Politecnico di Milano

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Scuola di Ingegneria Industriale e dell'Informazione

The Cauchy-Kowalevski Theorem and Its Consequences

Thesis by

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All his life – he had difficulty saying this, as he admitted, being always wary of too much enthusiasm – all his life he had been waiting for such a student to come into this room.
A student who would challenge him completely, who was not only capable of following the strivings of his own mind but perhaps of flying beyond them.

— Alice Munro, *Too Much Happiness*

Abstract

In 1874, Sofya Kowalevski, the first woman to obtain a doctorate in mathematics in Europe, brought to light the proof of the Cauchy-Kowalevski theorem (CKT), the first general result for the existence of local analytic solutions to partial differential equations (PDEs) with Cauchy data.

The thesis aims to present this milestone of mathematics, highlighting the depth of detail, consequences, and the simplicity of the ideas it brought to light. To this end, fundamental notions and results are frequently recalled to address the discussion, and all the main forms in which the CKT can be stated are treated.

Additionally, there is a section dedicated to three historically crucial examples for understanding PDEs and another dedicated to its two fundamental applications: the Holmgren theorem and the Cartan-Kähler theorem.

Keywords: PDEs, characteristics, analyticity/holomorphy, power series, majorants method, Cauchy-Kowalevski, Holmgren, and Cartan-Kähler theorems

Contents

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