Alessandro Candolini

Non-Relativistic Quantum Mechanics

Quantum Field Theory

July 31, 2015

“In the good old days, theorizing was like sailing between islands of experimental evidence. And, if the trip was not in the vicinity of the shoreline (which was strongly recommended for safety reasons) sailors where continuously looking forward, hoping to see land — the sooner the better.

Nowadays, some theoretical physicists (let us call them sailors) [have] found a way to survive and navigate in the open sea of pure theoretical constructions. Instead of the horizon, they look at stars, which tell them exactly where they are. Sailors are aware of the fact that the stars will never tell them where the new land is, but they may tell them their position on the globe.

Theoreticians become sailors simply bacause they just like it. Young people, seduced by capitans forming crews to go to a Nuevo El Dorando soon realize that they will spend all their life at sea. Those who do not like sailing desert the voyage, but for the true potential sailors the sea become their passion. They will probably tell the alluring and frightening truth to their students — and the proper people will join their ranks.”

— Andrei Losev

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PREFACE

To be written…

Trieste, July 31, 2015

Alessandro Candolini

Part I

THE BASIS

1 Schwinger's approach to quantum mechanics

> "I presume that all of you have already been exposed to some undergraduate course in Quantum Mechanics, one that leans heavily on de Broglie waves and the Schroedinger equation. I have never thought that this simple wave approach was acceptable as a general basis for the whole subject, and I intend to move immediately to replace it in your mind by a foundation that *is* perfectly general."
>
> J. Schwinger, *Quantum Mechanics. Symbolism of Atomic Measurements* **Schwinger:2001**.

1.1 Introduction

Compared with other traditional areas of physics, quantum mechanics is not easy. It often lacks physical intuitition and it relies on heavy mathematical background from the very beginning. As we shall see shortly, topics like uncertanty principle, the role of probability etc asks immediately for a theoretical framework/formalism... Phase space is not able to capture, it does not offer the tools. Naturally leading from the very beginning to adopt .

2 LINEAR OPERATORS IN HILBERT SPACES

Operator formulation of standard non-relativistic quantum mechanics heavily relies on the theory of linear operators in Hilber spaces. In particular, the spectral theory of self-adjoint operators (bounded and unbounded ones) is a key ingredient in formulating the basic rules of quantum mechanics. This chapter is aimed at providing the necessary mathematical background of functional analysis employed by non-relativistic quantum mechanics. It is a chapter on mathematics, not on quantum physics. The Reader interested in how functional analysis is applied to formulate quantum mechanics should jump to the next chapters.

2.1 BANACH AND HILBER SPACES

Unless stated otherwise, let $\mathbb{K}$ denote equivalently the field of real numbers $\mathbb{R}$ or the field of complex numbers $\mathbb{C}$. (It is possible to develop the theory also for the skew-field of quaternions, but this case will not be discussed here to avoid dealing with the non-commutativity of quaternionic product.)

DEFINITION 2.1 (NORM): Let V be any linear space over $\mathbb{K}$. A "norm" on V is any application $V \to \mathbb{R}$ denoted by $\|\cdot\|$ satifying the following properties:

(a) $\|\varphi\| \geqslant 0, \quad \forall \varphi \in V$,

(b) $\|\varphi\| = 0$ if and only if $\varphi = 0$,

(c) $\|\alpha\varphi\| = |\alpha|\|\varphi\|$ for all $\alpha \in \mathbb{K}$ and $\varphi \in V$,

(d) $\|\varphi + \psi\| \leqslant \|\varphi\| + \|\psi\|$, for all φ and ψ in V (this is called "triangle inequality")

DEFINITION 2.2 (NORMED LINEAR SPACE): A "normed linear space" is a pair $(V, \|\cdot\|)$ where V is a linear space and $\|\cdot\|$ is any norm on V.

THEOREM 2.1: Let $(V, \|\cdot\|)$ be any normed linear space. Let $d\colon V \times V \to \mathbb{R}$ be the function defined by

$$d(\varphi, \psi) = \|\varphi - \psi\| \,. \tag{2.1}$$

Then, (V, d) is a metric space. The metric in eq. (2.1) is called the "metric induced by the norm" on V.

Proof. We need to prove that eq. (2.1) defines a metric over V, i. e., we need to show that d satisfies:

(a) $d(\varphi, \psi) \geqslant 0, \quad \forall (\varphi, \psi) \in V \times V$,

(b) $d(\varphi, \psi) = 0$ if and only if $\varphi = \psi$,

(c) $d(\varphi, \psi) = d(\psi, \varphi), \quad \forall (\varphi, \psi) \in V \times V$,

(d) $d(\varphi, \psi) \leqslant d(\psi, \eta) + d(\eta, \psi), \quad \forall (\varphi, \psi, \eta) \in V \times V \times V$ (the "triangle inequality").

Item a follows from property a of the norm. Item b follows from property b of the norm, since $\|\varphi - \psi\| = 0$ if and only if $\varphi - \psi = 0$, i. e., if and only if $\varphi = \psi$. Item c follows from property c of the norm, since

$$d(\varphi, \psi) = \|\varphi - \psi\| = \|-(\psi - \varphi)\| = |-1|\|\psi - \varphi\| = \|\psi - \varphi\| = d(\psi, \varphi) \,.$$

Item d follows from property item d of the norm, since

$$d(\varphi,\psi) = \|\varphi-\psi\| = \|\varphi-\eta+\eta-\psi\| \leqslant \|\varphi-\eta\| + \|\eta-\psi\| = d(\varphi,\eta) + d(\eta,\psi)\,.$$

This completes the proof. ∎

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