MPC controller for unicycle robot

[Interactive lesson]

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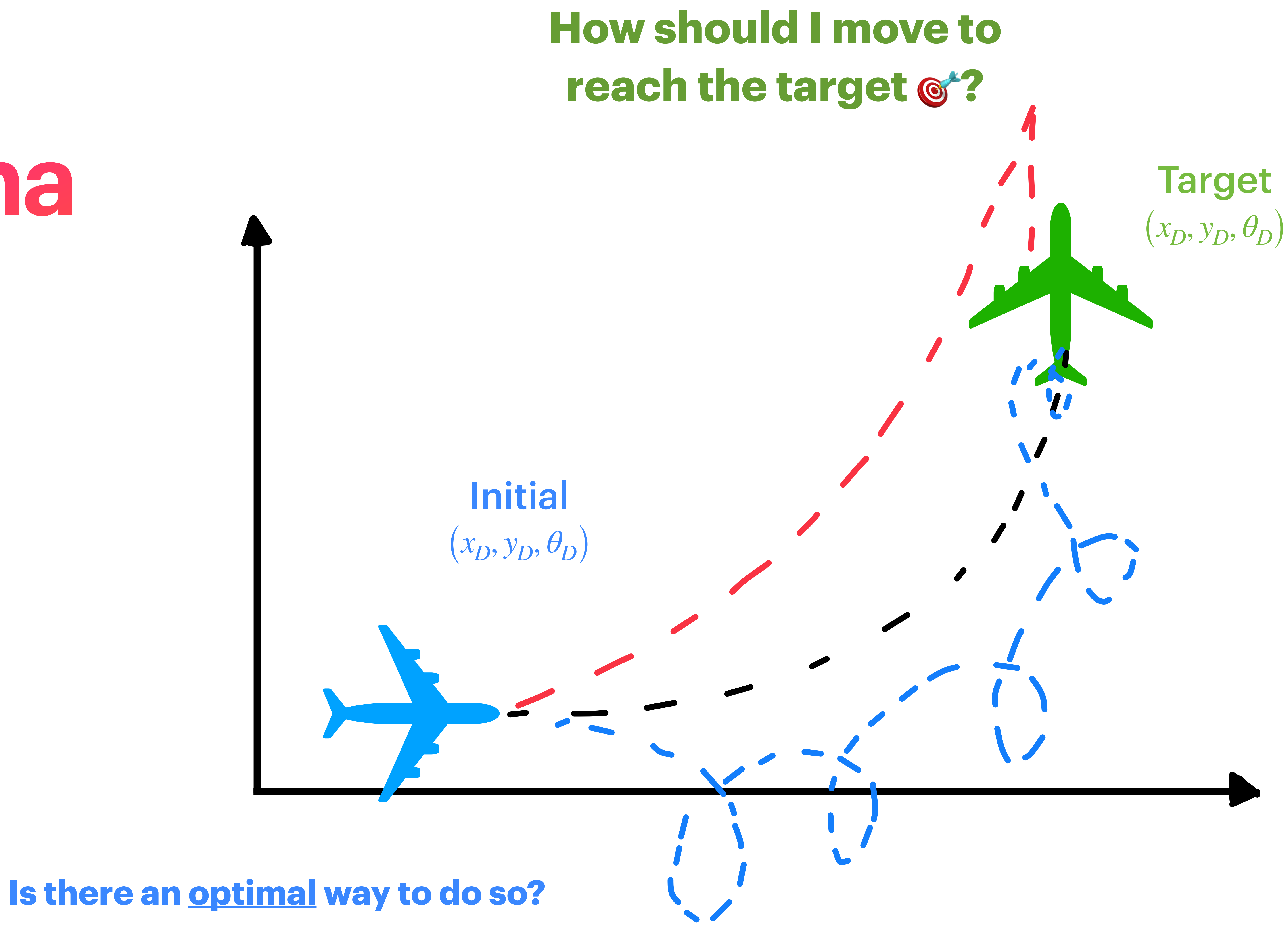
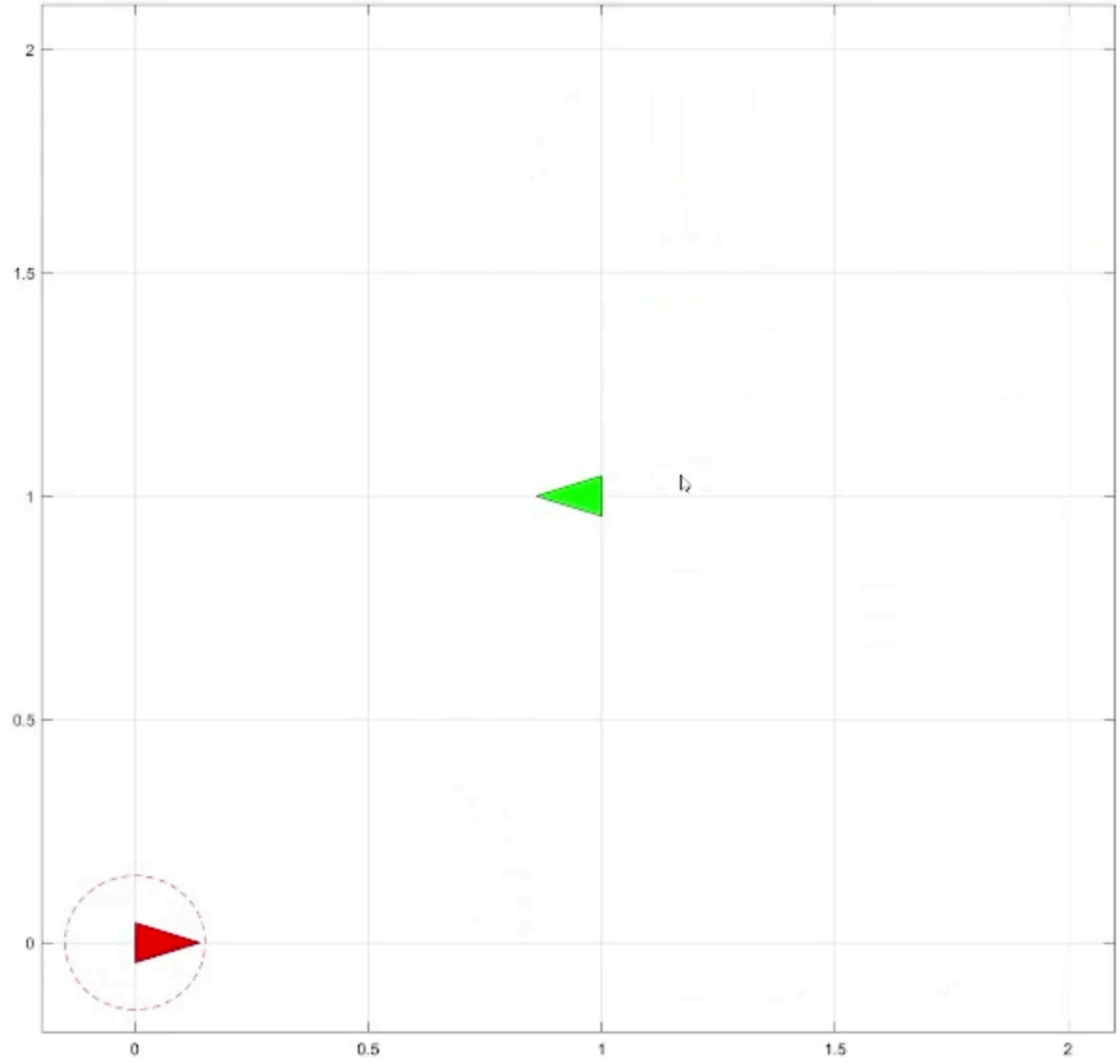
Task we are gonna cover

<u>Reaching a state</u>

Following a trajectory




2
1.5
1
0.5
0
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0.5
1
1.5
2

Program of the lecture

1. Introduction to optimal control and how can we solve it
2. Super fast overview of CasADi
3. Trajectory optimization
4. MPC

Introduction to OCP

- We are looking for an optimal control policy u(t) which optimally drives the system for the whole time horizon T
- x, u are trajectories, so infinitely dimensional
- Also the constraints are infinite

$$\min_{x,u} \int_0^T \varphi_R(x(t), u(t), t)dt + \varphi_F(x(T))$$

Subject to

$$x(0) = x_0$$

$$\dot{x} = f(x(t), u(t), t) \forall t \in [0,T]$$

$$g(x(t), u(t), t) \leq 0 \forall t \in [0,T]$$

Multiple shooting

This is a discretization of the previous problem

General form of a NLP

$$\min_{x,u} \sum_{t=0}^{N-1} \varphi_R\left(x_t, u_t\right) + \varphi_F\left(x_N\right)$$

Subject to

$$x_0 = x_0$$
$$x_{t+1} = x_t + f(x_t, u_t)dt$$
$$g(x_t, u_t) \leq 0$$

$$\min_{w} \varphi(w)$$

Subject to

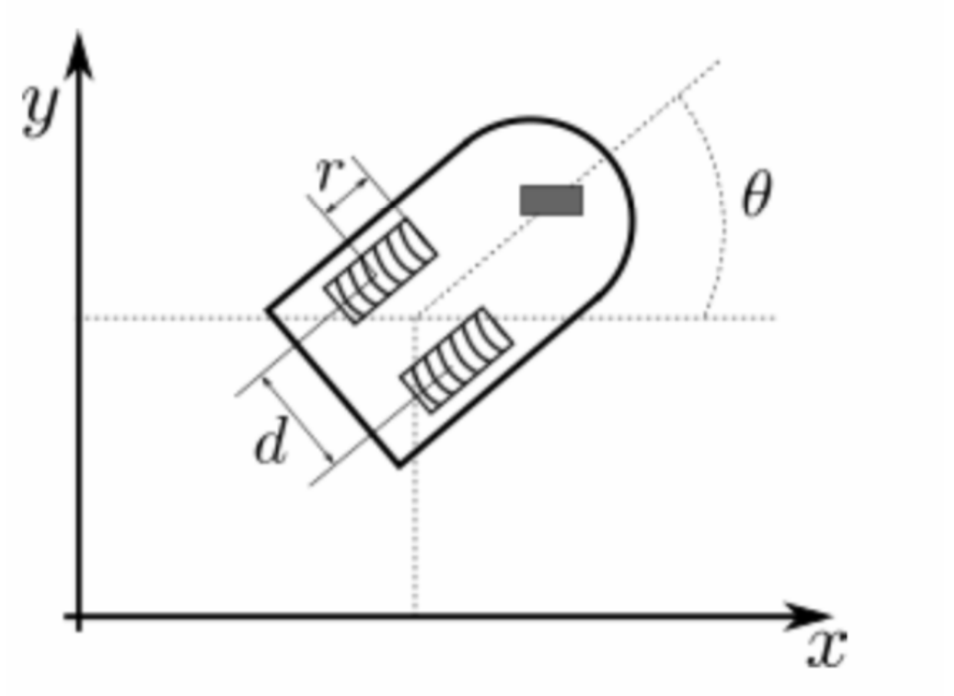
$$f(w) = 0$$
$$g(w) \leq 0$$

Now we have a NLP

We can use NLP off the shelves solver to find a <u>local optimum</u> 🙌🏽

What's about the unicycle

- We will focus only on the kinematics (no dynamics)
- We want to reach a reference state
- Also we want to keep the control low
- We may want to add velocity bounds



$$\begin{cases} \dot{x} = \dfrac{\omega_R + \omega_L}{2} r \cos(\theta) \\ \dot{y} = \dfrac{\omega_R + \omega_L}{2} r \sin(\theta) \\ \dot{\theta} = \dfrac{\omega_R - \omega_L}{d} r \end{cases}$$

$$\min_{z,u} \sum_{t=0}^{N-1} \| z_t - z_D \|^2 + \| u_t \|^2$$

Subject to
$$z_0 = z_0$$
$$z_{t+1} = z_t + f(z_t, u_t)dt$$
$$l_B \leq u_t \leq u_B$$

weights can be put inside or outside the norm

CasADi

You can find all the info here: https://web.casadi.org

Basically it's a library for automatic differentiation and it's very focused on optimal control. By installing it you are also installing some solvers. In particular Ipopt (ai pi opt for Italian speakers) is a very powerful one.

You can find a brief description of it here: https://drops.dagstuhl.de/volltexte/2009/2089/pdf/09061.WaechterAndreas.Paper.2089.pdf

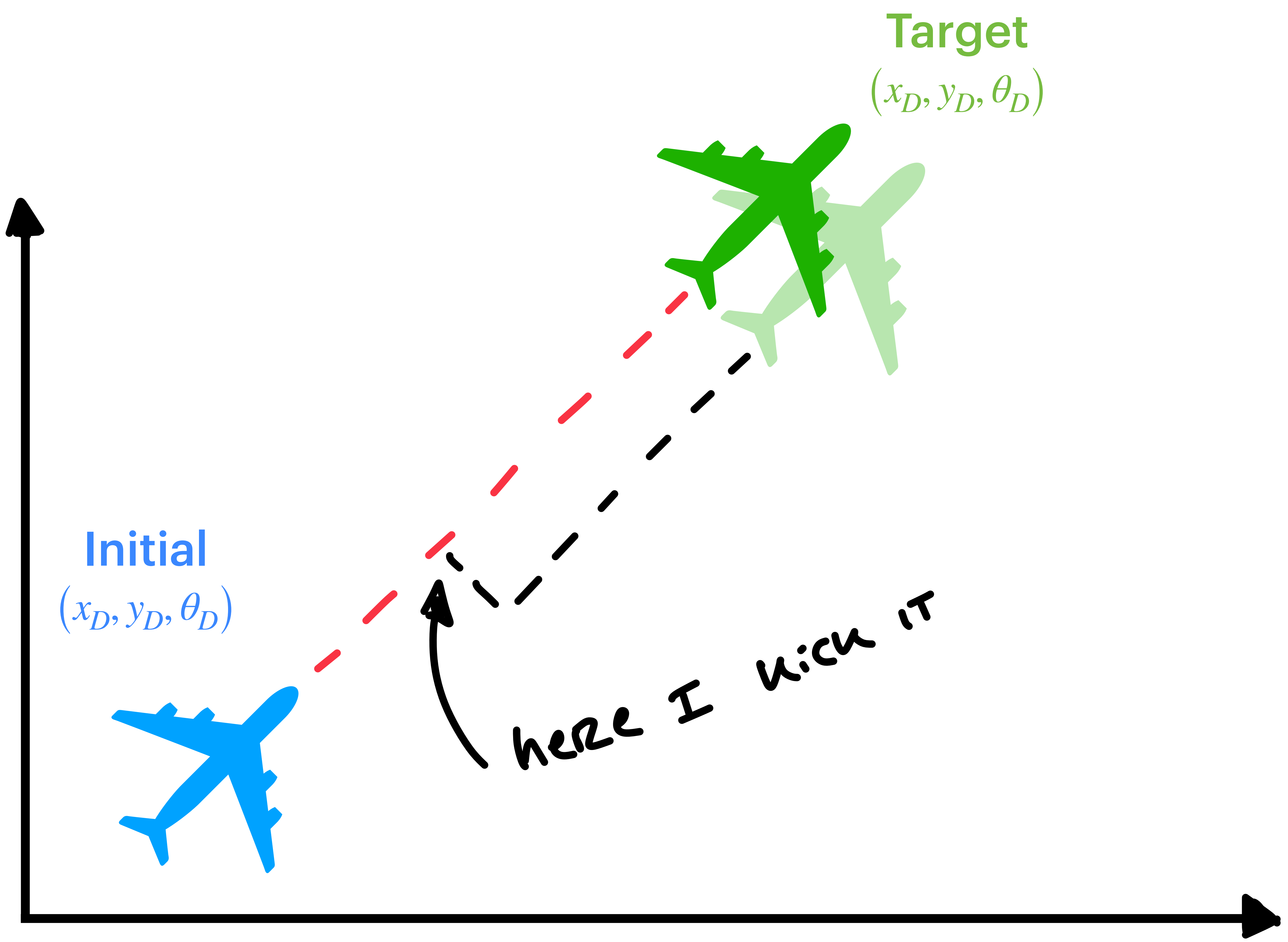
And a detailed one here: https://link.springer.com/article/10.1007/s10107-004-0559-y

How does it work is not the scope of this class and we will focus instead on how we can use it

Problem

If we apply directly u(t) we are gonna miss the target

We have to somehow close the loop

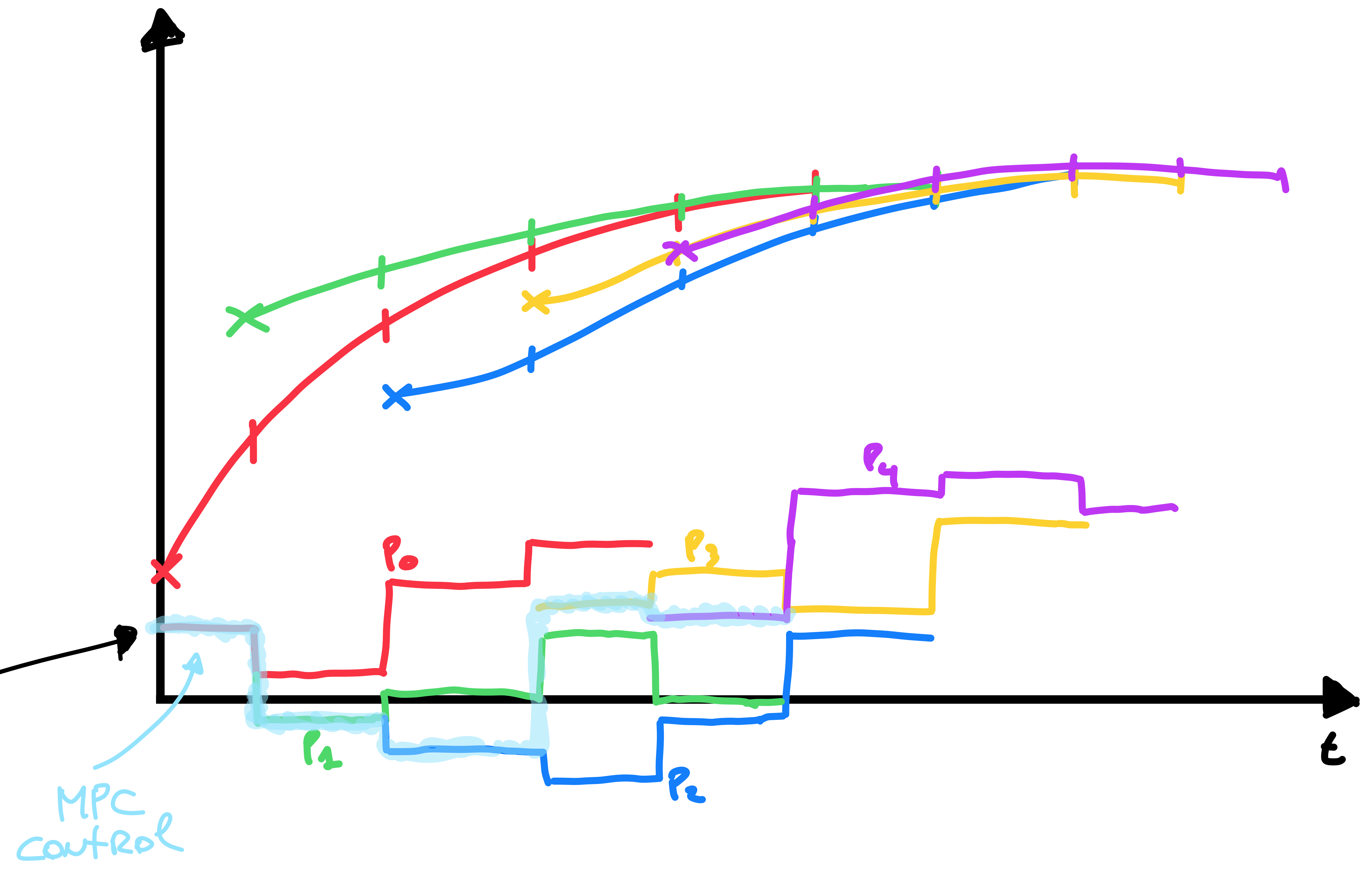


Solution

MPC is a way to use optimal control to control a robot.

It consist in solving an OCP from the initial state and apply the control only of the first time step while neglecting the others.

Then, another OCP is solved and it keeps running using only the first solution



From $U^* = \begin{bmatrix} u_0^* \\ u_1^* \\ \vdots \\ u_{N-1}^* \end{bmatrix}$

I only keep u_0^*

Thank you for your attention