::: FORMULAE :::

::: Introduction :::

- Input: $\{x_0, x_1, \ldots, x_n\}$ e $\{y_0, y_1, \ldots, y_n\}$ (points coordinates).
- $n+1$ points defining n intervals .

::: Polynomial interpolation :::

- Interpolating polynomial of degree n:

$$p(x) := \sum_{i=0}^{n} c_i \left[\prod_{j=0}^{i-1}(x - x_j)\right]$$

  Ex. if $n = 2$ (3 points), $p(x) = c_0 + c_1(x - x_0) + c_2(x - x_0)(x - x_1)$.
- Parameters $\{c_0, c_1, \ldots, c_n\}$ are obtained (in quadratic time and linear space) by the pyramid-shape computations.

::: Quadratic spline :::

- For $x \in [x_k, x_{k+1}]$, the parabolic curve rewrites as:

$$f_k(x) := y_k + z_k(x - x_k) + \frac{z_{k+1} - z_k}{2(x_{k+1} - x_k)}(x - x_k)^2$$

  for each $k = 0, ..., n-1$ (n parabolic curves overall).
- Parameters $z_0, \ldots, z_n$ (intepretation $z_k := f'(x_k)$) are obtained by an order-1 recursion (linear complexity).
- Starting value z_0 is given, for the others:

$$z_{k+1} = -z_k + 2\frac{y_{k+1} - y_k}{x_{k+1} - x_k}$$

::: Cubic spline (natural) :::

- With$x \in [x_k, x_{k+1}]$, the cubic interpolating function is:

$$f_k(x) = \frac{z_{k+1}(x - x_k)^3 + z_k(x_{k+1} - x)^3}{6h_k} + \left(\frac{y_{k+1}}{h_k} - \frac{h_k}{6}z_{k+1}\right)(x - x_k) + \left(\frac{y_k}{h_k} - \frac{h_k}{6}z_k\right)(x_{k+1} - x),$$

  for each $k = 0, ..., n-1$, where $h_k := (x_{k+1} - x_k)$ represents the width on the interval where the cubic function is defined.
- Parameters $z_0, \ldots, z_n$ (intepretation $z_k := f''(x_k)$) are obtained from a linear system of $n-1$ equations with respect to the $n-1$ unknowns $z_1, \ldots, z_{n-1}$, while $z_0 = z_n = 0$ because of the natural sp-line condition. The $n-1$ equations of the linear system are:

$$h_{k-1}z_{k-1} + 2(h_{k-1} + h_k)z_k + h_k z_{k+1} = 6\left(\frac{y_{k+1} - y_k}{h_k} - \frac{y_k - y_{k-1}}{h_{k-1}}\right),$$

  where $k = 1, \ldots, n-1$. If $h_k = 1$ for each k (that means the x of the points are at the same distance on the x-axis and at distance one), equations rewrite as:

$$z_{k-1} + 4z_k + z_{k+1} = 6(y_{k+1} - 2y_k + y_{k-1})$$

::: Trigonometric Interpolation :::

- Premise: in this formula we have n points (and not $n+1$ as in the other formulae) of coordinates $\{(x_i, y_i)\}_{i=0}^{n-1}$ and we assum n to be odd.
- The interpolating function is:

$$f(x) := C + \sum_{j=1}^{m} \left[a_j \cos(jx) + b_j \sin(jx)\right],$$

  and depends on n parameters (thus, $m = \frac{n-1}{2}$).
- If the points are splitting the interval $[0, 2\pi]$ in n parts of equal size, the following formulae hold:

$$a_j = \frac{2}{n} \sum_{i=0}^{n-1} \left[y_i \cos(jx_i)\right], \quad \forall j = 0, 1, \ldots, m$$

$$b_j = \frac{2}{n} \sum_{i=0}^{n-1} \left[y_i \sin(jx_i)\right], \quad \forall j = 1, \ldots, m,$$

  and $C = \frac{a_0}{2}$, where a_0 is obtained from the cosine formula (this giving $a_0 = \frac{2}{n} \sum_{i=0}^{n-1} y_i$, that means C is the average of the y) coordinates.

::: Linear Regression :::

- Given n points $\{(x_i, y_i)\}_{i=1}^{n}$ finding representative line of equation $y = mx + q$
- $m = \frac{\sum_i x_i y_i - n\overline{x}\,\overline{y}}{\sum_i x_i^2 - n\overline{x}^2}$ and $q = \overline{y} - m\overline{x}$ where $\overline{x} = \frac{\sum_i x_i}{n}$ and $\overline{y} = \frac{\sum_i y_i}{n}$

::: Numerical Integration :::

- Approximating integral $\int_a^b f(x)\mathrm{d}x$ by dividing interval $[a, b]$ in n parts by $n+1$ points $x_0, x_1, \ldots, x_n$ where $x_0 = a$ e $x_n = b$.
- Summing trapezoids areas $A_{tr} = \sum_{i=0}^{n-1} (x_{i+1} - x_i) \frac{f(x_i) + f(x_{i+1})}{2} = \frac{b-a}{n} \cdot \sum_{i=0}^{n-1} \frac{f(x_i) + f(x_{i+1})}{2}$