Some notes about subpixel accuracy

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Abstract

Some notes about subpixel accuracy.

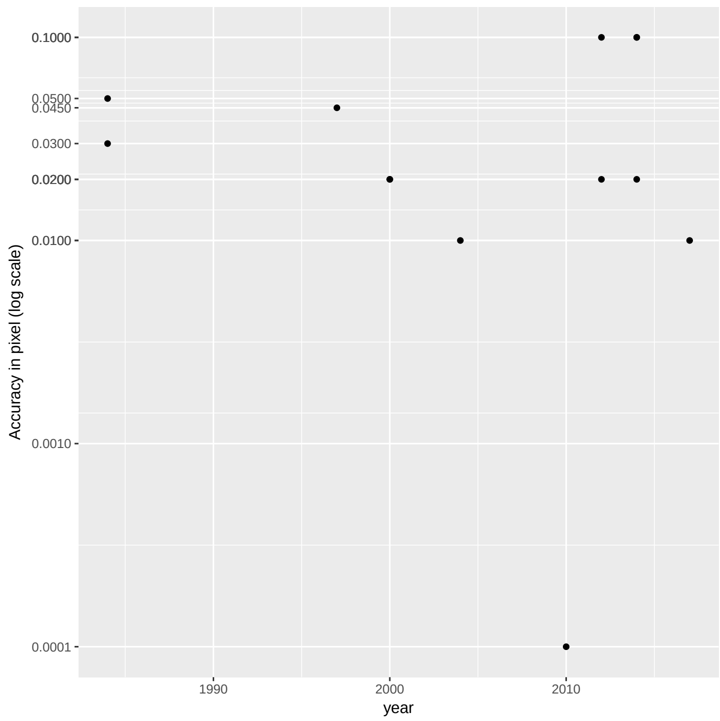

Figure 1: Some results.

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1 Some statements found in literature

In [8, section 8.2, p.270]:

> In order to transform images back from pixel coordinates to world coordinates, various transformations are required. In the simplest case, this includes only scaling, translation (shifting), and rotation of images. More general are affine (Section 8.3.1b) and perspective (Section 8.3.1c) transformations. For precise geometric measurements, it is also required to correct for the residual geometric distortion introduced by even well-corrected optical systems. Modern imaging solid-state sensors are geometrically very precise and stable. Therefore, **the potential of a position accuracy of better than 1/100 pixel distance is there**. To maintain this accuracy, all geometric transformations applied to digital images must preserve this high position accuracy. This demand goes far beyond the fact that no distortions are visible in the transformed images.

In [10]:

> HALCON's 3D calibration supports both area and line scan cameras and permits, for example, **subpixel-accurate measurements up to** $1\,\mu\mathrm{m}$ **in a field of view of 10 mm**.

In [5]:

> Modern CCD cameras are **usually capable of a spatial accuracy greater than 1/50 of the pixel size**. However, such accuracy is not easily attained due to various error sources that can affect the image formation process.

In [4]:

> A **commonly quoted rule of thumb is 0.1 pixel**, but lower is achievable, e.g. **about 0.02 pixel is shown for stripe position detection** in [1][1].

In [2]:

> 20.3.3 Differing Requirements for Size Measurement

[1] Cited here as [1].

Size measurement is important both in the food industry and in the automotive and small-parts industry. However, the problems in the two cases are often rather different. For example, the diameter of a biscuit can vary within quite wide limits ($\sim$5%) without giving rise to undue problems, but when it gets outside this range, there is a serious risk of jamming the packing machine, and the situation must be monitored carefully. In contrast, for mechanical parts, the required precision can vary from 1% for objects such as O-rings to 0.01% for piston heads. This variation clearly makes it difficult to design a truly general-purpose inspection system. However, the manufacturing process often permits little variation in size from one item to the next. Hence, it may be adequate to have a system that is capable of measuring to an accuracy of rather better than 1%, so long as it is capable of checking all the characteristics mentioned in Table 20.1. For cases where high precision is vital, it is important that accuracy of measurement is proportional to the resolution of the input image. Currently, images of up to 512 $\times$ 512 pixels are common, so accuracy of measurement is basically of the order of 0.2%. Fortunately, grayscale images provide a means of obtaining significantly greater accuracy than indicated by the above arguments, since the exact transition from dark to light at the boundary of an object can be estimated more closely. In addition, averaging techniques (e.g., along the side of a rectangular block of metal) permit accuracies to be increased even further–by a factor $\sqrt{N}$ if N pixel measurements are made. These factors permit measurements to be made to **subpixel resolution, sometimes even down to 0.1 pixels**, the limit often being set by variations in illumination rather than by the vision algorithms themselves.

In [6]:

Modern CCD cameras are **usually capable of a spatial accuracy greater than 1/50 of the pixel size**. However, such accuracy is not easily attained due to various error sources that can affect the image formation process. Current calibration methods typically assume that the observations are unbiased, the only error is the zero-mean independent and identically distributed random noise in the observed image coordinates, and the camera model completely explains the mapping between the 3D coordinates and the image coordinates. In general, these conditions are not met, causing the calibration results to be less accurate than expected.

In [3]:

> In special circumstances, measurements to subpixel accuracy can be achieved by interpolation between pixel values, and Reference 3[2] describes the **measurement of a knife edge to about 0.1 pixel accuracy**. As a rule of thumb, however, for size measurement applications, the sensor should have a number of pixels at least equal to twice the ratio of the largest to smallest object sizes of interest [4][3], and a lens is then selected to provide the required magnification and working distance.

In the abstract of [9]:

> **Accuracies achieved have reached to within 0.03–0.05 pixel**.

In [11]:

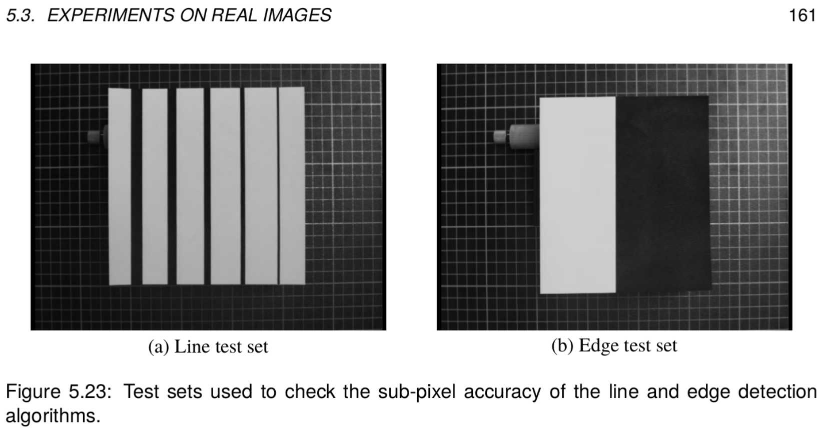
> In order to quantify the impact of the compensation method on the accuracy of the calibration, we measured the standard deviation of the calibration points before and after its adoption. **The standard deviation dropped from 0.081 pixel to 0.045 pixel**, which corresponds to an improvement of about 44 percent.

In [14]:

> Taking literally "plumb–line methods", we built a "calibration harp" instead of the classic flat patterns to obtain a high-precision measurement tool, **demonstrably reaching 2/100 pixel precisions**.

In [13]:

> We concluded by extensive numerical experiments that, although high degree polynomials were required to reach a **high precision of 1/100 pixels**, such polynomials were easily estimated and produced a precise distortion modeling without overfitting.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(a) Line test set

(b) Edge test set

Figure 5.23: Test sets used to check the sub-pixel accuracy of the line and edge detection algorithms.

Figure 2: Figure 5.23 from Steger's thesis [12].

1.1 Carsten Steger's thesis

In [12, p.1]

> Experiments on real images show that **sub-pixel accuracy better than one tenth of a pixel is possible in typical inspection tasks**.

In [12, Section 5.3.2 Sub-Pixel Accuracy of Line Position and Width, p.164]

> Therefore, **relative shifts of one tenth of a pixel can definitively be detected in real images.**
>
> As can be seen, the **absolute position errors are less than one fortieth of a pixel.**

In [12, Section 5.3.2 Sub-Pixel Accuracy of Line Position and Width, p.165]

> As can be seen, the variance is less than 0.0005 almost everywhere, i.e., **the standard deviation of the extracted line widths is less than one fortieth of a pixel.**

In [12, 5.3.3 Sub-Pixel Accuracy of Edge Position, p.166]

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[2]Cited here as [15].

[3]Cited here as [7].

However, the standard deviation of the edge positions is still very small, being approximately one twenty-fifth of a pixel. Therefore, with the same hypothesis test as used above, it can be shown that **edge shifts of one tenth of a pixel can be detected with better than 99.9% probability.**

> **The maximum absolute error is approximately one thirtieth of a pixel.** Therefore, edges can be extracted with very good absolute sub-pixel accuracy.

In [12, Chapter 6 Conclusions, p.168]

> For real images it is shown that **position shifts of one tenth of a pixel can be detected with a probability of more than 99.9%**, indicating that much better sub-pixel accuracy than one tenth of a pixel can be achieved for real images. Thus, it is shown that the line and edge extraction algorithms not only achieve sub-pixel resolution, but also sub-pixel precision and accuracy.

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