

# The Value of Cooperation: From AIMD to Flipped Classroom Teaching

Chee Wei Tan

This paper is in honor of Professor Dah-Ming Chiu on the occasion of his 70th birthday

November 12, 2021

# Talk Outline

- ▶ The Value of Cooperation in AIMD Abstraction
- ▶ Teaching Examples via Convex Optimization Theory and Perron-Frobenius Theory
- ▶ Integrate Classical Performance Evaluation and Modern Data Science Techniques
- ▶ A Parallel between AIMD and Flipped Classroom Teaching
- ▶ Technology-enabled Pedagogy and Blended Learning
- ▶ Thanks to Dah-Ming Chiu, Y. C. Tay, Vittoria de Nitto Persone, R. Srikant, Geoffrey M. Voelker, Lorenzo Alvisi, Mung Chiang, Steven Low and Stephen Boyd

# Abstraction and its Importance to Teaching

- ▶ Performance analysis often depends on a mathematical model.
- ▶ Elegant performance models such as the queuing models and stochastic models are good approximations of real systems
- ▶ Performance model choice often a trade-off between mathematical tractability and real-world relevance.
- ▶ Finding an appropriate model becomes even more crucial.
- ▶ Right “abstraction” helps to focus on most interesting and crucial aspect of a problem, and opens the door to a variety of performance analysis techniques.
- ▶ Allows one abstraction to relate to other abstractions, and to explore the trade-off between tractability and approximation.

## AIMD Abstraction

- ▶ The Additive-Increase-Multiplicative-Decrease (AIMD) algorithm abstraction proposed by Dah-Ming Chiu and Raj Jain<sup>1</sup> is influential in design of Transmission Control Protocol (TCP) protocol for Internet congestion control.
- ▶ In the early days of the Internet, many different models to study congestion control, some from the viewpoint of a single flow while others assume synchronous events or fluid flows.
- ▶ AIMD abstracts the congestion control problem as one with social consequences, whereby the parties encountering congestion work cooperatively to arrive at a good solution.

---

<sup>1</sup>Dah-Ming Chiu and Raj Jain. Analysis of the increase and decrease algorithms for congestion avoidance in computer networks. Computer Networks and ISDN Systems, 17(1):1–14, 1989.

## Illustrating an Abstraction

- ▶ Ingenuity behind the AIMD abstraction can be appreciated through a classical illustration of two flows (Figure 1)
- ▶ Usher in the development of robust TCP algorithms <sup>2</sup>
- ▶ Classical illustration found in a number of popular computer networking textbooks <sup>3 4</sup>
- ▶ Insightful illustration to accompany the right abstraction to a complex problem can be helpful to teaching
- ▶ Usher in new performance analysis techniques like convex optimization theory and Perron-Frobenius theory

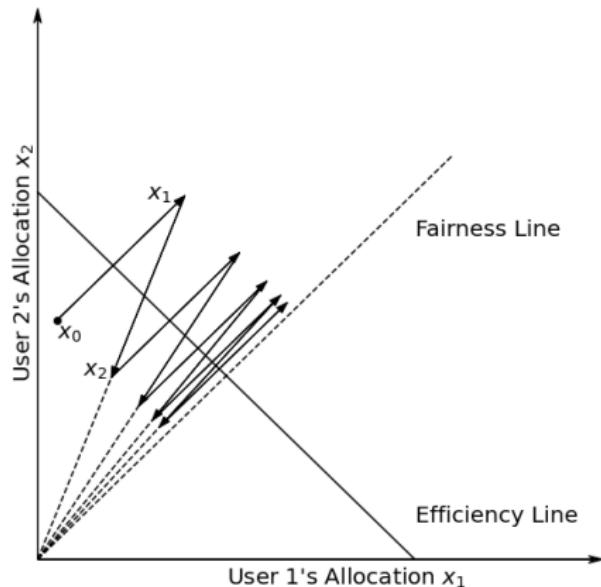
---

<sup>2</sup>Van Jacobson. Congestion avoidance and control. ACM SIGCOMM computer communication review, 18(4):314–329, 1988.

<sup>3</sup>Jim Kurose and Keith Ross. Computer Networking: A Top-Down Approach, 8th edition. Pearson, 2020.

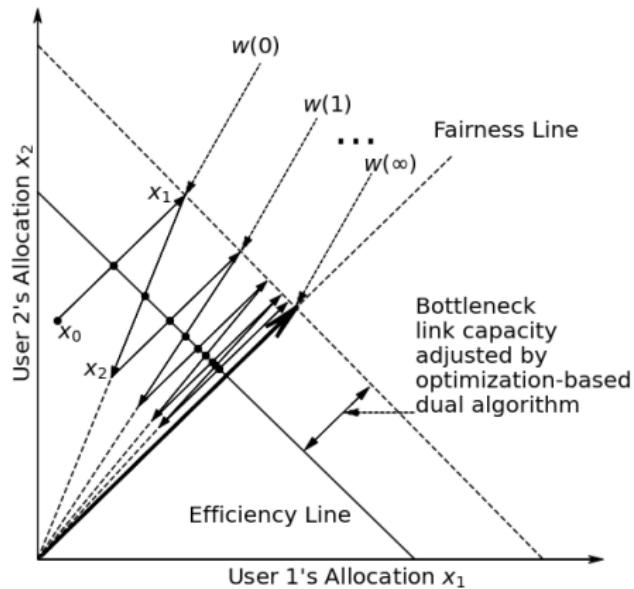
<sup>4</sup>Andrew Tanenbaum and David J. Wetherall. Computer Networks, 5th edition. Prentice Hall, 2010.

# Classical Visual Proof of AIMD



A visual proof of the AIMD abstraction

# Visualizing the Theories Behind AIMD



Teaching convex optimization theory and Perron-Frobenius Theory.

# An Optimization Theoretic Perspective of AIMD

- ▶ A theoretical framework called Network Utility Maximization (NUM) to analyze both the equilibrium and the dynamical nature of TCP algorithms<sup>5</sup> <sup>6</sup>
- ▶ Elegant framework brings tools and ideas from convex optimization theory to bear on the design of internet congestion control algorithms
- ▶ By leveraging Lagrange duality and gradient descent algorithms, TCP can be interpreted as a dual algorithm that maximizes the aggregate utilities in the network
- ▶ AIMD viewed as solving a network utility maximization where the sending source rates and network congestion measures are interpreted as primal variables and dual variables respectively

---

<sup>5</sup>Rayadurgam Srikant. The mathematics of Internet congestion control. Springer Science & Business Media, 2004.

<sup>6</sup>Steven H Low. Analytical methods for network congestion control. Synthesis Lectures on Communication Networks, 10(1):1–213, 2017. ▶ 🔍

# Network Utility Maximization(NUM)

- ▶ The basic network utility maximization of  $n$  sending sources can be formulated as follows:

$$\text{maximize} \sum_{s=1}^n U_s(x_s) \text{ subject to } Rx \preceq c, x \succeq 0,$$

where  $x_s$  is the sending rate of source  $s$  given a routing matrix  $R$  with entries  $R_{ls} = 1$  if source  $s$  uses link  $l$  with a link capacity  $c_l$  or 0 otherwise. At the  $t$ th iteration, source  $s$  solves:

$$x_s^*(q_s) = \text{argmax} \{ U_s(x_s) - q_s x_s \},$$

- ▶ The  $l$ th link runs the algorithm:

$$p_l(t+1) = \max \left\{ p_l(t) - \alpha(t) \left( c_l - \sum_{s: l \in L(s)} x_s^*(q_s(t)) \right), 0 \right\}.$$

- ▶ Using AIMD with an end-to-end marking probability  $q_s$  and a total delay  $D_s$ , TCP Reno has the utility function<sup>7 8</sup>:

$$\text{arctan utility : } U_s(x_s) = \frac{\sqrt{3/2}}{D_s} \arctan\left(\sqrt{2/3}x_s D_s\right)$$

- ▶ Mathematical basis for other fairness abstraction like proportional fairness in TCP Vegas and FAST TCP
- ▶ Other forms of TCP do not leverage Lagrange duality directly as they are instead interpreted as solutions to a penalty function formulation of the optimization problem

---

<sup>7</sup> Steven H Low. Analytical methods for network congestion control.

Synthesis Lectures on Communication Networks, 10(1):1–213, 2017.

<sup>8</sup> Rayadurgam Srikant. The mathematics of Internet congestion control.

Springer Science & Business Media, 2004.

# Teaching Optimization Theory

- ▶ Convex optimization theory important to networking applications<sup>9 10</sup>
- ▶ Teach students to optimize performance analysis and data science techniques, e.g., link capacity shaped by the abstraction of wireless network optimization<sup>11</sup>
- ▶ Guide students to design optimization algorithms, data analytics to reduce data-driven models into performance models with parameters to be optimized
- ▶ Teach students to appreciate how to connect abstractions across the network protocol layers (primal and dual decomposition in Lagrange duality theory)

---

<sup>9</sup>Stephen Boyd and Lieven Vandenberghe, Convex Optimization, Cambridge University Press, 2004

<sup>10</sup>Mung Chiang, Networked life 20 questions and answers - Cambridge University Press, 2012

<sup>11</sup>Chee Wei Tan. Wireless network optimization by Perron-Frobenius theory. Foundations and Trends in Networking, 9(2-3):107–218, 2015.

## Perron-Frobenius Theory Approach to AIMD

- ▶ Perspective via positive linear systems theory <sup>12 13</sup>
- ▶ Let  $w_s(k)$  denote the congestion window size of source  $s$  immediately before the  $k$ th network congestion event is detected by all the sources as shown in Figure 2.
- ▶ Let  $\alpha_s$  and  $0 < \beta_s < 1$  be the additive and multiplicative parameters of source  $s$  using the AIMD algorithm (that are conventionally set as 1 and 0.5) respectively
- ▶ Let  $q_{\max}$  and  $P$  be, respectively, the maximum queue length of the congested bottleneck link and the maximum instantaneous number of sent unacknowledged packets that are in transit (e.g.,  $P = q_{\max} + BT$  where  $B$  is the bottleneck link service rate in packets per second and  $T$  is the round-trip time)

---

<sup>12</sup>Abraham Berman, Robert Shorten, and Douglas Leith. Positive matrices associated with synchronised communication networks. *Linear Algebra and its Applications*, 393:47–54, 2004.

<sup>13</sup>Martin Corless, Christopher King, Robert Shorten, and Fabian Wirth. AIMD Dynamics and Distributed Resource Allocation. Society for Industrial and Applied Mathematics, 2016.

## Positive Linear System

- At the  $(k + 1)$ th congestion event, source  $s$ 's window satisfies

$$w_s(k + 1) = \beta_s w_s(k) + \left( \frac{\alpha_s}{\sum_{i=1}^n \alpha_i} \right) \sum_{i=1}^n (1 - \beta_i) w_i(k).$$

Let  $w(k) = (w_1(k), \dots, w_n(k))^T$  and write a positive system:

$$w(k + 1) = Aw(k),$$

where

$$\begin{aligned} A &= \begin{bmatrix} \beta_1 & 0 & \cdots & 0 \\ 0 & \beta_2 & 0 & 0 \\ \vdots & 0 & \ddots & 0 \\ 0 & 0 & \cdots & \beta_n \end{bmatrix} \\ &+ \frac{1}{\sum_{i=1}^n \alpha_i} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{pmatrix} (1 - \beta_1, \dots, 1 - \beta_n) \end{aligned}$$

## Positive Linear System

- ▶ The spectrum of the matrix  $A$  (e.g., the Perron-Frobenius eigenvalue and eigenvectors) provides insights on fairness, rate of convergence and transient response:<sup>14</sup>

$$\lim_{k \rightarrow \infty} w(k) = \left( \frac{\alpha_1}{1 - \beta_1}, \dots, \frac{\alpha_n}{1 - \beta_n} \right)^T,$$

which, if specialized to the case of  $\alpha_i = 1$  and  $\beta_i = 0.5$  for all  $i$ , is proportional to the all-ones vector as it should be

- ▶ Fairness line as Perron-Frobenius right eigenvector
- ▶ Classical power method algorithm can simulate AIMD and to visualize the iterates as shown in Figure 2

---

<sup>14</sup>Abraham Berman, Robert Shorten, and Douglas Leith. Positive matrices associated with synchronised communication networks. *Linear Algebra and its Applications*, 393:47–54, 2004.

# Teaching Perron-Frobenius Theory to Undergraduates

- ▶ Networking courses for undergraduates often use Figure 1 to show the fairness guarantee of AIMD
- ▶ Adapt curriculum to cover the classical linear Perron-Frobenius theory and the aforementioned eigenvector interpretation of the fairness line
- ▶ Combine performance analysis with data science techniques to study the transient behavior and convergence of flow rates in a bottleneck link
- ▶ Students can conduct experiments using data flows with different AIMD parameters  $\{\alpha_s, \beta_s\}$  for all  $s$  to compete with the conventional TCP sources to empirically deduce the *cooperative* configuration (i.e., “TCP-friendly”):

$$\alpha_s = 2(1 - \beta_s) \quad \forall s$$

# Teaching Perron-Frobenius Theory to Graduate Students

- ▶ Teaching curriculum can cover more general AIMD involving the nonlinear versions of the Perron-Frobenius theory<sup>15 16</sup>
- ▶ Design data analytics based on TCP data measurement and apply machine learning to data-driven models to create computer-generated TCP algorithms
- ▶ Data analytics to validate assumptions, e.g., single bottleneck link
- ▶ Machine learning to automate this process when the bottleneck link may shift around in the network due to variable traffic conditions

---

<sup>15</sup> Chee Wei Tan. Wireless network optimization by Perron-Frobenius theory. Foundations and Trends in Networking, 9(2-3):107–218, 2015.

<sup>16</sup> Roger D. Nussbaum and Bas Lemmens. Nonlinear Perron-Frobenius Theory. Cambridge University Press, 2012.

# Flipped Classroom Teaching

- ▶ In traditional university teaching, the teacher tends to just give lectures and hand out homework to students – there is less cooperation between teacher and students as well as between students
- ▶ A flipped classroom approach leverages the power of cooperation to improve the interaction between the teacher and students using feedback <sup>17</sup>
- ▶ Can we engineer flipped classroom teaching tools to enhance teacher-student interaction and cultivate the spirit of cooperation between the teacher and students?

---

<sup>17</sup> John Hattie and Helen Timperley. The power of feedback. Review of Educational Research, 77(1):81–112, 2007.

## A parallel between AIMD and Classroom Teaching

- ▶ Classroom flipping requires active feedback through cooperation
- ▶ In AIMD/TCP, the users may wish to send as much data packets as possible without knowing the link capacity ahead in time. In teaching, what is the "*content capacity*" (related to teachers asking "*am I teaching too much?*" or "*am I teaching this not enough?*")?
- ▶ Back-off when teachers realize that they are teaching too much content?
- ▶ In AIMD/TCP, the actual information of link capacity and degree of fairness are not known to the users sending the data packets. In teaching, what is the "*comprehension capacity*" (related to teachers asking "*are some students being overwhelmed and dropping behind?*" or "*is this material accessible to all the students?*")?
- ▶ Slow start when teachers introduce new or more advanced concepts in class?

# Technology-driven Pedagogy for Classroom Flipping

- ▶ Our mobile chatbot software technologies blend together Peer Instructions and Just-in-Time Teaching for in-person or remote instructions<sup>18</sup>
- ▶ Poll-quiz routine to regulate content delivery for the whole class to meet the “*content capacity*” and to use outside-class quizzes of varying difficulty levels to meet the “*comprehension capacity*” of individual students
- ▶ An optimization framework optimizes the frequency and difficulty levels of quizzes
- ▶ Data analytics collate the data from the series of poll-quiz routines to identify students’ weakness in learning and the instructors’ blind spots in online teaching

---

<sup>18</sup>Jingting Li, Lin Ling, and Chee Wei Tan. Blending peer instruction with just-in-time teaching: Jointly optimal task scheduling with feedback for classroom flipping. ACM Conference on Learning at Scale, 2021.

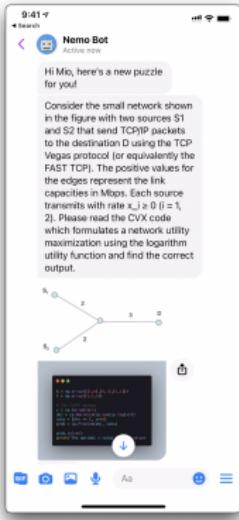
# Technology-driven Pedagogy for Classroom Flipping

- ▶ The teacher first issues a poll whose instantaneous response outcome can be observed by the entire class, and a short discussion (e.g., peer discussions) ensues
- ▶ Then conclude by a time-limited quiz whose content is related to the prior poll
- ▶ Polls and quizzes are typically multiple-choice questions but they can be enriched with multimedia contents
- ▶ Enrich Human-Computer Interaction via automated hints or interactive human-computer input like touch-screen annotation with auto-grading capability<sup>19 20</sup>

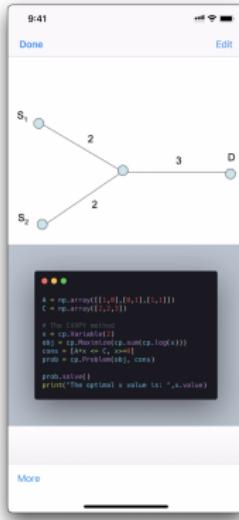
---

<sup>19</sup>Lin Ling and Chee Wei Tan. Human-assisted computation for auto-grading. IEEE ICDM Workshop on Machine Learning for Education, 2018.

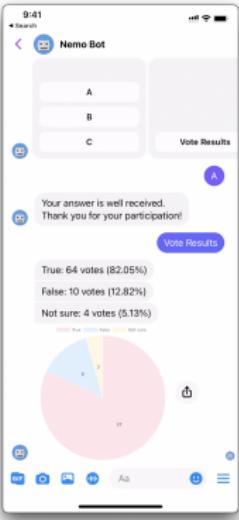
<sup>20</sup>Lin Ling and Chee Wei Tan. Peer-grading at scale with rank aggregation. ACM Conference on Learning at Scale, 2021.



(a)



(b)



(c)

**Figure:** Students receive a poll via a mobile chatbot software on the topic of network utility maximization in (a) testing their understanding of the CVX optimization software <sup>21</sup> in (b). In (c), the student once having voted on the poll gets to see the whole class response to the poll before engaging in peer discussions and answering a timed quiz.

<sup>21</sup> Michael Grant and Stephen Boyd. CVX: <http://cvxr.com/cvx>, Mar 2014



# Poll-Quiz Routine

- ▶ In-class synchronous Peer Instruction activity
- ▶ Process:
  - ▶ Poll: 5 minutes
  - ▶ Statistical visualization
  - ▶ Discussion: 10 to 20 minutes
  - ▶ Quiz with scoring (content related or identical to poll)

You get to play first in Tic-Tac-Toe. Which box would you put ✕ ?

A: Box in the center  
B: Any box at the corner  
C: Any box that is none of above

Tic Tac Toe

Vote Results

A  
B  
C

Your answer is well received.  
Thank you for your participation!

Any box at the corner : 78 votes (65%)  
Box in the center: 42 votes (35%)

Box in the center  
any box at the corner  
any box that is none of above

(a) Poll

Hi Mio, here's a new puzzle for you!

Alice 🧑 and Bob 🧑 play a game of Tic-Tac-Toe. In this game, the players try to get three O or three ✕ in a row (horizontal, vertical, or diagonal). They follow the following rules: A player always tries to win. A player always blocks the opponent from winning. Of course, the first rule has precedence over the second rule, because the game can be won in this way. In the game shown, six moves have been made. Alice plays with ✕ and Bob plays with O. However, we do not know who started the game. Who will win the game?

O O  
O X  
X X

A: Alice 🧑  
B: Bob 🧑  
C:  
D:

Please select the correct answer by clicking one of the buttons below 🤝

A B C

Your answer is well received!

(b) Quiz

# Online Classroom Flipping for Remote Teaching

The screenshot shows a Facebook Messenger interface. On the left, there's a sidebar with "Chats" and a search bar. The main area shows a message from "Nemo Bot" at 4:01 PM: "You get to play first in Tic-Tac-Toe. Which box would you put X?". Below this are three options: A: Box in the center, B: Any box at the corner, C: Any box that is none of above. To the right of the options is a Tic-Tac-Toe grid. At the bottom of the message area, there are buttons for attachments and a message input field. On the right side of the screen, there's a profile for "Nemo Bot" with a video thumbnail of a person, a blue circular icon, and the text "Active on Messenger". Below the profile are sections for "MORE ACTIONS", "MESSANGER LINK", "PRIVACY & SUPPORT", and "SHARED PHOTOS", which includes a collage of related images like pie charts and hand gestures.

Teaching an Online Course on Artificial Intelligence to 133 Freshmen

4:01 PM

You get to play first in Tic-Tac-Toe. Which box would you put X?

A: Box in the center  
B: Any box at the corner  
C: Any box that is none of above

Tic Tac Toe

A  
B  
C

Type a message...

Nemo Bot  
Active on Messenger

MORE ACTIONS

MESSANGER LINK

m.me/654163798317367

PRIVACY & SUPPORT

SHARED PHOTOS

A poll typically allows any student to respond intuitively or with the help of pre-class reading assignment.

# Instant Feedback to Students and Educator

The screenshot shows a Facebook Messenger interface. On the left, there's a sidebar for 'Chats' with a search bar and a message from 'Nemo Bot'. The main area displays a poll result with a pie chart. The text above the chart says: 'Any box at the corner : 72 votes (89.23%)' and 'Box in the center: 32 votes (30.77%)'. The pie chart has two segments: a large light blue segment labeled '72' and a smaller pink segment labeled '32'. Below the chart, a message says: 'You have already voted on this.' At the bottom, there's a message input field with placeholder text 'Type a message...' and various message icons.

Teaching an Online Course on Artificial Intelligence to 133 Freshmen

No unread messages Return to Chats

Nemo Bot Active on Messenger

Your answer is well received. Thank you for your participation!

Vote Results

Nemo Bot Active on Messenger

More Actions

Messenger Link

m.me/454163798317367

Privacy & Support

Shared Photos

Tic Tac Toe

2020-08-25 19:10:14

Any box at the corner : 72 votes (89.23%)

Box in the center: 32 votes (30.77%)

72

32

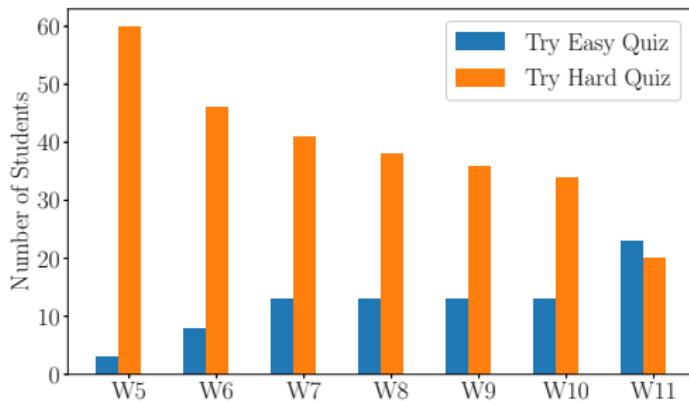
You have already voted on this.

Type a message...

A poll response that makes for great discussion among peers.

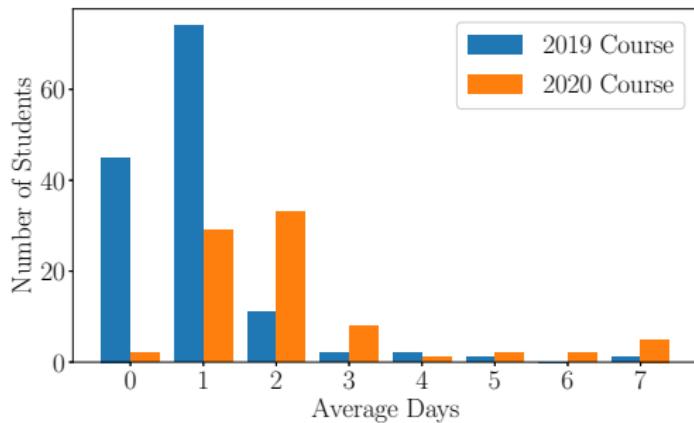
# Usability Profile on Quiz Difficulty Level

- ▶ Students are willing to opt for harder quizzes
- ▶ How to incentivize students is key, e.g., more points for a harder quiz
- ▶ Open up new avenues to auto-track students' abilities



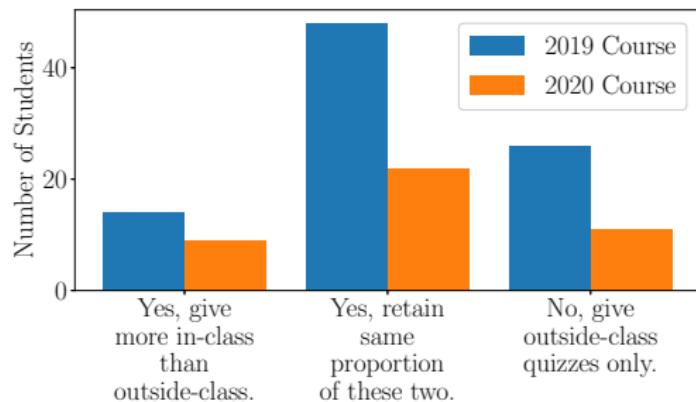
# Usability Profile on Pre-class Quiz Activities

- ▶ In 2019, 96% subscribers answer the quizzes within two days.
- ▶ In 2020, 99% subscribers answer the quizzes within three days.



# Students Vote on Blended Learning by End of Course

- ▶ More in-class quiz. (16%, 21%)
- ▶ Retain same proportion. (55%, 52%)
- ▶ Outside-class quiz only. (30%, 26%)



## Conclusions

- ▶ Teaching performance evaluation and data science via AIMD
- ▶ Nonlinear mathematics like optimization theory and Perron-Frobenius theory are fundamental to AIMD abstraction and can enrich teaching curriculum
- ▶ A parallel between AIMD/TCP and flipped classroom teaching
- ▶ Technology-enabled pedagogy with poll-quiz routines to enhance cooperation between teachers and students
- ▶ Allow teachers to find suitable operating points in the “*content capacity*” and to understand limitations due to the “*comprehension capacity*”
- ▶ Classroom flipping dataset available at Kaggle  
<https://bit.ly/3y4YKAZ>
- ▶ Useful for fully online instruction setting, e.g. during COVID-19 pandemic, to “read a room” and blended learning

22

---

<sup>22</sup>Maxwell Bigman and John Mitchell. Teaching online in 2020: Experiments, empathy, discovery. IEEE 7th Learning with MOOCs (LWMOOCs) 2020.

