Sequence Harmonization Factor

Alesandro Barro (alessandro1.barro@mail.polimi.it)

15 November 2023

1 Introduction

This article introduces the Sequence Harmonization Factor (SHF) H, an integer optimization tool. It converts real-valued scores to integer multipliers, optimizing sequences of weighted elements. H can be key in situations demanding discrete optimization, ensuring elements are proportionally and accurately represented. The ensuing definition details its mathematical framework.

2 Definition

Let Π be a set of values indicating a sequence of non-repeating elements $\pi_i \in \mathbb{Z}$, $\pi_i \neq \pi_j \ \forall i, j = 1, ..., n$

$$\Pi = \{\pi_i\}_{i=1}^{n} \tag{1}$$

Let $z \in \mathbb{Z}^+$, $z \neq 0$. Then we can indicate as $q \in \mathbb{Q}$ a rational partition of z

$$q = \frac{1}{z} \tag{2}$$

We can refer to a specific multiple of q with q_j where $j \in [1, z] \subseteq \mathbb{Z}$.

Let $\mathcal{V} : \pi_i \mapsto \mathbb{R} \ \forall i = 1, ..., n$ be the value function associating a real-valued score to each set's element. We can denote as w the normalized values

$$w_i = \frac{\mathcal{V}(\pi_i)}{\sum_i \mathcal{V}(\pi_i)} \tag{3}$$

This $\forall i = 1, ..., n$. Let q_j and q_{j-1} be the closest integer partitions to contain w_i. We need to find $q^* \in \mathbb{Q}$ such that

$$q_i^* = \min_j(|w_i - q_j|, |w_i - q_{j-1}|) \tag{4}$$

$\forall i = 1, ..., n$. Now let $h \in \mathbb{Z}$, $h \leq z$ be the harmonization multiplier defined as follows

$$h_i = \lfloor q_i^* z \rfloor \tag{5}$$

Again $\forall i = 1, ..., n$. Note that each element of the sequence will have a harmonization multiplier. If this one is zero-valued, than the corresponding element is deleted from the list. Simbolically, denoting Π^* as the final sequence, we can state that

$$\Pi^* = \{h_i \pi_i\}_{i=1}^n \tag{6}$$

$$\Pi^* = H\Pi \tag{7}$$

Where H is the Sequence Harmonization Factor, concluding our definition. □

3 Example Application

To illustrate the practical application of the SHF, consider a scenario involving a set of tasks, each with a different priority level. Our goal is to allocate resources to these tasks in a way that reflects their relative importance.

Suppose we have a set of tasks $\Pi = \{\pi_1, \pi_2, \pi_3, \pi_4\}$. Each task π_i is associated with a real-valued score, representing its priority, as determined by a value function $\mathcal{V}$. Let's assume the scores are as follows:

$$\begin{aligned}\mathcal{V}(\pi_1) &= 2.5,\\ \mathcal{V}(\pi_2) &= 3.5,\\ \mathcal{V}(\pi_3) &= 1.0,\\ \mathcal{V}(\pi_4) &= 2.0.\end{aligned}$$

We choose $z = 4$ as our partition number, yielding a partition value of $q = \frac{1}{4}$. Next, we compute the normalized scores w_i for each task:

$$\begin{aligned}w_1 &= \frac{2.5}{2.5 + 3.5 + 1.0 + 2.0} = \frac{2.5}{9.0},\\ w_2 &= \frac{3.5}{9.0},\\ w_3 &= \frac{1.0}{9.0},\\ w_4 &= \frac{2.0}{9.0}.\end{aligned}$$

We then determine the closest integer partitions (q_j and q_{j-1}) for each w_i and calculate the corresponding h_i:

$$\begin{aligned}h_1 &= \lfloor \frac{2.5}{9.0} \times 4 \rfloor = 1,\\ h_2 &= \lfloor \frac{3.5}{9.0} \times 4 \rfloor = 1,\\ h_3 &= \lfloor \frac{1.0}{9.0} \times 4 \rfloor = 0,\\ h_4 &= \lfloor \frac{2.0}{9.0} \times 4 \rfloor = 0.\end{aligned}$$

The final allocation of resources, as per the SHF, is as follows:

$$\Pi^* = \{h_1\pi_1, h_2\pi_2, h_3\pi_3, h_4\pi_4\} = \{\pi_1, \pi_2\}.$$

In this example, tasks π_1 and π_2 are selected for allocation based on their higher normalized scores. Tasks π_3 and π_4 are omitted due to their lower priority as indicated by their zero-valued multipliers.

This example demonstrates how the SHF can be applied to make discrete, integer-based decisions in resource allocation, effectively capturing the relative importance of tasks in a quantifiable manner.