Title of the Semester Project at QuDev

Research Project II

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Abstract

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Contents

Introduction

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Chapter 1

Experimental Setup

In this chapter we introduce the experimental setup used for our experiments. We start by presenting the design of our implementation of a qubit: the Transmon. Then we will show how the qubit is driven for our purposes, in particular how to perform readout and 2-qubit gates.

Finally we put everything together and present the chip used for quantum entangling and photon emission.

1.1 The Qubit

The basic idea behind the implementation of a quantum device for quantum information is realizing a 2-level quantum system. In the Quantum Device Lab this is done through superconducting circuits. The qubits are star-shaped transmon qubits [1]. They are characterized by a large capacitor and a SQUID (see fig. 1.1). The latter is flux-tunable and it is made out of two Josephson junctions in parallel.

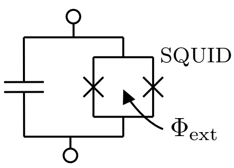

Figure 1.1: Circuit diagram of a transmon qubit

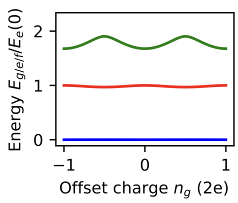

Figure 1.2: Energy spectrum relative to N_g

The Hamiltonian representing the dynamics of such a transmon is

$$\hat{H} = E_C(\hat{N} - N_g)^2 - E_{J_0,\max} \cos\left(\pi \frac{\Phi_{\mathrm{ext}}}{\Phi_0}\right) \cos\hat{\delta} \tag{1.1}$$

(a) Star-shaped transmon (b) Zoom-in of a SQUID loop

Figure 1.3: Images of the devices used in the lab

where $\hat{N}$ is the operator of number of Cooper pairs in the island and $\hat{\delta}$ is the phase operator of the transmon. This can be seen as a non-linear harmonic oscillator.

By changing the external magnetic flux through the SQUID Φ_{ext}, it's possible to change the energy splitting between the first two energy levels $|g\rangle$ and $|e\rangle$, and thus the frequency of the qubit. Furthermore, the anharmonicity allows us to access the first and second transition separately, since they have different energy gaps (see fig. 1.2). Nevertheless, it is important to mention that in our particular setup the qubits are not flux-tunable, because it is not useful for out purposes. We will present our setup in more detail in Section 1.4. The qubits are also coupled to a charge line, which is used to perform single-qubits rotations.

The qubits are star-shaped and made out of niobium(see fig. 1.3), fabricated on a silicon substrate.

1.2 Dispersive Readout

The readout mechanism allows us to measure the state of the qubit in a non-demolishing way [3]. In our experiments this is needed in order to characterize the device and for calibration purposes.

The transmon our capacitively coupled to a readout resonator, which is a waveguide on our device. By sending through this waveguide a signal far detuned from the qubit's frequency, its frequency will slightly change depending on the state of the qubit, without interfering with the state of the latter. This process ids described by the James-Cummins Hamiltonian

$$\hat{H} = \hbar\omega_r \hat{a}^\dagger \hat{a} + \frac{1}{2}\hbar\omega_{ge}\hat{\sigma}^z + \hbar g(\hat{a}^\dagger\hat{\sigma}^- + \hat{a}\hat{\sigma}^+), \tag{1.2}$$

which models the coupling between a 2-level system and a light mode in a cavity. Thus, by measuring the frequency shift of the readout resonator, we

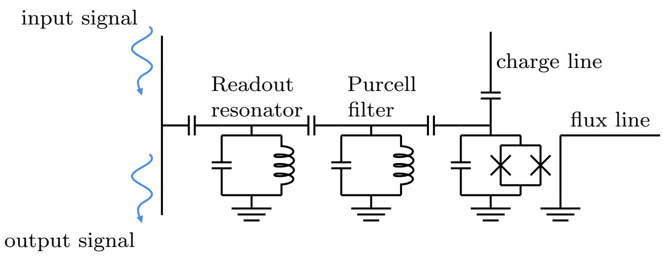

Figure 1.4: Diagram representing a storage qubit and their readout mechanism

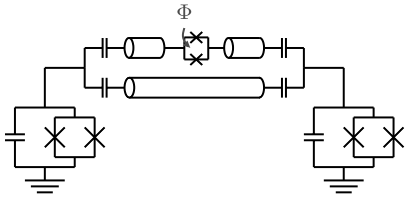

Figure 1.5: Schematic representation of two qubits connected by a tunable coupler

are able to infer the qubit's state.

In order to not let the qubit couple too strongly to the environment through the readout resonator, a Purcell filter is applied between the resonator and the qubit (see fig. 1.4). This allows to keep the lifetimes of the qubits longer, by suppressing spontaneous emission due to the Purcell effect [2].

1.3 Tunable Couplers

Qubits in our device are coupled through tunable couplers. They allow us to mediate interaction between the qubits and perform two-qubits gates. They are composed of two planar waveguide, one of them with a fixed coupling term J_{fixed}. The other is instead flux tunable, meaning that its coupling term $J_{\text{tunable}}(\Phi)$ can be tuned with the magnetic flux generated by a flux line and going into a SQUID loop placed on the waveguide as depicted in fig. 1.5.

The SQUID loop connected to the coupler allows us to control the interaction between the two qubits. To minimise the interaction between the two qubits and the ZZ crosstalk, the DC flux driving the SQUID loop has to be calibrated to the operation point.

To implement a two-qubit gate, a sinusoidal pulse is sent on top of the DC flux to the SQUID loop. If the modulation frequency of this pulse matches a transition of the two-qubit system, it will perform a two-qubit gate. In this way we are able to turn on interactions between specific levels of the qubits. Reference the next section maybe.

1.4 Our Chip

Our device consists of four star-shaped transmon qubits: two storage qubits and two emitter. All of them are flux-tunable and thus coupled to a flux line.

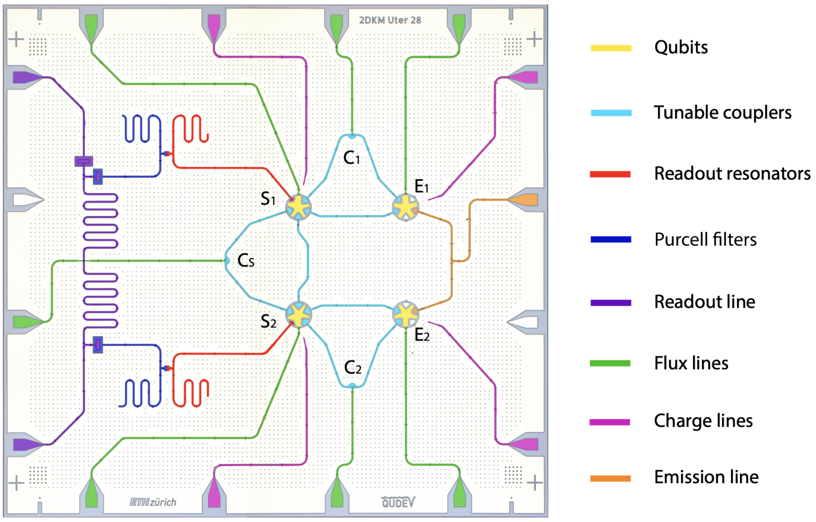

Figure 1.6: False-color micrograph of the employed four-qubit device

1.4.1 Storage Qubits

The storage qubits, depicted with an 'S' in fig. 1.6, are coupled to each other through a tunable coupler (C_S). They possess extended lifetimes, enabling us to execute multiple operations on them before their decay or undergo decoherence. The storage qubits are connected to both a readout resonator and a Purcell filter, utilized for calibration purposes.

1.4.2 Emitter Qubits

Each storage qubit is connected to an emitter qubit, denoted by an 'E' in the diagram, through tunable couplers (C_1 and C_2). They are characterized by a very strong coupling to a waveguide referred to as "emission line". This leads to a rapid decay of any excitation induced in them via the emission line, manifesting as emitted photons. This prevents us from conducting multiple operations on these qubits, since any excitation would be promptly released into the emission line. Finding a good DC flux for the tunable couplers linking storage and emitter qubits is crucial to avoid the decay of the former into the ladder and subsequently into the environment

Chapter 2

Protocols for 2 qubit gates

To execute experiments and induce the emission of entangled photons into the waveguide, conducting both single- and two-qubit gates on our device is essential. As previously explained in the preceding chapter, conducting single-qubit gates on the emitter qubits is not viable due to their short lifespans. This limitation also influences the type of two-qubit gate feasible between a storage and an emitter qubit. For instance, we are unable to perform a controlled-phase gate, or CZ gate, between a storage and an emitter.

In this chapter, we will discuss the types of two-qubit gates that we can perform between storage-storage and storage-emitter qubits. The gates we are able or unable to perform is the main limiting factor affecting the number of entangled photonic states we can create with our device.

2.1 Storage-Emitter

In fig. 2.1 we illustrate the level diagram for a storage-emitter (S-E) system.

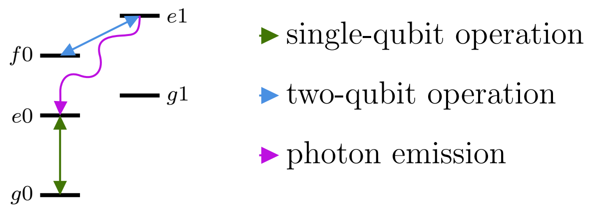

Figure 2.1: Level diagram of storage-emitter interaction

We label the levels of the storage qubit with $|g\rangle, |e\rangle, |f\rangle$ and those of the emitter as $|0\rangle, |1\rangle$. Due to the rapid emission of the emitter, the second excited level $|2\rangle$ is not relevant for our purposes. In addition, we cannot

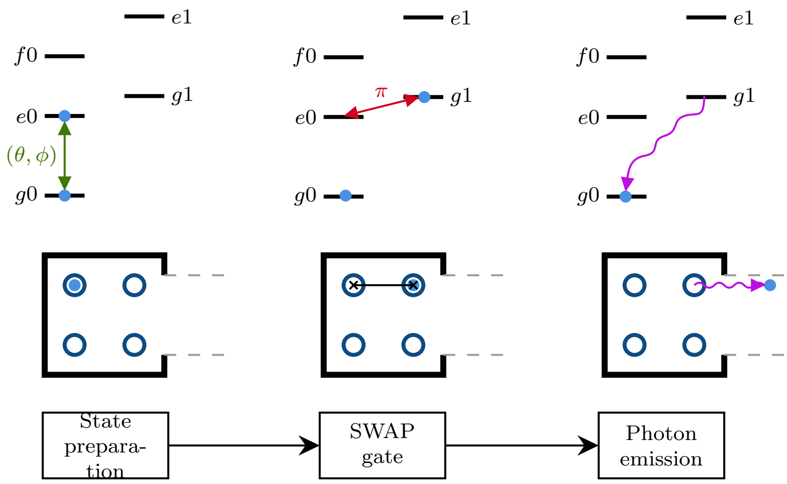

Figure 2.2: Protocol for executing an S-E SWAP gate

run single-qubit gates on the emitter. The only operations we can execute are single-qubits gate on the storage qubit and two-qubit gates that do not require single-qubit gates on the emitter. In other words, we can only transfer the excitation from the storage to the emitter.

The two-qubit gates we are able to execute considering the limitations of our device are SWAP and controlled emission (or CNOT).

2.1.1 SWAP

The simplest two-qubit gate executable on an S-E pair is the SWAP gate. It facilitates the mapping of the storage qubit's state onto the emitter qubit, consequently leading to photon emission through spontaneous decay into the emission line of the ladder.

Another application of this gate is device resetting. Given the emitter qubit's rapid decay, typically residing in the ground state, the SWAP gate reliably resets the storage qubit's state to the ground state.

The unitary matrix representing the SWAP gate is

$$U_{\mathrm{SWAP}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{2.1}$$

This implements the operation

$$|\psi\rangle_{\mathrm{S}} |0\rangle_{\mathrm{E}} \xrightarrow{U_{\mathrm{SWAP}}} |g\rangle_{\mathrm{S}} |\psi\rangle_{\mathrm{E}} \tag{2.2}$$

which transfer the quantum state of the storage in the emitter qubit, leaving the former in its ground state.

The depicted sequence shown in fig. 2.2 illustrates the operations required to execute the SWAP gate between a storage and an emitter. First, an arbitrary state is prepared on the storage qubit through single-qubit operations. Following this, a π rotation is induced within the $e0$-$g1$ manifold of the two-qubit system. Lastly, upon decay of the emitter qubit, a photon is emitted in the emission line, preserving the state initially encoded by the storage qubit.

2.1.2 CNOT

The other operation that we can run between storage and emitter qubits is a controlled emission, or CNOT. The unitary matrix representing this operation is

$$U_{\text{CNOT}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}. \tag{2.3}$$

A photon is emitted form the emitter qubit only if the storage qubit is in the excited state $|e\rangle$, otherwise no photon is emitted. Thus, it performs the operation

$$|\psi\rangle_{\text{S}} |0\rangle_{\text{E}} = (\alpha |g0\rangle + \beta |e0\rangle)_{\text{SE}} \xrightarrow{U_{\text{CNOT}}} (\alpha |g0\rangle + \beta |e1\rangle)_{\text{SE}}. \tag{2.4}$$

This observation highlights that, since the final state cannot be expressed as a tensor product of two individual pure states, the resulting state is entangled.

Due to the strong coupling between the emitter and the emission line, we cannot depend on operations that involve transferring population to the emitter qubit and its subsequent return. Essentially, our capability allows only for a π rotation between the storage and emitter qubits. Hence, in executing a CNOT gate within our protocol, the initial step involves population transfer from the $e0$ to the $f0$ state through a single-qubit π rotation. Subsequently, a π rotation within the $f0$-$e1$ manifold is performed, effectively transferring the state from the $e0$ level to the $e1$ level within the two-qubit system.

The protocol employed for executing the controlled emission is depicted in fig. 2.3. Within the diagram, the entanglement between the two qubits is denoted by a red line. We will discuss what we mean by entanglement when dealing with graph states in reference section.

Please note, this method effectively executes a CNOT gate within the S-E manifold, operating under the assumption that the emitter qubit is predominantly in its ground state $|0\rangle$, a condition frequently observed.

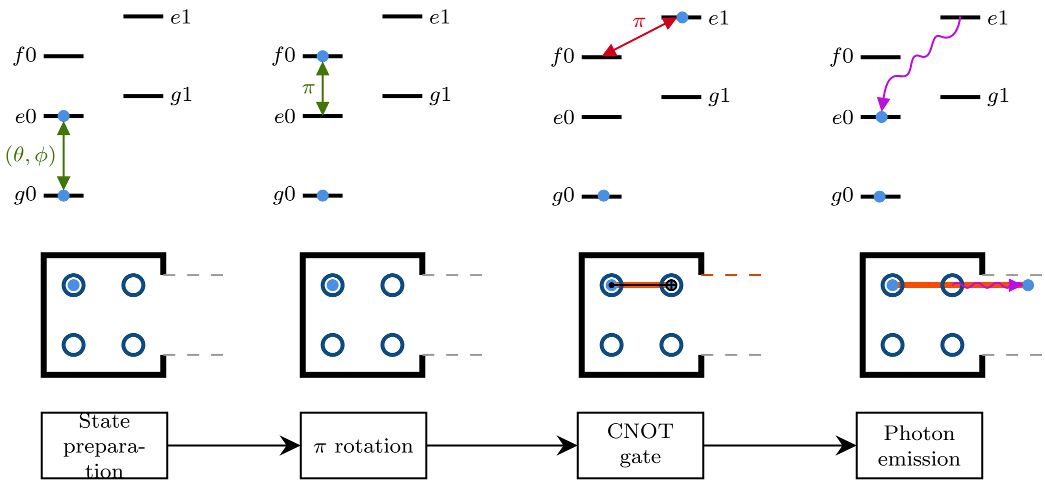

Figure 2.3: Protocol for S-E CNOT gate

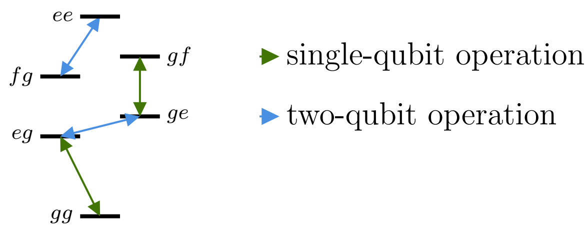

Figure 2.4: Level diagram of storage-storage interaction

2.2 Storage-Storage

In the S-S system, the extended lifetime of the two qubits allows us to execute single-qubit gates on them. Additionally, the capability to perform 2π rotations on the sidebands of this two-qubit system enables the implementation of a CZ gate within the S-S system.

In fig. 2.4, the energy levels of the two-qubits system is shown, together with some single- and two-qubit gates that we can run on them.

2.2.1 CZ

The unitary matrix representing a controlled-phase gate, or CZ, is

$$U_{\mathrm{CZ}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{2.5}$$

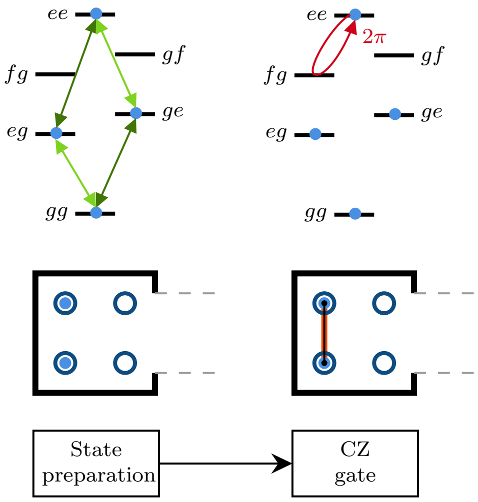

Figure 2.5: Protocol for S-S CZ gate

It performs the operation

$$(\alpha\,|gg\rangle + \beta\,|ge\rangle + \gamma\,|eg\rangle + \delta\,|ee\rangle)_{\mathrm{S_1S_2}} \xrightarrow{U_{\mathrm{CZ}}} (\alpha\,|gg\rangle + \beta\,|ge\rangle + \gamma\,|eg\rangle - \delta\,|ee\rangle)_{\mathrm{S_1S_2}}. \tag{2.6}$$

The operational sequence on our device involves transferring population between the states $|ee\rangle$ and $|fg\rangle$ within the S-S system. After a 2π rotation, the population returns to the $|ee\rangle$ state, but with an opposite phase.

This gate isn't feasible within the S-E system as it necessitates a full 2π rotation, which isn't reliably achievable. Moreover, it doesn't make sense as we can't distinguish between a CZ gate and an identity gate, given our inability to sustain the $|e1\rangle$ state.

Chapter 3

Three graph state

3.1 graph states

3.2 logical operatioin

3.3 How to build it

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