The book contains chapters on polygons, tessellations, finite polyhedra, space filling, and open packings. A chapter on the construction of geometric models is also included, because we believe that the physical manipulation of actual models is a useful learning experience.

Written to be understood at the high school and college level, *Polyhedra Primer* will appeal to both academic and general audiences. We think it will be especially meaningful for people in those professions—architecture, planning, engineering, industrial design, and art—where a knowledge of geometry provides a rich resource of form and spatial options and is useful as the basis for the formulation of new solutions to design problems. It is hoped that the book will find further audience among teachers, students, and practitioners of mathematics, crystallography, general morphology, and other areas of scientific endeavor that require a knowledge of spatial geometry.

We hope that beyond such pedagogical purposes, the reader will enjoy our little book and perhaps will be struck, as we have been, by the extraordinary spatial diversity that emanates from the sublime order and elegant simplicity that is exemplified by the subject of polyhedra.

1 Polygons

Points, Lines, and Angles

Points
Line

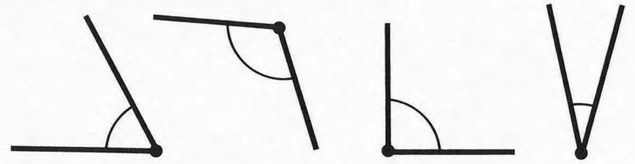
Two points can be connected by a straight line on a flat surface or plane.

Parallel

Lines are parallel if they lie in a common plane and do not intersect no matter how far they are extended.

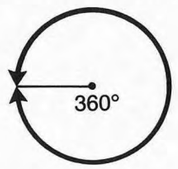
Angle

An angle is the figure formed by two lines meeting at the same point.

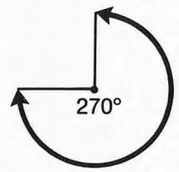


Degrees
Minutes

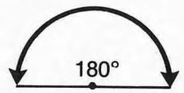
Angles are measured in degrees and minutes. There are 60 minutes in one degree. The symbol for degrees is (°). The symbol for minutes is (').



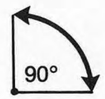
There are 360° 0' in a full circle.



There are 270° 0' in ¾ circle.



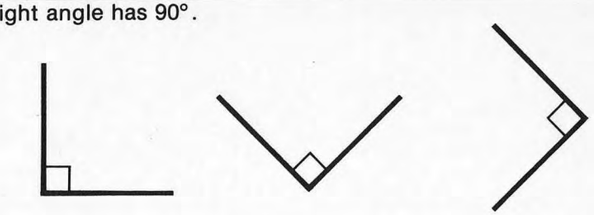
There are 180° 0' in ½ circle.



There are 90° 0' in ¼ circle.

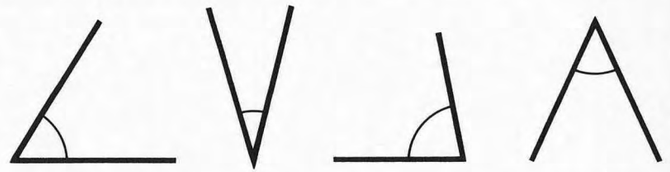
Right Angle

A right angle has 90°.



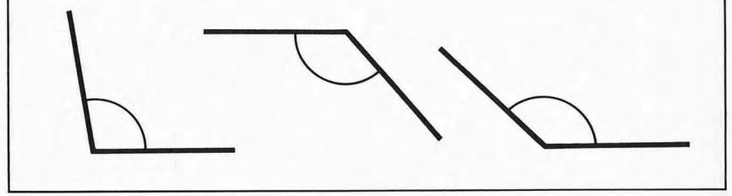
Acute Angle

An acute angle has less than 90°.



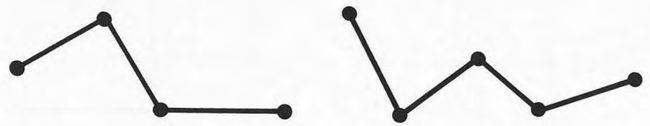
Obtuse Angle

An obtuse angle has more than 90° but less than 180°.

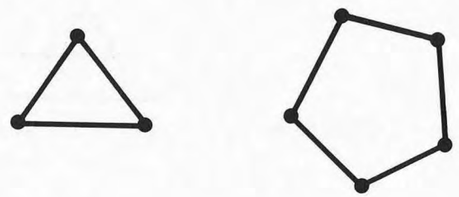


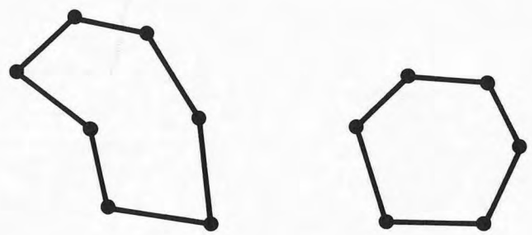
Polygons

Three or more points can be connected by a line.



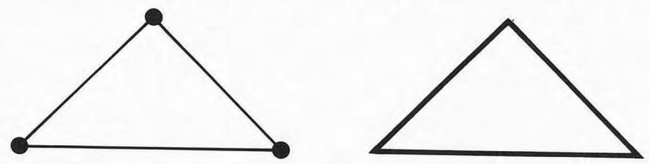
Three or more points can be connected by a line to form a closed loop. The closed loop is a polygon. A polygon is a portion of a plane bounded by three or more lines or segments.





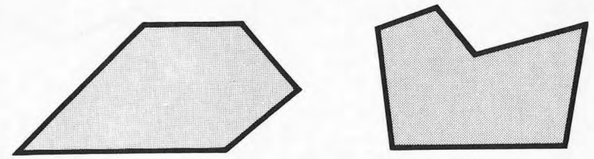
Vertex
Sides

A vertex is a corner of a polygon. (Plural: vertices). The sides of a polygon are the segments connecting the vertices.



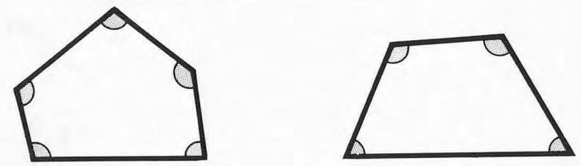
Interior

The interior of the polygon is the plane area bounded by the sides.

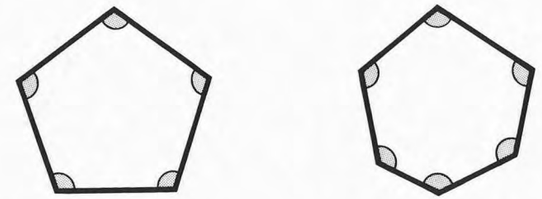


Interior or Face Angles

The interior or face angles are the angles formed by adjacent sides of a polygon and lying within the interior of the polygon.



If n is the number of sides of a polygon, the sum of the interior angles of a polygon is (n-2) × 180°.

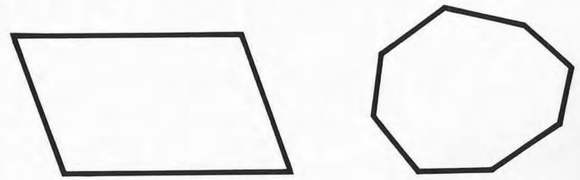


(5-2) × 180° = 540°

(6-2) × 180° = 720°

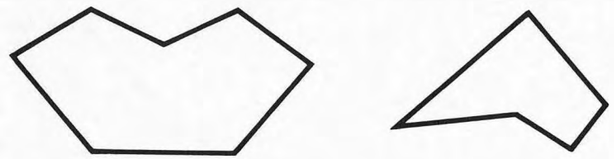
Convex Polygon

A polygon is convex if each interior angle is less than 180°.



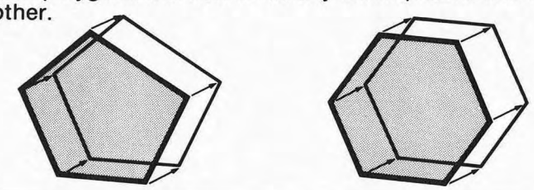
Concave Polygon

A polygon is concave if one of its interior angles is more than 180°.



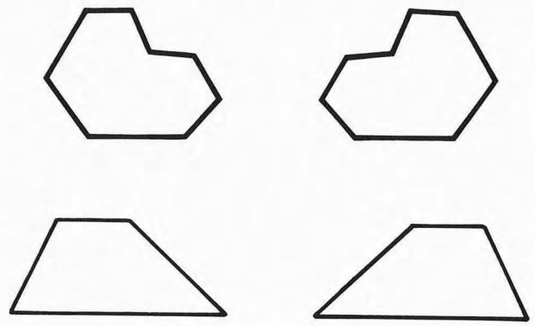
Congruent

Polygons are congruent if they are the same shape and size. Congruent polygons will match exactly when placed one on top of the other.



Enantiomorph

A polygon is an enantiomorph if it exists in a left- and right-hand version. Enantiomorphs have all the properties of congruence except for handedness.

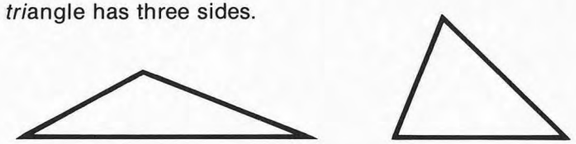


Naming Polygons

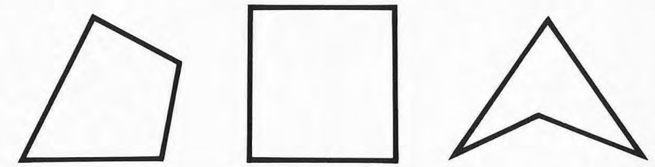
n-gon

Polygons are usually named by the number of sides they have. An n-gon is a polygon with an unspecified number of sides.

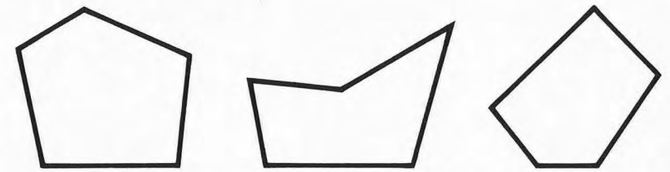
A *tri*angle has three sides.



A *quad*rilateral or *tetra*gon has four sides.

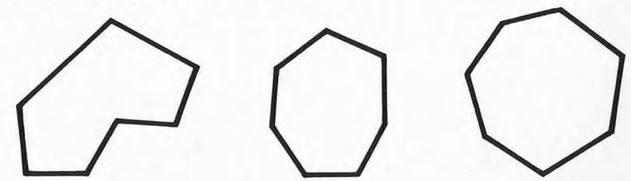


A *penta*gon has five sides.

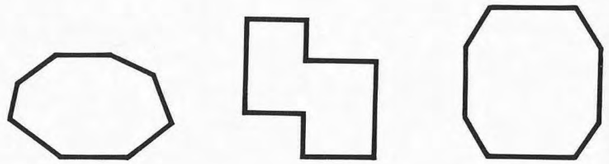


A *hexa*gon has six sides.

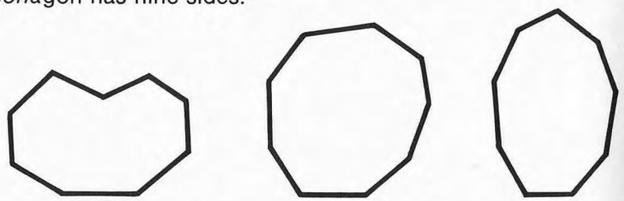
A *septa*gon has seven sides.



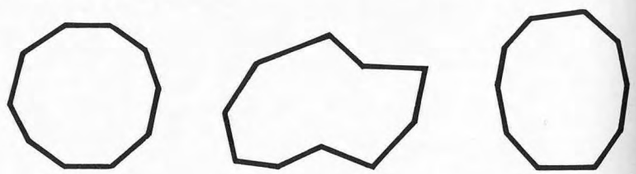
An *octa*gon has eight sides.



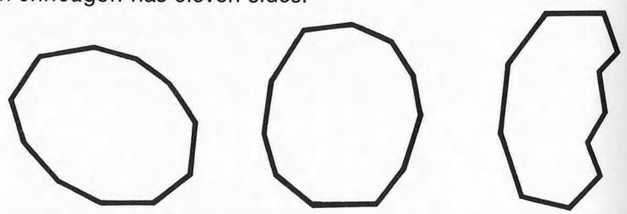
A *nona*gon has nine sides.



A *deca*gon has ten sides.



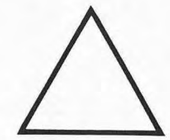
An *ennea*gon has eleven sides.



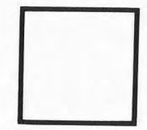
A *dodeca*gon has twelve sides.

Regular Polygons

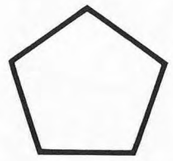
A regular polygon has equal interior angles and equal sides.



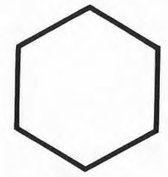
Regular Triangle



Regular Quadrilateral

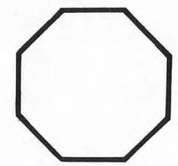


Regular Pentagon



Regular Hexagon

Regular Septagon



Regular Octagon

Regular Nonagon

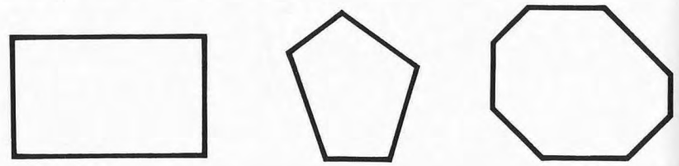
Regular Decagon

Regular Enneagon

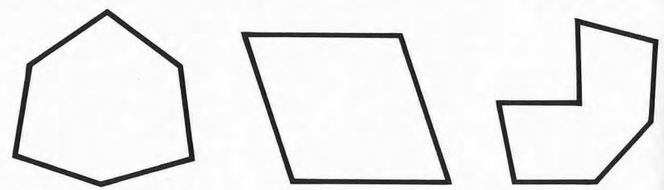
Regular Dodecagon

Nonregular Polygons

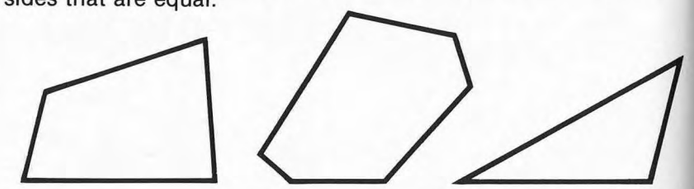
A polygon can have equal interior angles and unequal sides.



A polygon can have unequal interior angles and equal sides.

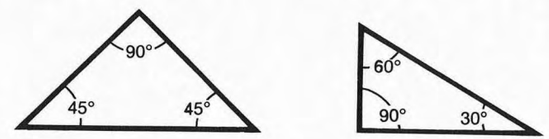


A polygon can have no interior angles that are equal and no sides that are equal.



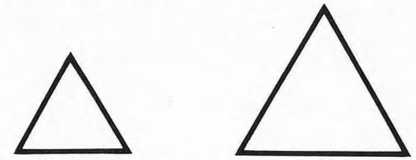
Types of Triangles

The sum of the face angles of any triangle is always equal to 180°. (3-2) × 180° = 180°.



Equilateral

Equilateral triangles have equal length sides and equal interior angles. They are regular polygons.



Isosceles

Isosceles triangles have two sides of equal length and a third side—the base—of different length. The two interior angles common to the base are equal.

Scalene

Scalene triangles have three sides of different length and three different interior angles.

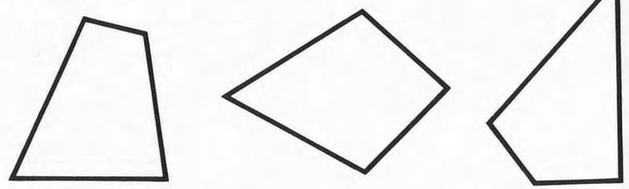
Right

Right triangles have one right (90°) angle. They can be isosceles or scalene. A right isosceles triangle has one right angle and two 45° angles.

Types of Quadrilaterals

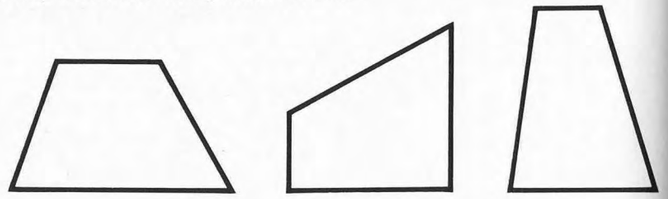
Trapezium

A trapezium has no parallel sides.



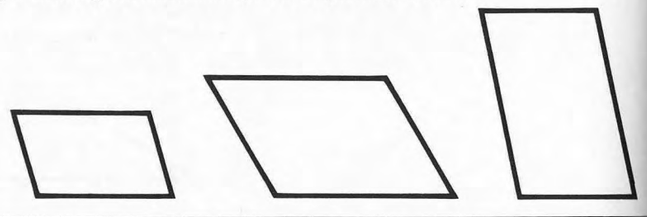
Trapezoid

A trapezoid has two parallel sides.



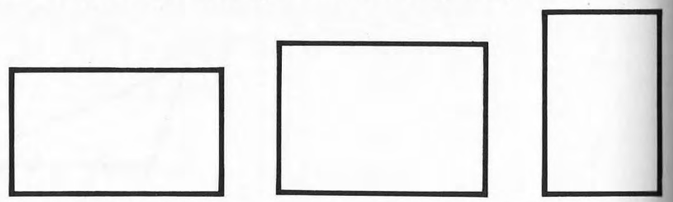
Parallelogram

A parallelogram has two pairs of parallel sides.



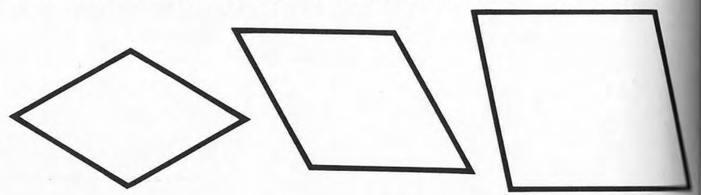
Rectangle

A rectangle is a parallelogram with four right angles.



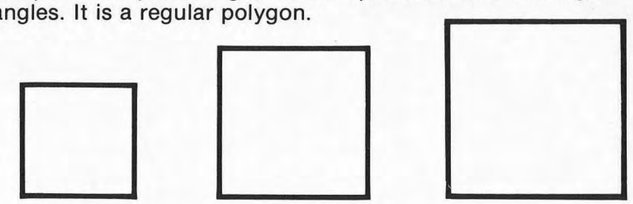
Rhombus

A rhombus is a parallelogram with equal sides.



Square

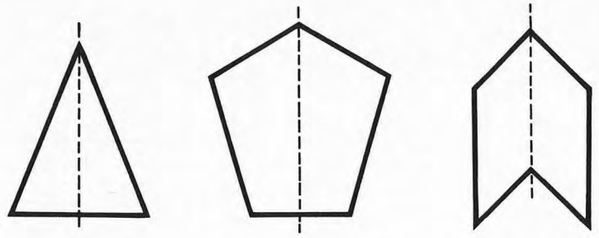
A square is a parallelogram with equal sides and four right angles. It is a regular polygon.



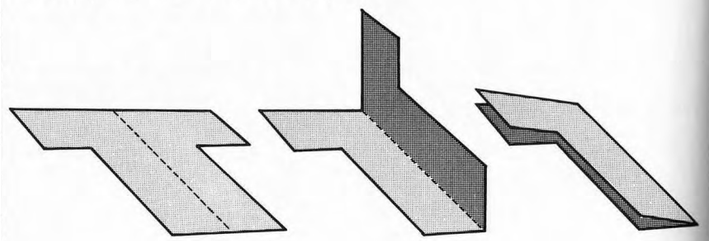
Symmetry and Polygons

Mirror Symmetry

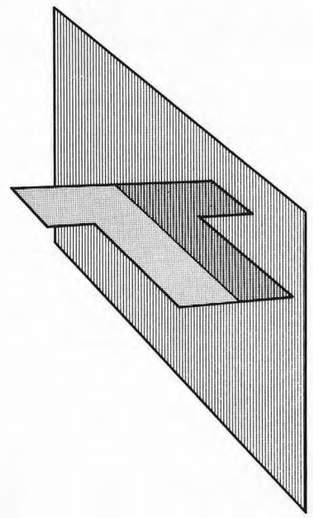
A polygon has mirror symmetry, or one mirror plane, if it is the same on either side of a line that divides it in half.



If a polygon with mirror symmetry is folded in half, the two folded parts will match exactly.

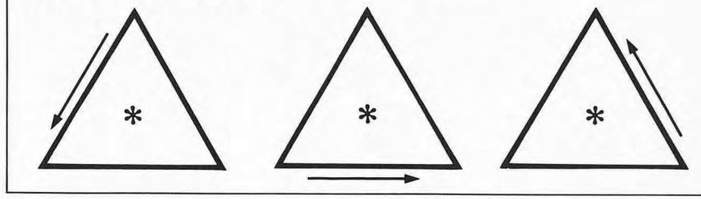


Or, if one half of a polygon with mirror symmetry is held up to a mirror, its reflection will look the same.



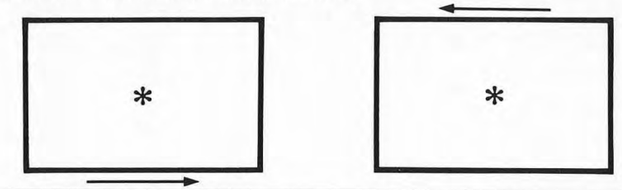
Rotational Symmetry

A figure has rotational symmetry if it repeats itself in one 360° revolution about an axis.



2-fold

A polygon has 2-fold rotational symmetry if it repeats itself twice in one 360° revolution about an axis.

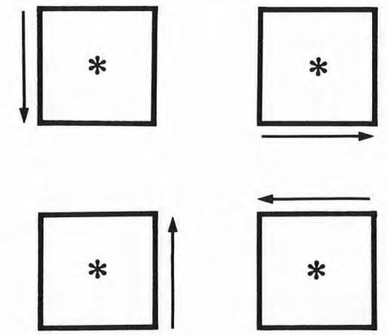


3-fold

A polygon has 3-fold rotational symmetry if it repeats itself three times in one 360° revolution about an axis.

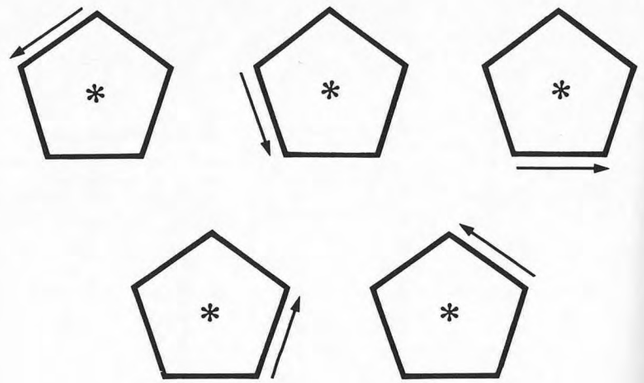
4-fold

A polygon has 4-fold rotational symmetry if it repeats itself four times in one 360° revolution about an axis.



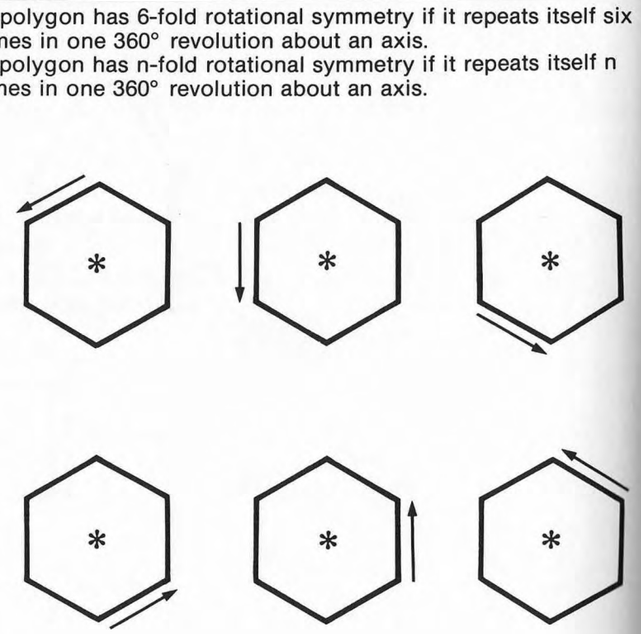
5-fold

A polygon has 5-fold rotational symmetry if it repeats itself five times in one 360° revolution about an axis.



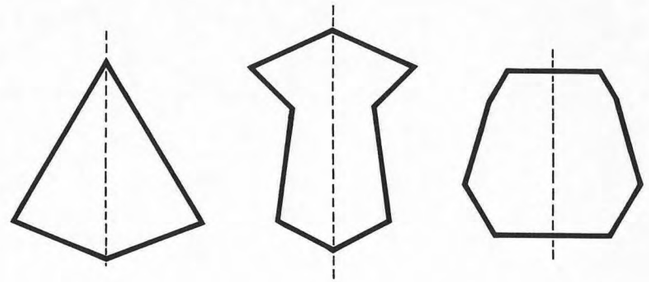
6-fold
n-fold

A polygon has 6-fold rotational symmetry if it repeats itself six times in one 360° revolution about an axis.
A polygon has n-fold rotational symmetry if it repeats itself n times in one 360° revolution about an axis.



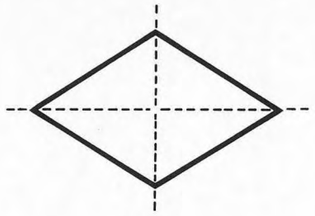
Combinations of Symmetries

A polygon can have mirror symmetry and no rotational symmetry.

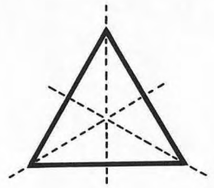


A polygon can have n-fold rotational symmetry and no mirror symmetry. In this case, the polygon will always exist as an enantiomorph.

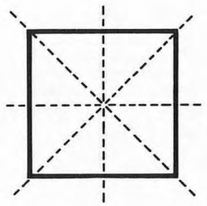
A polygon can have mirror symmetry and n-fold rotational symmetry. In this case, the polygon will always have the same number of mirror planes as n-fold symmetry.



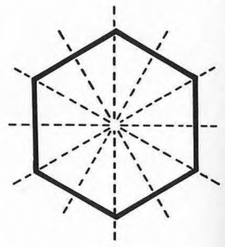
2-fold
2 mirror planes



3-fold
3 mirror planes



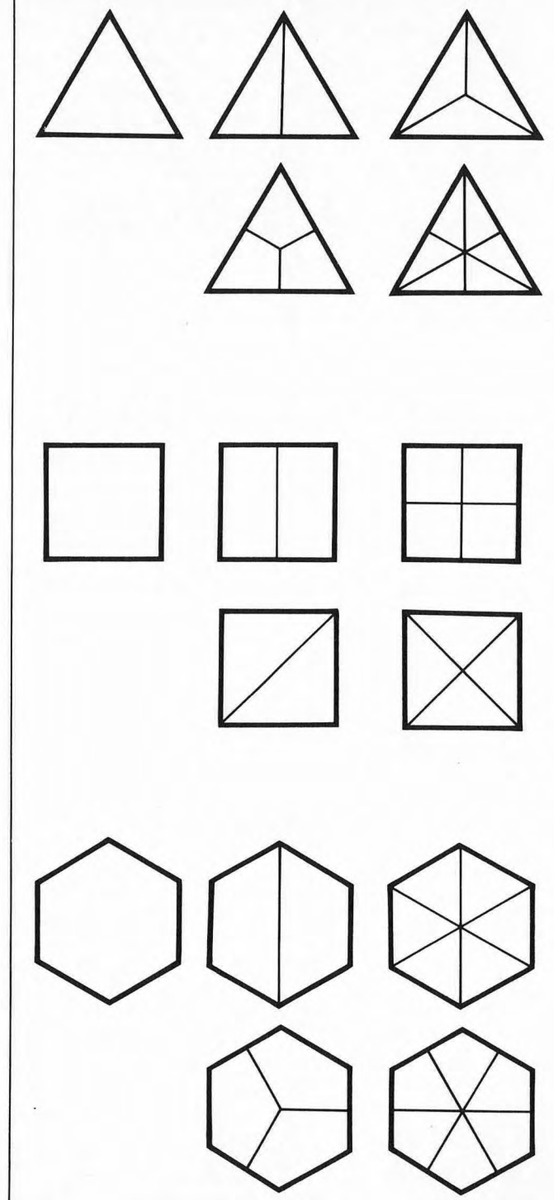
4-fold
4 mirror planes



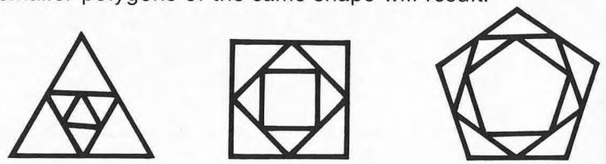
6-fold
6 mirror planes

Subdividing Polygons

Regular polygons can be subdivided into smaller congruent or enantiomorphic polygons.

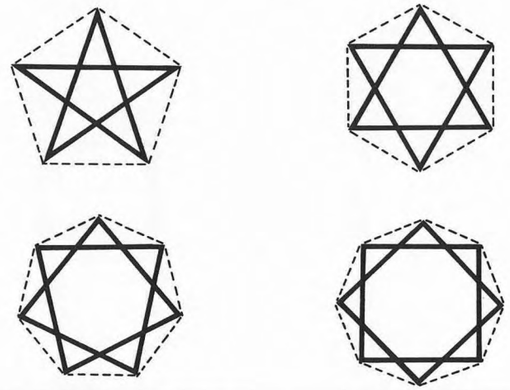


If the edges of a regular polygon are bisected, or cut in half, and the points of bisection joined between adjacent edges, smaller and smaller polygons of the same shape will result.



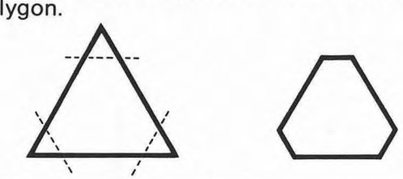
Star Polygons

Star polygons can be formed from polygons of more than five sides either by connecting alternate vertices of the polygon, or by extending the sides of the polygon until they intersect.

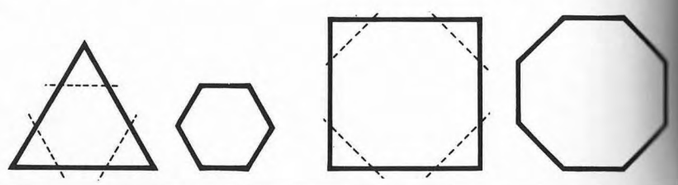


Truncating Polygons

A truncated polygon is one whose corners have been cut off. Truncation forms a new polygon with more sides than the original polygon.



A regular polygon can be truncated to form a new regular polygon.



Tessellations