Chapter 1: Introduction to Fiber Optics

Basics of Electromagnetic Waves

Maxwell's Equations (Time Domain)

Electromagnetic phenomena are described by **Maxwell's equations**, which relate the electric and magnetic fields and their variations in space and time:

$$\nabla \times \vec{e} = -\frac{\partial \vec{b}}{\partial t} \tag{1}$$

$$\nabla \times \vec{h} = \vec{j} + \frac{\partial \vec{d}}{\partial t} \tag{2}$$

$$\nabla \cdot \vec{d} = \rho \tag{3}$$

$$\nabla \cdot \vec{b} = 0 \tag{4}$$

where:

- $\vec{e}(\vec{r}, t)$ is the electric field [V/m],
- $\vec{d}(\vec{r}, t)$ is the electric flux density [$\mathrm{C/m^2}$],
- $\vec{h}(\vec{r}, t)$ is the magnetic field [A/m],
- $\vec{b}(\vec{r}, t)$ is the magnetic flux density [$\mathrm{Wb/m^2}$],
- $\rho(\vec{r}, t)$ is the charge density [$\mathrm{C/m^3}$],
- $\vec{j}(\vec{r}, t)$ is the electric current density [$\mathrm{A/m^2}$],
- $\vec{r} = (x, y, z)$ is the position vector.

The differential operator ∇ (nabla) is defined as:

$$\nabla = \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{bmatrix}$$

and acts as:

$$\nabla \cdot \vec{a} = \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z}, \qquad \nabla \times \vec{a} = \begin{bmatrix} \frac{\partial a_z}{\partial y} - \frac{\partial a_y}{\partial z} \\ \frac{\partial a_x}{\partial z} - \frac{\partial a_z}{\partial x} \\ \frac{\partial a_y}{\partial x} - \frac{\partial a_x}{\partial y} \end{bmatrix}$$

Two useful vector identities to remember:

$$\nabla \cdot (\nabla \times \vec{A}) = 0, \qquad \vec{A} \cdot (\nabla \times \vec{B}) = 0$$

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Time-Harmonic Fields

For time-harmonic fields, we can express a time-varying electric field as:

$$\vec{e}(t) = \text{Re}\left[\vec{E} e^{j\omega t}\right]$$

where $\omega = 2\pi f$ is the **angular frequency**.

Expanding:

$$\vec{E} = \vec{E}_R + j\vec{E}_I, \qquad e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$$

hence:

$$\vec{e}(t) = \vec{E}_R \cos(\omega t) - \vec{E}_I \sin(\omega t)$$

Taking the time derivative:

$$\frac{\partial \vec{e}}{\partial t} = \text{Re}\left[j\omega \vec{E} e^{j\omega t}\right]$$

Substituting into Maxwell's first equation:

$$\nabla \times \vec{e} = -\frac{\partial \vec{b}}{\partial t}$$

and using the harmonic representation:

$$\nabla \times \vec{E} = -j\omega \vec{B}$$

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Maxwell's Equations (Harmonic Domain)

In many practical cases, electromagnetic fields can be represented by **sinusoidal** (harmonic) signals. Using **complex Steinmetz vectors**, fields are expressed as:

$$\vec{e}(\vec{r}, t) = \mathrm{Re}[\vec{E}(\vec{r}) e^{j\omega t}]$$

Assuming no electric charges or currents ($\rho = 0$, $\vec{J} = 0$), Maxwell's equations simplify to:

$$\nabla \times \vec{E} = -j\omega \vec{B} \qquad \nabla \cdot \vec{D} = 0 \tag{5}$$

$$\nabla \times \vec{H} = j\omega \vec{D} \qquad \nabla \cdot \vec{B} = 0 \tag{6}$$

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Constitutive Relations

In dielectric (non-conductive) media, there are **no magnetic fields generated by conduction currents**. The relations between field vectors are:

$$\vec{B} = \mu_0 \vec{H}, \qquad \vec{D} = \varepsilon \vec{E}$$

where ε is the **dielectric permittivity** and μ_0 is the **magnetic permeability of free space**.

For **isotropic** materials:

$$\vec{D} = \varepsilon \vec{E}$$

For **dispersive** materials (frequency-dependent properties):

$$\vec{D}(\omega, \vec{r}) = \varepsilon(\omega, \vec{r}) \vec{E}(\omega, \vec{r})$$

with position vector

$$\vec{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

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[End of Chapter 1 excerpt]