 300 / 1000 [ 30%] (Warmup)
## Chain 4: Iteration: 400 / 1000 [ 40%] (Warmup)
## Chain 4: Iteration: 500 / 1000 [ 50%] (Warmup)
## Chain 4: Iteration: 501 / 1000 [ 50%] (Sampling)
## Chain 4: Iteration: 600 / 1000 [ 60%] (Sampling)
## Chain 4: Iteration: 700 / 1000 [ 70%] (Sampling)
## Chain 4: Iteration: 800 / 1000 [ 80%] (Sampling)
## Chain 4: Iteration: 900 / 1000 [ 90%] (Sampling)
## Chain 4: Iteration: 1000 / 1000 [100%] (Sampling)
## Chain 4:
## Chain 4: Elapsed Time: 0.17676 seconds (Warm-up)
## Chain 4: 0.101906 seconds (Sampling)
## Chain 4: 0.278666 seconds (Total)
## Chain 4:

```

```
precis(m2,depth=2)
```

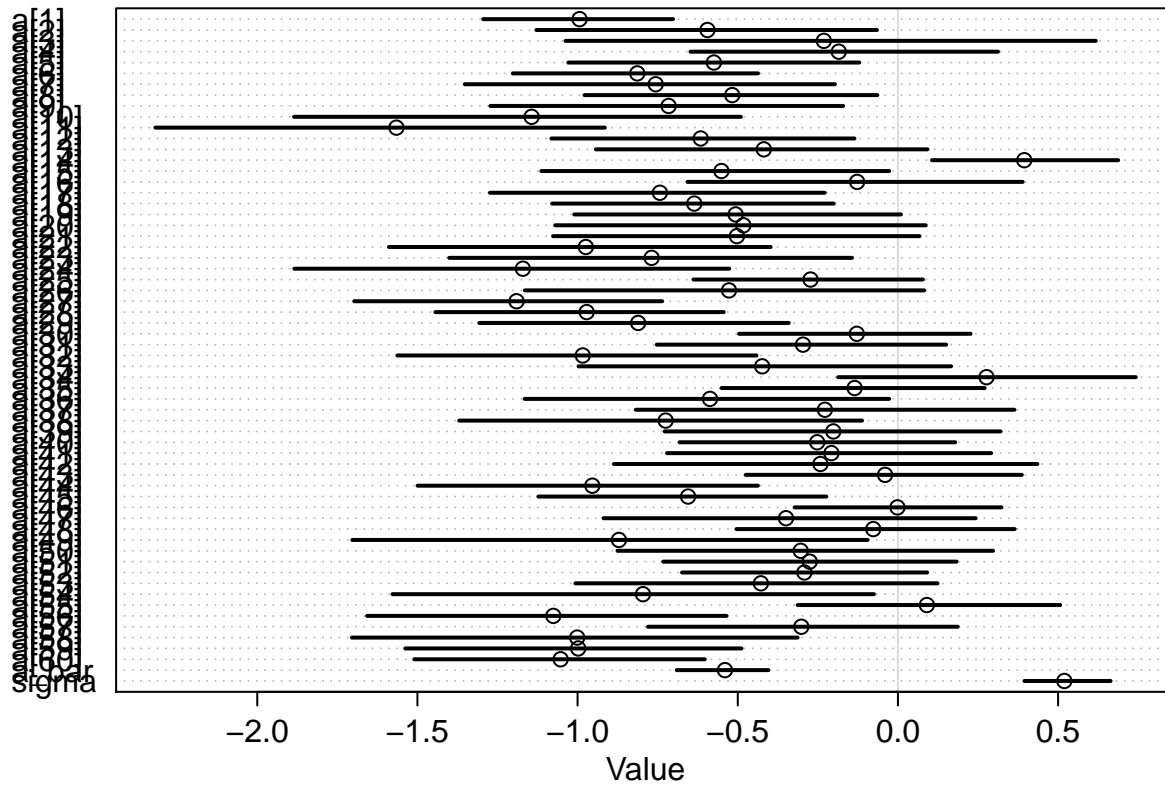
##		mean	sd	5.5%	94.5%	n_eff	Rhat4
##	a[1]	-0.993817813	0.18794025	-1.2947022	-0.701823196	3544.0195	0.9987209
##	a[2]	-0.594688227	0.33697611	-1.1287720	-0.065379120	3130.8953	0.9993047
##	a[3]	-0.231304361	0.52157369	-1.0375518	0.617651480	3360.8353	0.9990272
##	a[4]	-0.184715390	0.30514611	-0.6474008	0.312504002	3278.8542	0.9991729
##	a[5]	-0.574600060	0.28782543	-1.0290800	-0.120702535	3651.7666	0.9989587
##	a[6]	-0.813681904	0.24317103	-1.2023189	-0.436828229	2758.2063	0.9994750
##	a[7]	-0.756149907	0.36564908	-1.3519568	-0.196702971	4212.9147	1.0004378
##	a[8]	-0.517310477	0.28517730	-0.9784946	-0.064174412	3087.2662	0.9982998
##	a[9]	-0.715837687	0.34116149	-1.2722350	-0.171152369	3446.6954	0.9994635
##	a[10]	-1.143450856	0.43194208	-1.8853097	-0.490261543	2711.5817	0.9995631
##	a[11]	-1.565575059	0.44831699	-2.3181592	-0.914810431	1687.5083	1.0009902
##	a[12]	-0.615603374	0.30417345	-1.0820384	-0.135579272	3773.9790	0.9992532
##	a[13]	-0.418696615	0.32574946	-0.9431226	0.092308863	2790.3666	1.0004750
##	a[14]	0.394365456	0.17856567	0.1061670	0.687555242	3461.9779	0.9993302
##	a[15]	-0.550978033	0.34692867	-1.1130903	-0.027351932	4387.3693	0.9986528
##	a[16]	-0.127386319	0.33678657	-0.6567937	0.389456652	3918.4141	0.9989157
##	a[17]	-0.743070152	0.32830159	-1.2736606	-0.228840421	3379.1434	0.9990559
##	a[18]	-0.635856928	0.27697774	-1.0797422	-0.200436277	3835.3940	1.0001023
##	a[19]	-0.506648513	0.31964977	-1.0103745	0.009484768	2686.1769	0.9989728
##	a[20]	-0.482457431	0.36644518	-1.0699844	0.086844888	3036.0442	0.9996114

```

## a[21] -0.502650272 0.37376067 -1.0766354 0.067561302 2982.3655 0.9994195
## a[22] -0.974307253 0.37646722 -1.5890277 -0.397296492 2912.3626 0.9990445
## a[23] -0.768934179 0.39318975 -1.4007155 -0.143382261 2799.7903 0.9994921
## a[24] -1.170794378 0.42933190 -1.8842269 -0.526426947 1996.0510 0.9994757
## a[25] -0.272782059 0.22550136 -0.6383860 0.078073692 3649.4068 0.9988562
## a[26] -0.527358790 0.38821785 -1.1646848 0.081853627 3524.3913 0.9986763
## a[27] -1.190591874 0.29831108 -1.6972846 -0.735839469 2609.5301 0.9991062
## a[28] -0.971673888 0.28435626 -1.4439172 -0.543896484 2758.6831 0.9989562
## a[29] -0.810952174 0.30564317 -1.3068401 -0.340870641 3057.5856 0.9998674
## a[30] -0.128407649 0.22635000 -0.4961135 0.226622425 3493.2177 0.9983584
## a[31] -0.296316870 0.28309692 -0.7529505 0.150687684 3175.8442 0.9993601
## a[32] -0.983916143 0.35934110 -1.5629398 -0.441400486 2663.4469 1.0010611
## a[33] -0.423704209 0.36332250 -0.9970960 0.166044992 3309.6777 1.0002608
## a[34] 0.276534800 0.29431461 -0.1865165 0.742842689 3029.9400 0.9989316
## a[35] -0.135217301 0.25363904 -0.5506284 0.270635622 3669.9032 0.9995169
## a[36] -0.586782124 0.35632840 -1.1652832 -0.027299369 3332.0552 0.9988260
## a[37] -0.228026711 0.37073967 -0.8183873 0.363615165 3471.2894 0.9984415
## a[38] -0.725143207 0.38814131 -1.3695892 -0.111841158 2876.5718 0.9988531
## a[39] -0.201723634 0.33004064 -0.7281098 0.320610790 3455.6218 0.9994660
## a[40] -0.252064664 0.27263994 -0.6816980 0.178987025 3878.3956 0.9984592
## a[41] -0.207592088 0.31683279 -0.7208486 0.290996510 3262.0874 0.9994555
## a[42] -0.241879271 0.41124268 -0.8867779 0.435399028 3685.8049 0.9984863
## a[43] -0.040268891 0.26890554 -0.4752474 0.386770447 4424.5024 0.9987694
## a[44] -0.953957580 0.33339115 -1.4994976 -0.437000680 2448.7865 0.9994770
## a[45] -0.655114358 0.28632310 -1.1225562 -0.223323844 4361.8985 0.9993574
## a[46] -0.001539744 0.20412149 -0.3225511 0.323311199 3684.5162 0.9989460
## a[47] -0.349494094 0.36219345 -0.9192168 0.242863246 4133.5193 0.9991831
## a[48] -0.077019103 0.26711989 -0.5043203 0.364546341 4605.0131 0.9988252
## a[49] -0.870925182 0.51300048 -1.7031040 -0.094844373 2960.2142 1.0001170
## a[50] -0.303472046 0.35882797 -0.8755010 0.297015809 3668.5708 0.9995539
## a[51] -0.275459177 0.29056359 -0.7326520 0.183251832 2862.3020 0.9997387
## a[52] -0.291910080 0.23346002 -0.6739469 0.091338537 3887.4893 0.9994003
## a[53] -0.427875170 0.34564238 -1.0067723 0.123652942 3661.1211 0.9990256
## a[54] -0.795884536 0.47670205 -1.5778507 -0.074902286 3135.7027 1.0004905
## a[55] 0.090313125 0.25953739 -0.3129921 0.506384491 3103.8030 0.9992097
## a[56] -1.075763243 0.35203024 -1.6570358 -0.535303190 2975.6350 0.9989435
## a[57] -0.301491951 0.30450320 -0.7802890 0.186814472 3238.8983 0.9993358
## a[58] -1.001266086 0.43771668 -1.7046478 -0.313277649 2784.5245 1.0001488
## a[59] -0.998228826 0.32669169 -1.5379563 -0.488281114 3075.8017 0.9991543
## a[60] -1.052906672 0.28651457 -1.5098695 -0.603111762 3194.0566 0.9995379
## a_bar -0.540012743 0.09026923 -0.6902973 -0.404349199 1413.2628 0.9999215
## sigma 0.519469074 0.08453406 0.3948293 0.664016345 798.6412 1.0045030

```

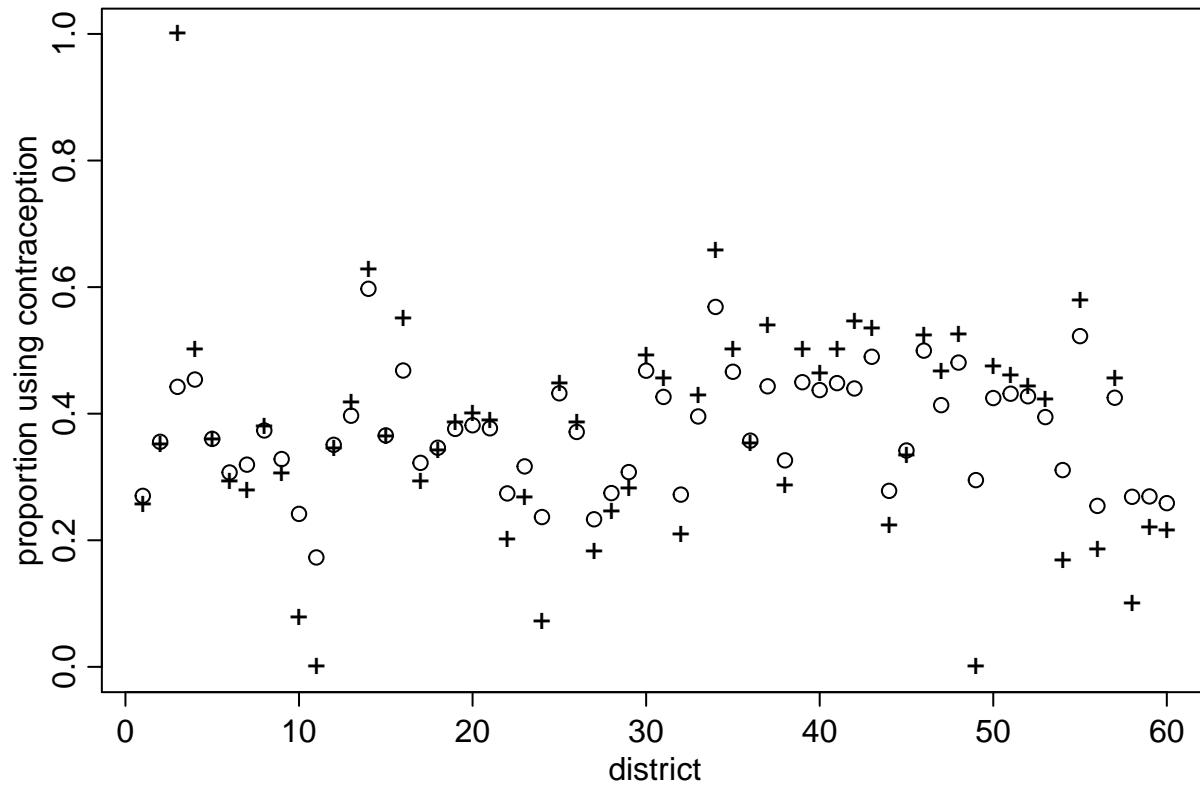
```
plot(precis(m2,depth=2))
```



```
post2 <- extract.samples(m2)
```

```
plot(logistic(apply(post2$a,2, mean)),ylim=c(0,1),main='varying intercepts, o predicted, + empirical',xlab='Value',ylab='District',pch=c(1,2))
```

varying intercepts, o predicted, + empirical



```
plot(logistic(apply(post2$a,2, mean)),ylim=c(0,1),main='o varying intercepts, x non varying',xlab='district')
points(logistic(apply(post1$a,2, mean)),pch='x')
```

o varying intercepts, x non varying

