Politecnico di Milano

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Scuola di Ingegneria Industriale e dell'Informazione

The Cauchy-Kowalevski Theorem and Its Consequences

Thesis by

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All his life – he had difficulty saying this, as he admitted, being always wary of too much enthusiasm – all his life he had been waiting for such a student to come into this room.
A student who would challenge him completely, who was not only capable of following the strivings of his own mind but perhaps of flying beyond them.

— Alice Munro, *Too Much Happiness*

Abstract

In 1874, Sofya Kowalevski, the first woman to obtain a doctorate in mathematics in Europe, brought to light the proof of the Cauchy-Kowalevski theorem (CKT), the first general result for the existence of local analytic solutions to partial differential equations (PDEs) with Cauchy data.

The thesis aims to present this milestone of mathematics, highlighting the depth of detail, consequences, and the simplicity of the ideas it brought to light. To this end, fundamental notions and results are frequently recalled to address the discussion, and all the main forms in which the CKT can be stated are treated.

Additionally, there is a section dedicated to three historically crucial examples for understanding PDEs and another dedicated to its two fundamental applications: the Holmgren theorem and the Cartan-Kähler theorem.

Keywords: PDEs, characteristics, analyticity/holomorphy, power series, majorants method, Cauchy-Kowalevski, Holmgren, and Cartan-Kähler theorems

Contents

Bibliography

[CE99]   Pierre A. Chiappori and Ivar Ekeland. Aggregation and market demand: an exterior differential calculus viewpoint. *Econometrica*, 67:1435–1458, 1999.

[CE06]   Pierre A. Chiappori and Ivar Ekeland. The micro economics of group behavior: General characterization. *Journal of Economic Theory*, 130:1–26, 2006.

[CE09a]   Pierre A. Chiappori and Ivar Ekeland. The economics and mathematics of aggregation: Formal models of efficient group behavior. *Foundations and Trends® in Microeconomics*, 5:1–2, 2009.

[CE09b]   Pierre A. Chiappori and Ivar Ekeland. The Microeconomics of Efficient Group Behavior: Identification. *Econometrica*, 77:763 – 799, 2009.

[Eke]   Ivar Ekeland. Some applications of the Cartan-Kähler theorem to economic theory.

[Eva10]   Lawrence C. Evans. *Partial Differential Equations*. American Mathematical Society, 2010.

[FMS20]   Nicola Fusco, Paolo Marcellini, and Carlo Sbordone. *Lectures on Mathematical Analysis 2* (in Italian). Zanichelli, 2020.

[Fol95]   Gerald B. Folland. *Introduction to Partial Differential Equations*. Princeton University Press, 1995.

[Hö63]   Lars Hörmander. *Linear Partial Differential Operators*. Spinger-Verlag, 1963.

[Joh82]   Fritz John. *Partial Differential Equations*. Springer-Verlag, 1982.

[Ken83]   Don H. Kennedy. *Little Sparrow: A Portrait of Sophia Kovalevsky*. Ohio University Press, 1983.

[Luc24]   Sandra Lucente. Cauchy-Kovalevskaja Theorem for Differential Equations (in Italian). *Le Scienze*, 2024. Collona *Rivoluzioni matematiche*.

[Ovs65]   L.V. Ovsyannikov. *Singular operators in Banach spaces scales* (in Russian). Doklady Acad. Nauk., 1965. p. 819–822.

[RF10]   Halsey L. Royden and Patrick M. Fitzpatrick. *Real Analysis*. Pearson, 2010.

[Rou80]   Delfina Roux. *Foundations of Advanced Analysis - PART I* (in Italian). Edizioni la Viscontea, 1980.

[Tre70]   François Treves. On local solvability of partial differential equations. 1970.

[Tre75]   François Treves. *Basic Linear Partial Differntial Equations.* Amacademic Press, 1975.

[Tre22]   François Treves. *Analytic Partial Differential Equations.* Springer Nature, 2022.