Numerics (Part IV)
[Lecture 4] Numerical Integration

Alessandro Antonucci
alessandro.antonucci@supsi.ch
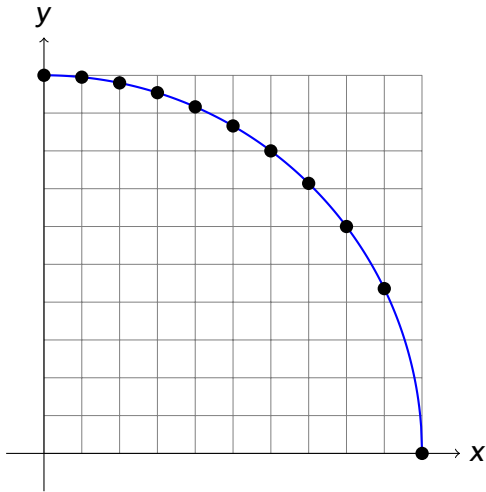
Numerical Integration

- Definite Integral of a function $\int_a^b f(x) \cdot \mathrm{d}x$?
- Given $F(x)$ such that $F'(x) = \frac{\mathrm{d}f}{\mathrm{d}x}$
- Then $\int_a^b f(x)\mathrm{d}x = F(b) - F(a)$
- Not every function can be integrated (ex. $f(x) = e^{-x^2}$)
- Idea: sampling f in $n + 1$ points, interpolating, and integrating the interpolating function
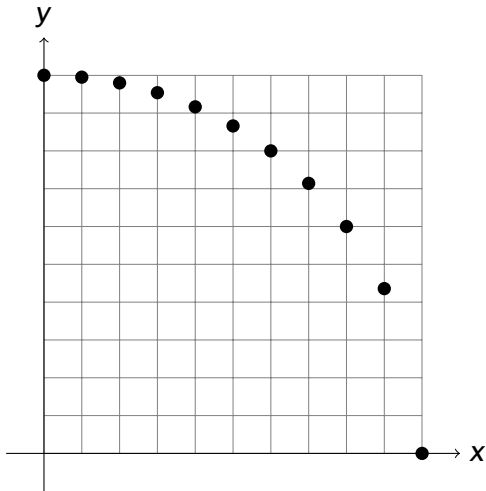- Trivial for interpolation based on linear sp-line
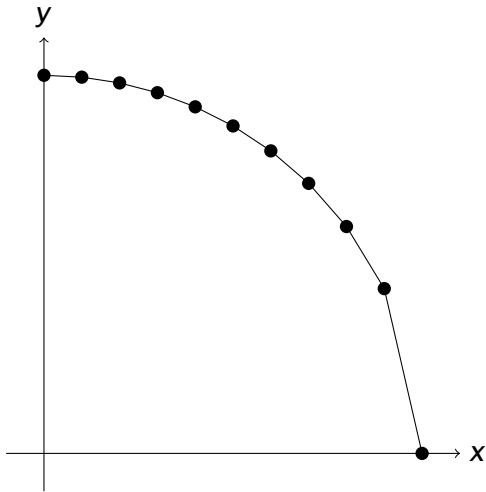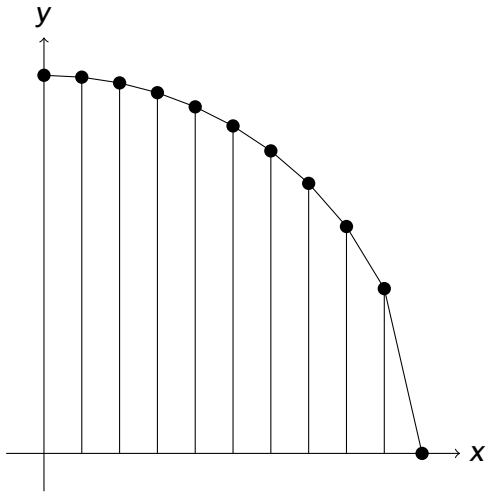
Trapezoidal Rule
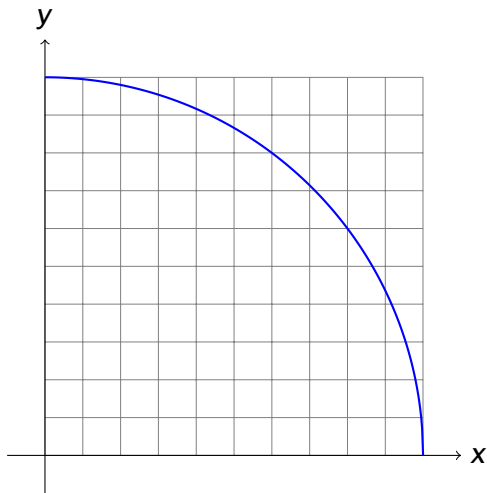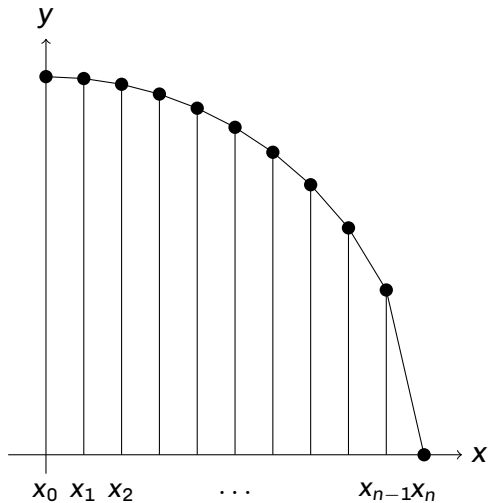
Trapezoidal Rule
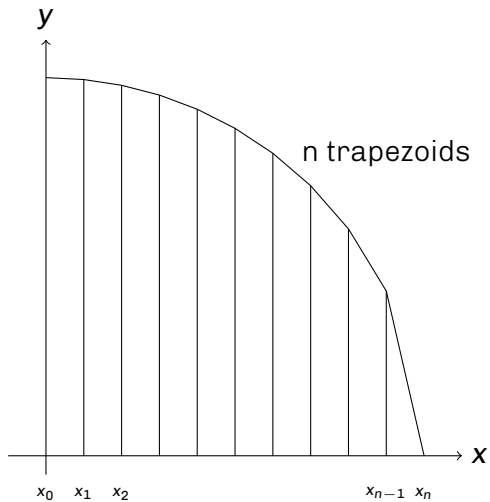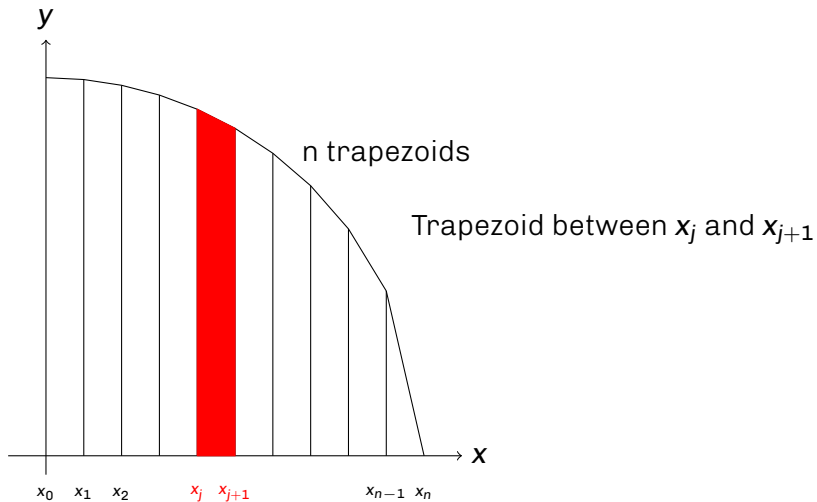
Trapezoidal Rule

Trapezoidal Rule

Trapezoidal Rule

Trapezoidal Rule

Trapezoidal Rule

Trapezoidal Rule

n trapezoids

Trapezoid between x_j and x_{j+1}

Trapezoidal Rule

y

$(x_j, f(x_j))$
$(x_{j+1}, f(x_{j+1}))$

n trapezoids

Trapezoid between x_j and x_{j+1}
bases are $f(x_j)$ e $f(x_{j+1})$

x

x_0 x_1 x_2 x_j x_{j+1} x_{n-1} x_n

Trapezoidal Rule

n trapezoids

Trapezoid between x_j and x_{j+1}
bases are $f(x_j)$ e $f(x_{j+1})$
altitude is $x_{j+1} - x_j$
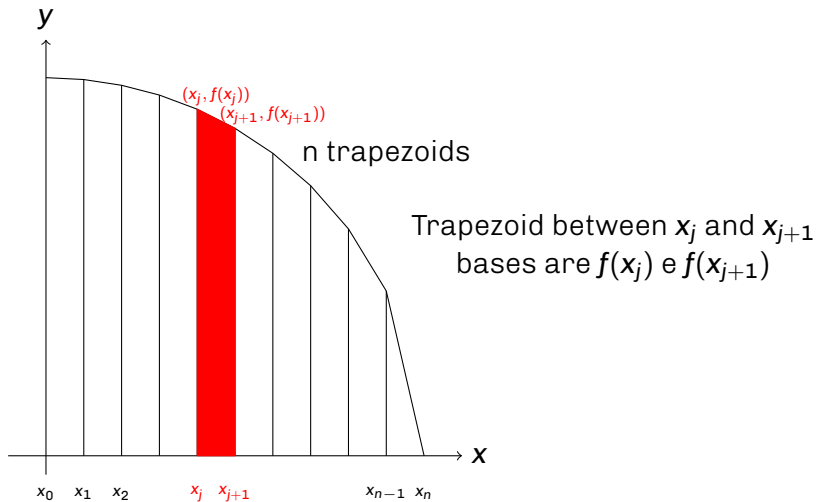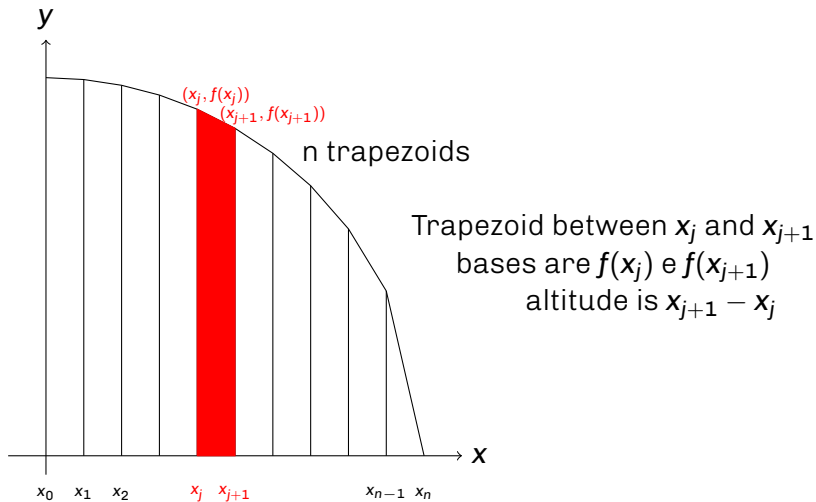
Trapezoidal Rule

n trapezoids

Trapezoid between x_j and x_{j+1}
bases are $f(x_j)$ e $f(x_{j+1})$
altitude is $x_{j+1} - x_j$

$$A_{\text{trapez}} = \frac{f(x_j) + f(x_{j+1})}{2}(x_{j+1} - x_j)$$
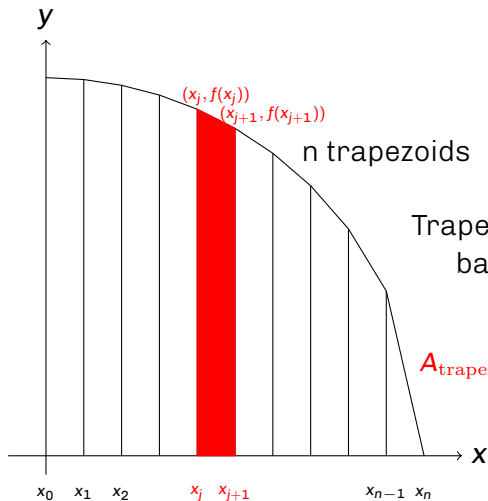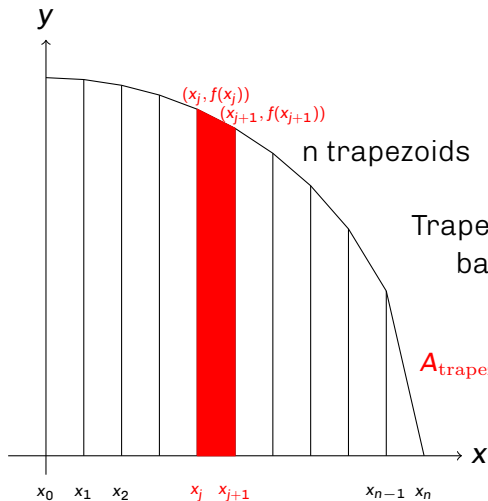
Trapezoidal Rule

n trapezoids

Trapezoid between x_j and x_{j+1}
bases are $f(x_j)$ e $f(x_{j+1})$
altitude is $x_{j+1} - x_j$

$$A_{\text{trapez}} = \frac{f(x_j) + f(x_{j+1})}{2}(x_{j+1} - x_j)$$

$$A_{\text{tot}} = \sum A_{\text{trapez}}$$

$$A_{\mathrm{tot}} = \sum_{j=0}^{n-1} \frac{f(x_j) + f(x_{j+1})}{2}(x_{j+1} - x_j)$$

$$A_{\text{tot}} = \sum_{j=0}^{n-1} \frac{f(x_j)+f(x_{j+1})}{2}(x_{j+1} - x_j)$$

If points are at same distance on the x-axis, all trapezoids have same altitude $\frac{x_n - x_0}{n}$
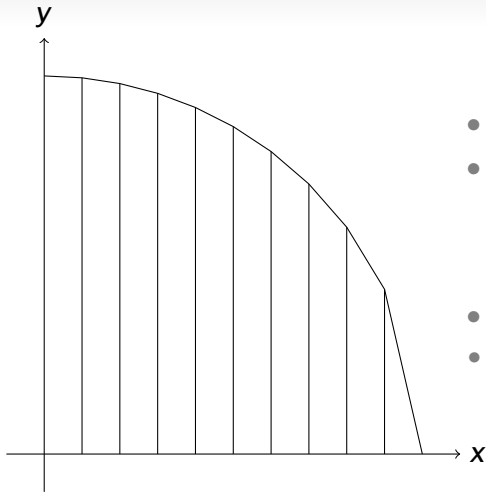
$$A_{\text{tot}} = \sum_{j=0}^{n-1} \frac{f(x_j)+f(x_{j+1})}{2}(x_{j+1} - x_j)$$

If points are at same distance on the x-axis, all trapezoids have same altitude $\frac{x_n - x_0}{n}$

$$A_{\text{tot}} = \frac{x_n - x_0}{n} \sum_{j=0}^{n-1} \frac{f(x_j)+f(x_{j+1})}{2}$$

- Trapezoid area?
- long basis plus short basis by altitude divided by two
- $\sum_i (x_{i+1} - x_i)\frac{f(x_i)+f(x_{i+1})}{2}$
- same distance x? same altitudes

$$A = \frac{x_n - x_0}{n} \sum_{i=0}^{n-1} \frac{f(x_i) + f(x_{i+1})}{2}$$

$$A = \frac{1}{10} \left(\frac{\sqrt{1.00} + \sqrt{0.99}}{2} + \frac{\sqrt{0.99} + \sqrt{0.96}}{2} + \right.$$

$$\frac{\sqrt{0.96} + \sqrt{0.91}}{2} + \frac{\sqrt{0.91} + \sqrt{0.84}}{2} +$$

$$\frac{\sqrt{0.84} + \sqrt{0.75}}{2} + \frac{\sqrt{0.75} + \sqrt{0.64}}{2} +$$

$$\frac{\sqrt{0.64} + \sqrt{0.51}}{2} + \frac{\sqrt{0.51} + \sqrt{0.36}}{2} +$$

$$\left. \frac{\sqrt{0.36} + \sqrt{0.19}}{2} + \frac{\sqrt{0.19} + \sqrt{0.00}}{2} \right) \simeq 0.7761$$

$$\frac{\pi}{4} \simeq 0.7854$$