```
R Notebook
                                                                                                                                                                                                                                                        Code ▼
                                                                                                                                                                                                                                                            Hide
    library(tidyverse)
    — Attaching core tidyverse packages —
                                   ----- tidyverse 2.0.0 --
    ✔ dplyr
                            1.1.0 ✓ readr
                                                                                2.1.4
    \checkmark forcats 1.0.0 \checkmark stringr 1.5.0
    ✓ ggplot2 3.4.1 ✓ tibble 3.2.0
    ✓ lubridate 1.9.2
✓ tidyr
                                                                              1.3.0

✓ purrr 1.0.1 — Conflicts —
                                                                               —— tidyverse_conflicts() —
    * dplyr::filter() masks stats::filter()
    # dplyr::lag() masks stats::lag()
    i Use the []8;;http://conflicted.r-lib.org/[conflicted package[]8;;[ to force all conflicts to become errors
 Ex. 1
                                                                                                                                                                                                                                                            Hide
    dfp <- function(k){</pre>
       if ( k==1 || k==2 || k==3 || k==4 || k==5 ){
            return(k/15)
        else{
             return(0)
        }
    cfp <- function(k){</pre>
        C <- 0
        for(i in 1:k){
         c = c + dfp(i)
        }
        return(c)
                                                                                                                                                                                                                                                            Hide
    library("tidyverse")
    x \leftarrow c(-1,0,1,2,3,4,5,5.01,6,7)
    for (i in 1:length(x)){
       if (i==1){
            pdf <- dfp(x[i])</pre>
            cdf <- cfp(x[i])</pre>
        }
        else{
            pdf <- append(pdf,dfp(x[i]))</pre>
             cdf <- append(cdf,cfp(x[i]))</pre>
    ggplot(mapping = aes(x=x,y=pdf)) + geom_point() + geom_line() + scale_x_continuous(breaks = scales::pretty_breaks(n)) + geom_point() + geom_line() + scale_x_continuous(breaks = scales::pretty_breaks(n)) + geom_line() + geom_
    = length(x))) +ggtitle("Probability density function")
              Probability density function
       0.2 -
   pd
       0.1 -
        0.0 -
                                                                                                                                                                                                                                                            Hide
    ggplot(mapping = aes(x=x,y=cdf)) + geom_point() + geom_line() + scale_x_continuous(breaks = scales::pretty_breaks)
    (n = length(x))) + ggtitle("Cumulative density function")
                Cumulative density function
        1.00 -
       0.75 -
b 0.50 →
       0.25 -
                                                                                                                                                                                                                                                            Hide
    cat("The mean value of the dist is", sum(x*pdf), "\n")
    The mean value of the dist is 3.666667
                                                                                                                                                                                                                                                            Hide
    cat("The variance of the dist is", sum(x*x*pdf)-sum(x*pdf)*sum(x*pdf),"\n")
    The variance of the dist is 1.555556
                                                                                                                                                                                                                                                            Hide
    Exp_val = sum(x*(6-x)*pdf)
    cat("The expected value of k(k-6) over the above density function is", Exp\_val)
    The expected value of k(k-6) over the above density function is 7
                                                                                                                                                                                                                                                            Hide
    rfp <- function(){</pre>
        x < - runif(1, 0, 1)
        v <- round(sqrt(30*x),0)</pre>
        if (v<1){
            return(1)
        else{
             return(v)
    probability <- c()</pre>
    for(i in 1:10000){
           probability <- append(probability,rfp())</pre>
    p <- data.frame(probability)</pre>
    orig_pdf <- data.frame(x=x,y=pdf)</pre>
    ggplot(p, aes(x=probability)) +
        geom\_histogram(aes(y=after\_stat(density)), breaks=x, alpha=0.6) + geom\_line(data= orig\_pdf, aes(x=x,y=pdf)) 
    gtitle("Sampled vs. Original Distribution")
               Sampled vs. Original Distribution
        0.3 -
        0.2 -
  density
        0.1 -
        0.0 -
                                                                                           2.5
                                                                                                                                             5.0
                                         0.0
                                                                                              probability
 Ex. 2
                                                                                                                                                                                                                                                            Hide
    a <- 0
    b <- 4
    c <- 3
    dtri <- function(x,a,b,c){</pre>
        if (x<c & x>0){
             return(2*(x-a)/((b-a)*(c-a)))
        else if ((x>c | x==c) & x<b) {</pre>
            return(2*(b-x)/((b-a)*(b-c)))
        else{
             return(0)
    xx <- seq(from = -1, to = 5, by = 0.1)
    tri <- c()
    for(i in 1:length(xx)){
        tri <- append(tri,dtri(xx[i],a,b,c))</pre>
    ggplot(mapping = aes(x=xx,y=tri)) + geom_point() + geom_line() + scale_x_continuous(breaks = scales::pretty_break)
    s(n = length(xx)/5)) + ggtitle("Probability density function")
               Probability density function
        0.5 -
        0.4 -
        0.3 -
   Ħ
        0.2 -
        0.1 -
                                                            0.5
                                                                                                     XX
                                                                                                                                                                                                                                                            Hide
    ctri <- function(a,b,c){</pre>
        y <- runif(1, 0, 1)
        if (y<c/b){
               k < - 2/((b-a)*(c-a))
               return((a*k+sqrt(a*a*k*k+2*k*y))/(k))
        else{
               k < -2/((b-a)*(b-c))
               y < -y+k*c*(b-c/2)-c/b
               return((b*k-sqrt(b*b*k*k-2*k*y))/(k))
    probtri <- c()</pre>
    for(i in 1:10000){
           probtri <- append(probtri,ctri(a,b,c))</pre>
    pt <- data.frame(probtri)</pre>
    orig_pdf <- data.frame(x=xx,y=tri)</pre>
    ggplot(pt, aes(x=probtri)) +
        geom\_histogram(aes(y=after\_stat(density)), breaks=xx, alpha=@.6) + geom\_line(data= orig\_pdf, aes(x=x,y=tri)) + ggom\_line(data= orig\_pdf, aes(x=x,y=tri))
    title("Sampled vs. Original Distribution")
               Sampled vs. Original Distribution
        0.5 -
        0.4 -
        0.3 -
   density
        0.2 -
        0.1 -
        0.0 -
                                                                                                  probtri
 Ex. 3
                                                                                                                                                                                                                                                            Hide
    x <- seq(from = 0, to = 200, by = 1)
    distprofit <- dexp( x , rate=1/30 )
    Empirical_distribution <- rexp(60, rate=1/30 )</pre>
    h <- hist(Empirical_distribution, breaks=50)</pre>
                                                        Histogram of Empirical_distribution
             5
  Frequency
             3
             2
                            0
                                                                              40
                                                                                                       60
                                                                                                                                                         100
                                                    20
                                                                                                                                 80
                                                                                                                                                                                  120
                                                                                  Empirical_distribution
                                                                                                                                                                                                                                                            Hide
    cat("The probability to wait less than 12 minutes is", pexp(12, rate=1/30), "\n")
    The probability to wait less than 12 minutes is 0.32968
                                                                                                                                                                                                                                                            Hide
    \verb|cat("The empirical average is ", mean(Empirical\_distribution) , "while the real average is", sum(distprofit*x)| \\
    ,"\n")
    The empirical average is 27.47466 while the real average is 29.70874
                                                                                                                                                                                                                                                            Hide
    cat("The probability to wait more than 1 hour is", 1-pexp(60, rate=1/30))
    The probability to wait more than 1 hour is 0.1353353
 Ex. 4
                                                                                                                                                                                                                                                            Hide
    p <- 1
    randp <- 0.2
    prior <- 0.7
    pro <- p*prior/(randp*(1-prior)+p*prior)</pre>
    print(pro)
    [1] 0.9210526
 Ex. 5
                                                                                                                                                                                                                                                            Hide
    unif_pdf <- function(x){</pre>
        if ( x>0 & x<60){
             return(1/60)
        else{
             return(0)
        }
    cunif <- function(x){</pre>
        if(x<= 60 & x>=0){
            return(x/60)
        else{
            return(0)
             }
    t <- c()
    for (i in 1:60){
       if (i<=15){</pre>
             t <- append(t, abs(15-i))
        else if(i<=45){
             t <- append(t, abs(45-i))
        else{
             t <- append(t, abs(75-i))
    cat("The prob to wait al most 10 minute is", 20/60,"\n")
    The prob to wait al most 10 minute is 0.3333333
                                                                                                                                                                                                                                                            Hide
    cat("The prob to wait at least 15 minute is", 30/60, "\n")
    The prob to wait at least 15 minute is 0.5
                                                                                                                                                                                                                                                            Hide
    cat("The mean time spent waiting is", mean(t))
    The mean time spent waiting is 14.5
 Ex. 6
                                                                                                                                                                                                                                                            Hide
    mean <- 0.10
    std <- 0.12
    sh <- 200
    price <- 85
    total <- sh*price
    x \leftarrow seq(from = 800, to = total*mean+total*mean*5, by = 100)
    distprofit <- dnorm(x, mean=total*mean, sd = total*std)</pre>
    barplot(distprofit, names=x, main = 'Part of the distribution to be integrate')
                                                      Part of the distribution to be integrate
```

Cat("The probability of a net profit of al least 800\$ is",1-pnorm(800, mean=total\*mean, sd=total\*std))

The probability of a net profit of al least 800\$ is 0.6704574

1700 2600 3500 4400 5300 6200 7100 8000 8900 9800

0.0000.0