Constellation Design for a Lunar Global Navigation Satellite System

Corso di Laurea Magistrale in Ingegneria Spaziale ed Astronautica
Facoltà di Ingegneria Civile e Industriale

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Introduction

GNSS = Satellite navigation system

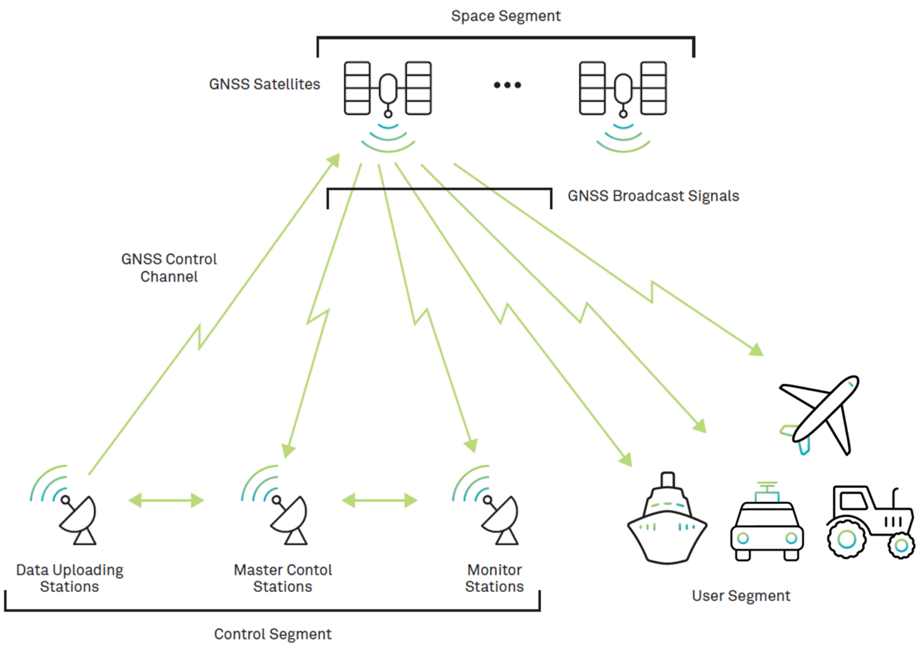
Location: determining the user's position

Navigation: identifying the best route

Tracking: monitoring an object's movement

Mapping: creating maps of a specific area

Timing: computing precise timing



Main objective

Design a lunar

GNSS constellation

Minimal constellation (low T, low P, low h)	Simultaneuous coverage of 4 satellites	Complete coverage of the surface
Mask angle of at least 5 deg	Good geometric properties	Orbital stability

Fundamentals

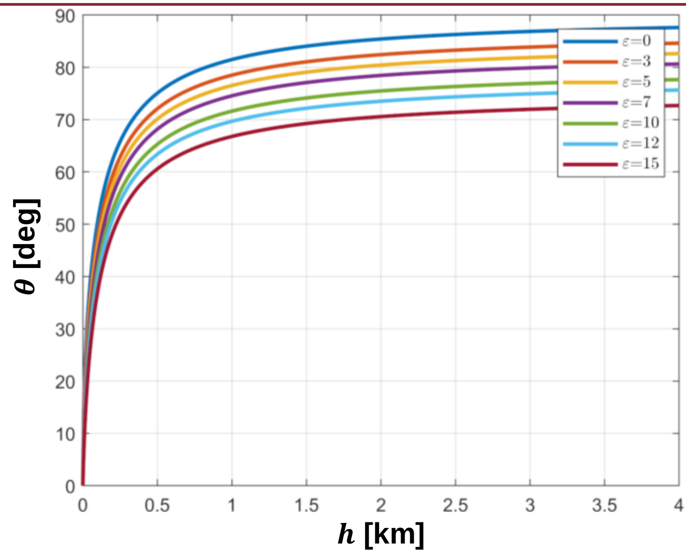
Navigation equations

Unknowns are: x, y, z, τ

$$PR_i = \sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_I - z)^2} + \tau$$

If the systems is 4-by-4, it can be solved for the user's position and clock offset.

→ 4-fold coverage required



Constellation coverage

$$\theta = \cos^{-1}\left(\frac{R_p}{R_p + h}\cos\varepsilon\right) - \varepsilon$$

for an **homogenous** and **circular** constellations:

$$a = a_i \qquad e = 0$$
$$i = i_i$$

→ coverage properties are fixed for all satellites and over time

$i_{max} \leq \frac{\pi}{2}$ to have prograde orbits

$i_{min} \geq \frac{\pi}{2} - \theta$ to reach the poles

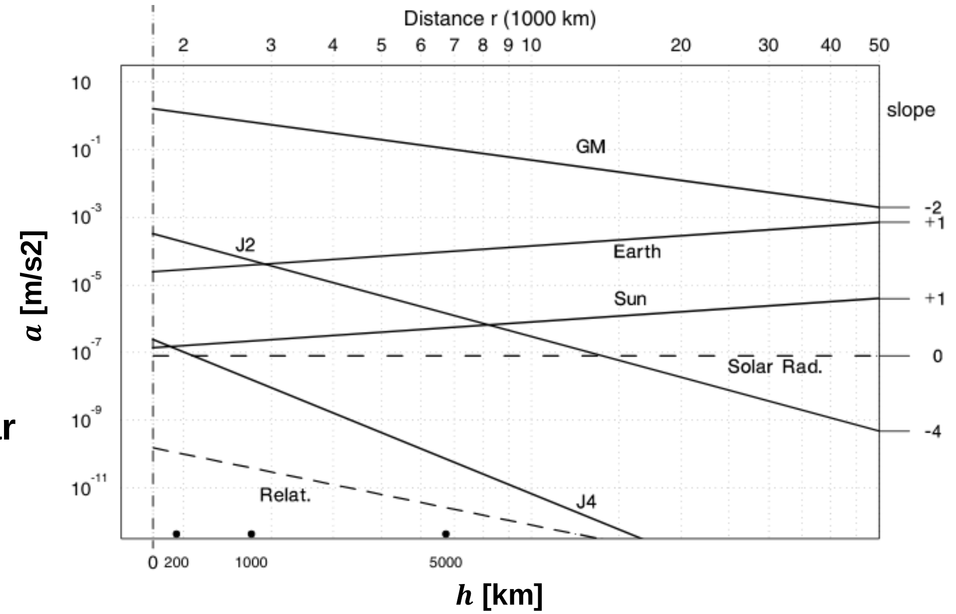
Orbital mechanics (1/2)

Perturbations

$$\vec{a}_{CE} = \mu_E \left(\frac{\vec{R}_{ME} - \vec{r}}{\left\| \vec{R}_{ME} - \vec{r} \right\|^3} - \frac{\vec{R}_{ME}}{R_{ME}^3} \right)$$

$$\vec{a}_{CS} = \mu_S \left(\frac{\vec{R}_{MS} - \vec{r}}{\left\| \vec{R}_{MS} - \vec{r} \right\|^3} - \frac{\vec{R}_{MS}}{R_{MS}^3} \right)$$

Terra-Solar potential

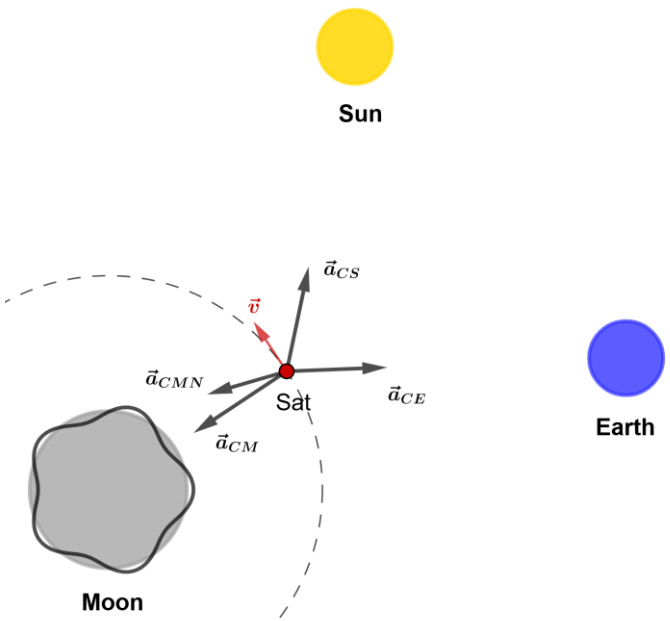


$$U = -\frac{\mu_M}{r} \sum_{n=2}^{\infty} \sum_{m=0}^{n} \left(\frac{R_M}{r} \right)^n \left[C_{nm} \cos(m\lambda) + S_{nm} \sin(m\lambda) \right] P_{nm(\sin\phi)}$$

$$\vec{a}_{CMN} = \nabla U$$

$$\vec{a}_{CM} = -\frac{\mu_M}{r^3} \vec{r}$$

Selenopotential



Dynamics equations ➡

$$\begin{cases} \dot{\vec{v}} = \vec{a}_{CM} + \vec{a}_{CMN} + \vec{a}_{CE} + \vec{a}_{CS} \\ \dot{\vec{r}} = \vec{v} \end{cases}$$

Orbital mechanics (2/2)

Repeating ground-track orbit

By considering only the J2 secular effects:

$$\dot{\Omega} - \frac{K_M}{a^{3.5}}(1-e^2)^{-2}\cos i$$

$$\dot{\omega} = \frac{K_M}{a^{3.5}}(1-e^2)^{-2}\left(2-\frac{5}{2}\sin i^2\right)$$

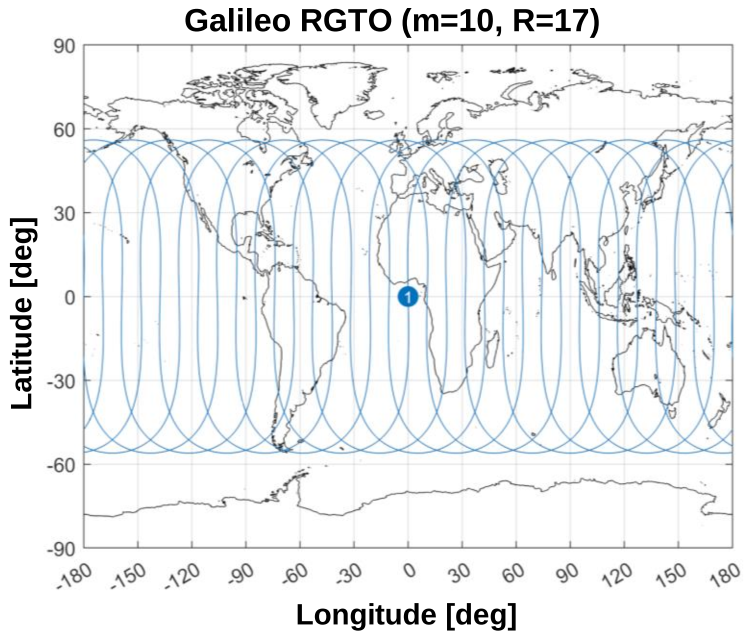
$$\dot{M} = \sqrt{\frac{\mu_M}{a^3}} + \frac{K_M}{a^{3.5}}(1-e^2)^{-1.5}\left(1-\frac{3}{2}\sin i^2\right)$$

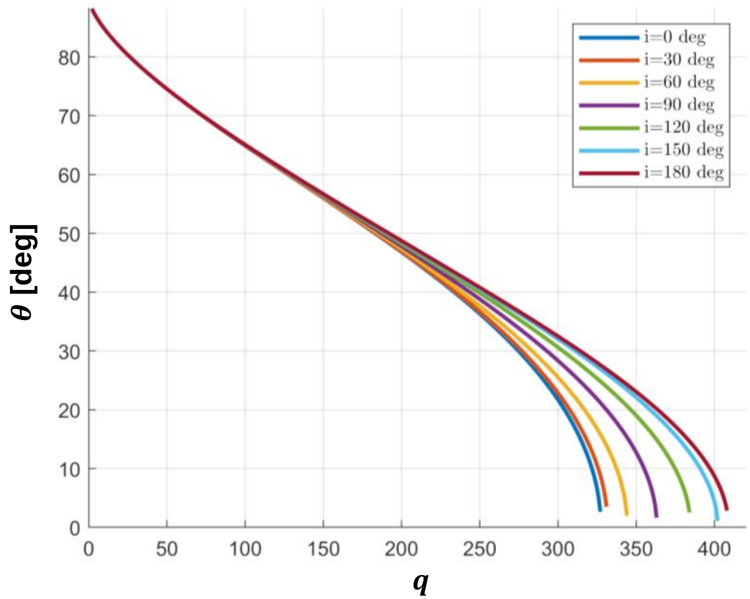
$$K_M = \frac{3}{2}J_{2M}R_M^2\sqrt{\mu_M}$$

$\boxed{mD_n = RT_n}$ conditions to have a RGTO

$$\boxed{a^{3.5} + b_1 a^2 + b_2 = 0}$$

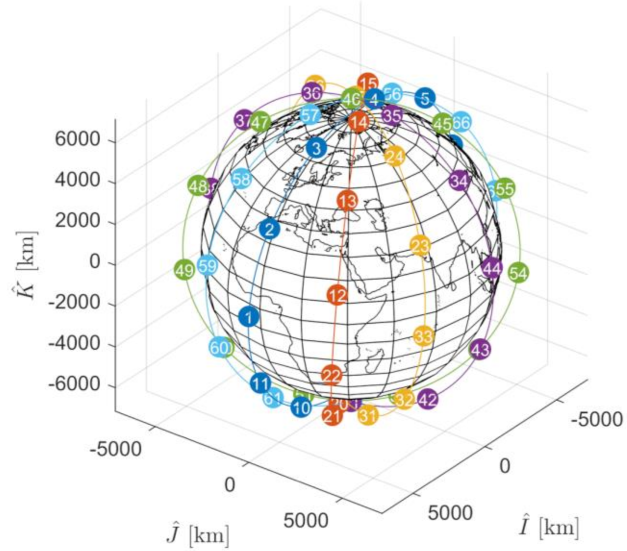
Only constellations based on RGTO with $\mathbf{m = 1}$ are considered, meaning that $\mathbf{q} = \boldsymbol{R}$





Constellation geometry (1/4)

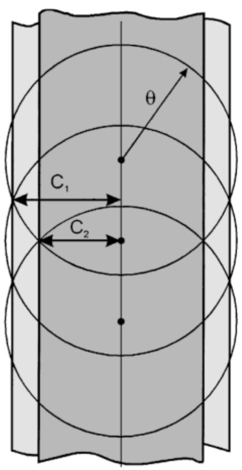
Streets-of-Coverage constellations



Uniform distribution of satellites on planes → **Non-symmetric** constellation

Handpicked distribution of orbital planes

SoC constellations are defined by 6 paramenters: $T, P, h, i, \Delta\Omega, \Delta M_{inter}$



$$c_j = \cos^{-1}\left(\frac{\cos\theta}{\cos(j\pi/N_p)}\right)$$

$$\Delta\Omega_{anti} = 2\pi\left\lfloor\frac{k}{2}\right\rfloor + 2(-1)^{k+1}\sin^{-1}\left(\frac{\frac{\sin[\pi[1-(k+1)\mathrm{mod}\ 2]-c_1-c_j]}{2(-1)^{k+1}}}{\sin i}\right)$$

$$\Delta\Omega_{co} = 2\sin^{-1}\left(\frac{\sin[(\theta+c_j)/2]}{\sin i}\right)$$

$$\Delta M_{inter} = \frac{j\,\pi}{N_p} - 2\cos^{-1}\left(\frac{\cos(\Delta\Omega_{co}/2)}{\cos[(\theta+c_j)/2]}\right)$$

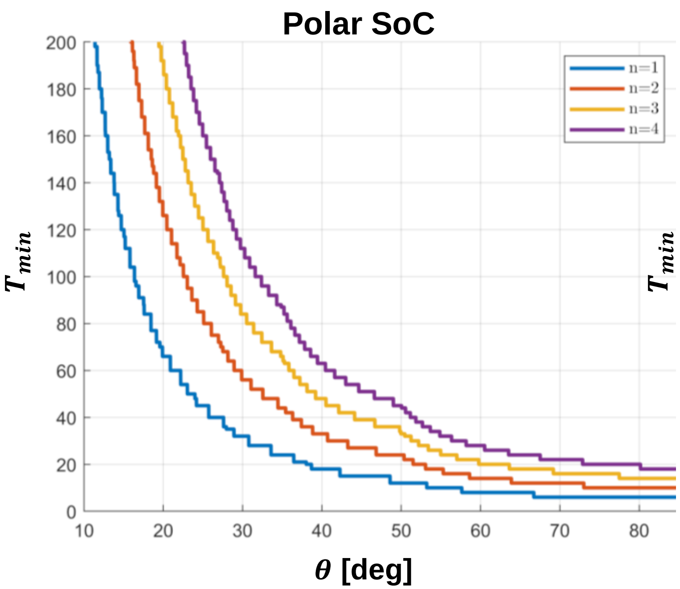
$n = jk$	$j = 1$	$j = 2$	$j = 3$	$j = 4$
$k = 1$	1	2	3	4
$k = 2$	2	4	6	8
$k = 3$	3	6	9	12
$k = 4$	4	8	12	16

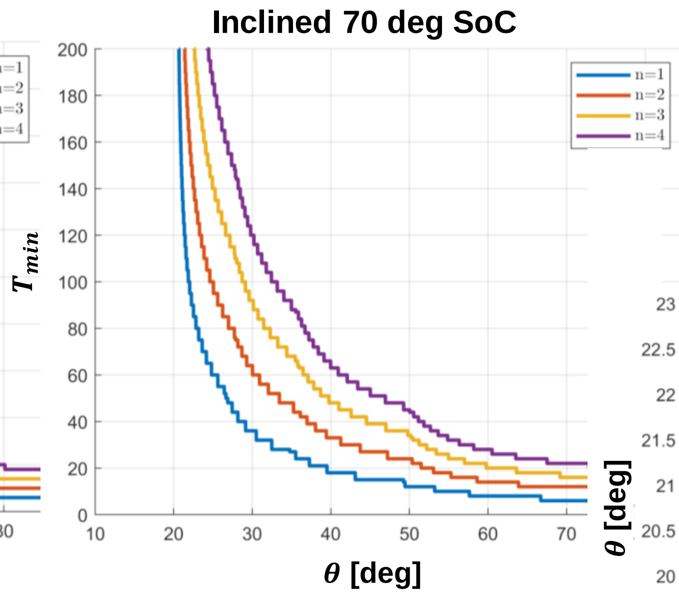
Constellation geometry (2/4)

Streets-of-Coverage constellations

coverage obtained analytically

Input: T, P, i → $\boxed{(P-1)\Delta\Omega_{co} - \Delta\Omega_{anti} = 0}$ → Output: $\theta, \Delta\Omega, \Delta M_{inter}$

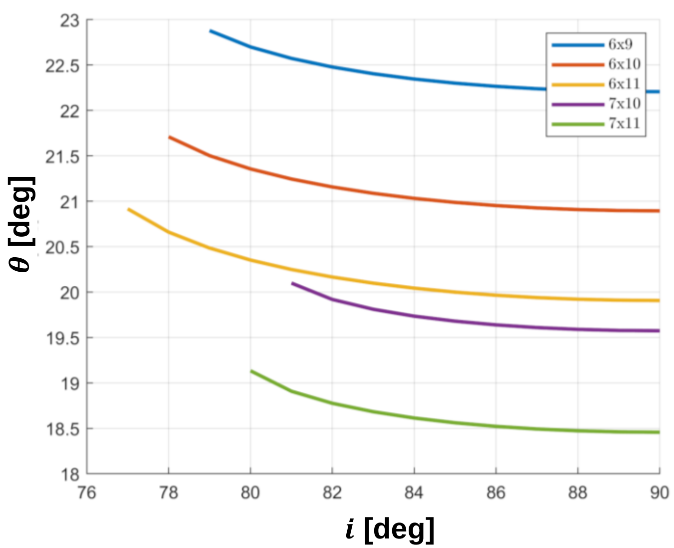




Minimum number of satellites: **Polar SoCs are more efficient**

Lowest altitude:

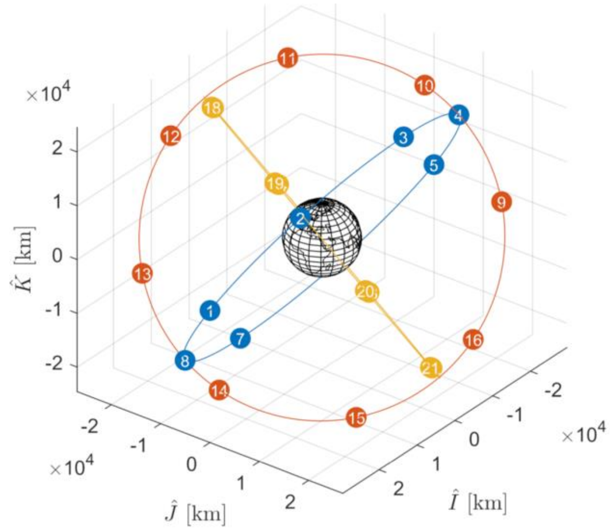
Polar SoCs are more efficient



Constellation geometry (3/4)

Waker-Delta constellations

Galileo constellation (WD 24/3/1)



Uniform distribution of satellites on planes

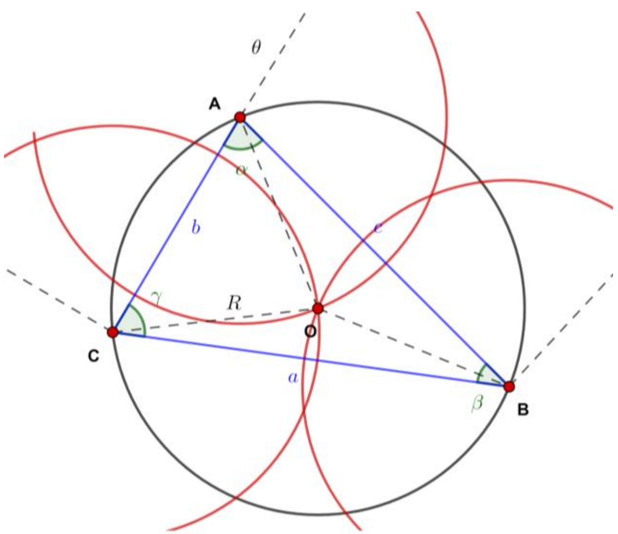
Uniform distribution of orbital planes

Symmetric constellation

Walker-Delta constellations are defined by 5 paramenters: T, P, F, h, i

$$Z(T) = \sum_{P \in D(T)} P$$

number of possible configurations given T satellites



$$R_{\Delta ABC} = \min\left\{ \max\{R_A, R_B, R_C\} \right\}$$

Furthest point from any vertex

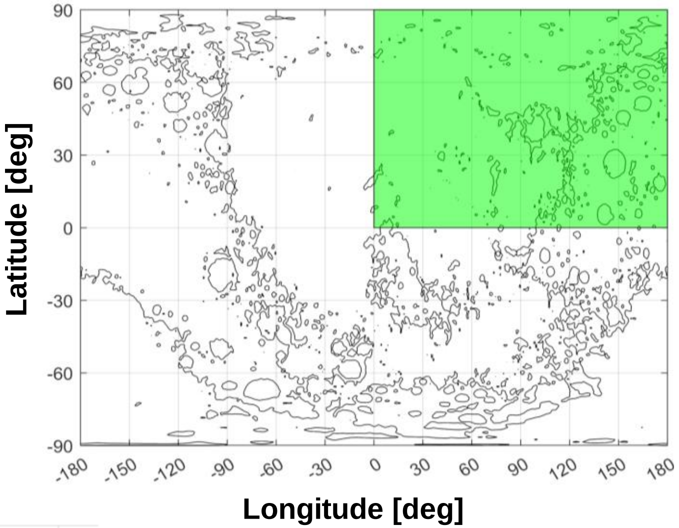
$$R_{Max} = \max_{t,j}\{R_{\Delta ABC}(t,j)\}$$

$$\theta = R_{Max}$$

R_{Max} is the minimum value that guarantee n-fold coverage

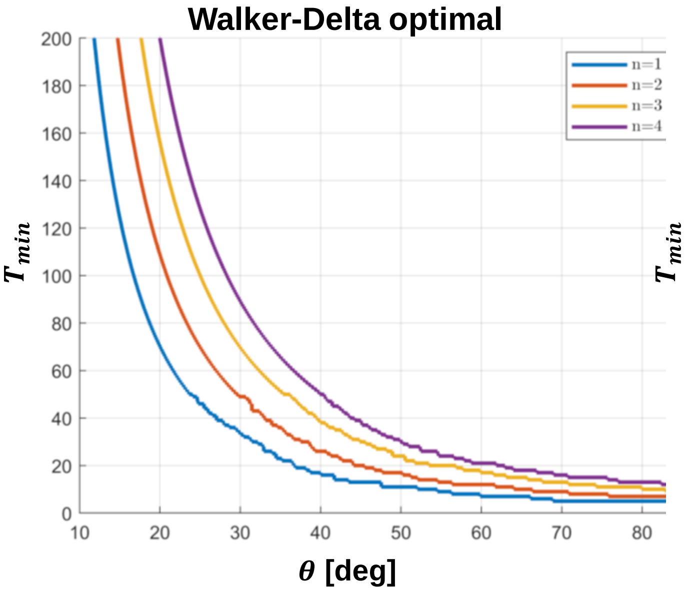
Constellation geometry (4/4)

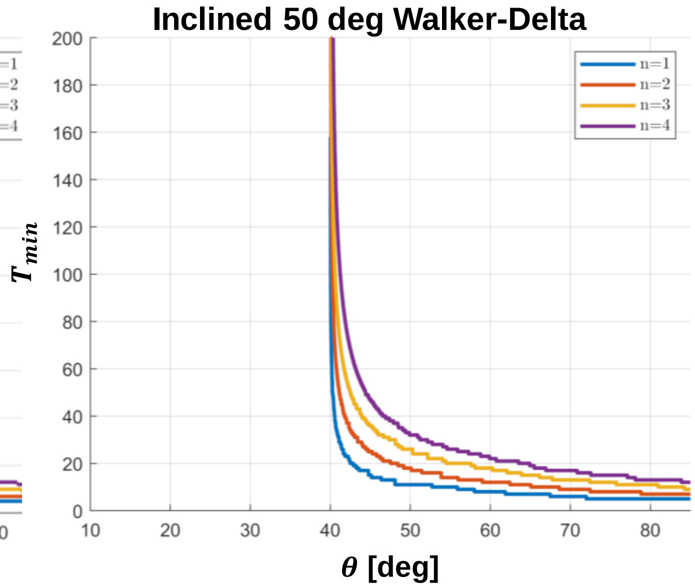
Walker-Delta constellations



coverage obtained through <u>numerical simulation</u>

Input: T, P, F, i → $\boxed{\theta = R_{Max}}$ → Output: θ





Simulation can be limited to a **quarter** of the surface due to **symmetry**

Optimal Walker-Delta constellations are found **with** $\boldsymbol{i = 45° \div 60°}$ →

$$T_{min} = \frac{4n}{[\theta(1 - p^{-nk})]^2} \qquad p = 0.26 \qquad k = 0.08$$

Efficiency parameters

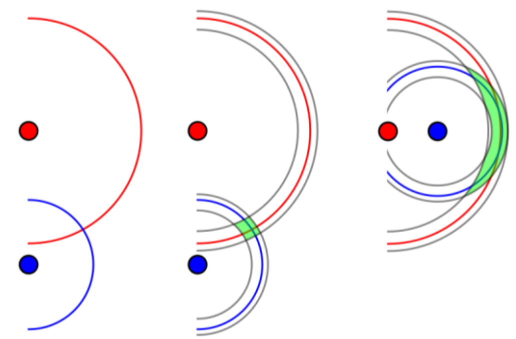
Excess coverage

GNSS	cov	GDOP
GPS	2.10	2.11
Galileo	2.03	2.23

Evaluates the total coverage available as a percentage of the total coverage required

$$cov = \frac{T\,(1 - \cos\theta)}{2n}$$

IDEAL VALUE = 1



Dilution of precision

By linearization of the navigation eqs:

$$\partial PR = A\,\partial X$$

Relation between uncertainties on pseudo-ranges and uncertainties on user's unknowns

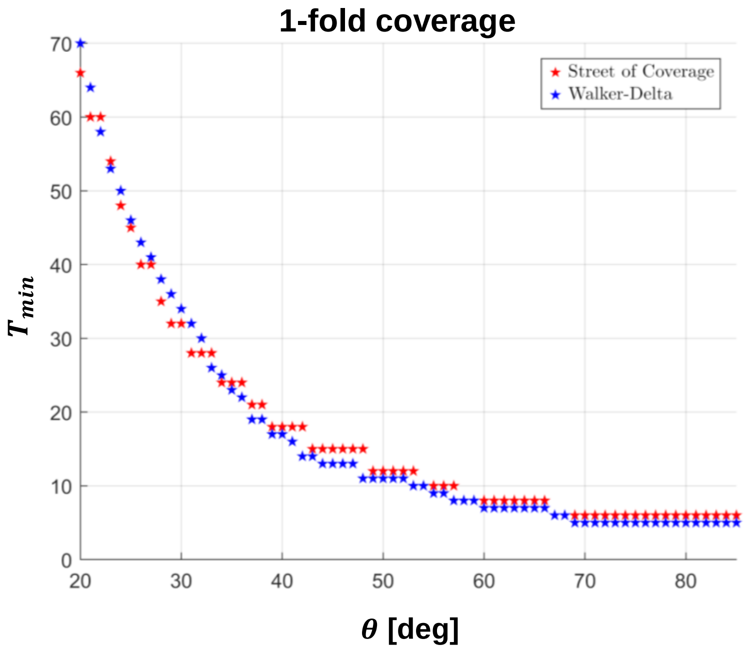
$$Q = (A^T A)^{-1} = \begin{bmatrix} \sigma_x^2 & \sigma_{xy} & \sigma_{xz} & \sigma_{x\tau} \\ \sigma_{yx} & \sigma_y^2 & \sigma_{yz} & \sigma_{y\tau} \\ \sigma_{zx} & \sigma_{zy} & \sigma_z^2 & \sigma_{z\tau} \\ \sigma_{\tau x} & \sigma_{\tau y} & \sigma_{tz} & \sigma_\tau^2 \end{bmatrix} \Rightarrow$$

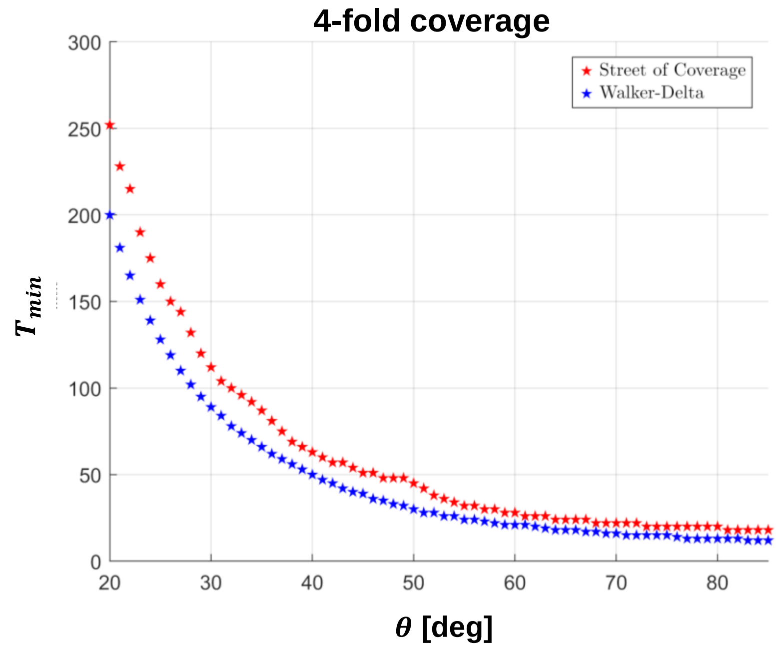
$$GDOP = tr(Q)$$
$$PDOP = tr(Q_{3x3})$$
$$TDOP = \sigma_\tau$$

Value	Interpretation
>20	Poor
10-20	Fair
5-10	Moderate
2-5	Good
1-2	Excellent
<1	Ideal

Results (1/2)

Walker-Delta vs Streets-of-Coverage



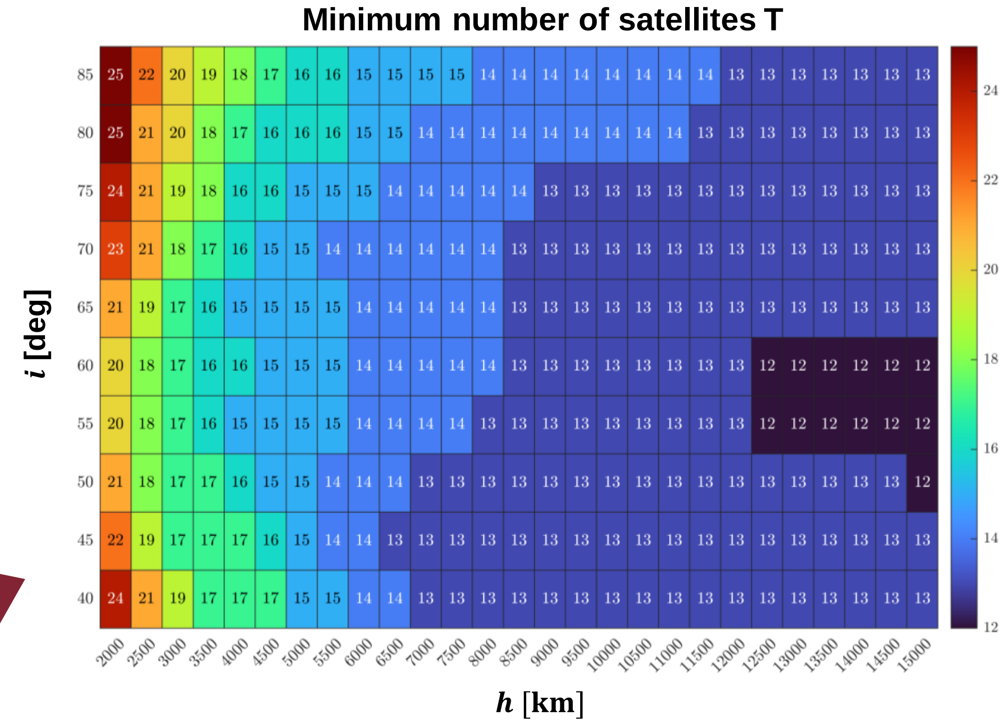


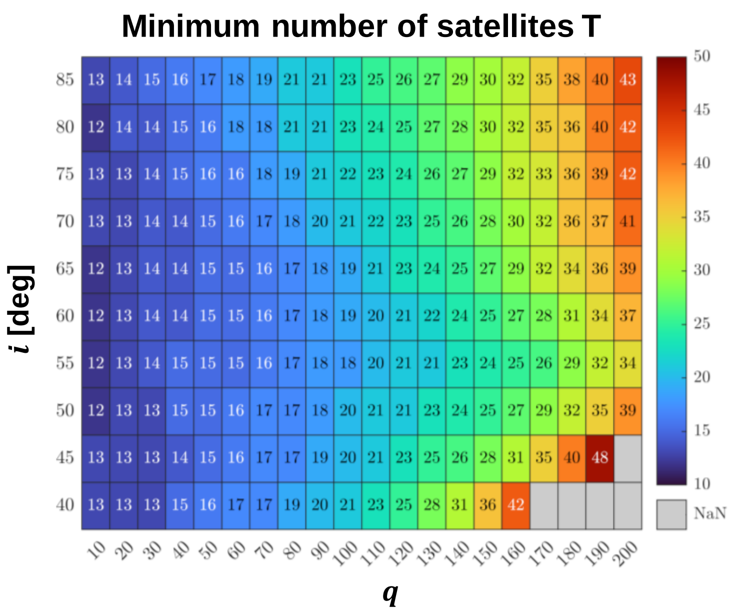
Class	1-fold $(T < 20)$	1-fold $(T \geq 20)$	2-fold	3-fold	4-fold
Walker-Delta	1st	2nd	1st	1st	1st
SoC	2nd	1st	2nd	2nd	2nd

Results (2/2)

Moon Walker-Delta tables

Tables that display the minimal constellations as a function of inclination and repeat factor (bottom) or altitude (right)

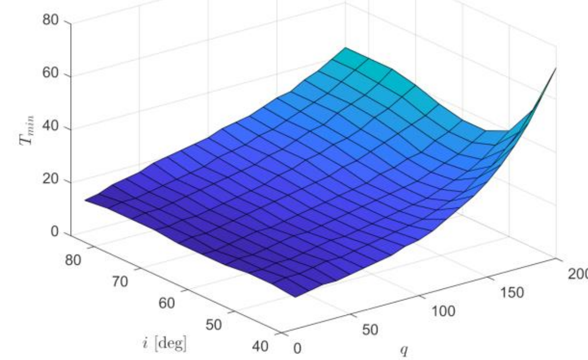




A fitting equation is useful to find more configurations

$$T_{min} = f(q, i)$$

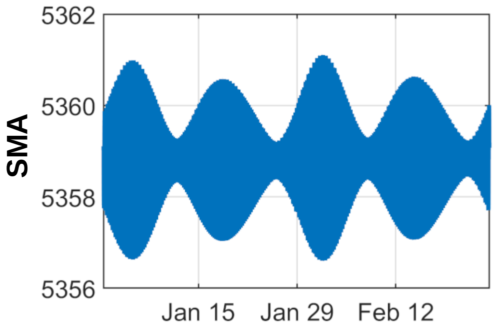
Polynomial surface fit55
($R^2 = 0.994$)

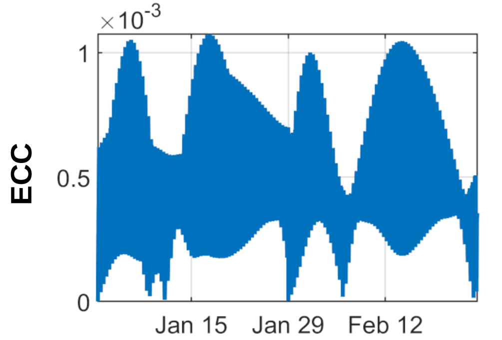


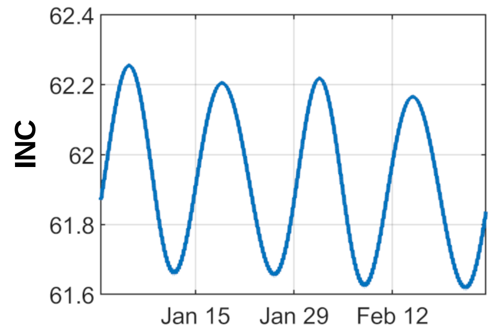
Lunar GNSS proposition (1/2)

ID	h [km]	i [deg]	T	P	F	R	θ [deg]	cov	GDOP	# sats
12	6529.00	56.26	15	5	1	35	72.91	1.32	4.51	4.26
16	3621.71	61.87	18	6	2	67	66.16	1.34	2.60	4.53
17	4302.94	51.65	18	6	2	56	68.35	1.42	2.43	4.83
26	3517.77	65.02	20	5	1	69	65.77	1.47	2.65	4.62

ID	Δa [km]	Δe	Δi [deg]	ΔV [m/s]	ΔV_{year} [m/s]
12	7.5861	0.0024	0.0040	4.47	58.16
16	1.2847	0.0005	0.0017	2.96	38.46
17	1.9226	0.0009	0.0027	15.87	206.35
26	1.1426	0.0006	0.0012	1.75	22.74







1 stationkeeping manuever

each repeat cycle (27.32 days)

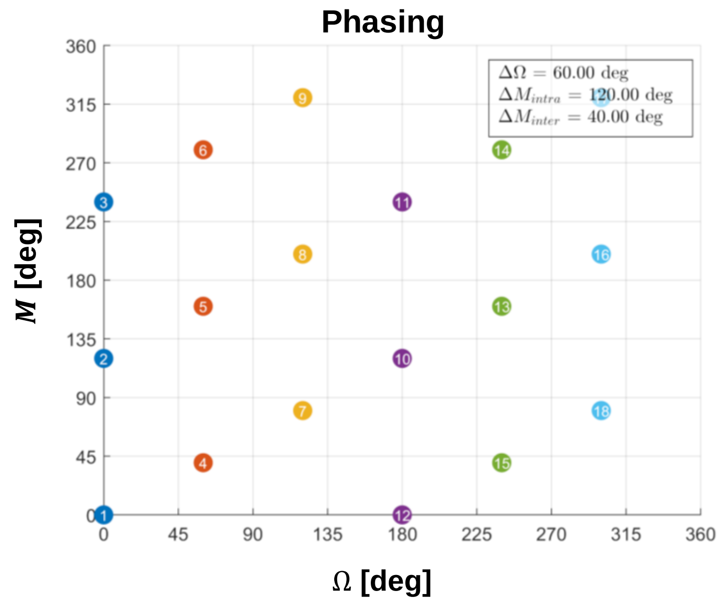
Kozai-Lidov mechanism:

It is easier to correct for e

$$C = \sqrt{1 - e^2} \cos i = const.$$

Lunar GNSS proposition (2/2)

Failure of a single satellite



Robustness: defined as the constellation performance to offer normal services, should one satellite fail.

Config	4-fold %	cov	GDOP	PDOP	TDOP	HDOP	VDOP	# sats
Mean	48.57	1.27	3.97	3.54	1.78	1.65	3.07	3.57
Worst	46.55	1.27	4.29	3.83	1.91	1.80	3.36	3.54
Best	52.71	1.27	3.76	3.35	1.67	1.57	2.90	3.61

SOLUTION 1

5-fold coverage,

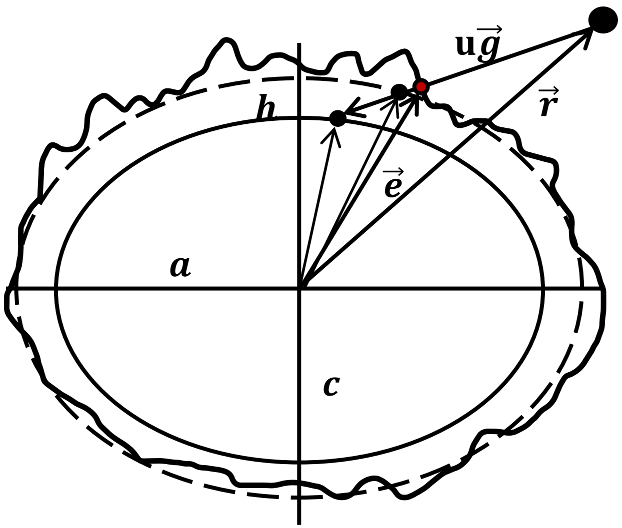
but requires to start over analysis

SOLUTION 2

spare in-orbit satellite,

but phasing manuevers need to be considered

LOS algorithm



Originally for **observation missions**

It obtains the intersection with an ellipsoid

$$A = (c+h)^2\left(g_x^2 + g_y^2\right) + (a+h)^2 g_z^2$$

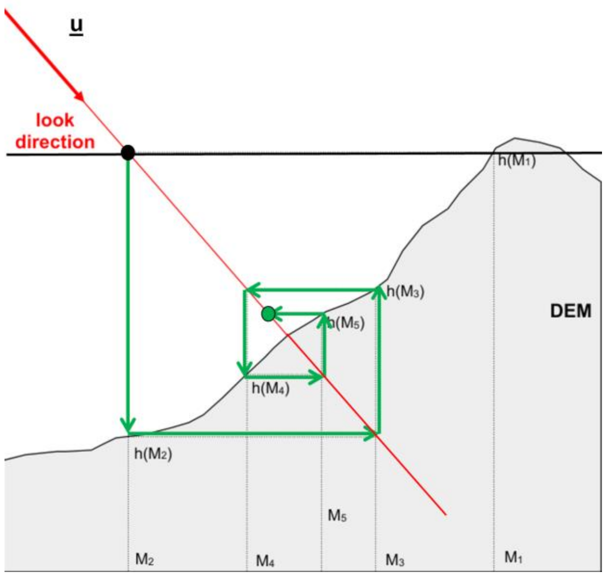
$$B = 2[(c+h)^2\left(r_x g_x + r_y g_y\right) + (a+h)^2 r_z g_z$$

$$C = (c+h)^2\left(r_x^2 + r_y^2\right) + (a+h)^2\left(r_z^2 - (c+h)^2\right)$$

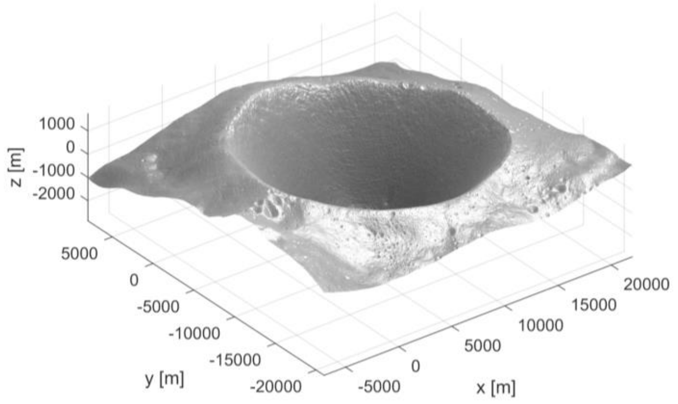
$$u = \frac{-B - \sqrt{B^2 - 4AC}}{2A}$$

$$\vec{e} = \vec{r} + u\vec{g}$$

If implemented along the **DEM**, it allows us to determine the <u>intersection</u> with the lunar surface

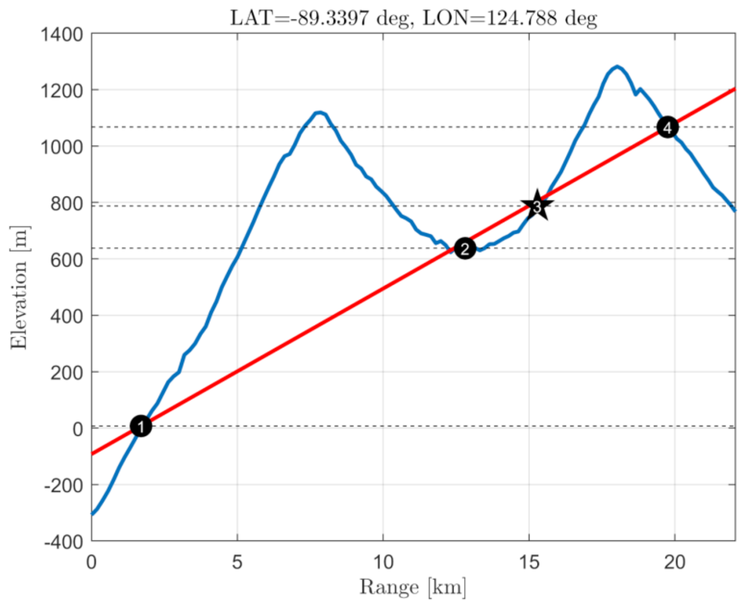


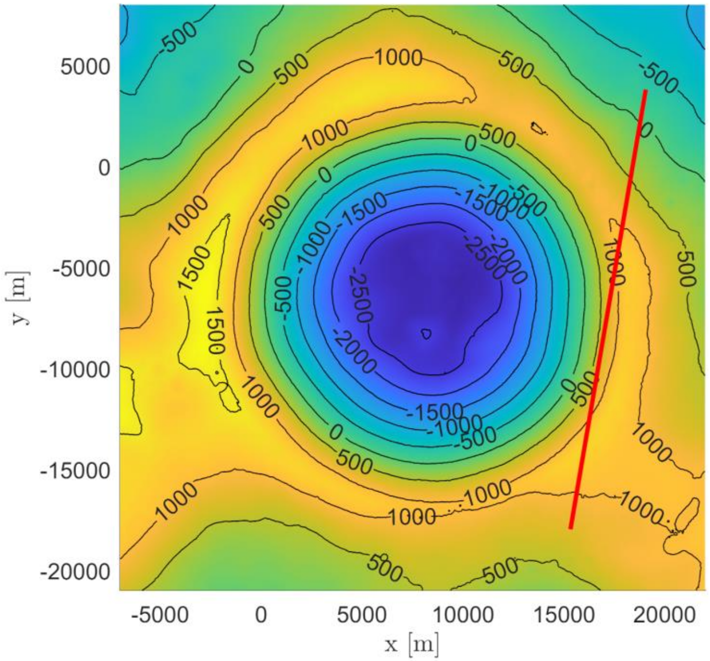
Case: Shackleton crater (1/2)



Shackleton crater: interesting crater for future human missions, located near the South Pole

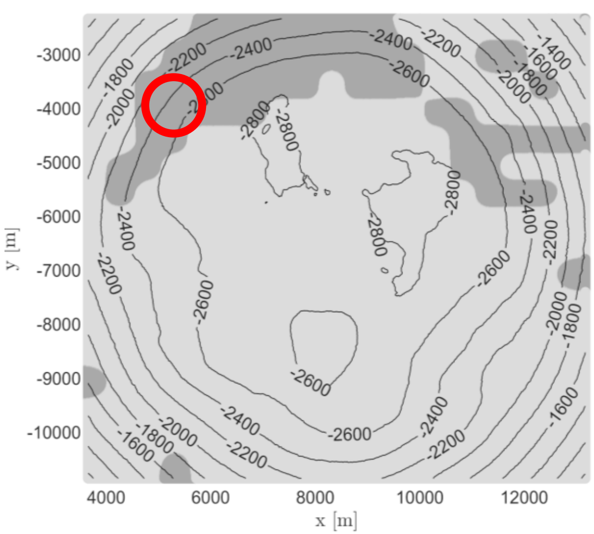
Intersections with the DEM → only the highest one is visible from the satellite





Case: Shackleton crater (1/2)

Grey areas determination



Grey areas: critical areas where a satellite is lost due to topography of the surface.

Presence of grey areas on the northern edge of the crater basin, only at times.

Grey area	Max Time [min]	Acc. Time %
Worst	10	2.59
Mean	4.29	0.27

Average coverage (over the entire repeat cycle):

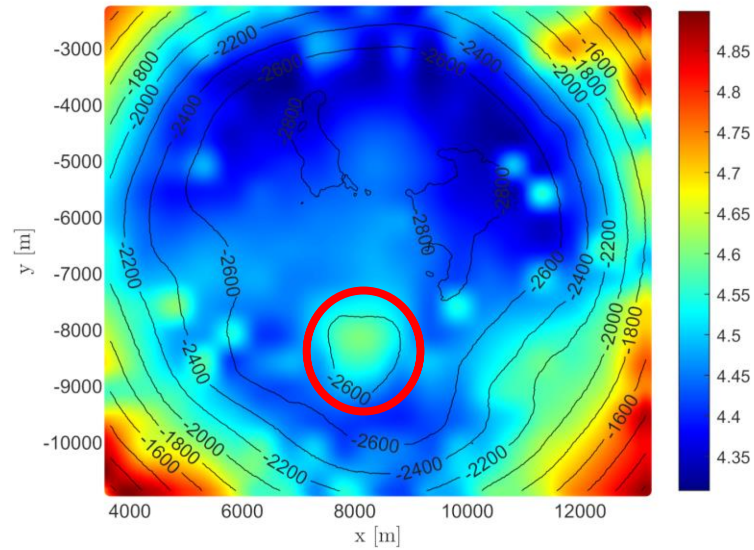
IDEAL COVERAGE

6.27

DETAILED COVERAGE

4.49

Area on the southern edge of the crater basin demonstrates an higher number of visible satellites on average: 4.6

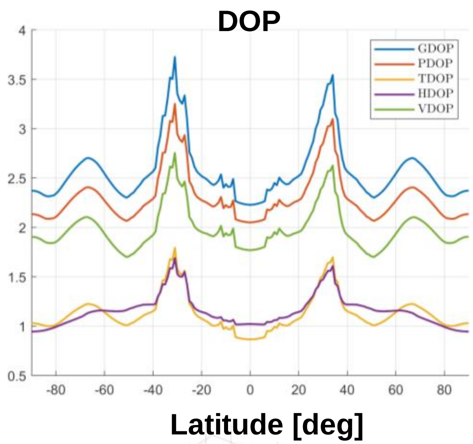


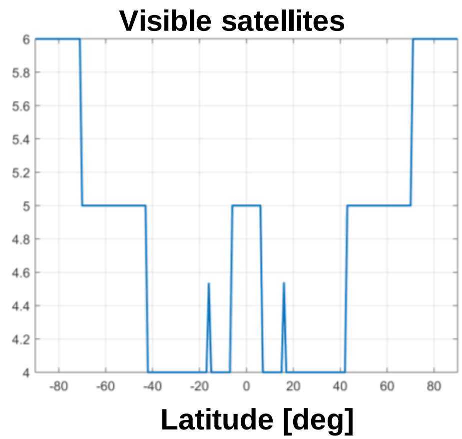
Conclusions

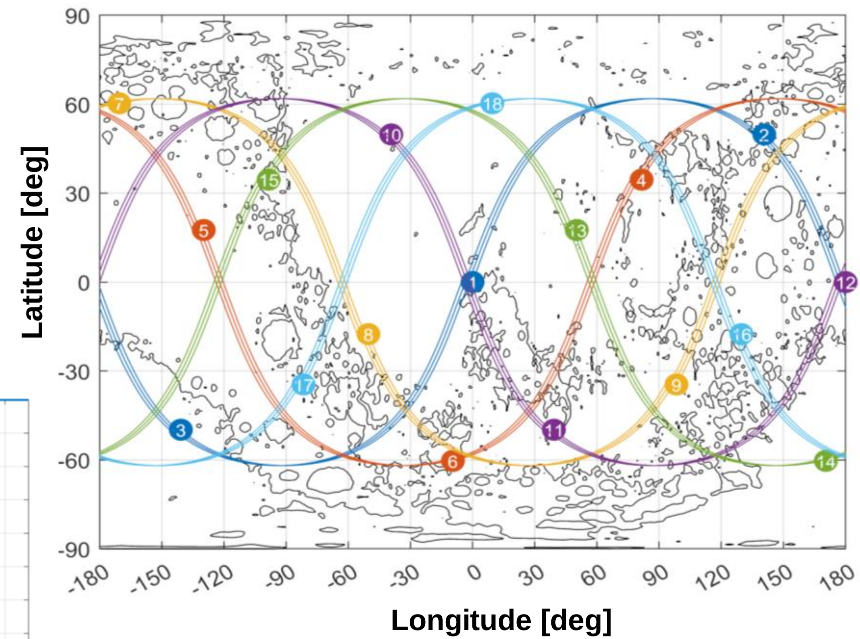
Walker-Delta 18/6/2 constellation

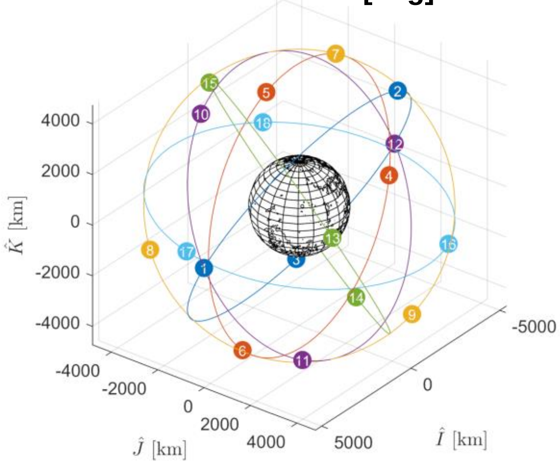
with $h = 3621.71$ km

and $\mathrm{i} = 61.87$ deg









Minimal constellation

4-fold global and continuous coverage

Mask angle of 5 deg

Good geometry

Relatively stable

18 satellites (suboptimal)

100% ideal, small gray areas on Shackleton crater

Elevation angle ≥ 5 deg

GDOP = 2.60

~ 13 stationkeeping manuevers per year