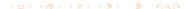
Formal Languages and Compilers - Exercises Lecture 6 Composite data types: vectors and matrices

03/04/2012

- 1 Definition
- 2 Arrays in crème CAraMeL
- 3 Bidimensional Matrices
- 4 Multidimensional matrices
- 5 Slices
- 6 Implementation



Data: "Container" for values (var or const)

Value: Something that is put in the data (everything that is

representable with a sequence of bits)

Data type (DT): Class for data and operations to manipulate it

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Data type (DT): Class for data and operations to manipulate it

- Basic data types: integers, floats, characters, enumerable types,...
- Structured data (data structures): matrices, records, lists,...

Specification

- Attributes: "technical" aspects for managing data
- Values: what you can put inside the data
- Operations: what you can do with that data

Implementation



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Specification

- Attributes: how it is represented in the internal memory
- Values: the maximum and minimum are defined as *MinInt*, *MaxInt*
- Operations: Sum, Multiplication, Subtraction, Division,...

- Attributes: decide at compile-time or at run-time
- Values: nothing to declare
- Operations: HW operations ADD, MUL,...or procedure:
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Attributes in the descriptor

Values like before

Operations access to the elements

V[0] V[1] V[n] Descr.



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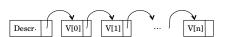


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Pros.

 $\Lambda \| \mathbf{V}[\mathbf{k}] \| = B + O(1)$

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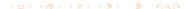
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 $\Lambda \| {\tt V[k]} \| = {\sf scanning \ the \ whole} \\ {\sf list}$

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Arrays in crème CAraMeL

- Data structure
- Homogenous (consists of elements of one type)
- Fixed length represented by a sequence

Descr.	V[0]	V[1]	•••	V[n]
--------	------	------	-----	------

Linear array: vector

Multidimensional array: matrix (remembered line by line)

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Linear array: vector

Multidimensional array: matrix (remembered line by line)

Vector in crème CAraMeL

Specification

Attributes

- Number of elements
- Type (dim.) of elements
- Component name = index

Values

v. number and type

Operations

- Access to the elements
- Creation/elimination of the vectors

Implementation

Attributos

- var V: array [LB...UB]
 of type
- type → Multiplier

Values

UB - LB + 1 elements of type type

- Declaration

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- $\mathbf{O}(\mathbf{k}) = \mathbf{M} \times \mathbf{k}$

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Address of the k-th element

$$\Lambda \|V[k]\| = \alpha + (k - LB) \times M
= (\alpha - LB \times M) + k \times M
= VO + k \times M$$

$$VO = \alpha - LB \times M$$
$$= \Lambda ||V[0]||$$



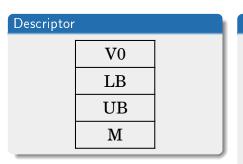
Address of the k-th element

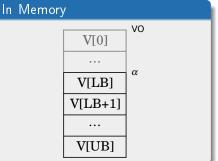
$$\Lambda \|V[k]\| = \alpha + (k - LB) \times M
= (\alpha - LB \times M) + k \times M
= VO + k \times M$$

$$VO = \alpha - LB \times M$$

= $\Lambda ||V[0]||$







Vectors: implementation

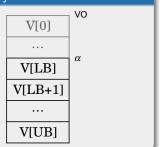
Simplification

By having M=1, we obtain $\Lambda ||V[k]|| = VO + k$ and $VO = \alpha - LB = \Lambda ||V[0]||$

Descriptor

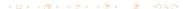
UB

In Memory



Outline

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Bidimensional Matrices

Definition

var V : array[LB₁ ..UB₁,LB₂..UB₂] of type

- Dimension of an element: M₂
- Dimension of a row: $M_1 = (UB_2 LB_2 + 1) \times M_2$

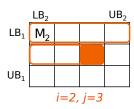
LB ₂			UB ₂	
LB_1	M ₂			
UB_1				

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■ Virtual Origin:

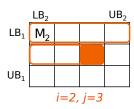
$$VO = \alpha - LB_1 \times M_1 - LB_2 \times M_2$$

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Multidimensional matrices

var V : array [LB₁...UB₁,...,LB_n..UB_n] of type

Multipliers

$$\mathbf{M}_n = \mathbf{M}$$

$$\mathbf{M}_i = (\mathbf{U}\mathbf{B}_{i+1} - \mathbf{L}\mathbf{B}_{i+1} + 1) \times \mathbf{M}_{i+1}, \quad i \in [1, n-1]$$

$$VO = lpha - \sum_{i=1}^n \mathtt{LB}_i imes \mathtt{M}_i$$
 $\wedge \|V[k_1,\ldots,k_n] = VO + \sum_{i=1}^n k_i imes \mathtt{M}_i$

Multidimensional matrices

var V : array [LB₁...UB₁,...,LB_n..UB_n] of type

Multipliers |

$$\mathbf{M}_n = \mathbf{M}$$

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$$VO = \alpha - \sum_{i=1}^{n} \mathtt{LB}_i \times \mathtt{M}_i$$

$$\Lambda \| V[k_1, \dots, k_n] = VO + \sum_{i=1}^{n} k_i \times \mathtt{M}_i$$

Multidimensional matrices

Attention

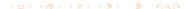
$$array[LB_1..UB_1, LB_n..UB_n]$$
 of $type$

 \sim

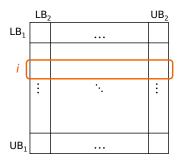
 $array[LB_1..UB_1]$ of $(array[LB_2..UB_2, LB_n..UB_n]$ of type)

Outline

- 5 Slices

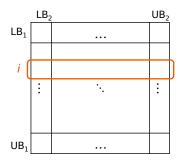


LB ₂			UB ₂
LB_1		•••	
		٠.	:
UB_1			



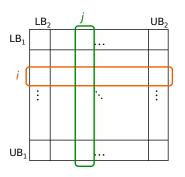
$$\begin{split} \mathbf{M} &= \mathbf{M}_2 \\ \mathbf{VO}_I &= \mathbf{VO}_V + i \times (\mathbf{UB}_2 - \mathbf{LB}_2 + 1) \times \mathbf{M}_2 \\ &= \mathbf{VO}_V + i \times \mathbf{M}_1 \\ \mathbf{LB} &= \mathbf{LB}_2 \\ \mathbf{UB} &= \mathbf{UB}_2 \\ \mathbf{\Lambda} \| I[k] \| &= \mathbf{VO}_I + k \times \mathbf{M} \end{split}$$

$$I = V[i][*] = V[i][LB_2], V[i][LB_2 + 1], ..., V[i][UB_2]$$



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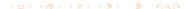


$$\begin{split} \mathbf{M} &= \left(\mathbf{UB}_2 - \mathbf{LB}_2 + 1\right) \times \mathbf{M}_2 = \mathbf{M}_1 \\ \mathbf{VO}_J &= \mathbf{VO}_V + j \times \mathbf{M}_2 \\ \mathbf{LB} &= \mathbf{LB}_1 \\ \mathbf{UB} &= \mathbf{UB}_1 \\ \mathbf{\Lambda} \|J[k]\| &= \mathbf{VO}_J + k \times \mathbf{M} \end{split}$$

$$I = V[i][*] = V[i][LB_2], V[i][LB_2 + 1], ..., V[i][UB_2]$$
$$J = V[*][j] = V[LB_1][j], V[LB_1 + 1][j], ..., V[UB_2][j]$$



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Syntax

lexer.mll: strings corresponding to new tokens

syntaxtree.ml: constructors

- Vector of bType * int * int for declaration var v:array [0 6] of int
- LVec of ide * aexp for the left side of the assignment
 - v[0]:=5;
- Vec of ide * aexp for expressions
 x:= v[2]:

parser.mly: productions for constructing new nodes of a.s.t



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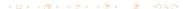
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parser.mly: productions for constructing new nodes of a.s.t.



- New value for the environment: Descr_Vector of loc * int * int (VO, LB, UB)
- Declaration with initialization to 0 (or 0.)
- Evaluation of expression (r-value)
- Evaluation of the address (I-value)

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