

# Cutting Boards



[Chinese Version](#)

[Russian Version](#)

Alice gives Bob a board composed of  $m \times n$  wooden squares and asks him to find the minimum cost of breaking the board back down into individual  $1 \times 1$  pieces. To break the board down, Bob must make cuts along its horizontal and vertical lines.

To reduce the board to squares,  $x_{n-1}$  vertical cuts must be made at locations  $x_1, x_2, \dots, x_{n-2}, x_{n-1}$  and  $y_{m-1}$  horizontal cuts must be made at locations  $y_1, y_2, \dots, y_{m-2}, y_{m-1}$ . Each cut along some  $x_i$  (or  $y_j$ ) has a cost,  $c_{x_i}$  (or  $c_{y_j}$ ). The total cost of a cut is  $n \times c$ , where  $n$  is the number of already-cut segments that the cut passes through.

The cost of cutting the whole board down into  $1 \times 1$  squares is the sum of the cost of each successive cut. Recall that the cost of a cut is multiplied by the number of already-cut segments it crosses through.

Can you help Bob find the minimum cost?

## Input Format

The first line contains a single integer,  $q$ , denoting the number of queries. The  $3 \cdot q$  subsequent lines describe each query over 3 lines according to the following format:

1. The first line has two positive space-separated integers,  $m$  and  $n$ , detailing the respective height ( $y$ ) and width ( $x$ ) of the board.
2. The second line has  $m - 1$  space-separated integers listing the cost,  $c_{y_j}$ , of cutting a segment of the board at each respective location from  $y_1, y_2, \dots, y_{m-2}, y_{m-1}$ .
3. The third line has  $n - 1$  space-separated integers listing the cost,  $c_{x_i}$ , of cutting a segment of the board at each respective location from  $x_1, x_2, \dots, x_{n-2}, x_{n-1}$ .

**Note:** If we were to superimpose the  $m \times n$  board on a 2D graph,  $x_0, x_n, y_0$ , and  $y_n$  would all be edges of the board and thus not valid cut lines.

## Constraints

- $1 \leq q \leq 20$
- $2 \leq m, n \leq 1000000$
- $0 \leq c_{x_i}, c_{y_j} \leq 10^9$

## Output Format

For each of the  $q$  queries, find the minimum cost (*minCost*) of cutting the board into  $1 \times 1$  squares and print the value of *minCost* %  $(10^9 + 7)$ .

## Sample Input 0

```
1
2 2
2
1
```

## Sample Output 0

```
4
```

### Explanation 0

We have a  $2 \times 2$  board, with cut costs  $c_{y_1} = 2$  and  $c_{x_1} = 1$ . Our first cut is horizontal at  $y_1$ , because that is the line with the highest cost (2). Our second cut is vertical, at  $x_1$ . Our first cut has a *totalCost* of 2, because we are making a cut with cost  $c_{y_1} = 2$  across 1 segment (the uncut board). The second cut also has a *totalCost* of 2, because we are making a cut of cost  $c_{x_1} = 1$  across 2 segments. Thus, our answer is  $\text{minCost} = ((2 \times 1) + (1 \times 2)) \% (10^9 + 7) = 4$ .

### Sample Input 1

```
1
6 4
2 1 3 1 4
4 1 2
```

### Sample Output 1

```
42
```

### Explanation 1

Our sequence of cuts is:  $y_5, x_1, y_3, y_1, x_3, y_2, y_4$  and  $x_2$ .

*Cut 1:* Horizontal with cost  $c_{y_5} = 4$  across 1 segment. *totalCost* =  $4 \times 1 = 4$ .

*Cut 2:* Vertical with cost  $c_{x_1} = 4$  across 2 segments. *totalCost* =  $4 \times 2 = 8$ .

*Cut 3:* Horizontal with cost  $c_{y_3} = 3$  across 2 segments. *totalCost* =  $3 \times 2 = 6$ .

*Cut 4:* Horizontal with cost  $c_{y_1} = 2$  across 2 segments. *totalCost* =  $2 \times 2 = 4$ .

*Cut 5:* Vertical with cost  $c_{x_3} = 2$  across 4 segments. *totalCost* =  $2 \times 4 = 8$ .

*Cut 6:* Horizontal with cost  $c_{y_2} = 1$  across 3 segments. *totalCost* =  $1 \times 3 = 3$ .

*Cut 7:* Horizontal with cost  $c_{y_4} = 1$  across 3 segments. *totalCost* =  $1 \times 3 = 3$ .

*Cut 8:* Vertical with cost  $c_{x_2} = 1$  across 6 segments. *totalCost* =  $1 \times 6 = 6$ .

When we sum the *totalCost* for all minimum cuts, we get  $4 + 8 + 6 + 4 + 8 + 3 + 3 + 6 = 42$ . We then print the value of  $42 \% (10^9 + 7)$ .