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Quantum Field Theory

Path Integral Methods and Renormalization

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April 27, 2015

“ In the good old days, theorizing was like sailing between islands of experimental evidence. And, if the trip was not in the vicinity of the shoreline (which was strongly recommended for safety reasons) sailors where continuously looking forward, hoping to see land — the sooner the better.

Nowadays, some theoretical physicists (let us call them sailors) [have] found a way to survive and navigate in the open sea of pure theoretical constructions. Instead of the horizon, they look at stars, which tell them exactly where they are. Sailors are aware of the fact that the stars will never tell them where the new land is, but they may tell them their position on the globe.

Theoreticians become sailors simply bacause they just like it. Young people, seduced by capitans forming crews to go to a Nuevo El Dorando [...] soon realize that they will spend all their life at sea. Those who do not like sailing desert the voyage, but for the true potential sailors the sea become their passion. They will probably tell the alluring and frightening truth to their students — and the proper people will join their ranks. ”

— Andrei Losev

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PREFACE

To be written…

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APHORISMS

The *lietemotiv* is *renormalization*. Let us summarize the key ideas. The section is mainly inspired by the introductory chapter of Zinn-Justin.

Relativistic quantum field theory (QFT) was originally developed to match quantum mechanics with the principles of special relativity. Now, it offers the most comprehensive theoretical framework to discuss elementary excitations above the ground state of physical systems with an infinite number of degrees of freedom.

In its various incarnations, QFT is the theoretical framework employed to describe all foundamental interactions but gravitations at microscopic scale — in the Standard Model of Particle Physics, the strong interactions are described by Quantum Chromodynamics (familiarly called QCD), which is an unbroken $\mathrm{SU}(3)_c$ locally-symmetric Yang-Mills gauge QFT, while the electroweak interactions are described by a spontaneusly broken $\mathrm{SU}(2)_w \times \mathrm{U}(1)_w$ Yang-Mills gauge QFT — and let to a deep understanding of the singular properties of a wide class of phase transitions at the critical point; furthermore, the statistical properties of some geometrical models (e. g., self-avoiding random walks, which are of practical interest) can be deal with by using the tools of QFT.

However, QFT in its most direct formulations, comes with a *severe conceptual drawback*, namely the appearance of infinities in the calculation of physical quantities.

To avoid the occurrence of infinities, an empirical, somewhat *ad hoc* (but systematic) procedure, which goes under the name of renormalization, was eventually developed which allows extracting from (meaningless) divergent mathematical expressions (meaningfull) finite numerical predictions to be compared with experiments.

The renormalization recipe works in this spirit: you have to carefully distinguish between *bare* quantities (e. g., mass, electric charge) and *effective* quantities. The latter ones are those which actually have to be related to experiments, while the former ones are additional auxiliary parameters of the physical model, which are necessary in order the renormalization recipe to work but whose physical meaning remains somewhat obscured. At this level, it might be said that renornalization is a way of organizing the calculations in order to get finite results from expressions which naively speaking would otherwise led to infinities.

From a pragmatic point of view, renormalization works: It has allowed and still allows calculations of increasing precision. In fact, the renormalization procedure would hardly have been convincing if the predictions were not confirmed with increasing precision by experiments.

Only later, did the procedure find a satisfactory physical interpretation which clarify the deep origin of renormalization and enlight the role of renormalizable QFTs. The problem of infinities is related to an unexpected phenomenon: *Renormalization is related to the non-decoupling of very different lenght scales* Today, Those who are familiar with Kenneth Wilson's ideas and the renormalization group, will immediately say that actually there is no divergence. More or less, the story goes like this: Every QFT requires an ultraviole completion (thus being an effective theory). The only difference between renormalizable and non-renormalizable QFTs lies in the fact that the former are *insensitive* to the ultraviolet data (which can be absorbed in a few low-energy parameters) while the latter depend on the details of the ultraviole completion.

Part I

COMPLEMENTS OF NON-RELATIVISTIC QUANTUM MECHANICS

1 SCHWINGER'S APPROACH TO QUANTUM MECHANICS

> "I presume that all of you have already been exposed to some undergraduate course in Quantum Mechanics, one that leans heavily on de Broglie waves and the Schroedinger equation. I have never thought that this simple wave approach was acceptable as a general basis for the whole subject, and I intend to move immediately to replace it in your mind by a foundation that *is* perfectly general."
>
> J. Schwinger, *Quantum Mechanics. Symbolism of Atomic Measurements* **Schwinger:2001**.

1.1 INTRODUCTION

THE

Part II

RENORMALIZATION

Part III

NON-ABELIAN GAUGE THEORIES

Part IV

APPENDICES

QUANTUM MECHANICS AND NUMBER THEORY

A

B INTERPLAY BETWEEN CLASSICAL AND QUANTUM MECHANICS