Hidden sector SUSY breaking I: Gauge mediation

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Contents

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1 Introduction

As we learned during this seminar, Supersymmetry must be softly broken, i.e. by terms that do not introduce quadratic divergences, to be in agreement with experiments. This breaking cannot occur however in the observable sector, since the supertrace theorem holds at tree level:

$$\mathrm{STr}\,\mathcal{M}^2 = \sum_J (-1)^{2J}(2J+1)\mathcal{M}_J^2 = 0, \tag{1}$$

where $\mathcal{M}_J$ denotes the mass of a particle with spin J. Eq (1) generically implies the existence of a superpartner lighter that the corresponding ordinary particle, in case of tree level communication.

However the theorem is not valid for non-renormalizable theories and at quantum level. It is then possible to construct theories where SUSY is broken in a so-called *hidden sector* that has no renormalizable tree-level coupling to the *observable sector*. This hidden sector is usually assumed to live at very high energies. The effective theory describing the observable sector is obtained by integrating out the hidden one, and should then have non-vanishing supertrace (at tree or quantum level).

Once SUSY is broken, we have to explain how the breaking is mediated to the MSSM. A possibility is to assume a non-renormalizable theory, such that Eq (1) does not hold. This can be achieved by supergravity. Another approach is to consider ordinary gauge interactions to mediate the breaking. Then the observable mass spectrum will have a vanishing supertrace at tree level, and splittings will arise only at quantum level.

Both possibilities have advantages and disadvantages, such that there is not a favourite candidate. Gravity mediation is problematic because it generally introduces new sources of flavour violation, since the soft terms are generated at the Planck scale. This necessarily at a scale larger or equal than Λ_F, defined as the scale above that unknown flavour-violation effects arise. On the other hand, gauge interactions are flavour blind, and soft terms are generated at a messenger scale S, which is a priori unrelated to Λ_F. If $S \ll \Lambda_F$, then the only flavour violation in these theories is given by Yukawa coupling effects, as in the Standard Model. This the main motivation for gauge-mediated SUSY breaking (GMSB).

Gauge mediation can be seen as a drawback from the unification of forces, but not dealing with quantum gravity makes things much easier, and GMSB models can be solved by means of ordinary quantum field theory.

Before getting started with the construction of gauge-mediated models, let us recall the SUSY Lagrangian density:

$$\begin{aligned}\mathcal{L} =& \mathcal{L}_{\text{chiral}} + \mathcal{L}_{\text{gauge}} \\ &- \sqrt{2}g\,(\phi^* T^a \psi)\,\lambda^a - \sqrt{2}g\lambda^{\dagger a}\left(\psi^\dagger T^a \phi\right) + g\,(\phi^* T^a \phi)\,D^a.\end{aligned} \tag{2}$$

Note that the second line in Eq (2) consists of interactions whose strengths are fixed to be gauge couplings by the requirements of supersymmetry, even though they are not gauge interactions from the point of view of an ordinary field theory. This terms be important when computing breaking observable masses.

2 Gauge-mediated breaking models: general features and the minimal model

A SUSY model with gauge-mediated breaking consists of three sectors:

- The *observable* sector: contains the usual particles and their supersymmetric partners
- The *hidden* or *secluded* sector: is where the breaking originates
- The *messenger* sector: contains new messenger chiral supermultiplets that communicate the breaking to the observable sector

2.1 The hidden sector

As already anticipated, the hidden sector does not couple directly to the MSSM. Here SUSY is somehow broken. How this breaking occurs is an open and separate problem that we will not address here. The content of this sector is still not specified, but for our purposes we will just assume that trough W_{breaking} a gauge-singlet chiral superfield S acquires the VEVs $\langle S \rangle$ for its scalar component S, and $\langle F_S \rangle$ for its auxiliary F-term component.

The case considered assumes that the goldstino field, that originates from the SUSY breaking, completely overlaps with the chiral superfield S. However this is just an assumption we did for simplicity, and more general models assumes that the goldstino is a linear combinations of different fields.

2.2 The messenger sector

This sector is responsible to mediate the breaking from the hidden to the observable sector. It is formed by some *messenger* superfields that transform under the gauge group as a real, non-trivial representation. These fields couples at tree level with the goldstino superfield (a single superfield S in our simple description of the hidden sector). This coupling generates masses of order $\langle S \rangle$ and mass squared splittings of order $\langle F_S \rangle$ inside the supermultiplets.

Also this sector is generally unknown, and is the main source of model dependence. We will first analyse the simplest possible model, and then we will extend it to a more general, yet still simple, case. We start by considering a set of "lepton-like" and "boson-like" left-handed chiral supermultiplets q, $\bar{q}$, ℓ, $\bar{\ell}$ transforming under $SU(3)_C \times SU(2)_L \times U(1)_Y$ as

$$q \sim \left(\mathbf{3}, \mathbf{1}, -\frac{1}{3}\right), \quad \bar{q} \sim \left(\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{3}\right), \quad \ell \sim \left(\mathbf{1}, \mathbf{2}, \frac{1}{2}\right), \quad \bar{\ell} \sim \left(\mathbf{1}, \mathbf{2}, -\frac{1}{2}\right). \tag{3}$$

Assume that these supermultiplets couple to the hidden sector via the superpotential

$$W_{\text{mess}} = y_2 S\,\ell\bar{\ell} + y_3 S\,q\bar{q}. \tag{4}$$

The fermionic part of the messenger fields $\psi_{\ell,\bar{\ell},q,\bar{q}}$ pair up to get mass terms

$$\mathcal{L} = -y_2\langle S\rangle\psi_\ell\psi_{\bar{\ell}} - y_3\langle S\rangle\psi_q\psi_{\bar{q}} + \text{ c.c.} \tag{5}$$

Meanwhile, their scalar messenger partners have a scalar potential given by[1]:

$$V = \left|\frac{\delta W_{\text{mess}}}{\delta\ell}\right|^2 + \left|\frac{\delta W_{\text{mess}}}{\delta\bar{\ell}}\right|^2 + \left|\frac{\delta W_{\text{mess}}}{\delta q}\right|^2 + \left|\frac{\delta W_{\text{mess}}}{\delta\bar{q}}\right|^2 + \left|\frac{\delta}{\delta S}\left(W_{\text{mess}} + W_{\text{breaking}}\right)\right|^2, \tag{6}$$

where we have denoted the scalar component with the same symbol of the corresponding superfield. Now, recalling what we have discussed for the hidden sector, suppose that, at the minimum of the potential,

$$\begin{gathered}\langle S\rangle \neq 0,\\ \left\langle\delta W_{\text{breaking}}/\delta S\right\rangle = -\langle F_S^*\rangle \neq 0\\ \langle\delta W_{\text{mess}}/\delta S\rangle = 0.\end{gathered} \tag{7}$$

Then, inserting the VEVs in Eq. (6), we obtain the quadratic scalar mass terms in the potential:

$$\begin{aligned}V = &|y_2\langle S\rangle|^2\left(|\ell|^2 + |\bar{\ell}|^2\right) + |y_3\langle S\rangle|^2\left(|q|^2 + |\bar{q}|^2\right)\\ &- \left(y_2\langle F_S\rangle\,\ell\bar{\ell} + y_3\langle F_S\rangle\,q\bar{q} + \text{ c.c. }\right) + \text{ quartic terms.}\end{aligned} \tag{8}$$

Now we will take a look at the generalised model, that consists of N_f flavours of chiral superfields Φ_i and $\bar{\Phi}_i$. Then the superpotential coupling hidden and messenger sector becomes:

$$W_{\text{mess}} = \lambda_{ij}\bar{\Phi}_i S\,\Phi_j. \tag{9}$$

In analogy with the simplest case discussed above, it is easy to believe that the spinor components of Φ_i and $\bar{\Phi}_i$ form Dirac fermions with masses $m_{\text{fermions}} = \lambda\langle S\rangle$ and that the scalar components have the squared mass matrix

$$\begin{pmatrix}\Phi^\dagger & \bar{\Phi}\end{pmatrix}\begin{pmatrix}(\lambda S)^\dagger(\lambda S) & (\lambda F_S)^\dagger\\ (\lambda F_S) & (\lambda S)(\lambda S)^\dagger\end{pmatrix}\begin{pmatrix}\Phi\\ \bar{\Phi}^\dagger\end{pmatrix} \tag{10}$$

Here we have once again denoted the scalar components with the same symbol of the respective superfield and we have suppressed flavour indices and VEV brackets.

When dealing with a single field S, the matrices λS and λF_S can be simultaneously made real and diagonal. Then it is easy to compute the eigenvalues of matrix (10):

$$m^2_{\text{scalars}} = (\lambda S)^2 \pm \lambda F_S \tag{11}$$

with the corresponding eigenvectors $\frac{1}{\sqrt{2}}(\Phi + \bar{\Phi}^\dagger)$ and $\frac{1}{\sqrt{2}}\left(\bar{\Phi} + \Phi^\dagger\right)$.

[1] Here we are neglecting D-term contributions, which do not affect the following discussion

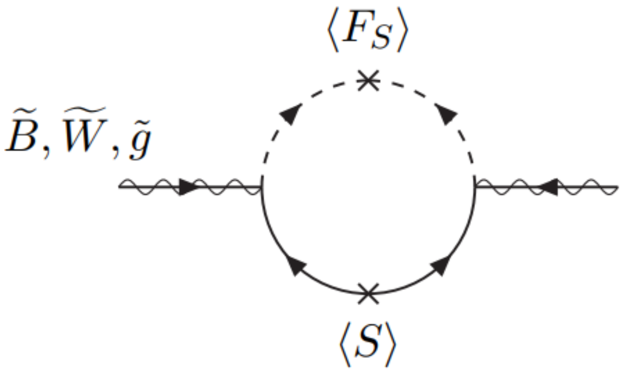

Figure 1: One loop order Feynman diagram for gauginos masses. The scalar and fermionic components of the messenger fields Φ are denoted by dashed and solid lines. Note that the interaction vertices are of gauge strength even if they do not involve gauge bosons.

As anticipated, we obtain masses of order $\langle S \rangle$ and squared mass splittings of order $\langle F_S \rangle$. Then, the mass scale $\sqrt{\langle F_S \rangle}^2$ is the measure of SUSY breaking in the messenger sector.

However we are more interested in the breaking in the observable sector. From now on, we will absorb the couplings λ in the definition of $\langle S \rangle$ and $\langle F_S \rangle$, and introduce the parameter $\Lambda \equiv \frac{\langle F_S \rangle}{\langle S \rangle}$.

2.3 The observable sector

This sector contains the SM particles along with their superpartners, and is referred as the Minimal Supersymmetric Standard Model (MSSM). Recall that this sector does not couple at tree level to the hidden superfield S, and thus particle supermultiplets are degenerate at tree level and the supertrace theorem holds. Splittings arise at quantum level, via gauge-interaction with the messenger fields. Vector bosons and matter fermions masses are protected by gauge invariance, but, once SUSY is broken, their superpartners can acquire mass consistently with gauge symmetry. In the next section we will see how these masses arise and what they turn out to be.

3 The mass spectrum

3.1 Gauginos and sfermions masses

Gauginos masses arise at one loop order via the Feynman diagram reported in Figure 1. This can be computed with an ordinary QFT calculation, and then RG evolved

[2]That is true only if $\langle F_S \rangle$ and $\langle S \rangle^2$ are of the same order. If $\langle F_S \rangle \ll \langle S \rangle^2$ is assumed, then the mass splitting is of order $\langle F_S \rangle / \langle S \rangle$ in a first order expansion.

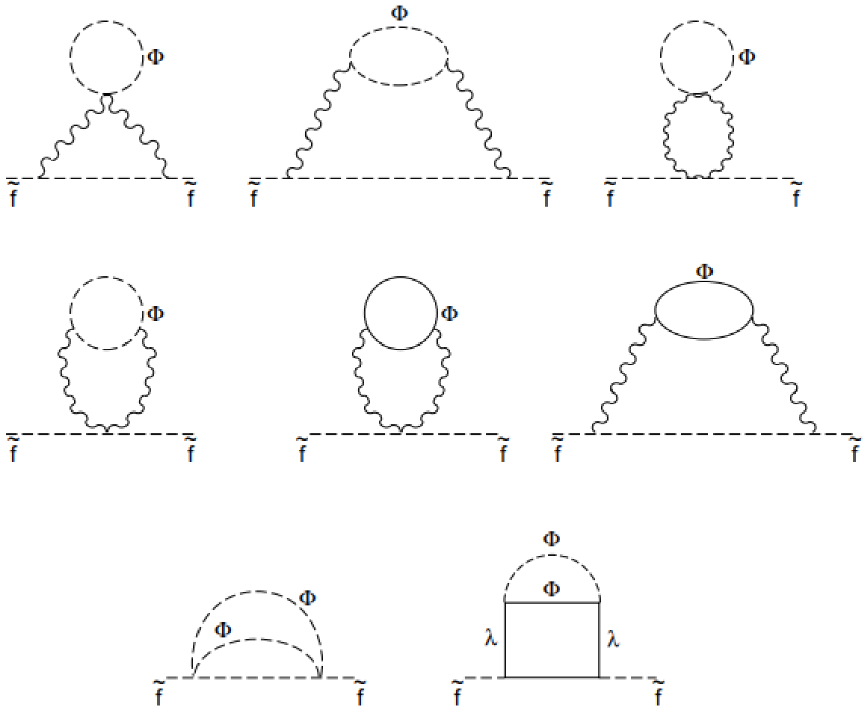

Figure 2: Two loops order Feynman diagram for sfermions masses. The wavy lines represent ordinary gauge bosons, while λ indicates a gaugino.

to the electroweak scale, giving the following result:

$$\begin{aligned} \tilde{M}_{\lambda_r}(t) &= k_r \frac{\alpha_r(t)}{4\pi} \Lambda_G \quad (r = 1, 2, 3) \\ \Lambda_G &= N\Lambda \left[1 + O\left(\frac{F_S^2}{S^4} \right) \right], \end{aligned} \tag{12}$$

where $k_1 = 5/3$, $k_2 = k_3 = 1$, and the gauge coupling constants are normalized such that $k_r\alpha_r$ are all equal at the GUT scale. Recall that $\alpha_r = \frac{g_r^2}{4\pi}$ and $t = \ln \frac{S^2}{Q^2}$, where Q is the low-energy scale at which the soft term are defined.

$N = \sum_{i=1}^{N_f} n_i$ is the *messenger index*, and n_i is twice the Dynkin index of the gauge representation **r** of the flavour index i.

Leaving out all this complications, note that the breaking effect is proportional to the gauge interaction strength and of order Λ.

Sfermion masses arise only at two loop order, and the relevant Feynman diagram are reported in Fig. 2. Neglecting Yukawa-couplings contributions, the sfermions squared masses turn out to be:

$$m_{\tilde{f}}^2(t) = 2\sum_{r=1}^{3} C_r^{\tilde{f}} k_r \frac{\alpha_r^2(0)}{(4\pi)^2}\left[\Lambda_S^2 + h_r \Lambda_G^2\right],$$
$$h_r = \frac{k_r}{b_r}\left[1 - \frac{\alpha_r^2(t)}{\alpha_r^2(0)}\right], \tag{13}$$
$$\alpha_r(t) = \alpha_r(0)\left[1 + \frac{\alpha_r(0)}{4\pi} b_r t\right]^{-1},$$

where $C_r^{\tilde{f}}$ is the quadratic Casimir of the $\tilde{f}$ particle and b_r are the β-function coefficients $b_3 = -3$, $b_2 = 1$, $b_1 = 11$. $\alpha_r(0)$ are the gauge coupling constants at the messenger scale $\langle S\rangle$. Assuming a single superfield S,

$$\Lambda_S^2 = N\Lambda^2\left[1 + O\left(\frac{F_S^2}{S^4}\right)\right]. \tag{14}$$

Note that again, despite the more complicated structure of Eq. (13), the breaking effect is of order Λ as for the gauginos. While in the latter case the masses depend only on $N\Lambda$, from Eq. (14) we can see that N and Λ must be treated as two independent parameters of the model.

3.2 Properties of the mass spectrum

In computing the soft masses $\langle F_S\rangle \ll \langle S\rangle^2$ was and will be assumed. $\langle F_S\rangle > \langle S\rangle^2$ is not permitted by requiring a positive messenger squared mass. In the case $\langle F_S\rangle \sim \langle S\rangle^2$ corrections to equations (12) and (13) can be computed. Note that Λ, and thus the sparticle soft masses in this approximation, does not depend on the couplings λ between the messenger fields and S. However the corrections mentioned above do, and then the choice of λ brings uncertainty to the mass spectrum predictions.

As already remarked in the introduction, the gauge-mediated mass spectrum respects flavour universality, which is guaranteed by the symmetry of gauge interactions, if gravity-mediated contributions do not reintroduce large flavour violations. Requiring that gravity accounts for not more than 1‰of soft-squared mass, since it generates soft terms with typical size F_S/M_P, the flavour criterion gives a rough upper bound on the messenger mass scale:

$$S \lesssim \frac{1}{10^{\frac{3}{2}}}\frac{\alpha}{4\pi} M_P \sim 10^{15}\text{GeV}, \tag{15}$$

where $M_P = (8\pi G_N)^{-1/2} = 2.4 \times 10^{18}$ GeV is the reduced Planck mass.

We have seen that gauginos mass-breaking term are generated at one-loop level, while sfermions ones at two-loop level. Combined with the different canonical dimensions of the two, this implies that all SUSY-breaking masses have the same scaling $\tilde{m} \sim \frac{\alpha}{\pi}\Lambda$. Within our simple model, the whole spectrum is determined by the effective breaking scale Λ, the messenger index N, the messenger mass S, and an additional parameter $\tan\beta$, that arises when Yukawa effects are accounted for. The high degree of predictability of GMSB is supposed to be helpful in distinguishing it to the more generic spectrum of gravity-mediated models in the case SUSY is discovered in experiments.

4 Summary and conclusions

In our brief discussion we have not touched many aspects of gauge-mediation. Many models that goes beyond our discussion have been constructed. A mention-worthy example are models with *dynamical* SUSY breaking. An important issue within GMSB models is the so-called μ-problem, i.e. the difficulty in generating the correct mass scale for the Higgs bilinear term in the soft superpotential. We also have not said anything about empirical consequences of the mass spectrum. The most remarkable fact is that most likely the LSP in these models is the gravitino: this leads to important implications in collider experiments and cosmology, as the LSP is usually a good candidate for dark matter.

Nonetheless we have outlined the main features of gauge-mediation and obtained important results. SUSY breaking is successfully transmitted from the hidden to the observable sector via the messenger particles in a renormalizable framework. The resulting spectrum is predictable, and most importantly, acceptable. It depends only on a few parameters and the model can be solved using standard field theory, without knowledge of quantum gravity. In contrast to gravity-mediated models, the flavour problem is naturally solved thanks to the flavour-blind nature of gauge interaction and the independence of SUSY and flavour breaking mass scales.

References