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Non-Relativistic Quantum Mechanics

Quantum Field Theory

July 31, 2015

“In the good old days, theorizing was like sailing between islands of experimental evidence. And, if the trip was not in the vicinity of the shoreline (which was strongly recommended for safety reasons) sailors where continuously looking forward, hoping to see land — the sooner the better.

Nowadays, some theoretical physicists (let us call them sailors) [have] found a way to survive and navigate in the open sea of pure theoretical constructions. Instead of the horizon, they look at stars, which tell them exactly where they are. Sailors are aware of the fact that the stars will never tell them where the new land is, but they may tell them their position on the globe.

Theoreticians become sailors simply bacause they just like it. Young people, seduced by capitans forming crews to go to a Nuevo El Dorando soon realize that they will spend all their life at sea. Those who do not like sailing desert the voyage, but for the true potential sailors the sea become their passion. They will probably tell the alluring and frightening truth to their students — and the proper people will join their ranks.”

— Andrei Losev

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THE BASIS

1 SCHWINGER'S APPROACH TO QUANTUM MECHANICS

> "I presume that all of you have already been exposed to some undergraduate course in Quantum Mechanics, one that leans heavily on de Broglie waves and the Schroedinger equation. I have never thought that this simple wave approach was acceptable as a general basis for the whole subject, and I intend to move immediately to replace it in your mind by a foundation that *is* perfectly general."
>
> J. Schwinger, *Quantum Mechanics. Symbolism of Atomic Measurements* **Schwinger:2001**.

1.1 INTRODUCTION

THE

2 LINEAR OPERATORS IN HILBERT SPACES

Operator formulation of non-relativistic quantum mechanics heavily relies on th theory of linear operators in Hilber spaces. In particular, the spectral theory of self-adjoint operators is a key ingredient in formulating the basic rules of quantum mechanics. This chapter is aimed at providing the necessary mathematical background of functional analysis employed by quantum mechanics. It is a chapter on mathematics, not on quantum physics. The Reader interested in how functional analysis is used to formulate quantum mechanics should jump to the next chapters.

2.1 BANACH AND HILBER SPACES

Unless stated otherwise, let $\mathbb{K}$ denote equivalently the field of real numbers $\mathbb{R}$ or the of complex numbers $\mathbb{C}$.

Definition 2.1 (norm): Let V be any linear space over $\mathbb{K}$. A norm on V is any application $\|\cdot\|: V \to \mathbb{R}$ satifying

(a) $\|\varphi\| \geqslant 0$ for any $\varphi \in V$,

(b) $\|\varphi\| = 0$ if and only if $\varphi = 0$,

(c) $\|\alpha\varphi\| = |\alpha|\|\varphi\|$ for all $\alpha \in \mathbb{K}$ and $\varphi \in V$,

(d) $\|\varphi + \psi\| \leqslant \|\varphi\| + \|\psi\|$, for all φ and ψ in V (this is called "triangle inequality")

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