Exercise 1 - Discrete random variable

- the probability distribution function of a discrete variable k is given by the following:

$$p(k) = \begin{cases} \dfrac{k}{15} & \text{with} \quad \text{k} = 1, 2, 3, 4, 5 \\ 0 & \text{otherwise} \end{cases}$$

1) write the R probability functions for the probability density and cumulative distribution functions, following the R naming convention

2) produce two plots showing the pdf and cdf, separately

3) compute, using R, the mean value and variance of the probability distribution

4) compute, using R, the expected value $E\left[k\left(6 - k\right)\right]$

5) write the R function that allows to sample random numbers from the probability distribution

6) using the implemented function (point (5)), sample 10^5 random numbers from this distribution and plot them in a graph showing the distribution of the numbers superimposed to the pdf (normalize properly the plots with random numbers)

Exercise 2 - Continuous random variable

- The triangular distribution, in the interval (a, b), is given by the following:

$$f(X) = \begin{cases} \dfrac{2(x - a)}{(b - a)(c - a)} & a \leq x < c \\[3mm] \dfrac{2(b - x)}{(b - a)(b - c)} & c \leq x \leq b \\[3mm] 0 & \text{otherwise} \end{cases}$$

where $c \in [a, b]$.

a) plot the function, given the interval (a, b)

b) and write an algorithm to generate random numbers from the triangular distribution

c) generate 10^4 random number from the distribution, show them in an histogram and superimpose the analytical curve

Exercise 3

- the waiting time, in minutes, at the doctor's is about 30 minutes, and the distribution follows an exponential pdf with rate 1/30

A) simulate the waiting time for 60 people at the doctor's office and plot the relative histogram

B) what is the probability that a person will wait for less than 12 minutes ?

C) evaluate the average waiting time from the simulated data and compare it with the expected value (calculated from theory and by manipulating the probability distributions using R)

B) what is the probability for waiting more than one hour before being received ?

Exercise 4 - Multiple choices exams

- the final exam of a course is given to the students in the format of a multiple choice written test: for each questions there are five possible alternatives

- a student either knows the answer, or selects randomly the answer among the five possible choices

- assuming $p = 0.7$ the probability that the student knows the answer, once a correct answer is given, what it the probability that the student really knew the correct answer ?

- write a small R program to provide the answer

Exercise 5 - Waiting time

- starting from 5:00 in the morning, every half an hour there is a train from Milano Centrale to Roma Termini. We assume there is always an available seat on a train leaving from Milano

- assuming a person arrives at a random time between 10:45 and 11:45 and compute the probability that she has to wait

a) at most 10 minutes

b) at least 15 minutes

c) what is the average time spent waiting ?

Exercise 6 - stock investment

- the annual return rate for a specific stock on the market is a normal variable with a 10% mean and a 12% standard deviation

- Mr X decides to buy 200 share of that specific stock at a price of 85€ per share

- what is the probability that after a year his net profit from the investment is at least 800€ ?

- (suppose there are no transaction costs and that there are no annual dividend for the stock)

References

[1] https://cran.r-project.org/web/packages/nycflights13/nycflights13.pdf