Tutorial 1

1. The time (in seconds) that a sample of employees took to complete a task is:

14 28 40 13 25 27 20 29 49 66

Find the following for this data:

- (a) Arithmetic mean
- (b) Median
- (c) Variance
- (d) Standard deviation
- (e) Coefficient of variation (L11QUE/MES students only)
- 2. Consider the following sample of prices of certain goods and quantities sold.

Price	Quantity	
£10	100	
£15	90	
£20	75	
£25	50	
£30	0	

- (a) Sketch a scatter plot of price (along the x-axis) and quantity (along the y-axis). Do you think these two variables will have a negative covariance, a positive covariance, or zero covariance?
- (b) Compute and interpret the covariance and the correlation coefficient.
- **3.** Out of a group of 20 people, 15 enjoy cycling and 8 enjoy reading. 2 of them enjoy neither cycling nor reading. How many people enjoy both cycling and reading?
- **4.** A committee of two members is to be chosen from a group of 4 male and 2 female candidates. If the choice is made randomly, what is the probability that both members are male? (Hint: define 'the first member is male' and 'the second member is male' as two events A and B).

Tutorial 2

- 1. The manager of a music store finds that 30% of customers entering the store ask an assistant for help, and 20% of customers entering the store make a purchase before leaving. Also, 15% of customers entering the store both ask an assistant for help and make a purchase before leaving.
 - (a) What is the probability that a customer entering the store will ask an assistant for help or make a purchase or both?
 - (b) What is the probability that a customer that asks an assistant for help will make a purchase before leaving?
 - (c) Consider the events "asks assistant for help" and "makes purchase"
 - (i) Are these events mutually exclusive?
 - (ii) Are they collectively exhaustive?
 - (iii) Are they independent?
- 2. Suppose there are 21 seats available in a classroom. It is known from past experience that the probability that a student who has registered for the module attends the class is 80%. There are 25 students registered.
 - (a) What is the probability that there won't be enough seats?
 - (b) What is the probability that there will be at least one empty seat?
 - (c) What are the assumptions behind your answer? Discuss how realistic they are for this particular case.
- **3.** Consider the joint probability distribution

		Y	
		1	2
X	0	0.30	0.20
	1	0.25	0.25

where X represents the number of exams a student has in a day during final examinations and Y represents the number of snacks eaten by the student during the same day.

- (a) Find the marginal probability distributions for X and Y.
- (b) Calculate the expected value and variance of X and Y.
- (c) Calculate the probability of having one exam conditional on eating two snacks.

Tutorial 3

1. X is a continuous random variable with probability density function (PDF)

$$f(x) = \frac{1}{9}x^2, \qquad 0 \le x \le 3$$

- (a) Find the following probabilities:
 - (i) $P(0 \le X \le 1)$
 - (ii) $P(0 \le X \le 2)$
 - (iii) $P(1 \le X \le 2)$
- (b) Find the expected value and variance of X.
- 2. The random variable Z has a standard normal distribution.
- (a) Find the following probabilities:
 - (i) P(0 < Z < 1.20)
 - (ii) P(-1.33 < Z < 0)
 - (iii) P(Z > 1.33)
 - (iv) P(-0.77 < Z < 1.68)
- (b) Find x given that P(x < Z < 1.68) = 0.2
- **3.** The tread life of a particular brand of tyre has a normal distribution with mean 35,000 miles and standard deviation 4,000 miles.
 - (a) What is the probability that a tyre of this brand will have a tread life between 35,000 and 38,000 miles?
 - (b) What is the probability that a tyre of this brand will have a tread life of less than 32,000 miles?

Tutorial 4

- 1. A random sample of 16 bags of a chemical were tested to estimate the mean impurity content. It is known that the impurity content is distributed normally. The sample mean impurity content was 20.4 grams, and the sample standard deviation was 6.4 grams. Find the 95% confidence interval for the population mean.
- 2. An auditor takes a random sample of 400 invoices relating to the activities of a company in a particular year. The sample mean of the invoices is £250 and the sample standard deviation is £64. Find a 95% confidence interval for the population mean of the company invoices in the same year.
- **3.** A production process produces bottles of shampoo. If the process is operating correctly, the contents weight of these bottles will have a mean of 20 ounces. A random sample of four bottles was drawn. The sample mean contents weight was 19.54 ounces, and the sample standard deviation was 0.52 ounces. It is known that the weight of the bottles follows a normal distribution. Test at the 5% significant level, against a two-sided alternative, the null hypothesis that the population mean is 20 ounces.
- 4. A telemarketing group claim that, after training, employees will earn an average of £1,500 in their first month of work. A random sample of 150 employees is drawn. Sample mean earnings for the first month of work were £1,262 and the sample standard deviation was £432. Test at the 5% significance level the null hypothesis that the population mean is £1,500, against the alternative that it is less than £1,500.
- **5.** Explain what the following hypothesis testing terms mean:
 - (a) Type I and Type II errors
 - (b) Test Power (L11QUE/MES students only)

Tutorial 5

1. X is weekly household income (in hundreds of £) and Y is weekly household spending (in hundreds of £). A researcher conducts a survey of 75 households and records the following information:

$$\overline{x} = 41$$
 $\overline{y} = 19$ $Var[X] = 34$ $Var[Y] = 28$ $Cov[X, Y] = 11$

The researcher wants to estimate the following linear regression

$$y = \beta_0 + \beta_1 x + \epsilon$$

- (a) Estimate the regression coefficients $\widehat{\beta}_0$ and $\widehat{\beta}_1$.
- (b) Interpret your estimates of both $\widehat{\beta}_0$ and $\widehat{\beta}_1$.
- (c) The researcher now conducts another sample, this time from 150 households. She obtains the following OLS estimation:

$$y = 5.0 + 0.25x + \hat{\epsilon}$$

where the estimate $\widehat{\beta}_0$ has a standard error of 1.0 and the estimate $\widehat{\beta}_1$ has a standard error of 0.05.

- (i) Test the claim that there is no relationship between weekly household spending and weekly household income at a 5% level of significance (you may assume the relevant critical value is 1.96).
- (ii) Test the claim that "households increase their expenditure by £20 for every £100 increase of their income" at a 5% level of significance (again assume the relevant critical value is 1.96).
- 2. Outline the role of \mathbb{R}^2 . How can it be calculated? (L11QUE/MES students only)