

Homework 1

Algebraic Topology 2024-2025

Due 26 February, 2025

EXERCISE 1

Let A be a topological space and $X, Y \subseteq A$ be subspaces such that $A = X \cup Y$; assume that X and Y are either both open or both closed in A . Let B be a topological space. Let

$$f_X: X \rightarrow B \text{ and } f_Y: Y \rightarrow B$$

be continuous maps that coincide on the intersection $X \cap Y$. Show that there exists a unique continuous map

$$f: A \rightarrow B$$

which restricts to f_X on X and to f_Y on Y .

EXERCISE 2

Let X be a topological space. Show that the following conditions are equivalent:

- There exists a point $x \in X$ such that X deformation retracts to x ;
- For any point $x \in X$, X deformation retracts to x ;
- X is homotopy equivalent to the one-point topological space;

In this case, we will say that X is contractible. Show that any convex space is contractible. Show that if X is contractible, for any space Y , $X \times Y$ is homotopy equivalent to Y .